Proper motions, mean parallaxes and space velocities of RR Lyrae variables

Herk, G. van

Citation

Herk, G. van. (1965). Proper motions, mean parallaxes and space velocities of RR Lyrae variables. *Bulletin Of The Astronomical Institutes Of The Netherlands*, *18*, 71. Retrieved from https://hdl.handle.net/1887/5728

1965, *Bull. Astr. Inst. Netherlands* **18** 71–105; *Communication from the Observatory at Leiden*

PROPER MOTIONS, MEAN PARALLAXES AND SPACE VELOCITIES OF RR LYRAE VARIABLES

G. VAN HERK

Received 13 May 1965

New proper motion data on RR Lyrae variables are presented in this paper; they are used in a rediscussion of the proper motions, using all the available material. The new data have been derived partly from the results of the Mount Wilson programme started by A. van Maanen, and partly from a comparison of positions obtained from plates of the Leiden photographic refractor with those in the *Carte du Ciel Catalogue*; some hitherto unpublished motions determined by S. A. Mitchell are also given. The Mount Wilson and the Leander McCormick proper motions belong to the most accurate motions ever published. A list is given containing data on the RR Lyrae variables used in determining the mean parallaxes (table 7). This list should be complete to about 1963. Some of the magnitudes used are published photo-electric measurements, others are obtained from unpublished observations by G. E. Kron; systematic corrections have been applied to magnitudes taken from the *General Catalogue of Variable Stars*. For stars observed photo-electrically, the absorption was derived from the colours, for the others it has been computed according to the model $0^{\mathrm{m}}.19$ cosec b $[1-\exp(-0.01\, r \sin b)]$. The value obtained for $(B-V)_0$ is $0^{\mathrm{m}}.19$. For the discussion of the parallaxes the material was subdivided first according to period alone, then according to Preston's ΔS alone, and finally in a joint classification according to both period and ΔS. The parallaxes were computed in three practically independent ways: from the parallactic motions and from the peculiar motions in τ and in v. The influence of the ellipsoidal character of the velocity distribution has been taken into account. The group with $P < 0^{\mathrm{d}}.1$ evidently differs considerably from the others. The results for each individual group are given (table 11), but it seemed legitimate to combine all results (except for $P < 0^{\mathrm{d}}.1$) into one general mean. Thus the parallax for stars reduced to $m_{\mathrm{pg}} = 11.0$ was found to be $0''.00097 \pm 0''.00010$ (m.e.), corresponding to a median photographic absolute magnitude $M_{\mathrm{pg}} = +0^{\mathrm{m}}.87 \pm 0^{\mathrm{m}}.22$ (m.e.), or $M_{\mathrm{v}} = +0^{\mathrm{m}}.68$ (using the mean colour given above). Space motions are given for the case that $M_{\mathrm{pg}} = +0^{\mathrm{m}}.9$ and $M_{\mathrm{pg}} = +0^{\mathrm{m}}.5$ (table 7). The dispersions in Π, Θ and Z and their mean values are listed in table 12; these were computed for $M_{\mathrm{pg}} = +0^{\mathrm{m}}.9$, and for the individual group results for M_{pg}. A list of previous results for M_{pg} and the mean parallaxes is given in section 17.

1. The material

After the first Groningen conference on co-ordination of galactic research in 1953, I offered to determine the proper motions of some brighter RR Lyrae variables. The necessary plates were made available through the kindness of the director and staff of the Mount Wilson Observatory. The programme had been started by A. van Maanen many years earlier, and at the time of his death, in 1946, the major part had been completed. The Mount Wilson staff and especially Mr. W. C. Miller, kindly completed the programme with a number of new plates in subsequent years, as for some stars the original interval in time was only about 10 years. All plates were taken at the Cassegrain focus of the 60-inch telescope, for which the average scale, 1 mm = $8''.227$, was adopted throughout the work.

Meanwhile Mr. G. Pels and the author had also begun to observe a number of RR Lyrae variables, some of which were not on Van Maanen's programme. They used the Leiden 13-inch refractor (1 mm = $40''$) and intended to derive proper motions through a comparison with the positions given in the *Carte du Ciel Catalogues.* The Leiden plates were all taken with a grating in front of the objective. We had, therefore, two different types of material on hand to be discussed, denoted by A when we had Mount Wilson plates exclusively, or B when we used the combination Carte du Ciel – Leiden.

At the Mount Wilson Observatory, Drs. R. E. Wilson and A. van Maanen had already derived proper motions for 23 variables. These results were kindly put at our disposal by Dr. I. S. Bowen. In addition, we had at our disposal, through the courtesy of Dr. S. A. Mitchell, the results for 20 stars from observations at the Leander McCormick Observatory.

Since the work was done as a part-time job for many years, the progress was very slow and in the mean time

TABLE 1

Observational data for material A and B

Star	n_W		Ref.st.		ΔT	<m>	n_L		n_C	Ref.	<ΔT>
	p	e	u	e	max.	ref.	p	e		st.	
SW And							2	4	2	28.5	$53\overset{y}{.}8$
XX ·	4	7	29	0	$16\overset{y}{.}0$	14.8	2	3	2	22.5	27.5
CC							2	4	2	24.5	62.9
SW Aqr	5	9	33	1	22.9	14.4	3	5	2	12.7	59.6
SX	5	8	28	0	23.8	14.8	2	4	2	20.0	46.4
BS							2	4	2	9.5	55.5
CY							2	4	2	13.5	46.8
AA Aql							3	6	2	14.0	61.9
X Ari	4	5	19	1	24.2	12.7	4	8	3	15.5	52.3
RV							2	4	2	13.5	59.4
RS Boo	2	4	20	2	29.8	14.1	3	6	2	7.7	60.9
RU	6	9	25	1	26.9	15.2					
ST							2	4	2	14.5	23.0
SV	4	6	25	0	11.2	15.3	1	2	1	13.0	58.9
SW	5	7	21	0	24.0	14.1	2	4	1	15.0	21.5
SZ							3	6	3	10.7	53.6
YZ							2	4	2	14.5	21.6
RW Cnc							2	4	2	12.5	54.0
SS							2	4	1	13.0	62.1
VZ							2	4	2	19.0	41.4
RU CVn	4	7	20	0	24.9	15.2	2	4	2	14.0	37.9
RV Cap							2	4	2	13.0	50.8
RZ Cep							1	2	1	29.0	61.0
RR Cet	4	7	20	0	21.1	14.1	2	4	2	13.0	55.0
S Com	4	6	21	2	20.8	15.3	2	4	2	16.0	57.5
U	5	8	15	0	22.0	14.4					
V	5	7	18	0	22.9	15.0	2	4	1	10.0	53.5
RT	4	5	14	1	18.9	15.4					
RV	3	6	19	0	18.9	15.4					
RV Crb							2	4	2	21.0	56.9
UY Cyg							2	4	2	37.0	55.0
XX	7	11	26	0	24.8	14.3	3	4	1	21.0	52.0
XZ							4	6	2	33.8	51.4
RW Dra							1	2	1	18.0	50.9
SU	8	14	18	0	32.2	14.6	3	5	2	19.0	60.0
SW							2	4	2	15.5	59.1
RR Gem	4	5	35	0	$13\overset{y}{.}1$	15.0	2	4	2	21.0	$55\overset{y}{.}0$
SZ	6	9	23	0	23.9	13.9	2	4	2	18.5	61.9
TW Her							2	2	1	23.0	53.9
VX							2	4	2	14.0	57.3
VZ							3	4	2	12.7	22.8
AF	6	10	25	1	23.8	12.0					
AG	5	8	23	0	23.8	14.9	1	2	1	15.0	63.7
AR	6	8	25	3	20.0	14.9	2	4	2	18.5	51.8
CE	6	8	33	1	23.1	13.9	2	4	2	21.5	56.4
DY							1	2	1	16.0	48.9
VX Hya							1	2	1	17.0	36.0
RR Leo							2	3	2	12.5	49.5
RV	5	9	27	1	24.1	12.7					
RX							1	2	1	16.0	58.0
SS	8	12	20	0	22.1	14.8	2	4	2	16.0	53.0
V LMi	5	6	19	4	20.8	14.5	2	4	2	15.5	52.0
X							2	4	2	15.5	43.5
EH Lib							2	4	2	18.0	59.5
RR Lyr							1	2	1	45.0	60.8
RZ							2	4	2	24.5	21.2
UX	5	8	41	0	23.8	15.1					
ST Oph	6	11	25	1	26.9	14.3	2	4	2	11.5	56.5
V567							2	4	2	22.0	46.9
BP Peg							5	5	2	22.4	63.2
DY							2	4	2	15.0	58.9
TU Per	4	5	29	0	16.0	14.0					
RU Psc	5	9	23	0	17.2	14.6	2	4	2	15.0	60.3
CW Ser							3	5	3	17.3	42.0
SS Tau	5	7	23	0		13.6	1	2	1	14.0	59.8
AH							2	4	2	44.0	59.8
U Tri	5	8	30	0	21.0	14.9	4	5	1	14.8	59.6
RV UMa							4	7	3	16.2	48.4
SX	5	7	21	1		14.4	3	6	3	11.0	43.3
ST Vir	3	4	29	1	19.9	14.6	2	4	2	12.5	62.9

other material became available, viz. the data obtained through a repetition of the Carte du Ciel at various other observatories. This was first made available to us through the kindness of Dr. L. Plaut. Part of these data have now been published. All results known to us by the middle of 1962 have been used in the final discussion.

Table 1 gives the more important observational data for material A and B. For A the columns give, respectively, the number of plates, the number of exposures measured, the number of reference stars used, the number of reference stars which had been measured but were excluded because their relative annual motions turned out to exceed 0″.050, the difference in epoch between the earliest and latest plate, and the mean photographic magnitude of the reference stars.

For material B the columns contain: number of Leiden plates and exposures, number of Carte du Ciel exposures, the average number of reference stars, and the average difference in epoch.

From this table the inhomogeneity of the material, especially for B, is evident. There also exist large differences in quality among the plates of A. Therefore we must expect large differences in accuracy for the individual proper motions.

Dr. C. H. Hoffmeister kindly helped with the identification of UX Lyr, as an error in the original description made the location of this star uncertain.

2. The measurements of the coordinates and their accuracy

All A plates were measured on the Gaertner-type measuring machine of the Leiden Observatory. The B plates, as well as some Oxford Carte du Ciel plates which were kindly loaned to us by the director of the Oxford Observatory, were measured mainly on the new Zeiss comparator. The usual precautions were taken in these procedures. The plates were measured in two directions. The Gaertner machine carries two measuring wires, and both were used throughout. The computed distance of the two measuring wires yielded a mean error for a complete measurement of one star position varying from $\pm 1.0\,\mu$m to $\pm 1.8\,\mu$m, depending on the quality of the plates. Ten plates (of SS Tau and SX UMa) were remeasured by Mr. J. Smit. The motions found by him were averaged with my own results, as there were no systematic differences.

3. The reduction of the A material

The Mount Wilson plates were taken at rather different epochs and it was obvious that the best results could be derived from one solution which made use of all plates instead of trying to find suitable pairs of plates.

The variables, which were always near the centre of the plate, were taken as the origin. The mean positions for all reference stars and the residuals with respect to these means were formed for each exposure. Transformation formulae with quadratic terms in the coordinates were used. The computations were carried out twice. After the first computations the stars which showed annual proper motions greater than 0″.050 were rejected. The first computations were carried out by the computing section of the Mathematical Centre in Amsterdam, for which we want to thank Professor A. van Wijngaarden and his staff. The second time the computing centre of the Mathematical Institute of the University of Groningen kindly performed this work. The limit of 0″.050 is, of course, arbitrary, but has been used in other instances. In the reduction to absolute motions this rejection has been taken into account by the procedure developed by OORT (1936).

The average mean error of one proper motion was found to be ± 0″.0016 and ± 0″.0018 for α and δ, respectively. For the plates measured and reduced at Mount Wilson these figures are ± 0″.0022 and ± 0″.0023, respectively. These mean errors apply to motions relative to the average of the comparison stars. In order to find the mean error of the absolute motions of the RR Lyrae stars we have to take into account the error caused by the random motions of the comparison stars. If the average random motion in one coordinate is μ, the mean error arising from this cause will be $1.25\ \mu/\sqrt{n}$, with n the number of reference stars used. The average proper motion was taken from BINNENDIJK's (1943) table 4. Table 3 lists the individual mean errors, increased by these "cosmical errors". For the material reduced by Drs. Wilson and Van Maanen, the average photographic magnitude for the reference stars was estimated at 13.0 and n was taken as 8.

In the comparison of the proper motions from different sources these increased mean errors were used. The relative motions were transformed to absolute ones by applying the parallactic motion (BINNENDIJK, 1943) and the differential galactic rotation. Throughout this article we have taken A to be $+15$ and B equal to -10 km.sec^{-1}.kpc^{-1} (*I. A. U. Information Bull.* No. 11 11). These corrections were also applied to the proper motions derived at Mount Wilson, as every indication pointed to the conclusion that the motions given were differential ones. In this latter case no corrections for rejection of stars were applied, as we believe that no stars had been rejected.

In order to correct the differential motions to absolute values, we need the magnitudes of the reference stars on the Mount Wilson plates measured in Leiden. The material did not fulfill the requirements for a rigorous determination of the magnitudes. When available we used the Leiden plates to derive rough magnitudes for as many stars as possible, which were then used as reference stars on the A plates. By this detour, magnitudes for all stars on the A plates could be found. When no Leiden plates were available the BD magnitudes of stars on the Mount Wilson plates were reduced to HD magnitude and were used for scale and zero-point. It is clear that this procedure does not yield good photometric results, but it may suffice for finding the mean magnitude of the comparison stars, with an uncertainty of, perhaps, about ±0.25 magnitude. Though a rotating sector had been used to reduce the brightness of the variable stars to that of the comparison stars, there are sometimes considerable deviations between the reduced magnitude of the variable and the magnitudes of the comparison stars, in some cases up to 4 magnitudes. This is partly due to the fact that, in order to increase the number of comparison stars, their average brightness has generally been fainter than what may have originally been intended. It is evident from this that magnitude errors may have played a part in reducing the accuracy of the proper motions derived from these plates. The magnitudes were determined with the Iris photometer at Leiden, mainly by Mr. M. Flohr.

4. The reduction of the B material

Material B consists of positions published in the *Carte du Ciel Catalogues* and new positions derived from Leiden plates. If there were two exposures on the latter plates, the two positions were averaged. In a few instances more than one Leiden plate was taken, which could each be combined with the same Carte du Ciel positions. In these cases two solutions were made, to get some insight into the part which the Leiden positions contributed to the final error. Because the Carte du Ciel plates overlap, there is nearly always the possibility to get two or sometimes even three solutions. These are partly correlated, as a fraction of the reference stars is common to the various plates.

The reference stars were chosen amongst those for which an absolute proper motion was known, or could be computed from existing meridian-circle material. We are grateful for the help we received from Professors J. Dick in Berlin and J. Haas in Bonn, who kindly sent us material from *Geschichte des Fixsternhimmels* in advance of publication. The very brightest stars were omitted.

The proper motions of the reference stars were derived originally in the GC system; they were reduced to the FK3 system with the aid of the tables given by Blaauw and Delhaye (1949). The proper motions were applied to reduce the positions on the Leiden plates to the Carte du Ciel positions. In the reduction of the new set of coordinates to the old set second-order terms in the coordinates were included. The coefficients of the quadratic terms exceeded 2.5 times their own mean errors in only 4 per cent of the cases. For the remaining 96 per cent of the material a solution was used with only linear terms in the coordinates. No terms depending on the magnitudes of the stars were included. In combining the results derived from the different pairs of plates weights of the form $\Delta T.f(d).\sqrt{n}$ were assigned. The weights depend on the interval ΔT between the old and new plates, on the number of comparison stars n, and on some function of the distance d of the variable star from the centre of the Carte du Ciel plate. The expression adopted for the relation $f(d)$ was of the form $p+qd^{-2}+rd^{-4}$; p, q and r being chosen such that at 8 cm from the centre the weight was still 0.8 times that at the centre while beyond this distance it dropped rapidly to 0.05 at $d=12.5$ cm. That the weights estimated in this way are correlated with the actual mean errors may be illustrated as follows. The absolute values of the residuals in the proper motions μ_α and μ_δ were computed from the average proper motion minus the individual value in the case of two or more pairs of plates. One expects the highest absolute residuals to occur with those cases where all individual plate pairs have low weight. If this happens one feels confident that the poor results are a consequence of known circumstances: too few reference stars, too short an interval in time, or the variable too far from the centre of the plate. Of the total of 274 residuals 54, or 20 per cent, were higher than 0″.010. Taking only the stars with an average weight of less than 1.0 for each pair of plates, we have a total of 44 residuals, of which 25, or 57 per cent, are higher than 0″.010. The motions in this group of low weight are clearly less accurate.

It is difficult to obtain a good insight into the errors introduced by the inaccuracy of the proper motions of the comparison stars, and to estimate the relative importance of these errors as compared to the errors of the measured positions on the Carte du Ciel and Leiden plates. An attempt to get some information on this was made by dividing the material into three groups in the following manner.

a) Two Leiden plates combined with two Carte du Ciel positions, with the number of reference stars in common less than 35 per cent of the total number.

b) Two Leiden plates combined with two Carte du Ciel positions, the number of reference stars common to the two pairs being between 35 and 70 per cent of the total.

TABLE 2

Weights and mean errors for proper motions of material B

Group	$\langle w \rangle$	m.e. unit weight μ_α (0″.0001)	m.e. unit weight μ_δ (0″.0001)	m.e. $\langle w \rangle$ μ_α (0″.0001)	m.e. $\langle w \rangle$ μ_δ (0″.0001)	per cent	n	$\langle \Delta T \rangle$ (years)	m
a	1.4	±112	±145	± 96	±124	20	17.2	48	22
b	1.7	180	125	140	97	50	17.1	53	27
c	1.7	89	59	68	45	87	16.5	54	17

c) Two Leiden plates combined with the same Carte du Ciel position; in this group an average of 87 per cent of the reference stars were in common.

Table 2 gives for each group the average weight for one pair of plates, the mean errors for unit weight for μ_α and μ_δ, the mean error for the average weight for one pair of plates, the average percentage of stars in common on the pairs of plates, the average number n of reference stars, the average interval in years, and the number m of motions on which the preceding figures are based.

