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COMMUNICATION FROM THE OBSERVATORY AT LEIDEN.

The effect of the second-order terms in the pulsation theory of Cepheid variation,

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1. In the pulsation theory of Cepheid variation, as set forth by Sir A. S. EDDINGTON [1]), the equations are simplified by development in series of powers of the amplitude of the pulsation, the square of the amplitude usually being neglected. As EDDINGTON has shown [2]), the second-order terms induce a forced vibration with twice the frequency of the fundamental oscillation, which makes, if its amplitude is sufficiently large, the radial velocity-curve markedly unsymmetrical. EDDINGTON has derived the equations with the inclusion of the second-order terms and made an estimate of the effect. Dr. WOLTJER suggested to me to compute it more exactly and indicated the method to be used.

This paper contains the calculations for one special case, the polytropic index being taken equal to 3 and the quantity $3-4/\gamma$, where γ is the ratio of specific heats for the mixture of matter and radiation, equal to 0·2. The equation based on the assumption of adiabatic oscillations has been used throughout the star, a procedure more or less justified by the small influence the departure from purely adiabatic conditions has been shown to have [1]).

The result is given in section 5.

2. The differential equation for the amplitude of the free oscillation is:

$$\text{(A)} \quad \frac{d^2\sigma_1}{dr_0^2} + \frac{4-\mu}{r_0}\frac{d\sigma_1}{dr_0} + \left\{\frac{\omega^2}{u} - \left(3-\frac{4}{\gamma}\right)\frac{\mu}{r_0^2}\right\}\sigma_1 = 0 \quad {}^{2}),$$

that for the amplitude of the forced oscillation:

$$\text{(C)} \quad \frac{d^2\sigma_2}{dr_0^2} + \frac{4-\mu}{r_0}\frac{d\sigma_2}{dr_0} + \left\{\frac{4\omega^2}{u} - \left(3-\frac{4}{\gamma}\right)\frac{\mu}{r_0^2}\right\}\sigma_2 = \sigma_1^2\left\{\frac{\mu}{r_0^2}\left(\frac{3}{\gamma} - \frac{21}{4} + \frac{9}{4}\gamma\right) - \frac{\omega^2}{u}\left(\frac{3}{2}\gamma - \frac{1}{2}\right)\right\} +$$
$$+ \sigma_1\frac{d\sigma_1}{dr_0}(1+\gamma)\left\{\frac{\mu}{2r_0}\left(3-\frac{4}{\gamma}\right) - \frac{\omega^2 r_0}{2u}\right\} + \left(\frac{d\sigma_1}{dr_0}\right)^2\left\{\frac{\mu}{4}(1+\gamma) - 1\right\} \quad {}^{3}),$$

where:

- r = distance from star's centre, expressed in EMDEN's unit,
- σ_1 = coefficient of cos nt in $\delta r_0/r_0$,
- σ_2 = ,, ,, cos $2nt$ in $\delta r_0/r_0$,
- n = frequency of fundamental oscillation,
- $\mu = g_0\rho_0 r_0/P_0$,
- g = acceleration of gravity,
- ρ = density,
- P = total pressure = gas pressure + radiation pressure,
- ω = frequency of fundamental oscillation, expressed in a suitable unit, which is different for different stars,
- $u = (\rho_0/\rho_c)^{1/3}$, ρ_c = density at star's centre,
- γ = ratio of specific heats for the mixture of matter and radiation.

Quantities with index zero are to be given the equilibrium values for a star with polytropic index equal to 3; the values of u and μ have been taken from the *B. A. Mathematical Tables, Vol. II.*

The coefficient of the first derivative in both equations is infinite for $r_0 = 0$ and $r_0 = R$, where R

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1) *M. N.* **79**, p. 2 and 177.
2) l. c., p. 183.

1) EDDINGTON, *M. N.* **87**, p. 539.
2) *M. N.* **79**, p. 11.
3) *M. N.* **79**, p. 185. EDDINGTON partly neglects radiation pressure and therefore his result is, as regards second-order terms, slightly different from the one found if radiation pressure is taken into account throughout. Since, however, this difference may be shown numerically never to amount to more than a few per cent, it has been neglected.

is the star's radius; the coefficient of σ_1 in (A), that of σ_2 in (C), and the right-hand side of (C) are infinite for $r_0 = R$, but finite for $r_0 = 0$. On account of these singularities the solution can not be found by quadratures only, but must be started with the aid of a series.

