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COMMUNICATION FROM THE OBSERVATORY AT LEIDEN.

Summary of the results derived from a discussion of the longitudes of Jupiter's satellites, with special reference to the rotation of the earth, by *W. de Sitter*.

The photographic plates taken at the Cape Observatory in 1924 complete the series of observations undertaken in accordance with the programme outlined in *History and Description of the Cape Observatory*, pp. xcviii to ci. After the reduction of these plates the discussion of the resulting corrections to the longitudes and the derivation of corrections to the adopted elements was taken in hand. This discussion was necessarily of a rather complicated nature, owing to the many entanglements of the different unknowns as a consequence of the mutual commensurabilities in the system. It is considered desirable to publish these discussions to some detail, more detail in fact than can be given in a number of the *B. A. N.* It was therefore decided to devote a separate publication in the *Annals* of the observatory to this discussion. The present paper accordingly only gives the results, with so much explanation of the manner in which they were derived as is necessary to enable the reader to form at least a superficial judgment regarding their trustworthiness.

The observational material used consists of:

1. The series of photographic observations just referred to, published in *M. N.* lxxvi, 448; *B. A. N.* **50**, **61**, **62** and **93**. Each of the twelve series gives corrections to the four mean longitudes.
2. Heliometer observations made at the Cape by Sir DAVID GILL and Mr. W. H. FINLAY in 1891, and discussed by me in *Cape Annals* XII, **1**.
3. Heliometer observations made at the Cape by BRYAN COOKSON in 1901 and 1902, and discussed by him in *Cape Annals* XII, **2**.
4. Photographic plates taken at Helsingfors by Prof. DONNER and at Pulkovo by Dr. KOSTINSKY from 1892 to 1898, measured by Mr. RENZ, and discussed by me in *Groningen Publications* **17**.
5. Photographic plates taken at the Cape in 1904, measured and discussed by me and published in *Gron. Publ.* **17**.
6. Micrometric observations made by various observers at Washington from 1903 to 1906, discussed by Dr. P. KREMER in his dissertation *).
7. Photographic plates taken at Pulkovo by Dr. KOSTINSKY from 1904 to 1910, measured by Mr. BALANOVSKI and discussed by me in *B. A. N.* **105**.

To these I add:

8. My discussion of the Harvard eclipses of Satellite IV, published in *M. N.* lxxi, 596, from which I have derived the mean longitude and an equation of condition between the corrections to the excentricity and the perijove.
9. My discussion of eclipses of Satellite III observed between 1770 and 1800, published in *B. A. N.* **4**.
10. The mean longitudes at the epoch 1750·0 taken from DAMOISEAU's tables.

In addition to these we have, of course, Dr. INNES's valuable series of observations of eclipses and other phenomena from 1908 to 1925, which will eventually contribute largely to the final determination of the elements. This series is being discussed by Mr. D. BROUWER, who hopes to derive his final results in the near future. In the mean time it appeared better to keep the discussion of the photographic, heliometric and micrometric observations separate.

The series of observations mentioned under 1. was primilary undertaken with a view to the determination of the libration and the inequalities of group II. These latter have periods between 460 and 490 days. In observations taken at successive oppositions of Jupiter, i. e. with an average interval of 400 days, they thus present themselves as inequalities with apparent periods between 6 and 8 years, and are consequently not separable from the libration. For this reason the observations of 1913 to 1924 were not taken at the

*) Discussie van Micrometerwaarnemingen van de Satellieten van Jupiter, Leiden, Ydo, 1923.

times of opposition, but about two months before and after these times, so that in the two epochs of the same year the inequalities of group II will have a difference of phase of about 90°.

The inequalities of group II are of the form

$$\Delta\lambda_i = \Sigma_j \varkappa_{ij}\, \varepsilon_j \sin \varphi_j \qquad \begin{pmatrix} i = 1, 2, 3 \\ j = 1, 2, 3, 4 \end{pmatrix}$$
$$\varphi_j = \upsilon + \varpi_j$$

Here ε_i and ϖ_i are the proper excentricities and perijoves, and $\upsilon = \lambda_2 - 2\lambda_3$, λ_i being the mean longitudes. The coefficients $\varkappa_{ij}$ depend on the masses. Since the publication of my theory of 1908 *) with which the observations have been compared, better values of the masses have become available, and also corrections to ε_3 and ϖ_3 were derived in *B. A. N.* **4**. Although we have not yet at our disposal sufficient data for a final determination of the masses, it still seemed desirable to introduce such corrections as appear plausible from the material now at hand. This includes the motions of the nodes derived in *B. A. N.* **102**, the motions of the perijoves ϖ_3 and ϖ_4, and the coefficient τ_{34} of the term in the equation of the centre of III depending on the mass of IV. From these data, together with the adopted coefficients of the great inequalities, the following corrections to the values of the masses adopted in *Annals* XII, **3** were derived:

$$(1) \qquad \begin{aligned} \lambda_0 &= -\cdot 011 \\ \lambda_1 &= +\cdot 150 \\ \lambda_2 &= -\cdot 025 \end{aligned} \qquad \begin{aligned} \lambda_3 &= -\cdot 025 \\ \lambda_4' &= +\cdot 025 \end{aligned}$$

These represent satisfactorily all the data, with the exception of the nodes of I and III. The motion of the node of I is, of course, rather weakly determined from the observations. In the case of the node of III there are some uncertainties in the theoretical expression which must be cleared up before a final judgment can be passed. For the present I have been content to adopt provisionally the corrections (1).

As a first step only the observed corrections to the libration

$$\vartheta = \Delta\lambda_1 - 3\,\Delta\lambda_2 + 2\,\Delta\lambda_3$$

were considered. This is independent of any corrections to the mean longitudes and the mean motions, and particularly the effect of the rotation of the earth is eliminated from it. The observed values, giving corrections to *El. and M.* are contained in the second column of Table **1**. To these must be applied corrections to take account of the effect of the corrections (1) to the masses on the inequalities of group II and of the corrections to ε_3 and ϖ_3. They then become ϑ_1, which is given in the third column of the table. It is at once evident that the major part of these residuals in not due to libration, but to the inequalities of group II. The motions of the angles φ_i

TABLE **1**. LIBRATION 1913—24.