These mean errors were obtained from the differences between the proper motions derived from the two pairs of plates. These differences should be smaller for the groups where the number of reference stars in common between the two pairs is larger. They should therefore be smaller for group b than for group a and still smaller for group c. In the latter the differences are still further decreased by the fact that the same Carte du Ciel positions were used in the two pairs. The table shows no appreciable difference between a and b, but a decided decrease for group c. In the following an average value of ± 140 was adopted for the mean error of unit weight, in accordance with the mean results for groups a and b. For these groups the influence of the common use of part of the reference stars was neglected in its effect upon the mean error. In the case of group c the decrease in mean error was not taken proportional to the root of the weights. Only the improvement due to the fact that more than one plate was taken at Leiden was taken into account. For unit weight this effect caused on the average a decrease of the square of the mean error by only 4×10^{-6}. This is based on a formal (and as explained above, incomplete) representation of the mean error of a proper motion by

$$\varepsilon_\mu^2 = \left(\frac{1}{\Delta T}\right)^2 (2\varepsilon_{\rm m}^2) + \frac{(n-p)}{n^2}\varepsilon_{\rm pm}^2, \qquad (1)$$

where n is the number of reference stars used, p the number of stars in common in two determinations, $\varepsilon_{\rm m}$ the mean error for the measurement of a position and $\varepsilon_{\rm pm}$ the mean error for an adopted proper motion of a reference star.

The final proper motions derived from the various sources, and their mean errors are given in table 3.

5. Results from other sources

Proper motions of the following sources have been used in combination with the Mount Wilson and Leiden results.

a) Leander McCormick proper motions, derived by the late Dr. S. A. Mitchell. The average internal mean error for one motion was given as ± 0″.0033, the average photovisual magnitude of the reference stars was 10.5, and the average number of these stars was 14.7 (highest number 20, lowest 8). The relative motions have been corrected for parallactic motion and for differential galactic rotation in the same way as was done for the Mount Wilson motions. The individual mean errors have been increased to allow for the effect of the average random motion of the reference stars.

b) Moscow proper motions, derived by PAVLOVSKAYA (1953a) who gave full details of the material and methods of reduction used. The mean errors were increased to allow for the random motions of the reference stars, the average number of which was ten. The Moscow and Leiden motions are not entirely independent, as the same Carte du Ciel positions were used in both. However, from the description of the methods used, it is clear that practically no reference stars will have been used in common. Mrs. Pavlovskaya

TABLE 3

The annual proper motions rounded off to three decimals in the unit of 0″.001, together with the mean errors

	μ_α (0″.001)	m.e.	μ_δ (0″.001)	m.e.
SW And				
MtW	- 2	4	- 19	4
LMcC	+ 8	5	- 29	5
Mosc	- 13	7	- 19	7
Lei	- 15	7	- 17	7
B and B	+ 6	11	- 12	11
m.c.	+ 4	23	- 20	8
XX And				
MtW	+ 52	3	- 26	5
LMcC	+ 68	4	- 34	4
Mosc	+ 59	12	- 62	12
Lei	+ 49	8	- 42	8
AC And				
Loz	- 11	9	- 8	9
Lur	- 8	8	+ 2	8
AT And				
Hels	- 2	2	+ 46	2
CC And				
m.c.	- 11	10	- 27	9
WY Ant				
Lou	+ 18	6	- 57	6
SW Aqr				
MtW	- 41	2	- 47	2
LMcC	- 43	4	- 54	4
Mosc	- 47	5	- 63	5
Lei	- 42	7	- 58	7
B and B	- 54	11	- 64	11
K and vR	- 50	7	- 44	7
SX Aqr				
MtW	- 50	2	- 50	2
Lei	- 30	8	- 20	8
B and B	- 31	13	- 41	13
K and vR	+ 7	7	- 31	7
TZ Aqr				
SF	+ 6	9	- 11	8
BR Aqr				
SF	+ 2	9	- 8	8
BS Aqr				
Lei	+ 27	9	- 7	9
m.c.	+ 22	9	- 2	10
Par 1)	+ 6	16	- 20	16
Lou	+ 30	6	- 22	6
SF	+ 40	9	- 11	8
BT Aqr				
SF	+ 6	9	- 5	8
CY Aqr				
Lei	+ 74	9	- 54	9
Loz	+ 53	15	- 60	15
AA Aql				
LMcC	- 3	4	- 18	4
Lei	- 4	7	- 8	7
B and B	- 4	11	- 3	11
SF	+ 3	9	0	8
S Ara				
Luy	- 15	13	- 20	13
Lou	- 25	6	- 14	6
Rus	- 25	6	- 15	6
X Ari				
MtW	+ 61	3	- 94	3
LMcC	+ 62	5	- 80	5
Mosc	+ 64	7	- 93	7
Lei	+ 84	9	- 101	9
B and B	+ 54	11	- 84	11
m.c.	+ 69	12	- 75	6
Tou	+ 77	4	- 93	4
RV Ari				
Lei	- 4	8	- 18	8
Loz	- 15	10	- 4	10
TZ Aur				
MtW	- 5	5	- 13	4
Luy	+ 15	20	- 1	20
RS Boo				
MtW	+ 26	3	- 5	4
Mosc	+ 7	7	+ 14	7
Lei	+ 18	8	+ 7	8
B and B	- 19	12	+ 32	12
m.c	+ 56	63	+ 36	28
K and vR	+ 1	9	- 12	9
RU Boo				
MtW	- 16	3	- 3	4
ST Boo				
MtW	- 22	6	+ 0	6
Lei	+ 25	12	- 1	12
B and B	- 16	12	+ 15	12
SV Boo				
MtW	- 1	4	- 12	4
Lei	+ 11	10	- 29	10
SW Boo				
MtW	- 26	4	+ 4	3
Lei	+ 11	10	- 46	10
B and B	- 60	13	+ 9	13
SZ Boo				
MtW	- 4	6	- 5	5
Lei	+ 7	9	+ 11	9
B and B	- 3	12	- 7	12
TV Boo				
Kur	- 25	8	- 22	8
Hels	- 9	5	- 30	5
TW Boo				
Mosc	+ 5	7	- 49	7
Hels	- 2	4	- 44	4
UY Boo				
Ros	- 15	7	+ 34	7
YZ Boo				
Lei	+ 9	16	- 22	16
RZ Cam				
Luy	+ 19	13	- 19	13
RW Cnc				
MtW	+ 4	4	- 38	4
Lei	+ 8	8	- 24	8
B and B	+ 9	14	- 58	14
SS Cnc				
MtW	+ 8	6	- 11	4
Mosc	+ 2	5	- 4	5
Lei	+ 1	10	+ 0	10
TT Cnc				
Mosc	- 48	5	- 42	5
VZ Cnc				
Lei	- 26	9	- 12	9
m.c. 2)	- 27	6	- 23	3
Tou	- 14	4	- 21	3
W CVn				
B and B	- 62	14	- 13	14
m.c.	- 29	10	- 2	9
K and vR	- 20	10	- 8	10
Z CVn				
Mosc	- 2	6	- 24	6
Mosc 3)	0	6	- 26	6
Luy	- 10	12	- 26	12
RR CVn				
B and B	- 9	13	+ 6	13
RU CVn				
MtW	- 41	4	- 3	4
Mosc	- 58	9	+ 2	9
Lei	- 51	11	- 14	11
B and B	- 33	13	+ 13	13
K and vR	- 29	10	- 12	10
SS CVn				
Mosc	- 1	6	- 38	6
Loz	+ 12	10	- 42	10
Hels	+ 6	6	- 48	6
ST CVn				
Loz	- 34	10	- 16	10
SV CVn				
Loz	- 20	10	- 30	10
AD CMi				
m.c.	- 13	7	+ 2	4
RV Cap				
LMcC	+ 33	5	- 106	5
Lei	+ 1	12	- 118	12
B and B	+ 40	10	- 121	10
YZ Cap				
Lou	- 23	6	- 9	6
IU Car				
Lou	- 9	6	- 27	6
BI Cen				
Lou	+ 6	6	+ 16	6
V499 Cen				
Lou	+ 32	6	- 18	6
Rus	+ 10	6	- 27	6
RZ Cep				
MtW	+ 105	3	+ 186	3
LMcC	+ 95	4	+ 190	4
Lei	+ 102	8	+ 197	8
m.c.	+ 88	14	+ 194	16
Shnir	+ 110	7	+ 200	7
Ray	+ 79	8	+ 187	8
Mit p	+ 90	10		
Grw p	+ 120	8		
Agh p	+ 92	8		
DQ Cep				
m.c.	+ 26	2	+ 19	2
RR Cet				
MtW	+ 15	4	- 47	4
Lei	+ 10	8	- 44	8
B and B	+ 24	13	- 51	13
m.c.	+ 37	62	- 63	40
K and vR	+ 29	8	- 79	8
RZ Cet				
SF	+ 12	9	+ 1	8
RY Col				
Lou	+ 36	6	+ 18	6
S Com				
MtW	- 10	3	- 9	4
Mosc	- 12	8	- 4	8
Mosc 3)	- 19	6	- 15	6
Lei	- 2	7	- 12	7
Luy	- 39	13	+ 39	13

TABLE 3 (*continued*)

	μ_α (0″001)	m.e.	μ_δ (0″001)	m.e.
U Com				
MtW	− 44	4	− 18	5
Mosc	− 59	8	− 18	8
B and B	− 48	13	− 18	13
V Com				
MtW	− 16	4	− 1	4
Lei	+ 3	11	− 11	11
Z Com				
MtW	+ 15	5	− 3	5
RT Com				
MtW	+ 14	5	− 17	4
RV Com				
MtW	− 4	4	− 4	4
ST Com				
Loz	− 38	10	− 16	10
Kar	− 33	7	− 43	7
RV CrB				
MtW	− 19	4	− 20	4
Lei	− 36	8	− 26	8
B and B	− 11	11	− 3	11
X Crt				
SF	+ 35	9	− 66	8
SW Cru				
Lou	+ 19	6	+ 8	6
UY Cyg				
MtW	+ 7	3	− 13	4
LMcC	− 0	4	− 10	4
Lei	+ 10	6	− 9	6
B and B	+ 1	11	+ 16	11
XX Cyg				
MtW	− 7	2	+ 1	2
Lei	+ 6	8	+ 23	8
B and B	+ 1	12	− 13	12
XZ Cyg				
MtW	+ 78	3	− 23	4
LMcC	+ 83	4	− 20	4
Lei	+ 80	7	− 36	7
B and B	+ 67	12	− 23	12
m.c.	+ 90	10	− 33	8
DM Cyg				
Loz	+ 11	9	− 1	9
Tor	− 1	9	+ 5	9
CK Del				
Tou	+ 13	4	+ 2	4
RW Dra				
MtW	− 2	5	− 26	7
Mosc	− 9	8	− 14	8
Lei	+ 15	13	− 2	13
B and B	− 8	11	+ 2	11
SU Dra				
MtW	− 38	4	− 72	4
Mosc	− 35	7	− 68	7
Lei	− 37	7	− 75	7
B and B	− 29	11	− 76	11
m.c.	− 49	14	− 80	18
SW Dra				
MtW	− 24	5	− 2	5
Mosc	− 14	8	− 11	8
Lei	+ 4	7	+ 1	7
B and B	− 27	12	− 7	12
XZ Dra				
m.c.	+ 29	12	− 8	50
RX Eri				
LMcC	− 38	4	+ 2	4
B and B	+ 1	13	+ 14	13
m.c.	− 42	12	− 17	18
Vyss 4)	+ 24	9	− 12	9
Lou	− 14	6	− 26	6
SW For				
Lou	− 1	6	+ 2	6
RR Gem				
MtW	+ 3	4	+ 3	4
LMcC	− 9	4	− 15	4
Lei	− 4	8	− 11	8
Luy	− 10	12	+ 14	12
Dzi	+ 3	10	+ 9	10
SZ Gem				
MtW	− 12	3	− 26	3
LMcC	− 8	5	− 24	5
Lei	− 8	7	− 23	7
RS Gru				
m.c.	− 65	5	− 17	20
Lou	− 56	6	− 23	6
SW Her				
MtW	− 7	4	+ 13	4
TW Her				
MtW	+ 1	4	− 7	5
LMcC	+ 8	5	− 12	5
Mosc	− 4	8	− 5	8
Lei	+ 24	10	+ 6	10
B and B	+ 4	11	− 28	11
VX Her				
MtW	− 49	5	+ 26	5
Lei	− 51	7	+ 23	7
B and B	+ 15	14	− 23	14
Mosc	+ 12	5	− 49	5
Vyss	− 3	9	+ 15	9
Mit p	− 50	10		
VZ Her				
MtW	− 21	4	− 16	5
LMcC	− 15	5	− 17	5
Lei	+ 11	13	− 27	13
B and B	− 26	11	− 30	11
AF Her				
MtW	− 15	3	− 12	2
Hels	− 18	3	− 2	4
AG Her				
MtW	− 3	3	− 17	3
Lei	+ 6	9	− 26	9
Hels	− 7	3	− 16	3
AR Her				
MtW	− 64	3	+ 12	3
Lei	− 50	7	+ 26	7
Loz	− 61	8	+ 13	8
Kats	− 52	8	+ 2	8
CE Her				
MtW	− 2	2	+ 0	2
Lei	+ 31	7	− 14	7
DY Her				
Mosc	− 2	6	+ 15	6
Lei	+ 11	11	+ 5	11
SV Hya				
B and B	− 10	13	+ 11	13
m.c.	− 81	3	+ 71	14
SZ Hya				
SF	0	9	− 48	8
UU Hya				
Tou	− 10	4	− 8	8
VX Hya				
Lei	− 6	12	+ 30	12
DG Hya				
SF	− 10	9	− 11	8
DH Hya				
SF	− 20	9	− 3	8
FY Hya				
Lou	− 52	6	+ 5	6
DE Lac				
Hels	+ 5	3	− 3	3
RR Leo				
Mosc	− 15	6	− 9	6
Lei	− 17	8	− 9	8
Luy	− 8	11	− 16	11
m.c.	− 17	39	− 21	21
RV Leo				
MtW	− 5	3	− 13	3
RX Leo				
MtW	+ 16	5	− 32	5
Mosc	+ 10	8	− 37	8
Lei	+ 12	10	− 20	10
B and B	+ 21	12	− 10	12
SS Leo				
MtW	− 16	4	− 15	4
Mosc	− 33	6	− 18	6
Lei	− 19	8	− 15	8
Luy 5)	− 27	19	− 14	19
ST Leo				
Mosc	− 18	7	− 58	7
Mosc 3)	− 15	9	− 25	9
Tou	− 3	6	− 34	4
SZ Leo				
Tou	− 18	6	− 30	4
UZ Leo				
Mosc	− 21	6	+ 12	6
m.c.	− 14	8	+ 4	8
WW Leo				
Tou	− 7	6	− 27	4
AA Leo				
Tou	− 3	4	− 26	4
V LMi				
MtW	+ 32	4	− 34	4
Lei	+ 53	8	− 32	8
Luy	− 17	15	+ 4	15
X LMi				
MtW	− 6	5	− 22	6
Mosc	+ 10	7	− 16	7
Lei	+ 16	8	− 24	8
B and B	+ 27	12	− 18	12
U Lep				
LMcC	+ 44	5	− 58	5
B and B	+ 49	11	− 51	11
Cape p	+ 40	8	− 46	8
TV Lib				
SF	+ 6	9	+ 8	8
UZ Lib				
SF	+ 33	9	− 3	8

TABLE 3 (*continued*)

	μ_α (0″001)	m.e.	μ_δ (0″001)	m.e.
EH Lib				
Lei	+ 1	7	- 7	7
m.c.	- 8	5	- 4	10
CW Lup				
Rus	- 9	6	- 15	6
Y Lyr				
MtW	- 3	4	+ 2	4
LMcC	+ 4	4	+ 2	4
B and B	- 5	12	+ 1	12
RR Lyr				
MtW	- 110	3	- 191	3
LMcC	- 108	4	- 192	4
Lei	- 110	7	- 194	7
m.c.	- 112	6	- 192	6
Kov	- 119	4	- 226	7
Agh p	- 106	8		
RZ Lyr				
MtW	+ 19	3	+ 24	4
Lei	+ 36	12	+ 68	12
B and B	+ 5	12	+ 34	12
UX Lyr				
MtW	- 1	3	- 1	2
FN Lyr				
Hels	+ 14	2	+ 11	2
LX Lyr				
Hels	- 13	2	- 20	2
BE Mon				
m.c.	- 18	23	+ 9	3
Tou	- 3	9	+ 10	6
UV Oct				
Lou	- 71	6	- 138	6
ST Oph				
MtW	+ 2	2	- 0	2
Lei	- 10	9	+ 2	9
B and B	+ 9	12	- 15	12
K and vR	- 8	6	+ 3	6
V445 Oph				
Mosc	+ 9	5	+ 16	5
V452 Oph				
Tou	+ 12	4	+ 14	4
V567 Oph				
Lei	- 25	8	+ 6	8
V784 Oph				
Tou	- 20	4	- 34	4
V816 Oph				
Tou	- 6	7	+ 10	6
TY Pav				
Lou	- 10	6	+ 1	6
DN Pav				
Lou	- 9	6	- 30	6
VV Peg				
MtW	+ 6	4	- 8	4
LMcC	+ 15	4	- 8	4
Mosc	+ 10	6	+ 8	6
B and B	+ 6	13	+ 30	13
BH Peg				
Loz	- 26	10	- 72	10
BP Peg				
Lei	- 12	6	+ 6	6
DH Peg				
Mosc	+ 24	5	+ 1	5
Tou	+ 17	4	0	4
DY Peg				
Mosc	+ 44	5	- 9	5
Loz	+ 59	5	- 10	5
Lei	+ 60	8	- 12	8
TU Per				
MtW	+ 15	2	- 5	2
B and B	+ 20	12	- 44	12
RV Phe				
Lou	+ 33	6	- 18	6
Rus	+ 50	6	- 19	6
SX Phe				
m.c.	+ 255	2	- 860	5
U Pic				
Rus	- 1	6	- 17	6
RU Psc				
MtW	+ 90	3	- 40	3
LMcC	+ 96	6	- 45	6
Lei	+ 111	8	- 48	8
Loz	+ 92	10	- 38	10
Arty	+ 114	7	- 30	7
RY Psc				
Mosc	+ 47	6	+ 2	6
V675 Sgr				
Lou	+ 0	6	+ 12	6
V1640 Sgr				
Lou	- 4	6	+ 9	6
V494 Sco				
Lou	- 20	6	+ 9	6
RU Scl				
m.c.	+ 25	24	- 14	24
VY Ser				
Mosc	- 113	7	+ 57	7
m.c.	- 72	23	- 26	25
AP Ser				
Mosc	- 54	8	- 44	8
Mosc 3)	- 48	8	- 31	8
Tou	- 39	4	- 37	4
AT Ser				
Mosc	- 10	6	+ 1	6
Tou	- 8	8	- 7	4
CW Ser				
Lei	- 4	8	- 0	8
Tou	- 13	6	+ 14	6
SS Tau				
MtW	+ 8	3	+ 1	2
Lei	+ 24	9	- 8	9
B and B	+ 30	12	+ 27	12
Tou	+ 10	7	- 5	6
AH Tau				
Lei	- 39	6	- 57	6
U Tri				
MtW	+ 4	2	- 6	2
LMcC	+ 26	4	- 21	4
Lei	+ 24	8	- 16	8
B and B	+ 20	12	- 48	12
K and vR	+ 23	10	- 14	10
W Tuc				
Lou	+ 3	6	+ 2	6
YY Tuc				
Lou	- 2	6	- 0	6
RV UMa				
Mosc	- 19	7	- 42	7
Lei	- 29	7	- 37	7
B and B	+ 13	13	- 18	13
SX UMa				
MtW	- 79	4	+ 13	4
Lei	- 68	8	+ 5	8
B and B	- 42	13	+ 2	13
TU UMa				
m.c.	- 86	14	- 52	1
Loz	- 77	8	- 51	8
AI Vel				
m.c.	+ 29	11	+ 36	8
CD Vel				
Rus	- 28	6	+ 30	6
ST Vir				
MtW	- 7	3	- 18	5
Mosc	+ 3	6	- 25	6
Lei	- 4	7	- 16	7
Luy	- 15	11	- 39	11
K and vR	+ 2	8	- 20	8
UU Vir				
Mosc	- 23	6	- 2	6
Luy	- 18	21	- 13	21
m.c.	- 30	2	+ 25	12
UV Vir				
Mosc	- 16	6	- 32	6
XX Vir				
Mosc	- 9	6	- 16	6
AD Vir				
SF	- 17	9	- 17	8
AF Vir				
Tou	- 60	4	+ 13	4
AS Vir				
SF	- 45	9	- 78	8
AU Vir				
SF	- 8	9	- 17	8
AV Vir				
Tou	+ 16	4	- 36	3
BB Vir				
Tou	- 45	4	+ 12	3
BC Vir				
Mosc	- 12	8	- 32	8
Tou	+ 32	7	- 34	6

Remarks to table 3.