Solutions of equations (A) and (C) are sought, which satisfy the boundary conditions that they are finite for $r_0 = 0$ and $r_0 = R$.

3. When a solution of the homogeneous equation (A) is started at the centre, such as to satisfy the first boundary condition, and continued outwards, it generally becomes infinite for $r_0 = R$, thus infringing the second condition; in the same way a solution started at the outer boundary, fulfilling the second condition, and continued inwards, will become infinite at the centre.

If, by the adjustment of the constant factors in these solutions, the two functions σ_1 are equated at an arbitrary argument r_0, then generally the two values of the first derivative are not equal. However, for a set of values of ω^2 the first derivatives are also equal and the two solutions actually form only one, satisfying both boundary conditions.

The smallest critical value of ω^2, which is the only one considered here, lies, as EDDINGTON has shown [1]), between ·055 and ·060; since for our purpose the solution is needed up to the boundary of the star and therefore ω^2 must be known more accurately, EDDINGTON'S calculations have been extended.

The solution starting at the centre with unit amplitude is given by EDDINGTON [1]) for $\omega^2 = {\cdot}055$ up to $r_0 = 4{\cdot}5$, for $\omega^2 = {\cdot}060$ as far as $r_0 = 5{\cdot}0$. At the outer boundary the required solution may be expanded in the series $b_0 \sum\limits_0^\infty k_i y^i$, where $y = (R - r_0)/R$; the coefficients k_i for the two values of ω^2 are found in columns 2 and 3 of Table 1. These series converge so rapidly that σ_1 and $d\sigma_1/dr_0$ can be computed with their aid as far inward as $r_0 = 4{\cdot}5$.

TABLE 1.

Values of the coefficients in the series for the solutions of equation (A).

i	k_i			k'_i
	$\omega^2 = {\cdot}055$	$\omega^2 = {\cdot}060$	$\omega^2 = {\cdot}058806$	$\omega^2 = {\cdot}058806$
0	+ 1·0000	+ 1·0000	+ 1·0000	+ 1·00000
1	— 2·0350	— 2·2382	— 2·1897	
2	+ 2·7105	+ 3·1391	+ 3·0338	+ ·98872
3	— 2·2840	— 2·8396	— 2·7004	
4	+ 1·6242	+ 2·1032	+ 1·9804	+ 1·1631
5	— 1·0601	— 1·4157	— 1·3235	
6	+ ·6463	+ ·8784	+ ·8174	— ·2901
7	— ·3644	— ·5155	— ·4758	
8	+ ·2023	+ ·2827	+ ·2611	+ ·707
9	— ·0930	— ·1438	— ·1304	

Now b_0 is determined by equating the two values of σ_1 at $r_0 = 4{\cdot}5$. Table 2 shows b_0, the corresponding value of $d\sigma_1/dr_0$, and the difference in $d\sigma_1/dr_0$ of the two solutions; the value of ω^2 for which this difference is zero has been found by linear interpolation to be equal to ·058835.

TABLE 2.

ω^2	σ_1 (outside)	σ_1 (inside)	b_0	$d\sigma_1/dr_0$ (outside)		$d\sigma_1/dr_0$ (inside)	difference in $d\sigma_1/dr_0$
·055	·5436 b_0	1·7349	3·1915	·11127 b_0	·3551	·5427	+ ·1876
·058835	·5152 b_0	1·6222	3·1487	·11653 b_0	·3669	·3655	— ·0014
·060	·5070 b_0	1·5879	3·1320	·11804 b_0	·3697	·3130	— ·0567
·058806	·5155 b_0	1·6231	3·1486	·11647 b_0	·3667	·3669	+ ·0002

One further approximation to ω^2 has been made, for which purpose the two solutions corresponding to $\omega^2 = {\cdot}058835$ have been computed. Quadratic interpolation, again at $r_0 = 4{\cdot}5$, fixed ω^2 at ·058806, which value has been used throughout the work; the corresponding value of b_0, the boundary amplitude of the free oscillation for unit central amplitude, is 3·149.