Epoch	ϑ	ϑ_1	φ_2	ϑ_2	R
1913·34	+ 0°·029 ± 0°·009	− 0°·028	26°	+ 0°·012	− 0°·011
13·69	+ ·179 ± 9	+ ·186	118	+ ·093	+ ·010
14·44	− ·087 ± 8	− ·125	332	− ·021	+ ·014
14.76	+ ·130 ± 10	+ ·089	63	+ ·067	+ ·025
15.58	− ·178 ± 8	− ·173	296	− ·082	+ ·011
15·88	− ·031 ± 10	− ·079	21	− ·031	− ·035
16·60	− ·070 ± 9	− ·020	226	− ·027	+ ·063
16.96	− ·097 ± 8	− ·099	328	·000	− ·014
18.86	+ ·008 ± 9	− ·006	148	− ·105	− ·113
19.14	− ·018 ± 15	+ .053	227	+ ·048	·000
22.35	+ ·036 ± 11	− ·029	60	− ·047	− ·015
24·56	− ·057 ± 8	− ·025	329	+ 0·74	+ ·034

being nearly equal, it is not possible from a short series to separate the four terms. I have therefore in this first approximation derived a correction with the argument φ_2 only. The values of φ_2 are given in the fourth column. By a graphical process the correction

$$\Delta\vartheta = +0^\circ\cdot 100 \sin(\varphi_2 - 50^\circ) = +0^\circ\cdot 064 \sin\varphi_2 - 0^\circ\cdot 077 \cos\varphi_2$$

was derived. This leaves the residuals $\vartheta_2 = \vartheta - \Delta\vartheta$, which are given in the fifth column of Table **1**. The last column gives the residuals of the final solution derived at the end of this paper. It will be seen that the values ϑ_2 represent a great improvement on ϑ or ϑ_1. The residuals R are still considerably better.

The effect on the libration of the inequality with argument φ_2 corresponding to the masses (1) *) is

$$(\alpha) \qquad -0^\circ\cdot 091 \sin \varphi_2.$$

Adding the correction $\Delta\vartheta$, this becomes

$$(\beta) \quad -0^\circ\cdot 027 \sin\varphi_2 - 0^\circ\cdot 077 \cos\varphi_2 = -0^\circ\cdot 082 \sin(\varphi_2 + 70^\circ).$$

Comparing this with (α) it is seen that the correction is equivalent to a correction of $+70^\circ$ to φ_2, or ϖ_2, and of $-0\cdot 10\,\varepsilon_2$ to ε_2.

A similar discussion was undertaken using all epochs mentioned above under 1 to 6 (the Pulkovo plates, 7, had at that time not yet been discussed), leading to the value

$$(\gamma) \qquad -0^\circ\cdot 084 \sin(\varphi_2 + 51^\circ),$$

agreeing well with (β).

*) On the masses and elements of Jupiter's satellites and the mass of the system, *Proceedings Amsterdam*, X, pp. 653—673 and 710—729. This will be quoted as *Elements and Masses*, or *El. and M.* It should be pointed out that the libration adopted in that paper is here omitted.

*) Strictly speaking in the first approximation the values $\lambda_0 = -\cdot 015$, $\lambda_3 = -\cdot 040$, $\lambda_4' = +\cdot 050$ were used instead of those given under (1), the values of λ_1 and λ_2 being the same.

About the time when this first approximation was made, Mr. BROUWER had also derived preliminary results from his discussion of Mr. INNES's observations, especially of Satellite II. He found for the term with argument φ_2 in the longitude of this satellite

$$+ 0^\circ{\cdot}013 \sin(\varphi_2 + 79^\circ)$$

The coefficient corresponding to the masses (1) is $+ 0^\circ{\cdot}028$, consequently the correction to ε_2 is $- 0{\cdot}53\,\varepsilon_2$, and to φ_2, or ϖ_2, $+ 79^\circ$.

For the equation of the centre be found

$$+ 0^\circ{\cdot}027 \sin(\lambda_2 - \varpi_2 - 35^\circ),$$

the tabular value being

$$+ 0^\circ{\cdot}034 \sin(\lambda_2 - \varpi_2)$$

The correction to ε_2 is thus now $- 0{\cdot}20\,\varepsilon_2$ and the correction to ϖ_2 is $+ 35^\circ$.

As a general mean we can adopt the following corrections for a mean epoch in the neighbourhood of 1917:

$$(2) \qquad \begin{aligned} \Delta\varpi_2 &= + 60^\circ \\ \Delta\varepsilon_2 &= - 0{\cdot}25\,\varepsilon_2 \end{aligned}$$

Further I take from *B. A. N.* 4, for the same epoch

$$(2') \qquad \begin{aligned} \Delta\varpi_3 &= - 7^\circ{\cdot}5 \\ \Delta\varepsilon_3 &= + 0{\cdot}095\,\varepsilon_3 \end{aligned}$$

It has been found convenient to express the time in units of 5000 days, counted from 1900·0 = J. D. 2415020, thus

$$\tau = \frac{t - 2415020}{5000}$$

The corrections (2) and (2′) are supposed to apply for the epoch $\tau = + 1{\cdot}25$, or

$$\text{J. D. } 2421270 = 1917{\cdot}11$$

The expression for the equations of the centre is

$$\Delta\lambda_i = 2\,\Sigma_j\,\tau_{ij}\,\varepsilon_j \sin(\lambda_i - \varpi_j)$$
$$\varpi_i = \varpi_{io} + \gamma_i\,(\tau - 1{\cdot}25)$$

The inequalities of group II, as has been already stated, are

$$\Delta\lambda_i = \Sigma_j\,\varkappa_{ij}\,\varepsilon_j \sin(\upsilon + \varpi_j)$$

The great inequalities are

$$\Delta\lambda_i = \mp X_i \sin(\lambda_i + \upsilon),$$

the lower sign being taken for satellite II.

The libration is

$$\Delta\lambda_i = Q_i\,\vartheta \qquad \vartheta = k \sin\psi$$
$$Q_1 - 3\,Q_2 + 2\,Q_3 = 1 \qquad \psi = \psi_0 + \beta\tau$$

The values of $\varkappa_{ij}$, τ_{ij}, γ_i, X_i, Q_i, β depend on the masses. Their expressions are given in Part 3 of Vol. XII of the *Annals* of the observatory.

The mean longitudes, to which these inequalities must be added, are, of course,

$$\lambda_i = \lambda_{io} + n_i\tau$$

The time used in astronomical computations, which I shall call the "astronomical time", is measured by the rotation of the earth. If, as has lately become more and more probable, this rotation is subject to secular and periodic changes, then this apparent time will differ from the true, or "uniform time", which is defined by its being the independent variable relatively to which the laws of motion are valid.

The angular momentum of the system Earth-Moon must remain constant. If the earth changes its velocity of rotation then either its own moment of inertia may change in the opposite direction, so as to keep the angular momentum of the earth constant, or the earth's moment of inertia may remain constant, the compensation being effected by the moon changing its angular momentum, or, of course, both methods of compensation may act at the same time. We put

C = the moment of inertia of the earth referred to its axis of rotation,
ω = the earth's velocity of rotation,
i = the inclination of the earth's equator on the moon's orbit, for which we can take its mean value, viz: the inclination of the ecliptic, $23^\circ{\cdot}45$,
μ = the moon's mass,
a = „ „ mean distance,
n_1 = „ „ mean motion,
e = „ „ excentricity.

The angular momentum of the system is then

$$C\omega \cos i + \mu\,a^2\,n_1\sqrt{1 - e^2}$$

The term due to the rotation of the moon can be neglected: it is of the order of 10^{-5} of the others. Most writers neglect the inclination putting $\cos i = 1$. It is however better to keep $\cos i$ in the formula.