1) Pavlovskaya has increased Parenago's value for μ_δ by $+23$. The original value is used here.
2) The value given by Pavlovskaya for the Moscow motion was determined from meridian-circle positions.
3) Two values, both determined at Moscow, were published by Pavlovskaya, the relation between them being unknown.
4) The relative motions given by Vyssotsky and Williams are entered in Pavlovskaya's publication. Her value taken from R. E. Wilson was based on meridian-circle positions.
5) Pavlovskaya does not mention Luyten's determination, but credits this same result to one of her students. I assume this is a misprint.

The abbreviations used in table 3 are the following.

Agh p	Allegheny parallaxes
Arty	N. M. Artyukhina, 1946, *Variable Stars* **6** 88
B and B	P. Fairfield-Bok and C. D. Boyd, 1933, *Astr. Obs. Harvard Bull.* No. 893
Cape p	Cape parallaxes
Dzi	R. M. Dzigvashili and D. S. Haftazi, 1951, *Comm. Sternberg Inst.* No. 56
Grw AC	*Greenwich Astrographic Catalogue*
Grw p	Greenwich parallaxes
Hels	V. R. Ölander, R. Lehti, G. Pipping and A. Savelius, 1959, *Soc. Sci. Fennicae Comm. Phys. Math.* **22** 37
K and vR	J. C. Kapteyn and P. J. van Rhijn, 1922, *Bull. Astr. Inst. Netherlands* **1** 37
Kar	D. K. Karimova, 1949, *Variable Stars* **7** 43
Kats	O. W. Katz, 1947, *Variable Stars* **6** 127
Kov	M. S. Kovalenko, 1936, *Astr. J.* **45** 94
Kur	I. A. Kurzemniece, 1950, *Pub. Phys.-Math. Inst. Latvian S.S.R. Sci.* **1** 119
LMcC	S. A. Mitchell, private communication to Dr. Oort
Lei	Leiden, motions discussed in this paper
Lou	J. v. B. Lourens, 1960, *Mon. Not. Astr. Soc. S. Africa* **19** 119
Loz	A. M. Lozinskij, 1951, *Comm. Sternberg Inst.* No. 56 19 A. M. Lozinskij, 1952, *Comm. Sternberg Inst.* No. 81 20 A. M. Lozinskij, 1953, *Variable Stars* **9** 324
Lur	M. A. Lurie, 1950, *Variable Stars* **7** 182
Luy	W. J. Luyten, 1927, *Astr. Obs. Harvard Bull.* No. 847
m.c.	meridian circle
Mit p	S. A. Mitchell, parallaxes from Leander McCormick Observatory
Mosc	E. D. Pavlovskaya, 1953, *Variable Stars* **9** 233
MtW	Mount Wilson, motions derived by R. E. Wilson, A. van Maanen and by the author
Par	P. P. Parenago, 1946, *Variable Stars* **6** 79
Ray	H. Raymond, 1940, *Astr. Obs. Harvard Bull.* No. 914
Ros	R. M. Rosenfeld, 1950, *Variable Stars* **7** 207
Rus	T. W. Russo, motions derived at the Cape Observatory, private communication to Dr. Plaut
SF	San Fernando (1950–1963, Serie A Nos. 1–4)
Shnir	P. G. Shnirelman, 1945, *Astr. Zhur.* **22** 34
Tor	A. Torondzjadze, 1948, *Variable Stars* **6** 328
Tou	1958, 1959, 1961, *Ann. Obs. Astr. Toulouse* **26**, **27** and **28**
Vyss	A. N. Vyssotsky and E. T. R. Williams, 1948, *Pub. Leander McCormick Obs.* No. 10

used faint comparison stars and therefore could not apply individual corrections for their proper motions.

c) Harvard proper motions, derived by Fairfield-Bok and Boyd (1933). The interdependence between these motions and those derived by Pavlovskaya will be greater than the one between the latter and the Leiden motions. However, it is impossible to estimate the amount of correlation, as we do not know for which stars the authors used Carte du Ciel positions.

d) Motions derived by Luyten (1927). We saw no reason to leave out a number of these (as was done by Pavlovskaya). Nothing is known about correlation between these motions and those derived elsewhere. The motions were made absolute with the aid of the values for various constants given earlier.

e) Motions derived by Kapteyn and Van Rhijn (1922). Again we used all values published, and the previous remark applies here also.

f) Motions derived by various astronomers, all mentioned in Pavlovskaya's paper, viz. Artynkhina, Dzigvashvili and Haftazi, Ikaunicks, Karimova, Kats, Kulikov, Kurzemniece, Lozinsky, Lurie, Parenago, Romanovsky, Rosenfeld, Safronov, Shnirelman and Torondzjadze. The reduction to absolute was again performed with the values mentioned before for the secular parallax and the constants of galactic rotation. Where we lacked the original publications, Pavlovskaya's values were used, with the differences in the rotation constants taken into account.

g) Motions derived through a repetition of the Carte du Ciel at various observatories:

Ölander *et al.* (1959), Helsingfors. The motions were reduced to absolute values (by Dr. L. Plaut); the mean errors were increased so as to include the cosmical error.

San Fernando (1950–1963, Serie A Nos. 1–4). The reduction to absolute proper motions was done as follows: The differences Yale 16 − SF, GC − Yale, FK3 − GC were added to the proper motions. The corrections were applied irrespective of the declinations. Though this procedure is not ideal, no better way could be found.

Paloque *et al.* (1958, 1959 and 1961), Toulouse. The differences between the motions published in volumes

26, 27 and 28 and those in the Yale catalogues were smoothed and added to the Toulouse motions, together with the GC—Yale and FK3—GC differences. The average mean errors were adopted.

h) Motions published by Cape observers:

LOURENS (1960). The motions were reduced to the FK3 system with the aid of the tables in *Bull. Astr. Inst. Netherlands* **10** 473.

RUSSO (1960). The reductions to absolute were performed with the latest adopted constants.

i) A few motions were found in publications dealing with parallax determinations. No effort has been made to obtain all published values.

j) Motions derived from published meridian-circle positions. The final system adopted is the FK3. Dr. R. H. Stoy kindly supplied the *Second Cape 1950 Catalogue* position for SX Phe in advance of publication.

6. Comparison of the motions from different sources

It has been possible to make 22 comparisons between nine sources with six or more stars in common. The values for Moscow and Lozinsky were taken together; this was also done for the two different parts of the material from Mount Wilson. Table 4 gives the details of the results. The unit is 0″.0001. The columns contain the abbreviations for the sources compared, the average difference in the proper motion in right ascension with its mean error, the same for the proper motion in declination, the number of stars involved and the average mean error for a difference in proper motion as expected from the adopted mean errors given by each source, as well as the mean error for the difference actually found. As some differences turned out to be exceptionally high, the formula 1.25 $\langle|d|\rangle$ was used to compute the mean errors tabulated as "observed". These high deviations thus carry less weight than those determined from the squares of the residuals.

A procedure of successive assumptions was used to derive the final weights for the various sources. As the motions derived from the Mount Wilson, the McCormick, and the Helsinki material are considerably more accurate than those derived from the other sources, and as the Helsinki results do not overlap sufficiently with those of the other two sources mentioned here, it

TABLE 4

Comparison of different sources

Sources	$\langle\Delta\mu_\alpha\rangle$ (0″.0001)	m.e.	$\langle\Delta\mu_\delta\rangle$ (0″.0001)	m.e.	n	m.e. of $\Delta\mu_\alpha$ exp. (0″.0001)	m.e. of $\Delta\mu_\alpha$ obs.	m.e. of $\Delta\mu_\delta$ exp. (0″.0001)	m.e. of $\Delta\mu_\delta$ obs.
1) MtW—LMcC	− 40	± 22	+ 28	± 20	16	± 56	± 81	± 58	± 78
2) MtW—Mosc	− 5	37	+ 35	44	21	84	136	86	153
3) MtW—Lei	− 91	22	− 0	23	44	94	151	94	138
4) MtW—B and B	− 39	34	− 3	38	32	128	161	129	208
5) MtW—m.c.	− 63	56	− 41	66	8	258	150	166	175
6) MtW—K and vR	− 86	87	+ 41	51	8	87	213	88	124
7) MtW—Luy	+150	93	−148	102	6	160	199	161	234
8) LMcC—Mosc	+ 63	32	+ 36	57	7	92	80	92	155
9) LMCc—Lei	− 11	40	+ 11	26	16	94	146	94	104
10) LMcC—B and B	+ 26	29	− 19	49	14	124	96	124	187
11) LMcC—m.c.	+ 28	33	+ 18	42	6	146	75	108	105
12) Mosc—Lei	− 54	35	− 43	34	25	113	156	113	135
13) Mosc—B and B	− 48	39	− 86	36	15	140	149	140	141
14) Mosc—m.c.	− 97	74	+ 56	109	9	236	219	163	258
15) Mosc—Luy	+ 52	54	− 33	96	6	156	143	156	195
16) Mosc—Tou	−130	70	− 4	27	6	83	134	76	52
17) Lei—B and B	+ 23	51	+ 6	43	31	150	274	150	236
18) Lei—m.c.	− 31	49	− 12	41	12	241	170	170	125
19) Lei—K and vR	− 74	62	+ 62	54	8	116	174	116	145
20) Lei—Luy	+206	116	−136	115	6	157	276	157	293
21) B and B—m.c.	− 27	144	− 21	83	9	339	405	232	208
22) B and B—K and vR	−125	70	+ 13	98	8	152	198	152	293

Table 5
Influence of different weights for the Mount Wilson and Leander McCormick proper motions

Source	$W_{\mathrm{MtW}} : W_{\mathrm{LMcC}} = 1 : 1$				$W_{\mathrm{MtW}} : W_{\mathrm{LMcC}} = 1 : 2$			
	m.e.	m.e. of m.e.	m.e.	m.e. of m.e.	m.e.	m.e. of m.e.	m.e.	m.e. of m.e.
	(0″.0001)		(0″.0001)		(0″.0001)		(0″.0001)	
MtW	±57	±7			±65	±8		
LMcC			±57	±7			±46	±6
	from MtW		from LMcC		from MtW		from LMcC	
Mosc	132	18	103	49	128	18	109	46
Lei	132	12	111	18	128	13	116	17
B and B	175	18	130	21	172	18	134	21
m.c.	152	32	70	26	148	32	77	23
K and vR	158	33			155	33		
Luy	208	48			206	49		

is practically impossible to obtain the true mean errors for Mount Wilson and McCormick from these comparisons. All one can do is to derive the mean error of the differences between the Mount Wilson and McCormick results. The mean errors for each of these two sources individually can then be found by making some assumption concerning their relative accuracy. As may be seen from the next to the last column of table 6 the mean errors from the measurements themselves, taking account of the errors caused by the unknown motions of the comparison stars, are approximately the same for the two sources. On the other hand, the numbers given in table 4, indecisive though they are, give a slight indication that the McCormick results have somewhat higher weight. I have made two solutions for the weight. For the first solution it was assumed that the weights of Mount Wilson and McCormick are equal, while in the second solution McCormick was given twice the weight of Mount Wilson. In the first case the mean error for a proper motion from each of the two observatories is ± 57, and in the second case it is ± 65 and ± 46. (The results for right ascension and for declination are from now on averaged, as there is no reason to keep them separated.)

If we use these values to derive the mean errors for the other sources we arrive at the values given in table 5.

Though the mean errors for both Moscow and Leiden agree better when derived with the last mentioned ratio of the weights for Mount Wilson and Leander McCormick than with the first ratio, the improvement cannot be considered significant, considering the mean errors. The improvement would become important only with an absurdly high ratio between the weights of the Leander McCormick motions and the Mount Wilson ones.

We consider first the mean errors for Moscow and Leiden. These turned out to be identical, and are higher than what we find from the direct comparison Moscow-Leiden. This is understandable, because the two results are not independent, as the same coordinates of the variables from the *Carte du Ciel Catalogues* were used. Once the mean errors for Moscow and Leiden are adopted, a second determination can be made of the mean errors of the other sources from the comparisons 12–20 (table 4). We can similarly use Fairfield-Bok and Boyd to obtain mean errors for m.c. and K and vR, though now the results become very uncertain as the mean error for Fairfield-Bok and Boyd is rather high.

The results of these consecutive steps are entered in table 6. The last two columns of this table show the average mean errors, as originally found by the various authors (increased as explained above), and the ratio between the finally adopted mean errors and the original values. The average ratio, 1.5 (omitting the meridian-circle result), is used to increase the given published mean errors from sources which we had not yet considered (see the bottom part of table 6).

It would have been more satisfying if we had had sufficient comparisons for each source to determine final mean errors in the manner just discussed, but as we lack these possibilities, the present method seemed the only way out of this problem.

TABLE 6

Adopted weights for different sources of proper motions

Source	From MtW and LMcC together m.e. (0″.0001)	From Mosc and Lei together m.e. (0″.0001)	From B and B m.e. (0″.0001)	Adopted m.e. (0″.0001)	Adopted relative weights	Original m.e. (0″.0001)	Adopted: original m.e.
MtW	± 57			± 57	3.0	± 38	1.5
LMcC	57			57	3.0	45	1.3
Mosc	126	±103		126	0.6	73	1.7
Lei	126	103		126	0.6	86	1.5
B and B	158	172		165	0.4	122	1.4
m.c.	117 ±20	131 ±54	±258 ±64	130	0.6	161	0.8
K and vR	158 33	99 49	182 62	147	0.5	83	1.8
Luy	208	184		196	0.3	146	1.3
SF				125	0.6	83	
Tou				75	1.8	50	
Lou				180	0.4	120	
Rus				141	0.5	94	
Hels				51	3.0	34	

The question is how much we gain in combining the different sources, when the first-epoch positions are all based on the Carte du Ciel. These sources are: Moscow, Leiden, part of Fairfield-Bok and Boyd, and Kapteyn and Van Rhijn. The first two, when taken together, will have a weight between one and two times their original weight. The fact that the reference stars were different for the two sets of determinations increases the weight for the first-epoch positions, but not by a factor 2, as the positions of the variables are the same. Doubling the weight of the position at the last epoch increases the weight of the proper motion by a factor 4/3. All in all, we have assumed that a combination of the Leiden and Moscow results corresponds with an increase in weight by a factor of 1.5. For the other sources the increase will be of a somewhat smaller order, as there will have been several reference stars in common; but as the weights of these other sources are already appreciably lower, the correct factor is of no great importance.

It is unlikely that there is much dependence between the Mount Wilson and the Leander McCormick results because the reference stars are in general quite different.

The relatively low weight for Luyten's values is mainly a consequence of the short interval in time between his first- and second-epoch plates. The relatively high weight for the Toulouse and Helsinki proper motions is due to the great number of plates taken, and, especially for Helsinki, to the great care with which the circumstances for the new plates were made equal to those of the first-epoch plates. Moreover, stars more than 4 cm from the centre of their plates were omitted from their observing list, and the plates were measured differentially with respect to the first ones.

Another point to be considered is whether the present material shows evidence of systematic differences between the results from the different sources. From table 4 one might get the impression that a difference exists between the results from Moscow and Leiden, but it is hard to tell to which source one would have to attribute the difference, and it is questionable whether the difference is significant. Accordingly we have decided not to apply any systematic corrections.

Table 7 is printed on opposite pages. The left-hand side contains the designation of the star, the new galactic longitude and latitude, the angle between star and antapex used in groups 3–8 of table 11, the mean photographic or B magnitude together with reference keys, the absorption, the period, the Bailey type, the metal index ΔS and the final means for the yearly proper motions in α and in δ with the mean errors. The right-hand side gives the designation of the star, the υ- and τ-components with respect to the apices derived for groups 3–8 of table 11, the radial velocities V with reference keys, the space velocities in km/sec with respect to the Sun (Π and Θ are the components parallel to the galactic plane, in the direction away from the galactic centre and in the direction of the galactic rotation, respectively; Z is directed towards the north galactic pole), and based on the adopted average absolute magnitude of $+0^m.9$ with the mean error of one component. The $\Delta\Pi$, $\Delta\Theta$ and ΔZ, added to the space velocities mentioned before will give the space velocity components for an average absolute magnitude of $+0^m.5$. The mean errors for μ_α are the same as for μ_δ except for RZ Cep, VX Her and SX Phe, where these are respectively ± 3, ± 4 and ± 2 in the units given.