Finally the solutions for $\omega^2 = {\cdot}058806$ have been computed too. For r_0 near R the solution has been expanded in the series $b_0 \sum\limits_0^\infty k_i y^i$, for r_0 near zero in the series $\sum\limits_0^\infty k_i' z^i$, where $z = r_0/R$. With the aid of this latter series a numerical integration has been started at $r_0 = 1{\cdot}0$, the results of which are shown in Table 3; the interval used in this and all other numerical integrations is 0·1. The coefficients k_i and k_i' may be found in columns 4 and 5 of Table 1; numerical details about the degree of equivalency of the two solutions are given in the last line of Table

1) *M. N.* **79**, p. 11.

1) l. c., p. 12.

TABLE 3. Solution of equation (A) for $\omega^2 = {\cdot}058806$, from $r_0 = 1{\cdot}0$ to $r_0 = 5{\cdot}0$.

r_0	σ_1	$\frac{d\sigma_1}{dr_0}$	$\frac{d^2\sigma_1}{dr_0^2}$	r_0	σ_1	$\frac{d\sigma_1}{dr_0}$	$\frac{d^2\sigma_1}{dr_0^2}$
1·0	+ 1·02130	+ ·043614	+ ·04767	3·0	+ 1·22744	+ ·178027	+ ·09425
1·1	1·02590	·048443	·04893	3·1	1·24572	·187628	·09780
1·2	1·03099	·053404	·05030	3·2	1·26497	·197591	·10148
1·3	1·03659	·058508	·05180	3·3	1·28525	·207930	·10531
1·4	1·04270	·063767	·05340	3·4	1·30657	·218656	·10927
1·5	1·04935	·069192	·05512	3·5	1·32899	·229788	·11338
1·6	1·05654	·074794	·05695	3·6	1·35254	·241338	·11765
1·7	1·06431	·080585	·05888	3·7	1·37727	·253323	·12206
1·8	1·07267	·086574	·06093	3·8	1·40322	·265755	·12663
1·9	1·08163	·092775	·06308	3·9	1·43044	·278655	·13138
2·0	1·09123	·099195	·06535	4·0	1·45897	·292037	·13630
2·1	1·10148	·105848	·06772	4·1	1·48886	·305921	·14141
2·2	1·11241	·112744	·07022	4·2	1·52017	·320325	·14670
2·3	1·12404	·119895	·07282	4·3	1·55295	·335267	·15218
2·4	1·13640	·127311	·07552	4·4	1·58724	·350770	·15789
2·5	1·14951	·135004	·07835	4·5	1·62312	·366852	·16379
2·6	1·16341	·142985	·08129	4·6	1·66063	·383535	·16992
2·7	1·17812	·151266	·08435	4·7	1·69985	·400844	·17630
2·8	1·19367	·159858	·08752	4·8	1·74082	·418802	·18290
2·9	1·21010	·168774	·09082	4·9	1·78363	·437433	·18977
3·0	+ 1·22744	+ ·178027	+ ·09425	5·0	+ 1·82833	+ ·456767	+ ·19697

4. The required solution of equation (C) must satisfy the conditions that it is finite at $r_0 = 0$ and $r_0 = R$.

The solution of (C) that satisfies the first boundary condition is equal to the sum of a particular solution of (C) and a solution of (B) – which is equation (C) with right-hand member zero – multiplied by a constant factor C_i, both of these having been chosen such as to be finite at $r_0 = 0$. In the same way the solution of (C) fulfilling the second boundary condition is equal to the sum of a particular solution of (C) and a solution of (B), multiplied by the factor C_e, both finite at $r_0 = R$. The constants C_i and C_e are determined from the condition that the function and its first derivative as given by the solution from the inside have the same values as those found in the solution from the outside; if this condition is satisfied for one value of r_0 the two solutions are really only one, which is finite at the centre as well as at the outer boundary.