The variations of a and n_1 are connected by Kepler's third law, by which $a^3\,n_1^2$ must remain constant. We have thus

$$(C d\omega + \omega\,dC)\cos i - \frac{1}{3}\,\mu\,a^2\sqrt{1 - e^2}\,dn_1 = 0\,,$$

or

$$\frac{d\omega}{\omega} + \frac{dC}{C} - \frac{1}{3}\,\frac{\mu\,a^2\sqrt{1 - e^2}}{C\cos i}\,\frac{n_1}{\omega}\,\frac{dn_1}{n_1} = 0.$$

To take account of the two possibilities mentioned above, I put

$$\frac{dC}{C} = -q\,\frac{d\omega}{\omega}\,,$$

q being a factor between 0 and 1. Introducing numerical values we then have

$$(1-q)\frac{d\omega}{\omega} = 1{\cdot}790\,\frac{dn_1}{n_1}.$$

Here n_1 is the *true* mean motion of the moon, referred to uniform time. The observed mean longitude, referred to astronomical time, corresponds to a mean motion n' of which the change is given by

$$\frac{dn'}{n'} = \frac{dn_1}{n_1} - \frac{d\omega}{\omega}.$$

Consequently

$$\frac{dn'}{n'} = -\frac{0{\cdot}790+q}{1{\cdot}790}\frac{d\omega}{\omega},$$

or

$$\frac{d\omega}{\omega} = -Q\,\frac{dn'}{n'}, \qquad Q = \frac{1{\cdot}790}{0{\cdot}790+q}.$$

Corresponding to the limits 0 and 1 for q, the value of Q may vary between 2·26 and 1.

The apparent irregularities in the motion of any other heavenly body, of which the mean motion is n, are given by

$$\frac{dn}{n} = -\frac{d\omega}{\omega}.$$

Consequently

$$dn = Q\,\frac{n}{n'}\,dn'.$$

If we assume the existence in the longitude of the sun of a term

$$\Delta\odot = +\,1''{\cdot}00\,(1+\varkappa)\,T^2,$$

where T is the time counted in centuries from 1900·0, and if this is ascribed to the retardation of the earth's rotation by tidal friction, then any other heavenly body, except the moon, must have a similar term

$$\Delta\lambda = +\,1''{\cdot}00\,(1+\varkappa)\,\frac{n}{n_0}\,T^2,$$

n_0 being the mean motion of the sun. In this case the moment of inertia of the earth is not affected, we have consequently $q = 0$, $Q = 2{\cdot}26$, and in the moon we have

$$\Delta☾ = +\,\frac{1''{\cdot}00}{Q}(1+\varkappa)\,\frac{n'}{n_0}\,T^2 = +\,5''{\cdot}92\,(1+\varkappa)\,T^2$$

In this case all writers agree that the maximum value of Q must be used. In the case of the great empirical term and of the minor fluctuations opinions differ.

GLAUERT, comparing the minor fluctuations in the moon with the sun, Mercury and Venus *) finds from two different discussions $Q = 1{\cdot}9$ and $Q = 3{\cdot}0$. JONES *), also using only the minor fluctuations, assumes a priori the maximum value of Q, for which, since he neglects the influence of the inclination, he takes $Q = 2{\cdot}55$. He states that this agrees well with the observations. The value corresponding to his "composite curve" of the sun, Mercury, Venus and Mars would be $Q = 2{\cdot}2$. Both these writers do not suppose that there is anything in the longitudes of the bodies investigated by them to correspond to the great empirical term; for this term they thus implicitly assume $Q = 0$. INNES **), BROWN ***) and FOTHERINGHAM ****) on the other hand treat the great empirical term and the minor fluctuations together, and assume the same value of Q for both. This is undoubtedly the more correct point of view. BROWN and INNES both assume *a priori*, and without any discussion, $Q = 1$; FOTHERINGHAM makes a determination of Q, from material largely coinciding with BROWN's, and finds $Q = 1{\cdot}37$. Anticipating the result of the present investigation we may state that Jupiter's Satellites give $Q = 2{\cdot}62$.

With a view to considerations like these it was decided to take account of the effect of irregularities in the earth's rotation, and to introduce Q as an unknow into the equations. The equation of condition for the longitude then becomes

$$\Delta\lambda_i = \delta\lambda_{io} + \tau\,\delta n_i + \frac{n_i}{n'}B'.Q + \frac{n_i}{n_0}S\,(1+\varkappa) + \Sigma_j\,\varkappa_{ij}\,\delta\,[\varepsilon_j \sin\varphi_j] + Q_i\,.\,\vartheta$$

The introduction of the terms with B', the fluctuations, and S, the quadratic term, necessitates corresponding corrections to $\delta\lambda_{io}$ and δn_i. These are taken into account automatically by putting

$$S = \tau^2 - 13{\cdot}69 + 9{\cdot}702\,\tau,$$

which makes $S = 0$ for 1750·0 ($\tau = -\,10{\cdot}957$) and 1917·1 ($\tau = +\,1{\cdot}25$). The factors n_i/n_0 corresponding to $1''{\cdot}00\,(1+\varkappa)\,T^2$ in the sun's longitude, and expressing the longitudes of the satellites in decimals of a degree, are

0·00108
0·00054
0·00027
0·00011

The term B' has been derived from BROWN's latest discussion. BROWN gives "*Th — Obs*", where *Th* repesents the moon's longitude by BROWN's tables

*) *M. N.* lxxv, p. 489, 1915.

*) *M. N.* lxxxvii, p. 4, 1926.
**) *Union Obs. Circ.* 65, p. 304, 1925.
***) *Transactions Yale Obs.* Vol. 3, p. 205, 1926.
****) *M. N.* lxxxvii, p. 142, 1927.

after taking out the great empirical term, and adding a quadratic term $4''{\cdot}79\ T^2$. The quadratic term corresponding to $1''{\cdot}00\ (1 + \varkappa)\ T^2$ in the sun, with $Q = 2{\cdot}26$, is $5''{\cdot}92\ T^2$. Calling B the difference "*Th — Obs*" taken from BROWN's paper, I put

$$B' = -B + 18{\cdot}0 + 1{\cdot}46\,\tau - 0{\cdot}021\ S$$

The correction $+ 18{\cdot}0 + 1{\cdot}46\,\tau$ makes $B' = 0$ for 1750·0 and 1917·1.

The coefficient of S is $(4{\cdot}79 - 5{\cdot}92)\ T^2/\tau^2$.

The factors n_i/n' to reduce to decimals of a degree in the longitudes of the satellites are

0·0043
0·0021
0·0011
0·00045

For the epochs mentioned at the beginning of this paper under 2 and 3 we have, in addition to the longitudes, also determinations of the equations of the centre and the large inequalities x_i, and at the epoch 8 we have one and at the epoch 9 two equations of condition for the equations of the centre. These have been treated in a similar way, but in the present abridged publication I will confine myself to the data referring to the longitudes. Also I shall not give the details of the discussion, but only indicate its general trend.