7. The radial velocities

Most velocities are from Joy (see the complete list below for further references), partly corrected by Mrs. Payne-Gaposchkin. The correction given by her to V LMi is so large that this has been taken to be a printing error, and Joy's original value was retained. There are 29 stars for which more than one author has given a determination of the radial velocity. A comparison of the different results for these 29 stars (60 residuals) yields a mean error of ± 23 km/sec. If we leave out the values derived by Colacevich, as reduced by Notni, we obtain from 19 stars (39 residuals) an r.m.s. error of ± 13 km/sec. In these comparisons all velocities except Joy's are determined from complete cycles, and the assumption underlying the computation of the r.m.s. error, namely that all determinations will have had the same weight, will not be true. As a mean error for one determination based on a few plates only, the value ± 18 km/sec was adopted.

No systematic difference could be found, either between the values given by Joy and the corrected values given by Mrs. Payne-Gaposchkin, or between Joy's values and the combined other determinations.

The key to the references given in table 7 for V follows here.

1. K. D. Abhyankar, 1959, *Ap. J.* **130** 834
2. H. A. Abt, 1955, *Ap. J.* **122** 390
3. W. P. Bidelman, 1947, *Ap. J.* **106** 135
4. W. K. Bonsack, 1957, *Ap. J.* **126** 291
5. A. Colacevich, 1935, *Lick Obs. Bull.* **17** 171
6. A. Colacevich, 1950, *Ap. J.* **111** 437
7. A. V. Farquahar, 1948, *Ap. J.* **107** 276
8. L. Gratton, 1953, *Bull. Astr. Inst. Netherlands* **12** 31
9. L. Gratton and C. J. Lavagnino, 1953, *Z. Ap.* **32** 69
10. G. H. Herbig, 1949, *Ap. J.* **110** 156
11. A. H. Joy, 1938, *Ap. J.* **88** 408
12. A. H. Joy, 1938, *Pub. Astr. Soc. Pacific* **50** 302
13. A. H. Joy, 1950, *Pub. Astr. Soc. Pacific* **62** 60
14. A. H. Joy and R. E. Wilson, 1950, *Pub. Astr. Soc. Pacific* **62** 58
15. A. H. Joy, 1955, *Pub. Astr. Soc. Pacific* **67** 420
16. T. D. Kinman, 1961, *Roy. Obs. Bull.* No. 37
17. G. Münch, 1951, *Ap. J.* **114** 546
18. P. Notni, 1956, *Mitt. Univ. Sternw. Jena* No. 26
19. G. F. Paddock and O. Struve, 1954, *Ap. J.* **119** 346
20. C. Payne-Gaposchkin, 1954, *Ann. Astr. Obs. Harvard* **113** 153
21. J. Ponsen, 1963, *Bull. Astr. Inst. Netherlands* **17** 29
22. G. W. Preston, 1959, *Ap. J.* **130** 507
23. G. W. Preston, H. Spinrad and C. M. Varsavsky, 1961, *Ap. J.* **133** 484
24. A. W. Rodgers, 1960, *Observatory* **80** 220
25. J. Sahade, O. Struve, O. C. Wilson and V. Zebergs, 1956, *Ap J.* **123** 399
26. R. W. Sanford, 1949, *Ap. J.* **109** 208
27. H. Spinrad, 1960, *Ap. J.* **131** 134
28. O. Struve, 1949, *Astr. J.* **54** 137
29. O. Struve and A. Blaauw, 1948, *Ap. J.* **108** 60
30. O. Struve and A. van Hoof, 1949, *Ap. J.* **109** 215
31. W. G. Tifft and H. J. Smith, 1958, *Ap. J.* **127** 591
32. C. M. Varsavsky, 1960, *Ap. J.* **131** 623
33. O. C. Wilson and M. F. Walker, 1956, *Ap. J.* **124** 325
34. L. Woltjer, 1956, *Bull. Astr. Inst. Netherlands* **13** 62

Note added after this work was completed:

Breger's (1964) article came too late to my attention to make use of his values of nine radial velocities. Some of his values show large differences with the values used in this article. We want to emphasize the great need for more determinations of this kind.

8. The magnitudes

The choice of the magnitudes $(B_{\max}+B_{\min})/2$ is important in connection with the determination of the absolute magnitude M. The magnitudes given in table 7 are based on many sources. Where two decimals are printed, a photo-electric or a good photographic magnitude was available; the references are numbered, and the key to these references is given below. Most of the accurate photo-electric determinations are due to the efforts made by Dr. G. E. Kron, who had set up a special programme for these stars. Where only one decimal is given, the values are taken from the *General Catalogue of Variable Stars* by Kukarkin *et al.*, with certain corrections. The number of photo-electric determinations by various authors was not large enough

1965BAN....18...71V

TABLE 7

Data for 210 RR Lyrae stars

Star	l	b	λ	$\langle m_{pg} \rangle$	Ref.	Abs.	P	t	ΔS	μ_α	μ_δ	m.e.
											(0".001)	
SW And	116°	-33°	135°	9.86	3,22,32	$0^{\mathrm{m}}.29$	$0^{\mathrm{d}}.442$	a	0	+1	-23	4
XX	128	-24	134	10.86	15,22	0.45	0.723	a	9	+59	-33	4
AC	107	-11	155	11.3	y	0.86	0.525	a		-10	-3	9
AT	110	-18	151	11.4	a	0.60	0.617	a	3	-2	+46	6
CC	121	-21	126	9.68	39	0.62	0.125			-11	-27	9
WY Ant	267	22	25	11.16	21	0.49	0.574	a	6	+18	-57	16
SW Aqr	51	-31	132	11.20	22	0.00	0.459	a	5	-43	-52	4
SX	58	-34	134	11.86	22	0.12	0.536	a	9	-42	-44	5
TZ	53	-44	124	12.7	yz	0.27	0.571	a	5	+6	-11	13
BO	55	-59	114	12.3	y	0.22	0.694	a	(6)			
BR	75	-65	111	11.8	y	0.21	0.482	a	3	+2	-8	13
BS	82	-66	104	9.70	21,32	0.49	0.198	c	0	+25	-12	7
BT	43	-31	128	11.8	y	0.37	0.407		2	+6	-5	13
CY	69	-47	117	11.01	15,20,23,30	0.20	0.061		2	+63	-57	11
AA Aql	43	-25	131	11.90	22	0.00	0.362	a	0	-3	-13	5
V341	46	-22	133	11.4	yz	0.50	0.578	a	3			
S Ara	343	-12	77	11.02	21	0.37	0.452	a	3	-22	-16	10
X Ari	169	-40	96	9.88	22,29,30	0.86	0.651	a	10	+65	-88	3
RV	149	-40	85	12.31	8,30	0.86	0.093			-9	-11	11
RW	150	-41	102	12.4		0.29	0.261	c				
TZ Aur	177	21	88	11.93	22	0.00	0.392	a	2	-3	-12	6
RS Boo	51	67	115	10.56	15,22,32	0.20	0.377	a	2	+22	+4	4
RU	31	64	107	13.8		0.21	0.493	a		-16	-3	6
ST	57	55	118	10.86:	15	0.08	0.622	a		-16	+1	5
SV	69	66	116	13.0		0.21	0.581	a		+1	-15	5
SW	63	68	113	12.50	22	0.04	0.514	a	(7)	-24	-1	5
SZ	42	66	109	12.8	y	0.21	0.523	a		-2	-3	5
TV	80	67	119	11.21	32	0.12	0.313	c	8	-12	-29	6
TW	71	63	118	11.23	15	0.20	0.532	a		-1	-45	6
UU	56	58	120	12.2	y	0.22	0.457	a				
UY	354	69	90	11.3		0.20	0.651	a	(10)	-15	+34	13
YZ	59	56	132	10.72	6,10,32	0.29	0.104		(0)	+9	-22	13
RZ Cam	148	23	117	13.0	yz	0.49	0.480	a		+19	-19	18
RW Cnc	197	44	77	11.6		0.27	0.547	a		+5	-38	5
SS	199	26	69	12.34	22	0.00	0.367	a	2	+6	-9	5
TT	212	28	61	11.4		0.39	0.563	a	7	-48	-42	13
VZ	216	29	55	7.96	2,13,15,30,33	0.49	0.178		0	-18	-20	5
W CVn	72	71	111	10.72	15	0.33	0.552	a	7	-34	-7	9
Z	124	73	105	11.5	b	0.20	0.654	a	(8)	-3	-25	9
RR	154	81	96	12.64	22	0.00	0.559	a	7	-9	+6	16
RU	54	75	105	12.1	y	0.20	0.573	a		-42	-3	5
RX	87	72	110	12.8	yc	0.20	0.540	a				
RZ	62	77	104	11.4		0.19	0.567	a				
SS	84	73	110	12.1	c	0.20	0.479	a		+6	-46	5
ST	47	75	108	11.6		0.20	0.329	c		-34	-16	13
SV	140	79	98	12.4	y	0.19	0.668	a		-20	-30	13
SW	135	80	99	12.3	d	0.19	0.442	a				

TABLE 7 (*continued*)

Star	υ	τ	V	Ref.	$\Pi_{0.9}$	$\Theta_{0.9}$	$Z_{0.9}$	m.e.	$\Delta\Pi$	$\Delta\Theta$	ΔZ
	(0".001)		(km/sec)		(km/sec)				(km/sec)		
SW And	+15	−17	−22	4,12	−18	−45	−34	11	−2	−6	−9
XX	+67	+5	−16	12	+150	−189	−86	15	+32	−36	−18
AC	−9	−5	−69	12,17	−58	−52	+16	34	−8	+3	+1
AT	−21	+41	−250	22	+6	−187	+274	27	+17	+8	+39
CC	+3	−29	−12	33	−25	−6	−49	18	−5	−2	−10
WY Ant	+35	+48	+208	16	−193	−259	−60	68	−41	−14	−27
SW Aqr	+47	−49	−5	12	−296	−213	+40	21	−61	−43	+7
SX	+35	−49	−220	15	−276	−353	+167	35	−75	−40	+9
TZ	+12	+3	+25	22	−12	−79	−93	123	0	−19	−16
BO			−55	13							
BR	+8	−2									
BS	+26	+11	+41	12,16	+38	−28	−59	15	+8	−8	−5
BT	+6	+5									
CY	+85	0	−30	13,28	+135	−278	−246	48	+26	−52	−54
AA Aql	+13	−4	−83	12	+20	−132	+8	38	−9	−14	−7
V341			−135	13							
S Ara	+24	−12	+185	16	−154	−156	+12	42	+4	−21	+11
X Ari	+109	−12	−40	12	−21	−220	−13	6	+1	−44	−7
RV	+1	−14	+35	13	−27	+6	−82	63	−11	−1	−12
RW			−60	13							
TZ Aur	+11	−5	+30	15,18	+51	−69	−34	45	+5	−14	−10
RS Boo	−20	+11	−9	12	−34	+61	−40	14	−8	+13	−6
RU	+14	−9	−60	15	+149	−222	+52	99	+26	−43	+21
ST	+12	−11	+21	13	+32	−35	+58	22	+7	−9	+8
SV	+9	+12	−160	13	−111	−164	−125	56	−27	−20	+5
SW	+18	−16	+15	12	+144	−155	+100	48	+30	−32	+17
SZ	+4	+1	−45	13	+5	−45	−34	48	−1	−6	+1
TV	+27	+16	−85	12	−53	−172	−25	31	−11	−28	+10
TW	+29	+34	−111	12	−156	−189	−59	30	−35	−29	+8
UU			+20	13							
UY	−16	−34	+143	7,13	+108	+82	+198	67	+33	+17	+13
YZ	+2	+24									
RZ Cam	+25	+9									
RW Cnc	+38	+8	−78	12	−110	−197	−65	29	−12	−43	−2
SS	+9	+6	+5	12	−49	−80	+28	45	−9	−16	+7
TT	+39	−50	+50	13,18	+111	−178	−244	64	+15	−31	−54
VZ	+24	−13	+26	2,14,27	+19	−28	−8	5	+1	−3	−4
W CVn	+29	−19	+22	12	+70	−93	+61	33	+15	−21	+9
Z	+22	+12	0	12	−60	−124	+41	51	−12	−25	+8
RR	0	−11	−10	12	+111	+6	−26	167	+23	+1	−4
RU	+31	−28	−55	12	+223	−229	+31	37	+43	−44	+17
RX			+5	13							
RZ			−15	12							
SS	+29	+36	−5	13	−262	−223	+55	38	−53	−44	+12
ST	+36	−11	−85	13	+104	−220	−28	78	+18	−41	+12
SV	+36	0	+12	12	+19	−309	+47	114	+4	−62	+7
SW			−36	13							

TABLE 7 (*continued*)

Star	l	b	λ	$<m_{pg}>$	Ref.	Abs.	P	t	ΔS	μ_α	μ_δ	m.e.
										(0".001)		
AD CMi	219°	14°	45°	9.42	1	$0^m.00$	$0^d.123$			- 13	+ 2	4
RV Cap	33	-36	117	11.3	y	0.33	0.448		6	+ 30	-109	5
YZ	35	-39	117	11.52	21	0.41	0.273	c	0-1	- 23	- 9	16
IU Car	270	-23	21	12.32	21	0.78	0.737	a	9	- 9	- 27	16
BI Cen	295	2	29	12.29	21	0.78	0.453	a	2	+ 6	+ 16	16
V499	315	18	52	11.38	21	0.41	0.521	a	6	+ 20	- 23	11
RZ Cep	109	5	153	9.89	16,22,32	1.11	0.309		5	+ 99	+189	4
DQ	94	6	152	7.58	38	0.45	0.079			+ 26	+ 19	2
RR Cet	144	-60	104	9.90	22,30	0.20	0.553	a	5	+ 19	- 52	5
RX	102	-78	100	11.6	y	0.19	0.574	a				
RZ	178	-60	87	11.7:	y	0.22	0.511	a	4	+ 12	+ 1	13
RY Col	246	-35	37	11.22	21	0.45	0.479	a	3	+ 36	+ 18	16
S Com	213	86	90	11.59	22	0.00	0.587	a	7	- 12	- 7	5
U	204	87	95	12.0		0.19	0.293	c	5	- 46	- 18	5
V	209	81	88	12.8		0.19	0.469	a		- 13	- 2	5
Z	328	81	88	13.4	y	0.19	0.547	a		+ 15	- 3	6
RT	335	78	88	14.0	y	0.19	0.565	a		+ 14	- 17	6
RV	340	80	94	14.2	y	0.19	0.350	c		- 4	- 4	6
RY	342	85	91	12.1	e	0.19	0.469	a	3:			
ST	348	81	91	11.6	y	0.19	0.599	a	5	- 36	- 30	11
RV CrB	49	45	132	11.56	22	0.00	0.332	c		- 20	- 19	5
W Crt	276	40	48	11.6	y	0.29	0.412	a	3			
X	279	49	52	11.2	y	0.25	0.733	a		+ 35	- 66	13
SW Cru	296	2	35	12.91	21	1.60	0.328	a	0	+ 19	+ 8	16
UY Cyg	74	-10	163	11.33	22	0.25	0.561	a	3	+ 4	- 10	4
XX	92	14	164	11.94	22	0.00	0.135			- 5	+ 2	5
XZ	88	17	166	9.79	15,22,30	0.12	0.467	a	6	+ 81	- 24	4
DM	79	-12	161	11.2	y	0.69	0.420	a	0	+ 5	+ 2	11
BX Del	60	-10		12.1		1.00	1.092		0			
CK	54	-15	143	12.4		0.73	0.443	a		+ 13	+ 2	7
DX	58	-19	145	10.36	29	0.94	0.473	a	2			
δ	60	-17	149	4.83	10	0.57	0.135			- 20	- 42	1
RW Dra	87	41	142	11.72	22	0.20	0.443	a	3	- 2	- 20	5
SU	133	48	119	9.96	15,22,34	0.12	0.660	a	10	- 39	- 73	5
SW	127	47	123	10.6	y	0.26	0.570	a	3	- 20	- 3	5
XZ	96	22	158	10.39	15	0.62	0.476	a	3	+ 29	- 8	13
RX Eri	214	-34	58	9.96	21,22,30	0.57	0.587	a	9-6	- 28	- 3	5
SV	194	-53	79	9.6		0.23	0.714	a	9			
BB	219	-34	55	12.1	yz	0.34	0.570	a	8			
BC	213	-34	54	10.7	f	0.33	0.264	c				
SS For	216	-73	77	9.7	y	0.20	0.495	a				
SW	243	-61		12.65	21	0.53	0.804	a	10	- 1	+ 2	16

TABLE 7 (*continued*)