The equations determining C_i and C_e can always be solved, unless 2ω is equal to the frequency of one of the overtones, in which case the determinant has the value zero. However, computations by Mr. J. A. EDGAR [1]) of the ratio of the first two critical values of the frequency for two different values of $3 - 4/\gamma$, gave reasons to expect that for $3 - 4/\gamma = 0{\cdot}2$ this ratio would not be too close to 2; afterwards the equations actually turned out to be sufficiently independent.

Equation (B) is:

$$\text{(B)} \quad \frac{d^2\sigma_2}{dr_0^2} + \frac{4-\mu}{r_0}\frac{d\sigma_2}{dr_0} + \left\{\frac{4\omega^2}{u} - \left(3-\frac{4}{\gamma}\right)\frac{\mu}{r_0^2}\right\}\sigma_2 = 0.$$

One solution has been started at the centre in the form of the series $\sum_0^\infty b_i' z^i$, and continued from $r_0 = 0{\cdot}9$ outwards by quadratures. Another solution was begun at the outer boundary with the series $\sum_0^\infty b_i y^i$; since this series converges less rapidly than in the case of the solution of (A), this solution has been carried on by numerical integration from $r_0 = 5{\cdot}9$ inwards. The coefficients b_i' and b_i are given in columns 3 and 2 of Table 4, the results of the numerical integrations, abbreviated to a convenient number of decimals, in Table 5.

TABLE 4. Values of the coefficients in the series for the solution of (B), and for the right-hand side and the particular solution of (C).

i	solution of (B)		right-hand side of (C)		solution of (C)	
	b_i	b_i'	f_i	f_i'	d_i	d_i'
0	+ 1·0000	+ 1·00000	— ·56954	— ·01090	·0000	·00000
1	— 9·3588		+ 3·8478		— 6·7728	
2	+ 39·0184	+ ·14956	— 13·035	— ·9141	+ 43·6566	— ·05185
3	— 101·726		+ 29·314		— 140·228	
4	+ 190·828	— 5·3555	— 49·605	+ ·0350	+ 298·810	— 1·7905
5	— 279·180		+ 68·312		— 476·763	
6	+ 336·65	— 18·340	— 80·732	— 19·45	+ 612·62	— 7·4954
7	— 348·62		+ 84·879		— 666·26	
8	+ 319·95	— 9·514	— 81·25		+ 636·26	— 10·672
9	— 266·40				— 547·85	

[1]) *M. N.* **93**, p. 430.

TABLE 5.

Solutions of equation (B).

Solution from the inside				Solution from the outside			
r_0	σ_2	$\frac{d\sigma_2}{dr_0}$	$\frac{d^2\sigma_2}{dr_0^2}$	r_0	σ_2	$\frac{d\sigma_2}{dr_0}$	$\frac{d^2\sigma_2}{dr_0^2}$
0·9	+ 1·00091	− ·0019	− ·0201	5·9	+ ·22361	+ ·38444	+ ·5135
1·0	1·00061	·0042	·0273	5·8	·18763	·33606	·4551
1·1	1·00004	·0074	·0358	5·7	·15621	·29321	·4030
1·2	·99911	·0114	·0455	5·6	·12883	·25528	·3564
1·3	·99772	·0165	·0568	5·5	·10501	·22175	·3149
1·4	·99577	·0228	·0698	5·4	·08434	·19216	·2780
1·5	·99311	·0305	·0847	5·3	·06646	·16602	·2451
1·6	·98960	·0398	·1017	5·2	·05104	·14301	·2158
1·7	·98508	·0510	·1210	5·1	·03777	·12276	·1897
1·8	·97934	·0641	·1430	5·0	·02640	·10497	·1666
1·9	·97218	·0797	·1678	4·9	·01670	·08936	·1461
2·0	·96333	·0978	·1958	4·8	·00847	·07566	·1278
2·1	·95252	·1189	·2273	4·7	+ ·00151	·06371	·1118
2·2	·93943	·1434	·2627	4·6	− ·00433	·05327	·0975
2·3	·92371	·1716	·3023	4·5	·00919	·04416	·0848
2·4	·90497	·2040	·3465	4·4	·01320	·03625	·0737
2·5	·88275	·2411	·3957	4·3	·01647	·02938	·0639
2·6	·85658	·2833	·4504	4·2	·01910	·02343	·0553
2·7	·82590	·3314	·5110	4·1	·02118	·01829	·0476
2·8	·79010	·3858	·5780	4·0	·02279	·01387	·0410
2·9	·74851	·4472	·6517	3·9	·02398	·01006	·0352
3·0	·70040	·5163	·7327	3·8	·02482	·00681	·0300
3·1	·64496	·5940	·8213	3·7	·02536	·00403	·0256
3·2	·58130	·6809	·9180	3·6	·02564	+ ·00168	·0217
3·3	·50845	·7778	1·0230	3·5	·02570	− ·00033	·0184
3·4	·42537	·8857	1·1365	3·4	− ·02558	− ·00202	+ ·0155
3·5	·33091	1·0054	1·2588				
3·6	·22386	1·1378	1·3896				
3·7	+ ·10291	1·2836	1·5286				
3·8	− ·03334	1·4437	1·6751				
3·9	− ·18634	− 1·6189	− 1·8281				