The corrections to the theory of *Elements and Masses* derived directly from the observations are given in Table **2**, under the head $\Delta\lambda_i$ (obs). To these the fluctuations B' were applied, putting $Q = 1$ and neglecting S. From the corrected values corrections $\delta\lambda_{io}$ and δn_i were derived. These were then introduced, as well as the corrections (2) and (2′) and the motions γ_i and coefficients $\varkappa_{ij}$ and τ_{ij} corresponding to the masses (1); further the term with B' for $Q = 1$ (so that the unknown in the equations becomes $\Delta Q = Q - 1$) and that with S transferring the coefficient of $\varkappa$ to the right hand member. The libration was still neglected. In this way the values are obtained which are given under the heading "1st Appr." It will be seen that these are very considerably smaller than the directly observed $\Delta\lambda_i$, especially for the series 1913—24 and for 1750. The improvement in the latter epoch is of course due entirely to δn_i, that in the epochs 1913—24 is due to the corrections $\delta\lambda_{io}$ and $\delta\,[\varepsilon_i \sin(\upsilon + \varpi_i)]$, and only in a very small measure to B'.

From this "first approximation" a determination of Q, and of further corrections to $\delta\lambda_{io}$ and δn_i was first made. For this purpose the observations were combined in several mean epochs: 1750, 1783, 1891, 1892—8, 1901—02, 1904—10 and 1913—24. By different systems of weighting these mean epochs, we found the following values of Q:

(3)	Satellite I	: $Q = + 1{\cdot}86$	$+ 2{\cdot}74$	$+ 2.42$
	„ II	: $+ 2{\cdot}57$	$+ 3{\cdot}75$	$+ 3{\cdot}37$
	„ III	: $+ 2{\cdot}16$	$+ 2{\cdot}93$	$+ 3{\cdot}19$
	„ IV	: $+ 4{\cdot}61$	.....	$+ 1{\cdot}85$

The values of $\delta\lambda_{io}$ and δn_i were practically the same for the different systems of weighting. They need not be given here. The last system appears to me to be the best. As a preliminary result I adopted

$$Q = + 2{\cdot}75 - 0{\cdot}55\,\varkappa$$

The coefficient of $\varkappa$ was also determined for each of the satellites separately in each of the three systems, giving very concordant results, of which the adopted value is an average.

This value of Q was now introduced, together with the adopted corrections to $\delta\lambda_{io}$ and δn_i, and from the remaining residuals, combined with those derived from the other equations given by the epochs 1891 (Cape); 1901, 02; 1783 and 1891 (Harvard eclipses) corrections were derived to ε_i, ϖ_{io}, and x_i. Corrections to γ_3 and γ_4, and a correction to the term in the equation of the centre of satellite III depending on the mass of satellite IV, were also determined, but not used. The corrections $\delta\varepsilon_i$, $\delta\varpi_{io}$ and $\delta\mathrm{x}_i$ were applied, and from the residuals thus found the libration was determined, both from the combination $\vartheta = \Delta\lambda_1 - 3\,\Delta\lambda_2 + 2\,\Delta\lambda_3$, and from each of the satellites separately. After applying this, the corrected residuals were used for a second determination of $\delta\lambda_{io}$, δn_i and Q. I here confine myself to stating the final results of these discussions, which were rather complicated and laborious, only giving some details about the determination of Q. The residuals remaining after substitution of the final values of the unknowns are given in Table **2** under the heading "Resid.".

The final values for the epoch 1900·0, with their probable errors as derived from the present investigation, are:

$\varepsilon_1 = 0^\circ{\cdot}0015$	$\pm\ ^\circ{\cdot}0010$	$\varpi_1 = 70^\circ$	$\pm\ 35^\circ$	$+ 742^\circ{\cdot}4\ \tau$
$\varepsilon_2 = 0{\cdot}0081$	$\pm\ 20$	$\varpi_2 = 149$	$\pm\ 12$	$+ 192{\cdot}45\,\tau$
$\varepsilon_3 = 0{\cdot}0902$	$\pm\ 20$	$\varpi_3 = 333{\cdot}3$	$\pm\ 1{\cdot}7$	$+ 33{\cdot}10\,\tau$
$\varepsilon_4 = 0.4249$	$\pm\ 10$	$\varpi_4 = 282{\cdot}82$	$\pm\ 0{\cdot}16$	$+ 9{\cdot}52\,\tau$

The motions given correspond to the masses (1).

$$\mathrm{x}_1 = 0^\circ{\cdot}4682 \pm {}^\circ{\cdot}0090$$
$$\mathrm{x}_2 = 1{\cdot}0718 \pm 55$$
$$\mathrm{x}_3 = {\cdot}0665\ (\pm 150)$$

TABLE 2.

Epoch	Satellite I			Satellite II		
	$\Delta\lambda_1$ (obs)	1st Appr.	Resid.	$\Delta\lambda_2$ (obs)	1st Appr.	Resid.
1750·0 1783·34	− 0°·024 ± °·004	+ 0°·002 + ·000κ	+ °·006 + ·000κ	− 0°·022 ± °·006	− 0°·009 + ·000κ	− °·002 + ·000
1891·75 1891·94	− 0·048 ± ·005	− 0·057 − ·021	− ·009 + ·005	− 0·011 ± ·003	− 0·042 − ·010	− ·009 + ·002
1892·82	− 0·054 ± ·008	− 0·081 − ·020	− ·026 + ·003	+ 0·016 ± ·005	− 0·036 − ·010	− ·012 + ·001
93·98	+ ·002 ± 14	− ·022 − ·019	+ 30 + 1	+ ·006 ± 9	− ·042 − ·010	− 15 + 1
95·06	+ ·006 ± 11	− ·007 − ·018	+ 37 0	+ ·007 ± 6	− ·015 − ·009	+ 23 0
96·15	+ ·001 ± 6	+ ·010 − ·018	+ 56 − 1	− ·035 ± 4	− ·019 − ·009	+ 26 0
97·27	− ·042 ± 11	− ·041 − ·017	+ 16 − 1	− ·035 ± 9	− ·008 − ·008	+ 27 − 1
98·30	+ ·006 ± 10	− ·007 − ·016	+ 53 − 1	+ ·012 ± 5	+ ·012 − ·008	+ 50 − 1
1901·60	+ 0·010 ± ·009	− 0·033 − ·014	− ·003 − ·001	+ 0·015 ± ·006	− 0·030 − ·007	+ ·014 ·000
02.62	+ ·003 ± 7	− ·027 − ·013	+ 6 − 1	− ·014 ± 4	− ·021 − ·006	+ 6 0
1904·89	+ 0·084 ± ·012	+ 0·059 − ·011	+ ·094 + ·000	− 0·007 ± ·007	− 0·001 − ·005	+ ·021 ·000
1903·89	+ 0·020 ± ·030	− 0·012 − ·012	+ ·021 ·000	+ 0·026 ± ·017	+ 0·011 − ·006	+ ·018 ·000
04·82	− ·024 ± 15	− ·050 − ·011	− 11 0	− ·029 ± 11	− ·023 − ·006	+ 1 0
05·92	− ·005 ± 18	− ·042 − ·010	− 13 + 1	− ·026 ± 9	− ·036 − ·005	+ 5 0
1904·92	− 0·016 ± ·013	− 0·042 − ·011	− ·009 ·000	− 0·013 ± ·010	− 0·011 − ·005	+ ·005 ·000
06·00	− ·003 ± 11	− ·037 − ·010	− 10 + 1	− ·026 ± 6	− ·032 − ·005	+ 10 0
08·25	+ ·063 ± 10	+ ·004 − ·008	+ 24 + 0	+ ·024 ± 7	− ·017 − ·004	+ 21 0
10·27	+ ·004 ± 11	− ·053 − ·006	− 32 0	+ ·047 ± 7	+ ·013 − ·003	+ 7 0
1913·34	+ 0·069 ± ·005	− 0.005 − ·004	+ ·012 + ·001	+ 0·028 ± ·004	− 0·001 − ·002	+ ·012 + ·001
13·69	+ ·063 ± 5	+ ·009 − ·003	+ 11 + 1	− ·020 ± 4	− ·031 − ·002	+ 2 + 1
14·44	+ ·076 ± 5	− ·011 − ·002	+ 4 + 2	+ ·059 ± 3	+ ·006 − ·001	− 4 + 1
14·76	+ ·073 ± 6	+ ·004 − ·002	+ 13 + 2	− ·001 ± 4	− ·018 − ·001	− 3 + 1
15·58	+ ·098 ± 6	+ ·008 − ·002	+ 15 + 1	+ ·094 ± 3	+ ·029 − ·001	+ 3 + 1
15·88	+ ·071 ± 6	− ·012 − ·001	− 3 + 2	+ ·042 ± 4	+ ·004 − ·001	+ 6 + 1
16·60	+ ·068 ± 5	− ·013 ·000	− 4 + 2	+ ·058 ± 4	+ ·007 ·000	− 20 + 1
16·96	+ ·081 ± 5	− ·013 ·000	− 16 + 2	+ ·062 ± 3	− ·005 ·000	− 4 + 1
18·86	+ ·052 ± 5	− ·017 + ·002	− 14 0	+ ·034 ± 4	+ ·026 + ·001	+ 30 0
19·14	+ ·060 ± 10	− ·026 + ·002	− 40 − 1	+ ·036 ± 6	− ·025 + ·001	− 13 0
22·35	+ ·052 ± 6	− ·035 + ·005	− 25 − 3	+ ·024 ± 5	+ ·002 + ·003	− 5 − 1
24·56	+ ·113 ± 6	+ ·010 + ·008	+ 13 − 8	+ ·066 ± 3	− ·018 + ·004	− 3 − 4