Star	υ	τ	V	Ref.	$\Pi_{0.9}$	$\Theta_{0.9}$	$Z_{0.9}$	m.e.	$\Delta\Pi$	$\Delta\Theta$	ΔZ
	(0".001)		(km/sec)		(km/sec)				(km/sec)		
AD CMi	- 1	- 13	+ 34	1	+ 36	- 8	- 17	10	+ 3	+ 1	- 4
RV Cap	+110	+ 25	-110	12	+ 6	-521	-217	25	- 14	- 96	- 56
YZ	+ 4	- 24	- 88	13,16	- 33	- 83	+124	83	- 19	- 8	+ 13
IU Car	- 18	+ 22	+307	16	-148	-244	-214	101	- 30	+ 8	- 19
BI Cen	- 5	- 16	+210	16	-120	-171	+116	101	- 6	+ 4	+ 21
V499	- 4	+ 30	+332	16	-300	-205	- 24	54	- 15	+ 4	- 25
RZ Cep	+172	+126	0	12	+301	-125	+210	7	+ 61	- 25	+ 42
DQ	+ 32	+ 6	- 22	25	+ 27	- 23	- 8	8	+ 5	0	- 1
RR Cet	+ 52	- 20	- 96	12	- 66	-165	+ 23	14	- 6	- 27	- 12
RX			- 75	12,18							
RZ	+ 7	+ 10	- 15	18	+ 49	- 40	+ 44	81	+ 11	- 8	+ 7
RY Col	+ 24	+ 32	+471	16	+229	-423	-137	71	+ 15	- 14	+ 27
S Com	+ 12	- 6	- 34	12	+ 41	- 78	- 40	33	+ 9	- 16	- 1
U	+ 43	- 24	+ 16	12	+214	-285	+ 2	36	+ 43	- 57	- 3
V	+ 8	- 10	0	15	+108	- 82	- 22	52	+ 22	- 16	- 4
Z	- 7	+ 14	- 50	15	-185	+ 89	- 69	83	- 38	+ 17	- 4
RT	+ 5	+ 22									
RV	+ 6	0									
RY			- 28	13							
ST	+ 46	- 10	-100	12	+ 84	-269	- 97	66	+ 15	- 55	0
RV CrB	+ 27	+ 4	-100	12	+ 3	-205	+ 4	33	- 10	- 29	+ 16
W Crt			+ 66	13							
X	+ 33	+ 67	+ 50	13	-305	-166	-115	63	- 61	- 27	- 31
SW Cru	- 19	- 7	- 23	16	-105	+ 60	+ 66	92	- 22	+ 12	+ 13
UY Cyg	+ 10	+ 3	- 5	12	+ 3	- 13	- 46	20	0	- 1	- 9
XX	0	+ 5	-150	12	+ 25	-154	- 2	37	+ 7	- 2	+ 7
XZ	- 82	- 22	-160	12	+ 34	- 88	-257	11	+ 6	+ 13	- 42
DM	+ 2	+ 5	- 49	12	+ 48	- 46	0	43	+ 8	+ 1	- 3
BX Del			+ 30	15							
CK	- 3	+ 13									
DX			- 45	4							
δ	+ 31	- 35	+ 9		- 12	+ 2	- 5	1	- 2	- 1	0
RW Dra	+ 6	+ 19	-124	12	- 89	-125	- 50	32	- 19	- 6	+ 6
SU	+ 83	+ 2	-175	12,32	- 14	-295	- 35	14	+ 13	- 43	+ 20
SW	+ 15	- 14	- 30	12	+ 55	- 60	- 28	18	+ 14	- 9	- 1
XZ	- 27	- 14	- 25	12	+ 16	+ 8	- 90	37	+ 4	+ 7	- 16
RX Eri	- 17	- 22	+ 70	12,16	+ 30	+ 3	- 92	11	- 4	+ 8	- 11
SV			+ 10	13,18							
BB			+240	13,18							
BC			+ 65	13							
SS For			-115	13							
SW			+167	16							

TABLE 7 (*continued*)

Star	l	b	λ	$\langle m_{pg} \rangle$	Ref.	Abs.	P	t	ΔS	μ_α	μ_δ	m.e.
											(0".001)	
RR Gem	187°	20°	78°	11.52	22	0^{m}.00	0^{d}.397	a	3	− 3	− 5	4
SZ	202	22	68	11.7	g	0.49	0.501	a		− 10	− 25	4
RS Gru	350	−48	85	8.38	11,21	0.37	0.147	a	0	− 61	− 19	10
SW Her	42	34	128	14.0	y	0.34	0.493	a		− 7	+ 13	6
TW	56	25	149	11.30	22	0.00	0.400	a	2	+ 5	− 9	4
VX	35	39	121	10.74	15,22	0.00	0.455	a	5	− 36	+ 15	5
VZ	60	35	140	11.50	22	0.00	0.440	a	4	− 16	− 18	4
AF	65	42	135	12.6	y	0.28	0.630	a		− 17	− 7	4
AG	64	41	135	12.6	y	0.29	0.649	a		− 4	− 17	4
AR	74	48	133	11.23	22	0.00	0.470	a	6	− 61	+ 13	5
AT	71	32	152	10.8	y	0.35	0.33					
CE	39	22		12.38	22	0.41	1.209	a	7	+ 4	− 2	5
DY	28	36	129	10.72	5,13,15,19,32	0.53	0.149	a		+ 5	+ 10	11
SV Hya	297	37	49	11.4	x	0.32	0.479	a		− 53	+ 47	10
SZ	240	26	39	11.5	yz	0.43	0.537	a		0	− 48	13
UU	230	38	53	11.7	y	0.31	0.524			− 10	− 8	7
VX	248	30	41	10.6		0.37	0.183			− 6	+ 30	13
WZ	254	34	38	11.0		0.34	0.538	a				
XX	245	21	32	11.4	y	0.52	0.508	a				
DG	234	25	42	12.3	y	0.45	0.430	a		− 10	− 11	13
DH	238	23	38	11.2	y	0.48	0.488	a		− 20	− 3	13
FY	319	31	59	12.79	21	0.37	0.637	a	(7)	− 52	+ 5	16
CZ Lac	100	− 5	165	11.7	h	1.29	0.432	a	1			
DE	93	−12	159	10.9	i	0.64	0.254	c		+ 5	− 3	6
RR Leo	208	53	74	10.89	15,32	0.29	0.452	a	8	− 15	− 14	7
RV	232	51	61	13.4	yzj	0.24	0.515	a		− 5	− 13	6
RX	209	70	82	12.3	k	0.20	0.653	a	(5)	+ 15	− 30	5
SS	265	57	59	10.8	l	0.23	0.626	a	8	− 19	− 15	5
ST	253	66	70	11.6	z	0.21	0.478	a	7	− 8	− 37	7
SZ	244	58	63	12.1	y	0.22	0.534			− 18	− 30	7
TV	263	49	55	11.4	ym	0.25	0.402		10:			
UZ	230	57	66	10.5	n	0.23	0.618			− 18	+ 8	9
WW	226	38	56	11.7	y	0.30	0.603	a		− 7	− 27	7
AA	254	66	69	11.8		0.21	0.599	a		− 3	− 26	7
V LMi	201	58	79	12.5	yzo	0.23	0.544	a	(4)	+ 31	− 30	5
X	183	54	89	11.6	y	0.24	0.684	a		− 0	− 21	5
U Lep	221	−35	53	10.6	y	0.33	0.581	a	9	+ 44	− 56	5
TV Lib	353	40	95	11.6		0.30	0.270	a	2	+ 6	+ 8	13
UZ	356	37	91	10.7		0.32	0.44			+ 33	− 3	13
EH	356	48	114	10.02	14,15	0.16	0.088			− 3	− 6	9
CW Lup	319	16	60	12.6	x	0.67	0.377			− 9	− 15	14
Y Lyr	73	21	157	13.0	yz	0.53	0.503	a	1	+ 0	+ 2	4
RR	75	12	165	8.00	7,15,17,22	0.41	0.567	a	6	−109	−193	4
RZ	62	16	152	11.58	22	0.00	0.511	a	9	+ 20	+ 30	5
UX	67	22	153	14.6	y	0.51	0.543	a		− 1	− 1	6
EZ	66	16	154	11.5	z	0.67	0.525	a	7			
FN	73	15	162	12.4	y	0.73	0.527			+ 14	+ 11	6

1965BAN....18...71V

TABLE 7 (*continued*)

Star	υ	τ	V	Ref.	$\Pi_{0.9}$	$\Theta_{0.9}$	$Z_{0.9}$	m.e.	$\Delta\Pi$	$\Delta\Theta$	ΔZ
	(0".001)		(km/sec)		(km/sec)				(km/sec)		
RR Gem	+ 4	- 4	+ 94	12	+ 92	- 29	+ 5	24	+ 1	- 3	- 5
SZ	+ 23	- 14	+332	12	+283	-226	+ 29	22	- 1	- 23	- 19
RS Gru	- 5	- 64	+ 81	16	-111	- 28	- 8	12	- 12	- 4	+ 11
SW Her	- 4	- 14	-130	15	+269	- 15	+ 80	102	+ 39	+ 11	+ 32
TW	+ 2	+ 10	- 15	12	- 15	- 19	- 45	23	- 6	0	- 7
VX	+ 15	- 36	-374	12	+328	-226	-102	22	+ 19	- 12	+ 26
VZ	+ 23	+ 6	-120	12	- 22	-184	+ 3	25	- 15	- 20	+ 14
AF	+ 18	- 3	-270	13	+ 63	-304	- 66	37	- 4	- 25	+ 23
AG	+ 12	+ 12	- 75	15	-109	-134	- 29	36	- 26	- 17	+ 4
AR	+ 48	- 40	-335	12	+253	-407	- 38	27	+ 39	- 38	+ 43
AT			- 60	15							
CE			-235	13							
DY	- 11	- 3	- 46	4,13	+ 57	+ 15	- 27	37	+ 4	+ 7	0
SV Hya	+ 8	- 70	+ 78	12	+293	- 78	+217	52	+ 65	- 4	+ 35
SZ	+ 48	+ 5	+100	12	- 92	-233	- 89	67	- 28	- 31	- 27
UU	+ 10	- 8	+300	13,18	+173	-223	+126	41	+ 5	- 9	- 12
VX	- 21	- 22	- 15	13	+ 56	+ 73	+ 39	45	+ 13	+ 11	+ 11
WZ			+315	13							
XX			- 10	13							
DG	+ 12	- 9									
DH	+ 4	- 20									
FY	+ 36	- 38	+ 80	16	+326	-331	+197	152	+ 76	- 59	+ 32
CZ Lac			-120	22							
DE	+ 6	- 1									
RR Leo	+ 17	- 11	+ 65	12	+ 67	- 83	+ 7	28	+ 7	- 13	- 9
RV	+ 14	- 1	0	15	- 18	-161	- 94	81	- 4	- 32	- 18
RX	+ 21	+ 26	-103	12	-235	-166	- 66	41	- 41	- 36	+ 7
SS	+ 22	- 9	+145	12	+ 45	-154	+ 71	20	+ 7	- 15	- 10
ST	+ 36	+ 11	+150	12,24	- 44	-260	+ 59	41	- 13	- 41	- 15
SZ	+ 35	- 4	+ 90	12	+ 35	-268	- 55	52	+ 3	- 45	- 26
TV			- 95	13,18							
UZ	- 2	- 20	- 25	13	+ 57	+ 20	- 44	32	+ 13	+ 1	- 5
WW	+ 28	- 4									
AA	+ 24	+ 10									
V LMi	+ 20	+ 38	- 85	12	-351	-182	+ 64	44	- 62	- 39	+ 27
X	+ 21	+ 4	+ 40	12	+ 7	-123	+ 41	29	- 3	- 24	+ 1
U Lep	+ 70	- 10	+111	12	- 54	-270	+ 3	18	- 25	- 43	+ 14
TV Lib	- 10	- 1	- 58	12	+ 40	+ 57	- 31	73	0	+ 11	0
UZ	- 20	+ 27									
EH	+ 6	+ 2									
CW Lup	+ 16	+ 7									
Y Lyr	- 1	- 2	-110	13	+ 88	- 84	- 30	39	+ 11	+ 3	+ 2
RR	+180	+129	- 71	12,26,29	-202	-129	- 3	4	- 44	- 12	+ 2
RZ	- 35	- 10	-231	12	+302	- 79	-104	33	+ 40	+ 24	- 9
UX	+ 1	0									
EZ			- 75	13							
FN	- 17	- 4									

TABLE 7 (*continued*)

Star	l	b	λ	$<m_{pg}>$	Ref.	Abs.	P	t	ΔS	μ_α	μ_δ	m.e.
										(0".001)		
KX Lyr	69°	20°	155°	11.1	y	$0^{m}.54$	$0^{d}.441$	a	0			
LX	70	19	156	12.1		0.58	0.545	a	8:	- 13	- 20	6
BE Mon	205	1		11.2	y	1.47	2.706	a	0	- 7	+ 10	6
UV Oct	308	-24	45	9.85	21	0.70	0.543	a	9	- 71	-138	16
ST Oph	23	17	116	12.34	22	0.62	0.450	a	6	+ 0	+ 0	5
V445	8	28	107	11.1		0.39	0.397	a	1	+ 9	+ 16	13
V452	32	26	123	12.2	y	0.44	0.557	a	5:	+ 12	+ 14	7
V453	21	18		11.5	yz	0.62	0.971		4			
V567	28	12	133	11.8	z	0.78	0.130	c		- 25	+ 6	13
V716	10	28		12.3	y	0.41	1.116		7			
V784	31	20		12.8	y					- 20	- 34	7
V816	29	18	127	12.4		0.62	0.273	a		- 6	+ 10	7
TY Pav	331	-17	63	12.95	21	0.78	0.710		10	- 10	+ 1	16
DN	333	-31	68	12.67	21	0.12	0.468	a	(8)	- 9	- 30	16
VV Peg	78	-30	146	11.98	22	0.00	0.488	a	9	+ 10	- 5	4
AO	70	-23	150	13.1	yz	0.49	0.547	a	1			
AV	77	-24	149	10.4	p	0.41	0.390	a	0			
BH	86	-38	140	10.90	32	1.11	0.641	a	5	- 26	- 72	13
BP	74	-21		12.26	4	0.20				- 12	+ 6	13
CG	77	-21	155	11.5	q	0.52	0.467		2			
DH	69	-39	133	9.82	15,32	0.62	0.256	c	0	+ 19	+ 0	7
DY	91	-39	120	10.52	15,18,24	0.16	0.073	c		+ 54	- 11	8
TU Per	143	- 4	125	13.2	r	1.72	0.607	a		+ 16	- 10	5
AR	155	- 2	111	11.02	15,30	1.72	0.426	a	0			
RV Phe	336	-64	79	12.18	21	0.62	0.596	a	(8)	+ 42	- 19	12
SX	341	-70	76	7.06:	9	0.08	0.055			+255	-860	5
U Pic	258	-40	38	11.8	x	0.30	0.442	a		- 1	- 17	14
RU Psc	130	-38	118	10.46	22	0.04	0.390	c	7	+ 95	- 42	4
RY	101	-63	114	11.9		0.21	0.530	a	7	+ 47	+ 2	13
SS	132	-41	115	11.5		0.29	0.288	c	2			
XX Pup	237	9	32	11.6	z	1.00	0.517	a				
BB	241	10	28	10.7	z	0.82	0.480	a	3:			
ρ	243	4	20	3.0	z,27	0.12	0.141			- 87	+ 48	1
V355 Sgr	15	- 2		9.8	y							
V440	15	-19	107	10.6	syz	0.54	0.477	a	5			
V675	358	- 8	90	10.68	21	0.53	0.642	a	11	+ 0	+ 12	16
V1640	0	-14	97	12.91	40	0.78	0.367	a	0	- 4	+ 9	16
V494 Sco	357	- 0	90	11.78	21	0.70	0.427	a	(2)	- 20	+ 9	16
V703	357	- 1	100	7.8	26	0.37	0.115	c		+ 26	+ 11	21
RU Scl	41	-79	95	10.9	x	0.19	0.493			+ 25	- 14	13
δ Sct	24	- 2	126	4.90	12	0.11	0.194			+ 7	+ 1	1
VY Ser	6	44	97	10.51	36	0.78	0.714	a	9	- 92	+ 16	9

TABLE 7 (*continued*)

Star	υ	τ	V	Ref.	$\Pi_{0.9}$	$\Theta_{0.9}$	$Z_{0.9}$	m.e.	$\Delta\Pi$	$\Delta\Theta$	ΔZ
	(0".001)		(km/sec)		(km/sec)				(km/sec)		
KX Lyr			-60	22							
LX	+20	+12									
BE Mon											
UV Oct	+126	+91	+109	16	+208	-250	-119	34	+54	-35	-15
ST Oph	0	0	-45	15	+40	-17	-13	34	0	0	0
V445	-18	-3	-15	13	+32	+73	+3	56	+3	+15	+3
V452	-18	0									
V453			-95	13							
V567	+5	-25									
V716			-230	15							
V784											
V816	-5	-10									
TY Pav	+5	-9	+255	16	-217	-159	+1	136	-1	-8	+15
DN	+31	-3	-95	16	+172	-254	+124	163	+20	-60	+15
VV Peg	+9	+7	+10	13	+39	-25	-74	31	+8	-6	-14
AO			+115	13							
AV			-98	13							
BH	+49	-59	-260	13	-153	-296	+61	37	-34	-19	-21
BP											
CG			+5	13							
DH	+9	+17	-56	4,12	+57	-48	+12	16	+8	-1	-4
DY	+51	+21	-25	3	+169	-107	-89	32	+35	-18	-22
TU Per	+19	+1	-380	12	-231	-322	+24	31	+14	-19	-1
AR			-6	12,18							
RV Phe	+36	+30	+87	16	+167	-227	-126	77	+41	-42	-10
SX	+734	-516	-32	33,34	-59	-685	+132	8	-15	-139	+20
U Pic	+4	-16									
RU Psc	+100	+30	-115	12	+196	-365	-23	16	+50	-60	-18
RY	+25	+40	+26	13	+287	-129	-67	89	+58	-28	-8
SS			+5	13							
XX Pup			+409	18							
BB			+255	18							
ρ	-23	-97	+46		+30	-37	-2	1	+2	+1	-1
V355 Sgr			+10	12							
V440			-50	13							
V675	-11	-5	-105	16	+100	+39	+33	54	0	+7	+4
V1640	-8	-6	-17	16	+3	+47	+60	134	-2	+10	+12
V494 Sco	+2	-22	+26	16	-26	-25	+113	83	0	-5	+23
V703	-20	+20	-35	21	+34	+23	-14	20	0	+4	-3
RU Scl	+25	+14	+30	12	+62	-99	-53	57	+13	-21	-4
δ Sct	-3	+7	-45	5,19	+43	-18	0	1	0	+1	0
VY Ser	+50	-79	-140	13,32	+245	-148	+66	24	+29	-28	+34

TABLE 7 (*continued*)

Star	l	-b	λ	$\langle m_{pg} \rangle$	Ref.	Abs.	P	t	ΔS	μ_α	μ_δ	m.e.
										(0".001)		
AN Ser	23°	45°	110°	11.32	31	$0^{m}.86$	$0^{d}.522$	a	0			
AP	13	52	107	11.38	36	0.20	0.254		8:	- 44	- 37	7
AR	8	44	105	11.2	y	0.27	0.330		8:			
AT	18	42	106	11.4	y	0.28	0.747	a	9	- 8	- 5	7
AV	11	37	103	11.3	z	0.32	0.488	a	(6)			
BF	23	55		11.9		0.23	1.165		6			
CW	15	42	118	11.7	y	0.28	0.189			- 11	+ 10	7
T Sex	236	41	53	10.28	35	0.20	0.325	c				
SS Tau	180	-39	80	12.80	22	0.62	0.370	a		+ 12	+ 0	4
AH	166	-23	93	12.2		0.48	0.333			- 39	- 57	13
U Tri	138	-27	122	12.82	22	0.00	0.447	a	2	+ 16	- 15	4
W Tuc	302	-54	59	11.66	21	0.29	0.642	a	(7)	+ 3	+ 2	16
YY	325	-52	70	12.24	21	0.29	0.635	a	(8)	- 2	- 0	16
RV UMa	110	62	118	10.77:	15,31	0.33	0.468	a	8:	- 15	- 34	9
SX	113	60	123	11.06	25,32	0.00	0.307	c	6:	- 75	+ 11	5
TU	199	72	86	9.99	15,28	0.16	0.558	a	6	- 81	- 51	9
AI Vel	261	- 6	7	7.0	z	0.25	0.112			+ 29	+ 36	13
CD	272	6	10	12.6	x	1.42	0.573	a		- 28	+ 30	14
ST Vir	346	54	92	11.7	yz	0.24	0.411	a		- 5	- 20	5
UU	281	61	64	10.5	y	0.22	0.476	a	2	- 25	+ 7	8
UV	287	62	66	12.2	ty	0.22	0.587	a		- 16	- 32	13
XX	338	51		12.0		0.24	1.348	a	9:	- 9	- 16	13
AD	333	51	77	12.8	y	0.24	0.552	a		- 17	- 17	13
AF	355	59	92	11.9	y	0.22	0.484	a		- 60	+ 13	7
AM	314	46	62	11.4	u	0.27	0.615	a				
AS	303	53	63	11.6	z	0.24	0.553	a		- 45	- 78	13
AT	305	57	67	11.6	yz	0.23	0.526	a				
AU	317	55	76	11.6	z	0.23	0.343	c	7:	- 8	- 17	13
AV	325	71	82	12.0	z	0.20	0.657	a	6	+ 16	- 36	7
BB	340	65	86	11.0		0.21	0.471	a		- 45	+ 12	7
BC	323	68	80	12.5	z	0.20	0.565	a		+ 21	- 34	7
BN Vul	59	3	152	11.6		1.41	0.594	a	6			

to allow an intercomparison, and simple means have been taken. The mean error of one determination of a mean magnitude was found to be $\pm\, 0^{m}.067$.