Before the solution of (C) could be found the right-hand side had to be considered. This, and therefore the solution of (C) too, is proportional to the square of the amplitude of the free oscillation; σ_1 has been substituted with the constant factor corresponding to unit central amplitude.

For r_0 near R and r_0 near zero the right-hand side has been developed in series, viz. $\frac{1}{y}\sum_0^\infty f_i y^i$ and $\sum_0^\infty f_i' z^i$, and in between it has been tabulated numerically; the values of the coefficients f_i and f_i' are given in columns 4 and 5 of Table 4, whereas Table 6 shows the numerical values of the right-hand member.

TABLE 6.

r_0	right-hand side of (C)	r_0	right-hand side of (C)	r_0	right-hand side of (C)	r_0	right-hand side of (C)
0·8	− ·023235	2·2	− ·118872	3·6	− ·53283	5·0	− 3·2709
0·9	·026544	2·3	·131857	3·7	·59851	5·1	3·7958
1·0	·030264	2·4	·146227	3·8	·67348	5·2	4·4199
1·1	·034415	2·5	·162164	3·9	·75920	5·3	5·1670
1·2	·039016	2·6	·179877	4·0	·85746	5·4	6·0661
1·3	·044092	2·7	·199609	4·1	·97031	5·5	7·1562
1·4	·049667	2·8	·22165	4·2	1·10026	5·6	8·4876
1·5	·055790	2·9	·24629	4·3	1·25023	5·7	10·1287
1·6	·062496	3·0	·27396	4·4	1·42372	5·8	12·1756
1·7	·069845	3·1	·30505	4·5	1·62502	5·9	14·7622
1·8	·077898	3·2	·34007	4·6	1·85914	6·0	18·083
1·9	·086732	3·3	·37965	4·7	2·13234	6·1	22·432
2·0	·096437	3·4	·42440	4·8	2·45210	6·2	− 28·278
2·1	− ·107110	3·5	− ·47515	4·9	− 2·82777		

One solution of (C) for r_0 near zero has been started in the form of the series $\sum_0^\infty d_i' z^i$, another one for r_0 near R as $\sum_0^\infty d_i y^i$; numerical integrations have been performed from $r_0 = 1{\cdot}0$ outwards and from $r_0 = 6{\cdot}1$ inwards, the results of which are shown in Table 7 with a convenient number of figures. The values of the coefficients d_i and d_i' are given in columns 6 and 7 of Table 4.

TABLE 7.

Solutions of equation (C).