The value of $\mathbf{x}_3$ is that corresponding to the masses (1), as no reliable correction could be derived within the estimated probable error given. The two others, with their probable errors, are the values resulting from the observations. The libration was found to be *):

$$\vartheta = (0^\circ\!\cdot048 \pm {}^\circ\!\cdot015)\ \sin\ (\psi - 3^\circ \pm 15^\circ)$$

*) It may be mentioned that from the residuals ϑ_2 given in Table 1 we found $\vartheta = 0^\circ\!\cdot030 \sin (\psi - 20^\circ)$.

The corrections to the values of λ_{i0} and n_i *) of *Elements and Masses* are:

$$\delta\lambda_{10} = +0^\circ\!\cdot0617 \pm {}^\circ\!\cdot0040 \qquad \delta n_1 = +0^\circ\!\cdot00890 \pm {}^\circ\!\cdot00085$$
$$\delta\lambda_{20} = +0\cdot0308 \pm 20 \qquad \delta n_2 = +\ \cdot00465 \pm 80$$
$$\delta\lambda_{30} = +0\cdot0154 \pm 10 \qquad \delta n_3 = +\ \cdot00252 \pm 40$$
$$\delta\lambda_{40} = +0\cdot0115 \pm 10 \qquad \delta n_4 = +\ \cdot00240 \pm 50$$

These do not include the corrections involved in the constant and linear terms of B' and S.

*) The time being counted in units of 5000 days (τ).

LONGITUDES.

och	Satellite III			Satellite IV		
	$\Delta\lambda_3$ (obs)	1st Appr.	Resid.	$\Delta\lambda_4$ (obs)	1st Appr.	Resid.
'50	− 0°·023 ± °·008	− 0°·016 + ·000 ϰ	− °·010 + ·000 ϰ	− 0°·023 ± °·010	− 0°·007 + ·000 ϰ	− °·009 + ·000 ϰ
'83	+ ·022 ± 2	+ ·017 − ·006	+ 3 + 13			
391	− 0·016 ± ·002	− 0·013 − ·005	− ·001 + ·001	+ 0·0017 ± ·0010	− 0·0024 − ·0022	+ ·0011 + ·0005
HE.				+ ·0046 ± 30	+ ·0012 − ·0021	+ 47 + 4
392·	− 0·009 ± ·003	0·000 − ·005	+ ·017 + ·001			
93	− ·020 ± 5	− ·013 − ·005	+ 5 0			
95	− ·018 ± 3	− ·021 − ·005	− 3 0			
96	+ ·014 ± 2	·000 − ·004	+ 16 0			
97	+ ·011 ± 4	− ·004 − ·004	+ 11 0			
98	+ ·026 ± 3	+ ·019 − ·004	+ 30 0			
)01	− 0·028 ± ·004	− 0·020 − ·003	− ·010 ·000	− 0·0056 ± ·0012	− 0·0129 − ·0014	− ·0101 − ·0001
'02	− ·005 ± 3	− ·018 − ·003	− 4 0	− ·0036 ± 11	− ·0107 − ·0013	− 80 − 1
)04	+ 0·028 ± ·004	+ 0·007 − ·003	− ·015 ·000			
)03	− 0·025 ± ·010	− 0·038 − ·003	− ·023 ·000	+ 0·001 ± ·007	− 0·007 − ·001	− ·005 ·000
04	+ ·011 ± 5	− ·008 − ·003	− 1 0	+ ·004 ± 4	− ·004 − ·001	− 2 0
05	+ ·011 ± 4	− ·004 − ·002	− 1 0	+ ·012 ± 5	+ ·005 − ·001	+ 6 0
)04	+ 0·018 ± ·006	− 0·003 − ·003	+ ·008 ·000	− 0·013 ± ·003	− 0·021 − ·001	− ·019 ·000
06	+ ·011 ± 4	− ·006 − ·002	− 3 0	+ ·002 ± 2	− ·006 − ·001	− 5 0
06	+ ·018 ± 4	+ ·012 − ·002	+ 15 0	+ ·001 ± 2	− ·007 − ·001	− 6 0
10	+ ·013 ± 4	− ·001 − ·002	+ 9 0	− ·002 ± 2	− ·012 − ·001	− 12 0
13I	+ 0·022 ± ·002	+ 0·009 − ·001	+ ·008 ·000	+ 0·0106 ± ·0014	− 0·0008 − ·0004	− ·0015 + ·0001
II	+ ·028 ± 2	+ ·002 − ·001	+ 1 0	+ ·0097 ± 12	− ·0020 − ·0003	− 29 + 1
14I	+ ·007 ± 2	− ·002 − ·001	0 0	+ ·0118 ± 8	− ·0002 − ·0003	− 14 + 2
II	+ ·027 ± 3	+ ·006 − ·001	+ 3 0	+ ·0121 ± 13	− ·0002 − ·0002	− 15 + 1
15I	+ ·003 ± 2	− ·007 ·000	− 2 0	+ ·0193 ± 13	+ ·0068 − ·0002	− 53 + 2
II	+ ·012 ± 3	− ·005 ·000	− 6 0	+ ·0161 ± 15	+ ·0033 − ·0001	+ 16 + 2
16I	+ ·018 ± 2	− ·001 ·000	+ 4 0	+ ·0104 ± 12	− ·0027 ·0000	− 45 + 1
II	+ ·004 ± 2	− ·006 ·000	− 6 0	+ ·0169 ± 11	+ ·0036 ·0000	+ 16 + 2
18	+ ·029 ± 3	− ·002 ·000	− 4 0	+ ·0112 ± 11	− ·0025 + ·0002	− 43 − 1
19	+ ·015 ± 3	− ·003 ·000	− 1 0	+ ·0179 ± 18	+ ·0041 + ·0002	+ 23 − 1
22	+ ·028 ± 2	+ ·002 + ·001	− 2 0	+ ·0124 ± 14	− ·0025 + ·0005	− 48 − 2
24	+ ·014 ± 2	+ ·004 + ·002	+ 5 − 2	+ ·0161 ± 9	+ ·0012 + ·0008	0 − 9