A comparison between the values as given by the GCVS and the photo-electric determinations revealed systematic errors which obviously depend on the declination. Accordingly the material was split into stars with δ above and below $-25°$, and into photographic or visual magnitudes with either one or two decimals given in the GCVS. Table 8 shows the results.

The few cases with two decimals in the GCVS are probably from photo-electric observations, although the origins could not always be traced. The comparison with the other photo-electric data collected is better than the value given above for the r.m.s. error would suggest, but this could be accidental. It is certain that the magnitudes of the stars with the lowest declinations require corrections of the order of $+0^{m}.78$. After the above calculation had been made, Dr. Kwee showed me his accurate photographic results for V1640 Sgr. His determination gave a correc-

TABLE 7 (*continued*)

Star	υ	τ	V	Ref.	$\Pi_{0.9}$	$\Theta_{0.9}$	$Z_{0.9}$	m.e.	$\Delta\Pi$	$\Delta\Theta$	ΔZ
	(0".001)		(km/sec)		(km/sec)				(km/sec)		
AN Ser			- 60	13							
AP	+ 57	- 5	- 91	13,32	+ 65	-321	- 14	38	+ 2	- 62	+ 12
AR			+100	13							
AT	+ 9	- 2	- 70	13	+ 50	- 62	- 30	37	+ 1	- 10	+ 3
AV			- 55	13							
BF			-175	15							
CW	0	- 15									
T Sex			+ 38	12,31							
SS Tau	+ 8	+ 9	- 50	15	+ 8	- 56	+ 90	35	+ 10	- 12	+ 12
AH	+ 18	- 67									
U Tri	+ 22	- 2	- 60	15	+ 89	-246	- 60	46	+ 26	- 42	- 18
W Tuc	+ 1	+ 4	+ 71	16	- 2	- 38	- 64	94	+ 4	0	- 1
YY	- 1	- 2	+ 31	16	- 28	- 5	- 19	123	- 3	+ 1	+ 1
RV UMa	+ 34	+ 15	-178	12	- 63	-200	- 91	34	- 7	- 24	+ 14
SX	+ 53	- 54	-135	12	+319	-256	- 91	26	+ 70	- 39	+ 5
TU	+ 81	- 51	+ 92	12,23	+175	-233	+ 18	26	+ 30	- 46	- 14
AI Vel	- 22	- 41	+ 15	8,9,10	+ 9	- 16	+ 30	9	+ 1	0	+ 6
CD	+ 18	- 37									
ST Vir	+ 18	+ 10	- 35	12	- 23	-111	- 75	31	- 8	- 23	- 10
UU	+ 9	- 24	- 17	12	+ 83	- 14	- 18	28	+ 17	- 4	- 1
UV	+ 35	+ 6	+ 95	12	- 26	-290	- 43	100	- 3	- 50	- 25
XX			- 55	13							
AD	+ 24	- 1									
AF	+ 31	- 53									
AM			+105	13							
AS	+ 88	+ 18	+ 85	13	- 6	-492	-209	77	+ 5	- 91	- 56
AT			+358	13,18							
AU	+ 18	+ 6	+125	13,18	- 69	-150	+ 52	76	- 3	- 20	- 11
AV	+ 18	+ 35	+ 35	13	-240	-142	- 59	50	- 46	- 27	- 19
BB	+ 21	- 42	- 5	13	+182	- 85	+ 61	32	+ 36	- 18	+ 14
BC	+ 13	+ 38	0	13	-311	-125	-134	63	- 62	- 25	- 27
BN Vul			-235	13							

tion of $0^m.71$ to the value given in the GCVS, in perfect agreement with what was found before. The southern magnitudes were corrected by $0^m.78$ with the exception of V703 Sco. In this case the author on whose results the GCVS value is based (PLAUT, 1948) has determined the photometric zero-point from SA 157 and 40. The adopted mean error for a mean magnitude for stars with $\delta < -25°$ (except of course the well determined cases) is $\pm 0^m.50$. It is difficult to say, whether the stars north of $\delta = -25°$ need corrections to their magnitudes as published by the GCVS, without making further investigations. The literature was searched to find out from which sources the published data were taken. If these sources did not have their photometric zero-point determined in a satisfactory way, a correction of $0^m.2$ was applied to the values printed in the GCVS. The mean error for $\langle m_{pg} \rangle$ for these stars was taken to be $\pm 0^m.25$ or $\pm 0^m.30$ for original pg or v magnitudes respectively. All stars whose $\langle m_{pg} \rangle$ given in table 7 differs from the value printed in the GCVS are noted below. The magnitudes in table 7, though called m_{pg}, do not form a homogeneous group.

TABLE 8

Differences Δm in mean magnitudes of different sources. $\langle|r|\rangle$ is the average absolute residual, σ is 1.25 $\langle|r|\rangle$

GCVS	Δm(pe−pg)	n	$\langle\lvert r\rvert\rangle$	σ	Δm(pe−v)	n	$\langle\lvert r\rvert\rangle$	σ
				$\delta<-25°$				
1 dec.	$+0^{\mathrm{m}}.78$	14	$0^{\mathrm{m}}.40$	$\pm 0^{\mathrm{m}}.50$	$+0^{\mathrm{m}}.58$	3	$0^{\mathrm{m}}.64$	$0^{\mathrm{m}}.80$
2 dec.					+0.75	2	0.31	0.39
				$\delta>-25°$				
1 dec.	+0.24	42	0.20	0.25	+0.39	13	0.16	0.20
2 dec.	+0.06	5	0.04	0.05	+0.33	4	0.19	0.24

List of notes and references for $\langle m_{\mathrm{pg}}\rangle$.

x A correction of $+0^{\mathrm{m}}.78$ was added to $\langle m_{\mathrm{pg}}\rangle$ taken from B. V. KUKARKIN, P. P. PARENAGO, YU. I. EFREMOV and P. N. KHOLOPOV (1958, *General Catalogue of Variable Stars*)

y A correction of $+0^{\mathrm{m}}.2$ was added to $\langle m_{\mathrm{pg}}\rangle$ taken from B. V. KUKARKIN *et al.* (1958, *General Catalogue of Variable Stars*)

z A value of $+0^{\mathrm{m}}.25$ was added to transfer $\langle m_{\mathrm{v}}\rangle$ from B. V. KUKARKIN *et al.* (1958, *General Catalogue of Variable Stars*) to $\langle m_{\mathrm{pg}}\rangle$

a KIPPENHAHN's value for $\langle m_{\mathrm{pg}}\rangle$ was used (1953, *Astr. Nachr.* **281** 153)

b Value taken from 1952, *Ann. Astr. Obs. Harvard* **118** 99

c In accordance with the latest value given by PARENAGO (1930, *Astr. Nachr.* **240** 321)

d In accordance with 1938, *Pulkovo Circ.* No. 24, figure 1, plus the correction mentioned under y

e Value from 1927, *Kleine Veröff. Univ. Sternw. Berlin-Babelsberg* No. 4 19, corrected with $+0^{\mathrm{m}}.24$ (see note 1930, *Astr. Nachr.* **240** 321)

f An average of the values given in 1946, *Astr. J.* **52** 56, and 1958, *Stalinabad Trud.* (7) **76** 91, corrected for z

g The visual magnitude from 1953, *Variable Stars* **9** 330, corrected for z

h From 1940, *Variable Stars* **5** 258

i From 1948, *Tashkent. Bull.* **2** 499

j From 1923, *Astr. Nachr.* **218** 313

k From 1929, *Kleine Veröff. Univ. Sternw. Berlin-Babelsberg* No. 6 28, with corrections from 1930, *Astr. Nachr.* **240** 321

l From 1941, *Tadjik Ann.* **1** Part 3 58

m From 1935, *Variable Stars* **5** 89

n From 1950, *Astr. Nachr.* **279** 247, plus a correction of $+0^{\mathrm{m}}.29$, derived from photo-electric observations used in this article for four stars in common with Kühn's work

o From 1941, *Tadjik Ann.* **1** Part 3 54

p From 1952, *Ann. Astr. Obs. Harvard* **118** 8 and 19

q From the average of the corrected values from 1941, *Ann. Astr. Obs. Harvard* **111**, and 1935, *Astr. Nachr.* **255** 420, plus correction mentioned under y

r G. E. Kron has observed V_{min} $0^{\mathrm{m}}.68$ fainter than the value entered in *Variable Stars*, with a $(B-V)_{\mathrm{min}} = +1.00$. The star appears to be much fainter than Kukarkin's *et al.* values seem to indicate. C. R. D'ESTERRE (1915, *Mon. Not. Roy. Astr. Soc.* **75** 292) observed a photographic range of $+1^{\mathrm{m}}.3$. Adopted: D'Esterre's range and Kron's minimum

s From 1933, *Variable Stars* **4** 256 (corrected for x and z)

t From 1935, *Tadjik Ann.* **1** Part 1 33

u From 1931, *Variable Stars* **3** 111

1. K. D. ABKYANKAR, 1959, *Ap. J.* **130** 834
2. H. A. ABT, 1955, *Ap. J.* **122** 390
3. J. BALÁZS and L. DETRE, 1957, *Mitt. Sternw. Ungarischen Akad. Wiss. Budapest* No. 34
4. P. BROGLIA, 1959, *Mem. Soc. Astr. Italiana* **30** 142
5. P. BROGLIA and A. MASSANI, 1955, *Mem. Soc. Astr. Italiana* **26** 66
6. P. BROGLIA and A. MASSANI, 1957, *Mem. Soc. Astr. Italiana* **28** 102
7. P. BROGLIA and A. MASSANI, 1957, *Mem. Soc. Astr. Italiana* **28** 105
8. P. BROGLIA and E. PESTARINO, 1955, *Mem. Soc. Astr. Italiana* **26** 429
9. O. J. EGGEN, 1952, *Pub. Astr. Soc. Pacific* **64** 305
10. O. J. EGGEN, 1955, *Pub. Astr. Soc. Pacific* **67** 354
11. O. J. EGGEN, 1956, *Pub. Astr. Soc. Pacific* **68** 142
12. E. A. FATH, 1935, *Lick Obs. Bull.* **17** 175 and **18** 77
13. W. S. FITCH, 1955, *Ap. J.* **121** 690
14. W. S. FITCH, 1957, *Astr. J.* **62** 108
15. E. H. GEYER, 1961, *Z. Ap.* **52** 229
16. E. H. GEYER, 1958, *Z. Ap.* **44** 98
17. R. H. HARDIE, 1955, *Ap. J.* **122** 256
18. R. H. HARDIE and C. D. GEILKER, 1958, *Ap. J.* **127** 606
19. R. H. HARDIE and S. H. LOTT, 1961, *Ap. J.* **133** 71
20. R. H. HARDIE and C. R. TOLBERT, 1961, *Ap. J.* **134** 581
21. T. D. KINMAN, 1961, *Roy. Obs. Bull.* No. 37
22. G. E. KRON, or G. E. KRON and S. N. SVOLOPOULOS (unpublished)
23. D. H. MCNAMARA, G. AUGASON, R. HUERTA and W. MURRI, 1961, *Pub. Astr. Soc. Pacific* **73** 340
24. A. MASSANI and P. BROGLIA, 1954, *Mem. Soc. Astr. Italiana* **25** 53
25. P. NOTNI, 1962, *Astr. Nachr.* **262** 72
26. J. PONSEN, 1962, *Bull. Astr. Inst. Netherlands* **17** 29
27. J. PONSEN, 1962, *Bull. Astr. Inst. Netherlands* **17** 44
28. G. W. PRESTON, H. SPINRAD and C. M. VARSAVSKY, 1961, *Ap. J.* **133** 484
29. G. W. PRESTON, 1961, *Ap. J.* **134** 633
30. H. J. SMITH, 1955, *Harvard Dissertation*
31. H. SPINRAD, 1959, *Pub. Astr. Soc. Pacific* **71** 542
32. H. SPINRAD, 1959, *Ap. J.* **130** 539
33. H. SPINRAD, 1960, *Ap. J.* **131** 134
34. H. SPINRAD, 1961, *Ap. J.* **133** 479

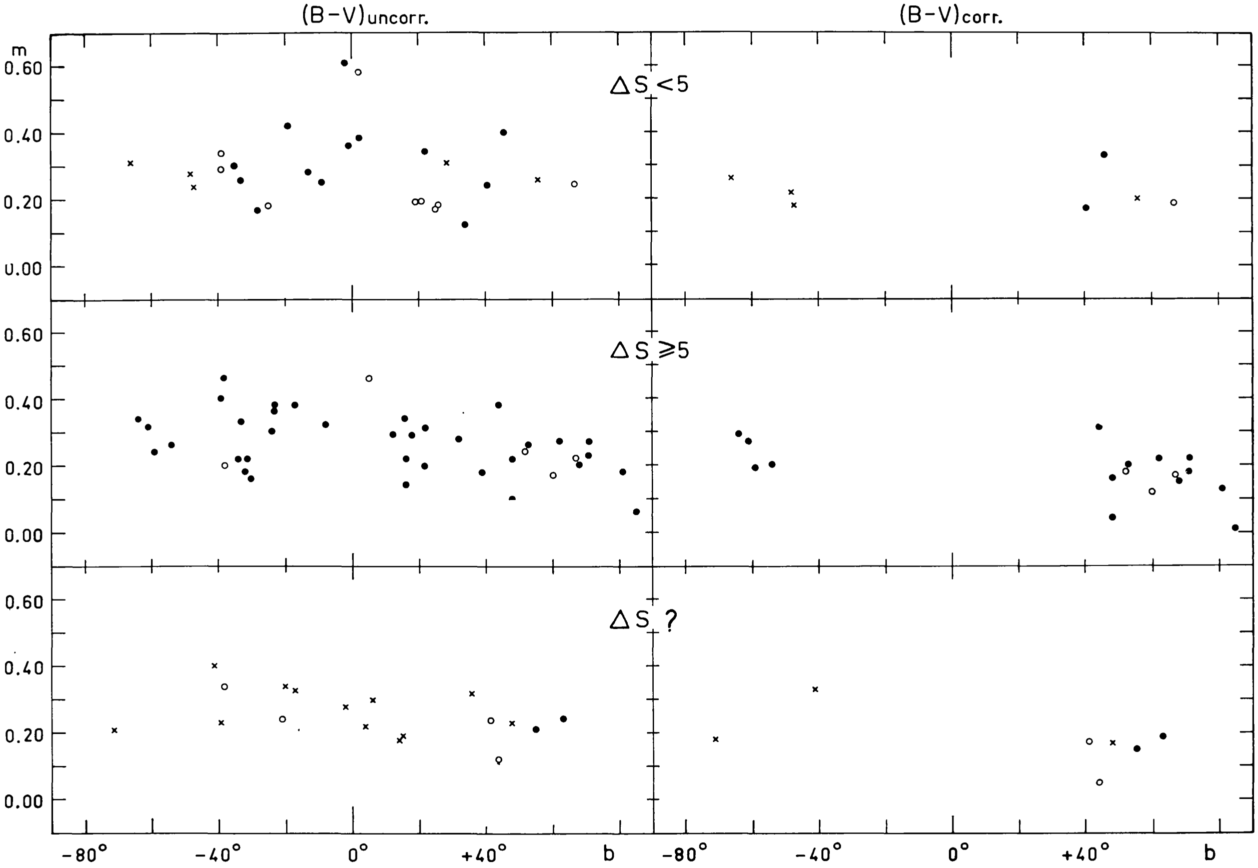

Figure 1. Left-hand side: $B-V$ observed photo-electrically; right-hand side: $B-V$ corrected for selective absorption for galactic latitudes $|b| > 40°$. Filled circles refer to $P > 0^d.42$, open circles to $0^d.2 < P \leq 0^d.42$, and crosses to $P \leq 0^d.2$.

35. W. G. Tifft and H. J. Smith, 1958, *Ap. J.* **127** 591
36. C. M. Varsavsky, 1960, *Ap. J.* **131** 623
37. M. F. Walker and O. C. Wilson, 1956, *Ap. J.* **124** 325
38. M. F. Walker, 1952, *Pub. Astr. Soc. Pacific* **64** 192
M. F. Walker, 1953, *Pub. Astr. Soc. Pacific* **65** 39
39. O. C. Wilson and M. F. Walker, 1956, *Ap. J.* **124** 325 (Lindblad and Eggen's values for CC And were rejected)
40. K. K. Kwee, 1962, *Ann. Sterrew. Leiden* **22** 3

9. The amplitudes

As the amplitudes will be used to subdivide the material, the question of the relation between B and V amplitudes is important. Preston (1959) takes a standard difference of $+0^m.30$ between these two amplitudes; Geyer (1961) assumes a linear relation with log P. From the photo-electric material at hand the following table was constructed. A subdivision according to ΔS showed no trace of a dependence on ΔS, except for the group with $0^d.3 < P < 0^d.4$, but this may well be spurious.