For Q finally we found from the four satellites:

$$(3') \qquad \begin{array}{lll} \text{Satellite I} & : & Q = +2{\cdot}79 \\ \quad ,, \quad \text{II} & : & \quad\;\; +2{\cdot}59 \\ \quad ,, \quad \text{III} & : & \quad\;\; +3{\cdot}26 \\ \quad ,, \quad \text{IV} & : & \quad\;\; +1{\cdot}85 \end{array}$$

The value for satellite IV is the same as before, since the inequalities of group II and the libration are absent, and the corrections $\delta\varepsilon_i$, $\delta\varpi_i$ and ϑ consequently do not affect the longitude of this satellite. Taking the mean with equal weights we find

$$Q = +2{\cdot}62 \pm 0{\cdot}20 - 0{\cdot}55\,\varkappa$$

The probable error has been derived from the deviations of the individual values from the mean. The probable error derived from the weights and the probable errors of the original observations would be about $\pm$ 0·16. The coefficient of $\varkappa$, of course, is the same as in the first solution. Comparing the values (3′) with (3) it will be seen that the effect of the corrections to the inequalities of group II and the libration has been to bring the values for I and II nearer to the mean. The result for III has been very little altered, as was to be expected. The mean has also remained practically the same.

The residuals of this final solution are given in Table 2 under the heading "Resid."

The determinations of the unknowns $\delta\varepsilon_i$, $\delta\varpi_{io}$, δX_i, ϑ on the one, and Q, $\delta\lambda_{io}$, δn_i on the other hand, are very largely independent of each other. Table 3 contains the equations from which the latter group of unknowns has been determined. Under the heading "1st Appr." are given the means by weights of the figures in the same column of Table 2. The column headed Q contains the factor of Q in the equation of condition. By the introduction of the constant and linear terms in B' this is made zero for the epochs 1750 and 1913—24. Consequently the unknowns $\delta\lambda_{io}$ and δn_i are practically determined from these extreme epochs, and Q depends on the intermediate epochs. Of these the epochs 1892—8 and 1904—10 are for several reasons *a priori* less reliable than the others, and in the adopted solution the first of these has been rejected, and the second has been given half weight. For satellite IV the epoch 1901—02, which evidently is affected by some mistake, has been rejected. It will be seen that the representation by $Q = 2{\cdot}62$ ("Resid.") is enormously better than that by $Q = 1$ ("1st Appr."). For the epochs 1891 and 1901—2 practically the same improvement could have been obtained by adopting a large positive value of $\varkappa$, somewhere in the neighbourhood of $\varkappa = 3$, but the epoch 1783 shows that this is impossible. Moreover so large a value of $\varkappa$ is extremely improbable a priori. JEFFREYS considers $1 + \varkappa = 0{\cdot}9$ to be an upper limit; FOTHERINGHAM and SCHOCH find from observations of old eclipses $1 + \varkappa = 1{\cdot}50$, and it does not appear probable that the true value can be much larger than this. The large value of Q thus appears to be well established. From the small coefficients of $\varkappa$ in the final residuals it is evident that a determination of this quantity from the present material is not possible. It would have to depend almost entirely on the epoch 1783.

TABLE 3. MEAN EPOCHS.

Epoch	τ	1st Appr.	Q	Resid.	1st Appr.	Q	Resid.
		Satellite I			Satellite III		
1750	− 10·96	+ 0°·002 ± °·004 + ·000 κ	+ ·0004	+ 0°·006 + 000 κ	− 0°·016 ± °·008 + ·000 κ	+ ·0001	− 0°·010 + 000 κ
1783	− 8·52				+ ·023 ± 2 + 6	+ ·0117	+ ·009 + ·013
1891	− 0·60	− ·057 ± 5 + ·021	− ·0290	− ·009 + ·005	− ·0130 ± ·0018 + ·0052	− ·0070	− ·0011 + ·0014
1892—8	− 0·32	− ·023 ± 4 + ·018	− ·0324	+ ·029 ·000	− ·0022 ± 13 + ·0045	− ·0081	+ ·0136 + ·0001
1901—2	+ 0·16	− ·029 ± 5 + ·013	− ·0247	+ ·002 − ·001	− ·0191 ± 23 + ·0032	− ·0061	− ·0064 − ·0003
1904—10	+ 0·48	− ·021 ± 5 + ·010	− ·0165	+ ·008 ·000	− ·0039 ± 16 + ·0023	− ·0035	+ ·0033 − ·0001
1913—24	+ 1·25	− ·0081 ± 16 ·0000	+ ·0001	− ·0043 ·0000	− ·0004 ± 6 − 0·001	·0000	+ ·0002 − ·0001
		Satellite II			Satellite IV		
1750	− 10·96	− 0°·009 ± °·006 + ·000 κ	+ ·0002	− 0°·002 + ·000 κ	− 0°·007 ± °·010 + ·000 κ	·0000	− 0°·009 + ·000 κ
1891	− 0·60	− ·042 ± 3 + ·010	− ·0145	− ·009 + ·002	− ·0015 ± ·0011 + ·0022	− ·0031	+ ·0020 + ·0005
1892—8	− 0·32	− ·017 ± 2 + ·009	− ·0162	+ ·018 ·000			
1901—2	+ 0·16	− ·025 ± 3 + ·007	− ·0124	+ ·010 ·000	− ·0117 ± ·0008 + ·0013	− ·0026	− ·0090 − ·0001
1904—10	+ 0·48	− ·013 ± 3 + ·005	− ·0084	+ ·012 ·000	− ·0084 ± 11 + ·0010	− ·0017	− ·0064 ·0000
1913—24	+ 1·25	− ·0004 ± 10 − ·0001	+ ·0001	+ ·0004 ·0000	+ ·0005 ± 3 ·0000	·0000	+ ·0003 ·0000

It would, of course, be extremely desirable to have one or more epochs in the long interval between 1783 and 1891. The only observations that can at all be taken into consideration are those by BESSEL with the Königsberg heliometer from 1832 to 1839 and by SCHUR at Strasburg, with different heliometers, from 1874 to 1880. In these observations the satellites were connected with the limbs of the planet, in consequence of which they are affected by large systematic errors. They have therefore not been used for the derivation of the above results. I have, however, computed the residuals which they give if the corrections to the elements here derived are applied. *) I find:

Sat.	BESSEL 1836·0 $Q=1$	$Q=2{\cdot}62$	p.e.	SCHUR 1879.0 $Q=1$	$Q=2{\cdot}62$	p.e.
I	+ 0°·050	+ 0°·001	± °·021	− 0°·047	− 0°·036	± °·029
II	+ ·004	− ·020	± 11	+ ·008	+ ·014	± 20
III	+ ·008	− ·004	± 7	+ ·008	+ ·011	± 9
IV	+ ·012	+ ·007	± 4	+ ·016	+ 0·18	± 6

*) The observed values are taken from SCHUR's discussion, *Nova Acta Leop. Carol.* XLV, p. 103, 1882.

The residuals for $Q = 2{\cdot}62$ are on the whole better than those for $Q = 1$, but the evidence is not very strong, owing to the large probable errors, especially for SCHUR.

If we assume the possibility of deviations from uniformity in the rotation of the earth, then the astronomical, or apparent, time requires a correction to reduce it to true, or uniform, time. This correction can be derived from the observations of any heavenly body, excepting the moon, by the formula

$$\Delta t = \frac{\omega}{n_i}\,(O - C)$$

For the moon the factor Q must be added.

Here O is the mean longitude in the orbit as derived from the observations, C is the mean longitude according to purely gravitational theory. In order to separate the irregular fluctuations from the secular variation, we can include the latter in C, by adding a quadratic term to the purely gravitational mean longitude. We then have

$$\Delta t = \frac{\omega}{n_i}\,(O - C_1) + 24^{s}{\cdot}4\,(1 + \varkappa)\,T^2,$$

T being the time counted in centuries from 1900·0, and the factor corresponding to a secular term $1''{\cdot}00\,(1 + \varkappa)\,T^2$ in the longitude of the sun. We then have, for Jupiter's satellites:

C_1 = mean longitude according to *Elements and Masses* + corrections to inequalities of group II and libration $+ A + BT + CT^2$

The values of A, B, C are

Satellite I :	$A = +0^{\circ}{\cdot}1369 - {}^{\circ}{\cdot}0148\,\varkappa$,	$B = +0^{\circ}{\cdot}2610 + {}^{\circ}{\cdot}0762\,\varkappa$,	$C = +0^{\circ}{\cdot}0573\,(1 + \varkappa)$
II :	+ ·0684 − ·0074	+ ·1319 + ·0381	+ ·0287
III :	+ ·0342 − ·0037	+ ·0674 + ·0191	+ ·0143
IV :	+ ·0195 − ·0015	+ ·0382 + ·0080	+ ·0060

The factors ω / n_i are for the four satellites, if the mean longitudes are expressed in degrees:

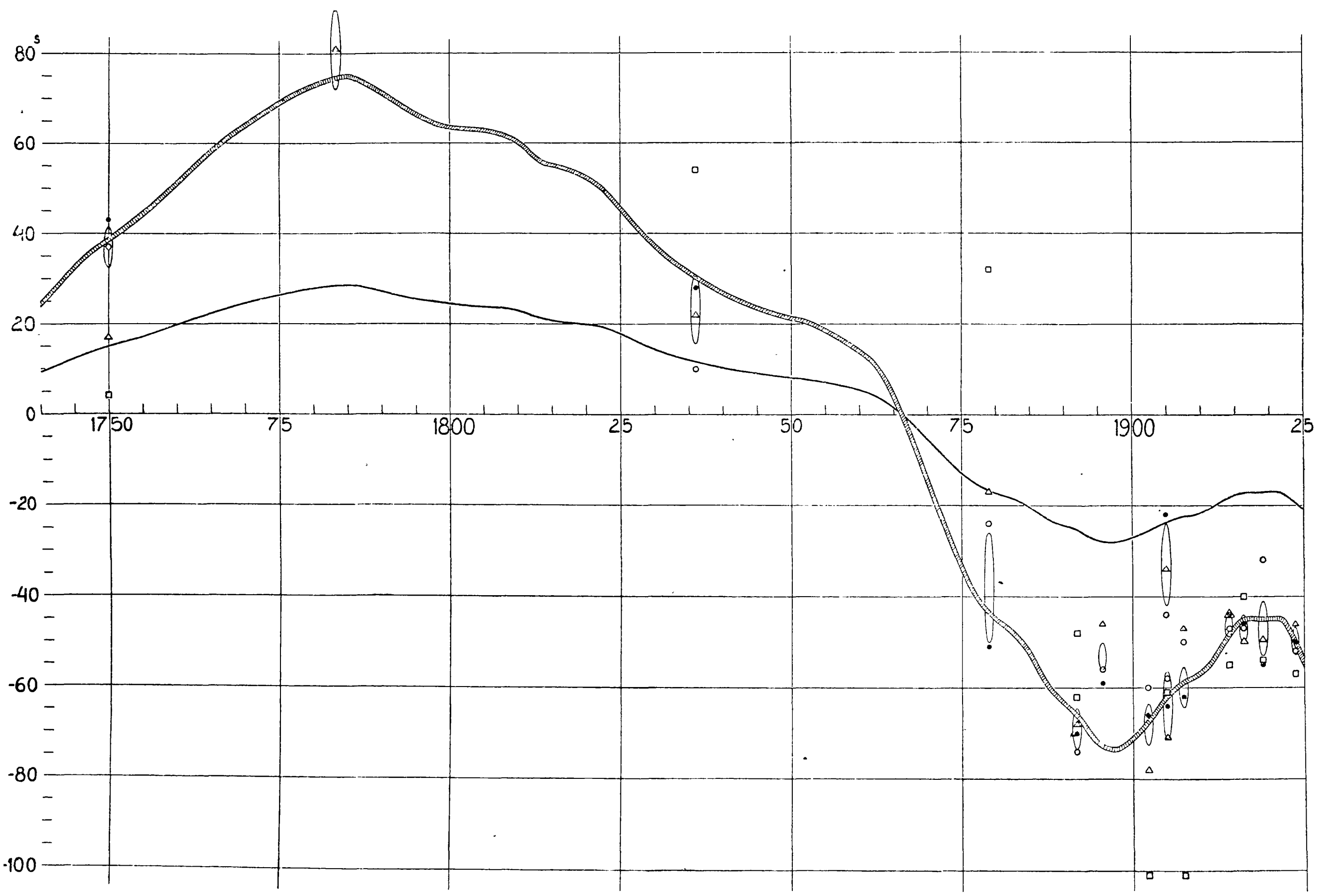
425^{s}
852
1717
4005

In Table 4 the corrections $\Delta_1 t = \omega / n_i\,(O - C_1)$ derived in this way have been given, taking $\varkappa = 0$. The first column gives the epoch, the second contains the reference to the enumeration of the observational data given at the beginning of this paper. *B.* refers to BESSEL and *S.* to SCHUR. The last column contains the weighted mean of the four satellites, the relative weights being different for different epochs. The values derived from the moon for the same epochs are also

TABLE 4. CORRECTIONS FROM ASTRONOMICAL TO UNIFORMLY ACCELERATED TIME.