P	n	ampl.$_B$−ampl.$_V$
$< 0^d.1$	6	$0^m.14$
0.1–0.2	10	0.15
0.2–0.3	4	0.14
0.3–0.4	13	0.29
0.4–0.5	21	0.32
0.5–0.6	17	0.27
0.6–0.7	8	0.24
0.7–0.8	4	0.28
0.8–0.9	1	0.25

In our work two values for the difference between the two amplitudes were used: $+0^m.14$ when $P < 0^d.3$ and $+0^m.29$ for $P > 0^d.3$.

10. The colours and the absorption

The observers of the photo-electric magnitudes have also given $B-V$. The material is insufficient to determine systematic differences between the results of

TABLE 9

Averages for $B-V$ and $(B-V)_0$ for different galactic zones

b	n	$\langle B-V \rangle$ obs.	$\langle \lvert r \rvert \rangle$	$\langle (B-V)_0 \rangle$	b	n	$\langle B-V \rangle$ obs.	$\langle \lvert r \rvert \rangle$	$\langle (B-V)_0 \rangle$
−90° to −80°					+90° to +80°	2	$0^{m}.12$		$0^{m}.07$
−80 −70	1	$0^{m}.21$		$0^{m}.18$	+80 +70	2	0.25		0.20
−70 −60	3	0.32		0.27	+70 +60	6	0.22	$0^{m}.027$	0.17
−60 −50	3	0.25		0.20	+60 +50	4	0.24		0.18
−50 −40	3	0.31		0.24	+50 +40	8	0.24	0.074	0.18
−40 −30	14	0.28	$0^{m}.071$		+40 +30	4	0.22		
−30 −20	7	0.28	0.073		+30 +20	7	0.24	0.066	
−20 −10	4	0.35			+20 +10	8	0.23	0.058	
−10 0	5	0.36	0.096		+10 0	5	0.39	0.106	

various authors. Giving all values equal weight, one arrives at a mean error of one determination of $\pm 0^{m}.057$ from an intercomparison of the results for the same stars by different observers. All photo-electrically observed $B-V$ are shown in figure 1, on the left-hand side. They were subdivided according to ΔS, and plotted against the galactic latitude. At a first glance one gets the impression that there is a tendency to have higher values for $B-V$ at negative latitudes than at positive latitudes. It may have been caused by a difference in zero-point. However, the reality of this tendency is doubtful and in what follows no distinction has been made between positive and negative latitudes, nor between the subdivisions with respect to ΔS and period. The averages of $B-V$ are given in table 9 for different galactic zones. It goes without saying that it is out of the question to subdivide the material according to galactic longitude as well. Where five or more values are given per zone of 10°, the average absolute residual with respect to the zone mean is entered under $\langle \lvert r \rvert \rangle$. The average r.m.s. deviation for an uncorrected $B-V$ is found to be $\pm 0^{m}.09$. The values of $B-V$ will be influenced by selective absorption. The amount of absorption was estimated by assuming a smooth model for the distribution of the absorbing material; a factor 4.1 was adopted for the ratio between the photographic absorption A_{pg} and the colour excess E_{B-V}. It was decided that only stars with $|b| > 40°$ should be used to compute $(B-V)_0$, stars in the zones $20° \leq |b| \leq 40°$ serving as a check. The intrinsic colours $(B-V)_0$ thus derived are given in table 9. Five different models were tried. They represented the given data about equally well, and the absorption model $A_{pg} = 0^{m}.19$ cosec b $[1 - \exp(-0.01\, r \sin b)]$ was retained. With this model the absorptions were computed where no photo-electrically measured colours were available. The value of $+0^{m}.19$ was thus obtained for $(B-V)_0$. The values for $B-V$, corrected for selective absorption, are plotted on the right-hand side of figure 1. The tacit assumption that our sample of RR Lyrae stars possesses one definite value for $(B-V)_0$ need not be correct, but can not be avoided.

11. The subdivisions and their dynamical characteristics

It is well known that the RR Lyrae variables are a mixture of stars with widely varying dynamical characteristics. It is obvious that for the determination of absolute magnitudes it will be of advantage to divide them into groups which are as homogeneous as possible in this respect. We cannot use the motions themselves for segregating them, because these motions are to be used to determine their mean parallaxes; we have to take recourse to some intrinsic property. The most obvious quantity is the metal abundance. PRESTON (1959) has shown how this can be determined by measuring certain line intensity ratios during the minimum phase of the variable. They are expressed in a quantity which he called ΔS. The determination of ΔS requires fairly accurate photometric measurements, at a phase where the stars are faint. As a consequence, values of ΔS are known for only about 58 per cent of our stars, and in many cases the measures must be uncertain. However, Preston has shown that there is a fair relation between ΔS, period and amplitude (cf. figure 5 in his 1959 article). We can make use of this relation for a rough ΔS classification of the stars for which ΔS has not been measured. In the following we have divided the stars into two groups, according to whether ΔS is 5 or larger (low metal content), or smaller than 5 (high abundance of metals). The variables for which ΔS is unknown were put in the first group if they belonged to Bailey's type a, and their periods were larger than or equal to $0^d.50$, or if they had periods between $0^d.44$ and $0^d.50$ and amplitudes larger than $1^m.5$. According to Preston's figure 5, 79 per cent of the stars fulfilling these conditions have $\Delta S \geq 5$. The rest of the stars of Bailey type a were considered as belonging to the group with $\Delta S < 5$. In Preston's sample 87 per cent of these actually have $\Delta S < 5$.

It has long been known that the dynamical properties are correlated with the periods, the stars of longer periods having the higher velocities. We have therefore also made subdivisions according to period. This division in different period groups has also been made for the stars for which ΔS is available. It is interesting to note that within each of the two ΔS groups there is still a pronounced change in dynamical properties with period. A subdivision according to period and Bailey type led to a slight decrease in the number of stars which were available for $P > 0^d.42$ but with no significant differences in the results. For the period interval $0^d.42 \geq P \geq 0^d.2$ the few c-type stars showed a higher solar velocity than the few a-type stars but it is questionable whether this is significant; the results are not given.

Because the numbers of known radial velocities in the various subdivisions are too small to get meaningful determinations of the complete solar motion vector, it has been assumed that the systematic motion with respect to the centre of the Galaxy was always a rotation around the galactic axis, and only the amount of the rotational velocity was determined from the radial velocities. The latter were first reduced to the local standard of rest by correcting for a standard solar motion of 20 km/sec towards $\alpha = 270°$, $\delta = +30°$. The average value of Θ relative to the local standard of rest was then computed from $V_{\text{corr}} = \langle\Theta\rangle \sin l \cos b$. These values of $\langle\Theta\rangle$ are given in column 2 of table 12. The total solar motion and the apex were then found by vectorially adding the standard solar motion. The results are shown in table 11, under $V_{\odot}$, A and D, the numbers of radial velocities being added.

This way of arriving at the apex and the solar motion can be criticized when the averages $\langle\Pi\rangle$ or $\langle Z\rangle$ are not small compared with $\langle\Theta\rangle$. We have adhered in our solution to the a priori point of view that the stars are so well mixed that the rotation should be the main component, and that values for $\langle\Pi\rangle$ or $\langle Z\rangle$ which turn out to be comparatively large reflect statistical fluctuations. However, for the few subdivisions with non-negligible $\langle\Pi\rangle$ or $\langle Z\rangle$, the classical solutions with an independent determination of the apex, are also given in tables 11 and 12 under subdivisions 14 and 15.

12. The mean parallaxes π_1 derived from the parallactic motion

Mean parallaxes can be derived in three practically independent ways, viz. from the weighted algebraic average of the υ-components in the direction of the antapex (mean parallax π_1), from the average without regard to sign of the τ-components perpendicular to υ (mean parallax π_2), and from the average without regard to sign of the residuals in υ which remain after subtracting the projection of the reflected solar motion in angular measure (mean parallax π_3). The proper motions were reduced to what they would have been if the stars had a distance corresponding with a mean magnitude 11.0 by multiplying by a factor $10^{0.2(m-11.0)}$, where m is the mean photographic magnitude corrected for absorption (both taken from table 7). The reduced motions are indicated by the suffix 11.

The mean parallax π_1 is found from $\upsilon_{11} = \pi_1 V_{\odot} \sin\lambda/4.74$, where λ is the distance from the antapex. The weight of υ_{11} is determined by the mean error of the reduced proper motions ε_{11}, and the dispersion of the velocities V_υ in the direction of the antapex. The total mean error ε_{tot} of the left-hand member of the equation of condition is, therefore, given by $\varepsilon_{\text{tot}}^2 = \varepsilon_{11}^2 + (\pi \text{ disp } V_\upsilon)^2/(4.74)^2$. Here, π is the assumed mean parallax for the RR Lyrae variables with $m = 11.0$. The error we make in the assumed value will have, in this case, a negligible error in the final result. The dispersions have been taken from preliminary reductions, which are sufficiently accurate for the purpose. The results are given in table 11 under π_1. Any error in the adopted $V_{\odot}$ enters fully into π_1.

13. The mean parallaxes π_2 derived from the components τ_{11}

In each division the stars were divided into subgroups according to the mean error of the reduced proper motions. The averages of the absolute values of the reduced components have to be corrected for the statistical error from the uncertainty of the motions. One cannot correct each subgroup for its own statistical error without running into a number of cases with imaginary results. If one would reject these cases as impossible ones, one would systematically cut off the accidentally low values of $\langle|\tau_{11}|\rangle$, and never the accidentally high values. To avoid this, a plausible mean value for $\langle|\tau_{11}|\rangle$ was derived first, which was then substituted for each individual mean. The mean for a subgroup was corrected for statistical error when the average error, η, of the motions of this subgroup was smaller than $0.75\langle|\tau_{11}|\rangle$. In cases where it was larger, the subdivision was rejected.

This plausible mean value was found in a somewhat arbitrary way by assigning weights to the means for each subgroup which for the best determined groups diminished very slowly with average mean error, and which for the worst determined groups were roughly proportional to the inverse of the average mean error of the motions for the group. The total number of stars in a subgroup was, of course, also taken into account in applying the final weights. After the corrections for statistical error in each subgroup had been applied, the final mean of the corrected $\langle|\tau_{11}|\rangle$ was computed with weights given by the formula

$$\varepsilon_{\tau,c} = 0.75\,\langle|\tau_{11}|_c\rangle\,[1+(\eta/\langle|\tau_{11}|_c\rangle)^2]/\sqrt{n}, \qquad (2)$$

derived from the general statistical formulae $\varepsilon_\tau = 0.75\,|\tau|/\sqrt{n}$ and $\tau = (\tau_c^2+\eta^2)^{\frac{1}{2}}$.

The errors in the factors by which the observed motions have been reduced to $m = 11.0$ form another cause which has made the dispersion in the τ_{11} components too large. The mean error for a value of m was adopted to be $\pm\,0^{\mathrm{m}}.25$, and the same value was taken for the mean error in the absorption. From this we arrive at a mean error in the reduced motion due to these causes alone of ± 16 per cent. This error has caused the values of $\langle|\tau_{11}|\rangle$ to be too large by about 1 per cent, a small, but systematic error, which has been corrected for.

For determining $\langle|V_\tau|\rangle$, the radial velocities were first corrected for solar motion, using $V_\odot$, A and D as given in table 11. It will be noted that errors in the solar motion have only a negligible influence in this case. The mean random radial velocity has to be corrected for statistical error, which correction, due to the large values of the velocities and the relatively small mean error of a radial velocity (± 18 km/sec), is usually small.

The mean parallaxes π_2 are derived from $\pi_2 = 4.74\,\langle|\tau_{11}|_c\rangle\,/\,\langle|V_\tau|\rangle$ where $\langle|\tau_{11}|_c\rangle$ is the mean τ corrected for the effect of accidental errors, and $\langle|V_\tau|\rangle$ is the average linear velocity in the direction of the τ-components. A difficulty was encountered in deriving the latter from radial velocities, in that it appeared that the stars were distributed in such a way that on the average the radial components made much larger angles with the major axis of the velocity ellipsoid (the Π-axis) than the τ-components. A sample of the distribution of these angles for V, τ and υ is shown in table 10. As a consequence of these systematic differences in distribution no reliable values of $\langle|V_\tau|\rangle$, or $\langle|V_\upsilon|\rangle$, could be obtained from the radial velocities

TABLE 10

Frequency distribution of angles between Π, Θ, Z and the υ-, τ- and V-components for six subgroups taken together

	υ, Π	υ, Θ	υ, Z	τ, Π	τ, Θ	τ, Z	V, Π	V, Θ	V, Z
0°–10°	0	37	1	39	0	4	4	2	17
10 –20	1	65	3	73	0	20	1	13	27
20 –30	4	71	13	45	0	27	17	36	47
30 –40	12	44	17	49	0	41	13	31	34
40 –50	17	36	36	43	0	40	42	39	30
50 –60	27	21	40	39	0	44	46	37	59
60 –70	63	42	73	28	7	44	64	71	58
70 –80	80	15	80	18	92	52	73	58	47
80 –90	141	14	82	11	245	73	85	58	26

without making an assumption concerning the ratios of the axes of the velocity ellipsoid. We have attempted to overcome this difficulty by excluding the τ-components making the smallest angles with the Π-axis and the radial velocities making the largest angles with this axis, so that for the remaining stars the average angles for the τ-components and the radial velocities are as nearly equal as feasible. It appeared, however, that so many stars had to be rejected in order to obtain even an approach to equality that the weights of the results were too seriously reduced. This was very evident by excluding only one or two stars more. A new solution would yield results which could vary by almost 50 per cent. This is another argument to show that the uncertainties in the motions are of great importance. We have therefore made a second solution in which we used all the motions, and reduced the radial velocities to τ-components by adopting the axial ratios of the velocity ellipsoid as found from the space velocities. This is evidently objectionable, because the axial ratios so determined are themselves influenced by the inhomogeneous distribution of the stars just described, but we saw no better way. The systematic errors which this procedure introduces into the mean parallaxes may have been partly counterbalanced by applying the same procedure to the υ-residuals (cf. section 14).

It was noted that the parallaxes derived from the entire material have a tendency to be lower than those based on the material selected according to the direction cosines with the Π-axis. The former agree better with the values π_1.

To allow for the differences in the averages of the squares of the direction cosines, $\langle|V|\rangle$ was multiplied by a factor f given by

$$f = \frac{\langle\cos^2(\tau,\Pi)\rangle \ \mathrm{disp}\,\Pi + \langle\cos^2(\tau,\Theta)\rangle \ \mathrm{disp}\,\Theta + \langle\cos^2(\tau,Z)\rangle \ \mathrm{disp}\,Z}{\langle\cos^2(V,\Pi)\rangle \ \mathrm{disp}\,\Pi + \langle\cos^2(V,\Theta)\rangle \ \mathrm{disp}\,\Theta + \langle\cos^2(V,Z)\rangle \ \mathrm{disp}\,Z}, \tag{3}$$

where cos (τ, Π) is the direction cosine of a τ-component relative to the Π-axis. In order to determine f we need the axial ratios disp Θ/disp Π and disp Z/disp Π. These were obtained from the space velocities computed with the aid of the final mean parallaxes. The dispersions used are given in table 12. In the second solution, where all stars were used irrespective of the orientations of τ and V relative to the axes of Π, Θ and Z, the factors f are sometimes quite large, and the results must be considered influenced by the uncertainty of these factors.