Epoch.	Obs.	Moon $Q = 1$	Moon $Q = 2{\cdot}62$	Sat. I	Sat. II	Sat. III	Sat. IV	Mean
1750·0	10	$+15^{s}$	$+38^{s}$	$+43^{s} \pm 2^{s}$	$+37^{s} \pm 5^{s}$	$+17^{s} \pm 14^{s}$	$+4^{s} \pm 40^{s}$	$+37^{s} \pm 3^{s}$
83·3	9	+28	+74			+81 ± 6		+81 ± 6
1836·0	*B.*	+12	+30	+28 ± 9	+10 ± 9	+22 ± 12	+54 ± 16	+23 ± 5
79·0	*S.*	−16	−43	−51 ± 12	−24 ± 17	−17 ± 15	+32 ± 24	−38 ± 8
1891·8	2	−25	−67	−70 ± 5	−74 ± 5	−68 ± 4	−62 ± 7	−69 ± 3
91·9	8	−25	−67				−48 ± 22	−48 ± 22
95·6	4	−28	−72	−59 ± 4	−56 ± 3	−46 ± 3		−53 ± 2
1902·2	3	−26	−67	−66 ± 6	−60 ± 4	−78 ± 6	−101 ± 5	−68 ± 3
04·9	5	−24	−63	−22 ± 12	−44 ± 10	−34 ± 10		−33 ± 6
05·0	6	−24	−62	−64 ± 11	−58 ± 9	−71 ± 9	−61 ± 21	−64 ± 5
07·4	7	−23	−59	−62 ± 6	−50 ± 5	−47 ± 6	−101 ± 8	−60 ± 3
14·1	1	−18	−48	−44 ± 3	−47 ± 3	−44 ± 3	−55 ± 4	−46 ± 2
16·2	1	−17	−45	−46 ± 3	−47 ± 2	−50 ± 3	−40 ± 5	−47 ± 2
19·0	1	−17	−46	−55 ± 5	−32 ± 4	−50 ± 5	−54 ± 6	−47 ± 4
23·7	1	−19	−49	−50 ± 5	−52 ± 3	−46 ± 4	−57 ± 6	−50 ± 2

given for the two hypotheses $Q = 1$ and $Q = 2{\cdot}62$, in both cases, however, adopting $Q = 2{\cdot}26$ for the term with T^2. It will be seen that, with the exception of the epochs 1895·6 and 1904·9, the values derived from the satellites agree very well with those found from the moon with the factor $Q = 2{\cdot}62$. These two epochs, those of the old Helsingfors and Pulkowa plates, and of the Cape plates for 1904 which were primarily taken for the determination of the inclinations, are, as has already been remarked, subject to considerable a priori uncertainty, in consequence of which their probable errors cannot be taken as a true measure of their accuracy. The data contained in Table 4 are represented in the diagram. The two curves from the moon, corresponding to the values $Q = 1$ and $Q = 2{\cdot}62$, are represented by a thin and a broad line respectively. The values derived from Satellite I are represented by dots, those from Satellite II by circles, from III by triangles and from IV by squares. The means of the last column are represented in the diagram by the centres of ellipses of which the semi-major axis is the mean error (or,



more exactly, one and a half times the probable error). It will be seen that in all cases, excepting the two already mentioned, these ellipses intersect the curve for the moon for $Q = 2{\cdot}62$. If the fluctuations in the earth's rotation were not real, in other words if the mean longitudes of the satellites were purely linear functions of the apparent time corrected only for the quadratic term $24^s{\cdot}4\ T^2$, then all these means ought to lie on a straight line connecting the epochs 1750·0 with the weighted mean of the four last ones, 1913—24. If this line is drawn it will appear that it intersects none of the ellipses for the intermediate epochs, with the exception of 1879·0 and 1904·9, most of the others remaining free of it by several times their semi-major axes. There can thus hardly be any reasonable doubt regarding the reality of fluctuations in the earth's rotation agreeing with those observed in the moon multiplied by a factor of the order of 2·6, or, if a positive value of $\varkappa$ is adopted, 2·5 or 2·4.

The accuracy with which the correction Δt can be derived from the observations of any heavenly body depends on the one hand on the mean motion of that body, and on the other hand on the accuracy with which its mean orbit longitude can be determined from the observations. This latter factor includes, of course, the uncertainty of the elements used in deriving the mean longitude from the true longitude, which is found directly from the observations. From the second point of view the moon is by far the most favourable of all heavenly bodies, from the first point of view Jupiter's satellites are only surpassed by those of Mars, which, however, are entirely ruled out by the second consideration.

From one good series of photographic plates like those of 1913—24 we can determine Δt with a probable error of about $\pm 3^s$. To this must still be added about $\pm 2^s$ on account of uncertainty of the elements*) making in all an uncertainty of about $\pm 3^{s}{\cdot}5$ or $\pm 4^s$. The probable error of one yearly mean for the moon's longitude, as derived from the Greenwich observations, may be estimated at $\pm 0''{\cdot}07$ or $\pm 0''{\cdot}08$. This would, with the factor $Q = 2{\cdot}6$, give a probable error of Δt of $\pm 0^{s}{\cdot}4$. The uncertainty of the elements will be relatively unimportant in this case. If there were no doubt on the amount of the factor Q the determination from the moon would thus be very much better than from the satellites, so much so as to make observations of the satellites for this purpose of very little value. The uncertainty of Q, however, even assuming it to be so small as to correspond to a probable error of $\pm 0{\cdot}20$, will introduce into the value of Δt derived from the moon a probable error of 1/13 of its amount, i. e. at the present time about $\pm 4^s$, and at the maxima more than $\pm 5^s$. The accurate determination of the factor Q thus becomes of very great importance. This determination should be based on comparison with the *total* fluctuations of the moon, not with the minor fluctuations alone.

For this comparison evidently the most promising results may be expected on the one hand from the transits of Mercury, and on the other hand from the sun. A long series is, however, required in order to separate the effect of the irregularities in the measure of time from corrections to the mean motion and secular acceleration. Unfortunately the observations of the sun previous to about 1835 appear to be very uncertain. The reduction of HORNSBY's observations between 1774 and 1803 and those of his successors ROBERTSON and RIGAUD from 1811 to 1838 will be of very great value to settle this uncertainty.

*) This is *not* included in the probable errors given in Table 4, and in the diagram.