14. The mean parallaxes π_3 derived from the residuals in the υ_{11}-components. Relative weights of π_1, π_2 and π_3

The mean parallax being known, the residual component υ'_{11} is found by the relation $\upsilon'_{11} = \upsilon_{11}(\mathrm{obs}) - \pi V_\odot \sin\lambda/4.74$. One can substitute π_1, or a properly weighted mean of π_1 and π_2 for π; the latter course was adopted. The reductions were made in the same way as in section 13. One needs the relative weights for π_1 and π_2 for the second solution. Mean errors derived from the material turn out to scatter tremendously which can only mean that they are highly unreliable. The relative weights can be computed with the aid of formulae published by Russell (1921). However, Russell in those days still neglected the effect of an error in the adopted value for the solar motion, which we now know to contribute substantially to the error in π_1. In addition one may doubt whether the corrections for statistical error are known sufficiently well not to influence the error in π_2 or π_3. We have used the following formulae derived by Oort (unpublished) for the relative mean errors for π_1 and π_2

$$\frac{\text{m.e. of } \pi_1}{\text{m.e. of } \pi_2} = \frac{1.26\,\pi_1 \ \mathrm{disp}\,V}{n^{\frac{1}{2}} V_\odot} \cdot \frac{n^{\frac{1}{2}}}{0.72\,\pi_2} = = 2.19 \frac{\langle|V_\tau|\rangle}{V_\odot}. \tag{4}$$

In applying these formulae we took care to take into account the difference in numbers of stars with known radial velocities and with a known tangential velocity. The factor π_1/π_2 was taken as 1 in computing the above ratio. The weights for π_3 were taken 30 per cent lower than those for π_2, because in π_3 the uncertainty in $V_\odot$ has a greater influence and an additional error comes in through the uncertainty in the value of π used in computing the residuals υ'_{11}. These relative weights are given in table 11, the weight for π_1 being taken as unity. The mean errors ε for π_1 and π_2 are given with

TABLE 11

Group parallaxes and absolute magnitudes.
The explanations for the columns π_1, π_2, π_3 and $\langle\pi\rangle$ are given in sections 12, 13, 14 and 15 respectively; those for the different weights and mean errors in sections 14 and 15, and that for $\langle M\rangle$ in section 15

		$V_\odot$	n_V	A	D	π_1	n_υ	ϵ_π	π_2 comp.	n_τ	π_2 corr.	ϵ_π	wt.	π_3 comp.	n_υ	π_3 corr.	ϵ_π	wt.	$\overset{\prime}{\langle\pi\rangle}$	$\langle M\rangle$	ϵ_M
1	$\Delta S<5$	96	59	306°.5	46°.4	75	50	14	115	40	105	11	1.5	102	32	106	13	1.0	97	+ 0.9	0.16
2	$\Delta S\geq5$	208	97	312.4	47.4	77	87	8	115	73	88	7	0.7	113	73	106	8	0.5	87	+ 0.7	0.11
3	$P\leq0^{d}.1$	28	5	281.8	37.5	754	6	366	495	6	791	244	2.5	743	6	702	305	1.6	756	+ 5.4	0.49
4	$0^{d}.1 <P<0.2$	43	13	293.8	42.7	18	16	8	39	9	36	8	1.9	31	4	32	10	1.2	30	− 1.6	0.36
5	$0.2 \leq P\leq0.42$	72	33	302.9	45.6	158	30	51	149	25	168	22	1.7	205	30	170	29	1.1	166	+ 2.1	0.22
6	$0.42<P\leq0.5$	154	40	310.6	47.2	82	36	16	134	32	127	15	1.1	134	32	134	19	0.7	113	+ 1.3	0.18
7	$0.5 <P<0.6$	179	52	311.6	47.3	85	47	15	112	43	75	8	1.0	101	43	94	10	0.7	84	+ 0.6	0.15
8	$0.6 \leq P<0.8$	242	29	313.1	47.6	50	27	7	100	21	88	13	0.4	86	21	82	15	0.3	65	+ 0.1	0.21
9	$\Delta S<5$ $0.1 <P\leq0.42$	58	19	299.4	44.7	25	18	7	100	11	100	18	1.2	106	5	110	22	0.8	78	+ 0.5	0.18
10	$0.42<P<0.62$	125	26	308.6	47.1	67	17	22	96	13	90	15	1.6	68	13	69	19	1.1	78	+ 0.5	0.29
11	$\Delta S\geq5$ $0.2 \leq P\leq0.42$	74	9	303.4	45.7	378	7	309	169	7	149	38	5.8	199	7	228	47	3.8	199	+ 2.5	0.32
12	$0.42<P<0.6$	193	35	312.0	47.4	86	32	14	127	29	110	14	0.8	147	29	149	18	0.5	108	+ 1.2	0.17
13	$0.6 \leq P<0.81$	249	20	313.3	47.6	49	18	7	126	12	95	17	0.5	107	12	112	22	0.3	72	+ 0.3	0.19
14	$0.2 \leq P\leq0.42$	82		289	43	163					183					169			170	+ 2.2	0.22
15	$\Delta S\geq5$ $0.2 \leq P\leq0.42$	153		281	49	194					192					63			146	+ 1.8	0.25

TABLE 12

Mean space velocity components and dispersions.
The subdivisions 1–15 are the same as in table 11; they are explained in section 11. The different columns are explained in section 16 with the exception of column 2 for which the explanation is given in section 11

	< θ >	< Π >	ε	< θ >	ε	<Z>	ε	disp		disp		disp		< Π >	ε	< θ >	ε	<Z>	ε	disp		disp		disp	
	from V	from space velocities with individual group M						Π	ε	θ	ε	Z	ε	from space velocities with adopted <M>= + 0.9						Π	ε	θ	ε	Z	ε
1	- 80	+ 9	13	- 46	11	- 13	9	86	11	73	9	55	7	+ 2	14	- 47	11	- 13	9	87	11	71	9	58	8
2	-192	+ 43	19	-196	13	- 26	9	168	14	112	10	79	8	+ 34	18	-184	12	- 25	9	155	13	105	9	77	8
3	- 9	+ 13	4	- 35	13	0	9	11	4	33	10	23	7												
4	- 26	+ 22	24	- 28	16	- 19	23	77	19	50	18	74	26	+ 19	7	- 20	5	- 4	3	23	5	15	4	8	3
5	- 56	+ 63	12	- 76	14	- 1	8	64	9	74	11	44	6	+103	23	-114	23	+ 4	8	122	18	123	18	44	7
6	-138	+ 33	21	-160	20	- 48	14	121	18	117	17	81	12	+ 33	24	-177	23	- 49	15	137	20	131	20	86	12
7	-163	+ 21	28	-160	16	- 19	12	183	22	104	12	77	11	+ 23	25	-147	14	- 16	11	164	19	94	11	76	12
8	-226	+ 29	34	-229	26	+ 9	19	161	26	123	20	89	17	+ 14	26	-172	21	+ 7	17	123	20	99	16	80	15
9	- 41	- 1	20	- 18	16	- 22	10	67	18	54	15	34	9	- 1	17	- 17	14	- 20	9	55	11	46	12	30	8
10	-110	+ 3	31	- 88	26	- 9	22	116	24	99	20	83	17	+ 1	26	- 79	25	- 10	19	98	20	93	19	70	14
11	- 58	+ 78	28	-129	18	- 6	31	74	20	49	13	82	22	+179	60	-230	35	+ 17	30	159	42	93	25	80	21
12	-177	- 7	27	-144	19	- 53	16	148	20	106	14	86	12	- 12	31	-155	23	- 57	16	172	23	124	17	90	12
13	-234	+ 42	45	-247	30	- 14	21	170	35	114	23	77	16	+ 34	35	-197	26	- 10	18	132	24	96	20	68	14
14		+ 61	9	- 75	14	0	8	50	8	72	10	42	7												
15		+112	38	-163	24	+ 2	26	100	26	64	17	69	18												

the values for π taken from π_1 and π_2 respectively. The mean error for π_3 is computed with the aid of the estimated weight and the mean error of π_2.

15. The final results

The final results are given in table 11. The first columns show the criteria used for grouping the stars, the solar velocity $V_\odot$, with the number of radial velocities from which it was derived, and the assumed coordinates A and D of the apex (cf. section 11). For π_2 and π_3 two different values are given: π (comp) is the value found before the factor f (section 13) was applied, π (corr) is the final value after the correction for the different influences of the velocity dispersion was taken into account. It has often been found that a large systematic difference between π_1 on the one hand, and π_2 on the other existed (WILSON, 1939; PAVLOVSKAYA, 1953b; MISSANA and PLAUT, 1963). These authors did not take into account the effect of the ellipsoidal velocity distribution. NOTNI (1957) took this effect into account and obtained small differences between the various sets of parallaxes. In our results the differences between π_1 and π_2 or π_3 are present to a small extent; considering the mean errors, the difference is not excessive.

The absolute magnitudes shown in column $\langle M \rangle$ were computed from the final mean parallaxes; the mean errors were found from the final average mean error of $\langle \pi \rangle$. There does not seem to be any systematic dependence of $\langle M \rangle$ on metal content or period, except for the small group of stars with periods less than one tenth of a day, which appear to be about $4^{m}.5$ fainter than normal RR Lyrae variables. These stars have been excluded in the following discussion of the absolute magnitudes. The group of stars with periods between one and two tenths of a day also seem to be exceptional, but this is mainly suggested by the appearance of the only negative M; we have considered this as an accidental deviation. In no other subdivision is there any suggestion of similar behaviour for this type of stars.

The averages of the absolute mean magnitudes were computed by combining the results from groups 1 and 2, 4 to 8, and 9 to 13, respectively yielding 90, 100 and 100 for the mean parallaxes in units of $0''.00001$, corresponding to absolute magnitudes of $+0^{m}.78$, $+1^{m}.01$ and $+1^{m}.00$. The differences between the three averages are due to differences in the total numbers of stars used. As final mean parallax I have adopted $\langle \pi \rangle = 0''.00097 \pm 0''.00010$ (m.e.), corresponding to $\langle M \rangle = +0^{m}.93 \pm 0^{m}.22$ (m.e.). The mean errors were computed from the mean errors given for π_1, π_2 and π_3 in table 11 for groups 1 and 2 (the other groups, containing mostly the same stars, cannot add to the final weight). The mean errors given for π_1, π_2 and π_3 were checked by comparing them with the differences between the individual values of π_1, π_2 and π_3 in table 11 and the final average of 97×10^{-5}, taking the relative weights into account. The agreement was satisfactory. The above value of $\langle M \rangle$ still requires a small correction because it was computed from $\langle M \rangle = m+5+5 \log \langle \pi \rangle$, while, strictly, it should have been computed from $\langle M \rangle = m+5+5 \langle \log \pi \rangle$. The difference between $\langle \log \pi \rangle$ and $\log \langle \pi \rangle$ depends on the dispersion in $\log \pi$ due to the dispersion in absolute magnitude and the accidental errors in the apparent magnitudes and the absorptions. An application of the formulae given by STRÖMBERG (1936) yielded a very wide dispersion in the values of his correction C^2, which can only have the significance that his formulae cannot be applied successfully in our case. If we estimate the dispersion in M, due to the causes mentioned above, to be $\pm 0^{m}.5$, then our estimate for the dispersion in $\log \pi$ is $\sigma = \pm 0.1$. With $\log \langle \pi \rangle - \langle \log \pi \rangle = 1.15\,\sigma^2$ we obtain a correction of $-0^{m}.06$ to $\langle M \rangle$. Our final result for the mean absolute magnitude is, therefore, $+0^{m}.87 \pm 0^{m}.22$ (m.e.).

16. Space motions

The parallaxes for the different groups described in the preceding sections and given in table 11 under $\langle \pi \rangle$, were used to compute space velocity components for all stars with a known radial velocity and proper motion. These components are Π and Θ, parallel to the galactic plane, in the direction opposite from the galactic centre and in the direction of the galactic rotation, respectively, and Z, directed towards the north galactic pole. The mean errors for these components were supposed to stem only from the mean errors in the proper motion components. Within each group the weighted average value for Π, Θ and Z was formed. These are shown in the columns 3–8 of table 12. The values for $\langle \Theta \rangle$ as computed from the radial velocities alone are entered as well; there is a good agreement in general. The dispersions in these velocity components were found by taking 1.25 times the average absolute residuals with respect to the means, after a suitable correction for the observational errors had been applied. This correction was applied in a manner completely similar to that described in section 13. The dispersions are given in the columns 9–14 of table 12. If we use as mean absolute magnitude for all stars, except the group with periods less than $0^{d}.1$, the value $+0^{m}.9$, we find different values for the space velocities. Their means and dispersions are found in table 12, columns 15–26. In table 7, the latter space velocity components are entered under the headings Π, Θ and Z. Furthermore, the differences between the space velocities computed from the assumptions $\langle M \rangle = +0^{m}.5$ and $\langle M \rangle = +0^{m}.9$ are given under $\Delta\Pi$, $\Delta\Theta$ and ΔZ. The results for $\langle \Theta \rangle$ and the velocity dispersions fully confirm what has been found by previous authors concerning the relation between ΔS and period on the one hand and the dynamical characteristics on the other. However, because of the new data on proper motions the present results are of greater weight. It should be noted that within each of the two ΔS groups there is a very pronounced increase of $-\langle \Theta \rangle$ and of the velocity dispersions with increasing period. The group of stars with a directly measured value of $\Delta S \geq 5$ and periods between $0^{d}.6$ and $0^{d}.8$ have the most extreme halo population II characteristics, with $\langle \Theta \rangle = -240 \pm 28$ km/sec and dispersion $\sigma = 170 \pm 35$ km/sec (m.e.). A preliminary discussion of the motions of the RR Lyrae variables, based on our provisional results, has been given elsewhere (Oort, 1965).

17. Comparisons with previous results

In this section we will give a few of the modern determinations of the parallax and the mean absolute magnitude of the RR Lyrae stars.

As was already remarked in section 15, we believe that the discrepancies found by several authors between π_1 and π_2 are mainly to be attributed to the neglect of the ellipsoidal character of the velocity distribution. Notni has applied the appropriate corrections; we do not, however, fully agree with all of his selection criteria, especially since he used kinematical criteria to select stars for different groups, while all our knowledge of the parallaxes depends on these same kinematical data. Another source of differences between our results and the ones published before is our treatment of the apparent magnitudes. For the globular-cluster RR Lyrae variables, Sandage and Katem (1964) found $M_v = +0^{m}.42 \pm 0^{m}.08$ (m.e.) exclusing M 13 where the only two RR Lyrae variables observed are about half a magnitude brighter. They used accurate photo-electric photometry of many stars in M 3, M 13, M 15 and M 92, applied Hoyle's evolutionary corrections as well as reddening and blanketing corrections and fitted their data to the Hyades main sequence, resulting in distance moduli for the various clusters investigated.

Arp (1962) has found for the cluster variables in M 5, $M_v = +0^{m}.6 \pm 0^{m}.2$ (m.e.). He also used accurate photometric data and fitted the M 5 main sequence to a Hyades main sequence.

Eggen and Sandage (1959), from four variables, derived $M_v = +0^{m}.6$. They made use of the presence of RR Lyrae variables in stellar groups for which a

Author	π_1	π_2 (0″.00001)	π_3	m_0	$\langle M_{pg} \rangle$	
Wilson (1939)	80	135		10.5	+0.15	$P < 1^{d}$
Pavlovskaya (1953 b)	80	123		10.9	+0.52	$P < 1^{d}.4$
Notni (1957)	163	164	134	10.0	+0.87	Large-velocity stars
				10.0	−0.24	$0^{d}.1 < P < 0^{d}.2$
	650	820		10.0	+4.55	$P < 0^{d}.1$
Missana and Plaut (1963)	97	150		11.26	+0.38	$0^{d}.2 < P \leq 0^{d}.4$
	80	131		11.03	−0.06	$0^{d}.4 < P \leq 0^{d}.5$
	77	105		11.30	−0.06	$0^{d}.5 < P \leq 0^{d}.6$
	50	115		11.25	−0.76	$P > 0^{d}.6$

parallax could be derived from proper motions and radial velocities.

Results from other authors are given in the table, with m_0 the mean photographic magnitude towards which the proper motions were reduced.

Acknowledgements

I wish to thank Professor Oort for his encouragement and advice throughout this work.

Note

After our manuscript had been completed, we received an advance copy of a paper by C. Sturch, which is to appear shortly in the *Astrophysical Journal.* In this paper, Sturch has introduced a new parameter $\delta(U-B)$, which can serve as a substitute for Preston's ΔS. For those stars which our lists have in common the following results are found from the space motions derived from $M_{\rm pg} = +0.9$.

n	$\langle P\rangle$	$\langle\delta(U-B)\rangle$	$\langle\Delta S\rangle$	n	$\langle\Pi_{0.9}\rangle$	disp	$\langle\Theta_{0.9}\rangle$	disp	$\langle Z\rangle$	disp
							(km/sec)			
13	$0^{\rm d}.61$	$0^{\rm m}.041$	8.3	11	−10	91	−157	130	−12	118
33	0.56	0.088	6.5	29	+13	174	−170	110	−20	100
16	0.46	0.140	3.7	13	+ 8	108	−103	95	−22	88
7	0.39	0.194	1.3	7	+ 7	69	− 47	91	− 6	44

The first two lines yield a result comparable with lines 11–13 of our table 12. The third line is comparable with line 10, the fourth with line 9. It is tempting to make a subdivision according to period as well and for this purpose the material of the first two lines was taken together. The relative error in the $\delta(U-B)$ will be comparatively large for these stars so that a combination seems advisable to increase the number of stars. The results are given below. It will be noticed that the outcome is now different from the trend shown in lines 12 and 13 of table 12.

n	P	$\langle\delta(U-B)\rangle$	$\langle\Delta S\rangle$	n	$\langle\Pi_{0.9}\rangle$	disp	$\langle\Theta_{0.9}\rangle$	disp	$\langle Z\rangle$	disp
							(km/sec)			
16	$>0^{\rm d}.6$	$0^{\rm m}.058$	8.3	15	+23	135	−139	108	+27	112
30	<0.6	0.098	6.2	25	− 2	160	−181	119	−42	95

References

H. ARP, 1962, *Ap. J.* **135** 311

L. BINNENDIJK, 1943, *Bull. Astr. Inst. Netherlands* **10** 9

A. BLAAUW and J. DELHAYE, 1949, *Bull. Astr. Inst. Netherlands* **10** 473

M. BREGER, 1964, *Mon. Not. Astr. Soc. S. Africa* **23** 112

O. J. EGGEN and A. R. SANDAGE, 1959, *Mon. Not. Roy. Astr. Soc.* **119** 255

P. FAIRFIELD-BOK and C. D. BOYD, 1933, *Astr. Obs. Harvard Bull.* No. 893

E. H. GEYER, 1961, *Z. Ap.* **52** 229

J. C. KAPTEYN and P. J. VAN RHIJN, 1922, *Bull. Astr. Inst. Netherlands* **1** 37

J. v. B. LOURENS, 1960, *Mon. Not. Astr. Soc. S. Africa* **19** 119

W. J. LUYTEN, 1927, *Astr. Obs. Harvard Bull.* No. 847

N. MISSANA and L. PLAUT, 1963, *Contr. Oss. Astr. Torino* Nuova Serie No. 33

P. NOTNI, 1957, *Wiss. Zts. Univ. Jena* **6** *Math. Nat. Reihe* **3/4** 145

V. R. ÖLANDER, R. LEHTI, G. PIPPING and A. SAVELIUS, 1959, *Soc. Sci. Fennicae Comm. Phys. Math.* **22** 37

J. H. OORT, 1936, *Bull. Astr. Inst. Netherlands* **8** 75

J. H. OORT, 1965, *Stars and Stellar Systems*, ed. G. P. Kuiper and B. M. Middlehurst (Univ. of Chicago Press, Chicago) **5** chapter 21

E. PALOQUE, H. BERTHOMIEU, P. PRETRE and M. REYNIS, 1958. *Ann. Obs. Astr. Toulouse* **26** 5

E. Paloque, P. Pretre and M. Reynis, 1959, *Ann. Obs. Astr. Toulouse* **27** 31

E. Paloque, P. Pretre and M. Reynis, 1961, *Ann. Obs. Astr. Toulouse* **28** 5

E. D. Pavlovskaya, 1953a, *Peremennye Zvezdy* **9** 233

E. D. Pavlovskaya, 1953b, *Peremennye Zvezdy* **9** 349

L. Plaut, 1948, *Ann. Sterrew. Leiden* **20** 3

G. W. Preston, 1959, *Ap. J.* **130** 507

H. N. Russell, 1921, *Ap. J.* **54** 140

T. W. Russo, 1960, private communication

A. R. Sandage and B. Katem, 1964, *Year Book* 63 (Carnegie Inst. Washington), *Rep. Mount Wilson and Palomar Obs.* 22

G. Strömberg, 1936, *Ap. J.* **84** 555

R. E. Wilson, 1939, *Ap. J.* **89** 218