Solution from the inside				Solution from the outside			
r_0	σ_2	$\frac{d\sigma_2}{dr_0}$	$\frac{d^2\sigma_2}{dr_0^2}$	r_0	σ_2	$\frac{d\sigma_2}{dr_0}$	$\frac{d^2\sigma_2}{dr_0^2}$
1·0	− ·001953	− ·00578	− ·01388	6·1	− 3·6819	+ 1·1732	+ 5·428
1·1	·002605	·00732	·01694	6·0	3·7735	·6741	4·572
1·2	·003427	·00919	·02055	5·9	3·8194	+ ·2550	3·827
1·3	·004456	·01145	·02480	5·8	3·8269	− ·0945	3·177
1·4	·005732	·01417	·02976	5·7	3·8025	·3833	2·612
1·5	·007307	·01743	·03556	5·6	3·7520	·6193	2·124
1·6	·009239	·02131	·04232	5·5	3·6802	·8102	1·702
1·7	·011595	·02593	·05019	5·4	3·5913	·9618	1·339
1·8	·014453	·03139	·05932	5·3	3·4889	1·0796	1·026
1·9	·017906	·03784	·06990	5·2	3·3763	1·1686	·759
2·0	·022059	·04543	·08216	5·1	3·2560	1·2327	·533
2·1	·027036	·05434	·09633	5·0	3·1304	1·2763	·341
2·2	·032978	·06477	·11270	4·9	3·0014	1·3020	·180
2·3	·040048	·07696	·13157	4·8	2·8705	1·3131	+ ·047
2·4	·048437	·09118	·15334	4·7	2·7392	1·3121	− ·065
2·5	·058362	·10774	·17840	4·6	2·6084	1·3009	·156
2·6	·070074	·12698	·20723	4·5	2·4792	1·2815	·230
2·7	·083862	·14933	·24040	4·4	2·3524	1·2554	·288
2·8	·100058	·17523	·27852	4·3	2·2283	1·2243	·332
2·9	·119044	·20522	·32227	4·2	2·1076	1·1894	·365
3·0	·141259	·23990	·37255	4·1	1·9906	1·1516	·389
3·1	·167204	·27998	·43023	4·0	1·8774	1·1119	·405
3·2	·197460	·32623	·49642	3·9	1·7682	1·0708	·413
3·3	·232687	·37958	·57238	3·8	1·6632	1·0295	·415
3·4	·273648	·44108	·65948	3·7	1·5623	·9880	·413
3·5	·321214	·51191	·75940	3·6	1·4656	·9471	·405
3·6	·376386	·59345	·87406	3·5	1·3729	·9071	·393
3·7	·440313	·68728	1·00562	3·4	1·2841	·8685	·379
3·8	·514313	·79522	1·15668	3·3	− 1·1991	− ·8314	− ·363
3·9	·599898	·91937	1·33018				
4·0	·698806	1·06212	1·52964				
4·1	·813036	1·22629	1·75911				
4·2	·944886	1·41511	2·02346				
4·3	− 1·097004	− 1·63234	− 2·32839				

As the particular solutions of (C) have been chosen such as to be zero at the centre and at the outer boundary, the constants C_i and C_e represent the central and boundary amplitudes of the forced vibration.

The two values of the function and the first derivative at $r_0 = 3{\cdot}4$ have been equated, which gives

$$-{\cdot}2736 + {\cdot}42537\, C_i = -1{\cdot}2841 - {\cdot}02558\, C_e$$
$$-{\cdot}4411 - {\cdot}8857\, C_i = -\ {\cdot}8685 - {\cdot}00202\, C_e,$$

with the result:

$$C_i = +\ 0{\cdot}3780,$$
$$C_e = -\ 45{\cdot}78.$$

Table 8 summarizes the numerical values of the final solution of (C) throughout the star; in the upper part of the table the solutions starting at the centre have been used, in the lower part those starting at the outer boundary. It is seen that, whereas the amplitude of the forced vibration is practically constant for $r_0 < 2{\cdot}5$, it increases rapidly after the node at about $r_0 = 3{\cdot}2$.

TABLE 8.

Numerical values of the final solution of (C).

r_0	partic. sol. of (C)	sol. of (B)	C_i × sol. of (B)	final sol. of (C)
0·0	·0000	+ 1·0000	+ ·3780	+ ·3780
0·5	— ·0003	1·0006	·3782	·3779
1·0	·0020	1·0009	·3783	·3763
1·5	·0073	·9931	·3754	·3681
2·0	·0221	·9633	·3641	·3420
2·5	·0584	·8828	·3337	·2753
3·0	·1413	·7004	·2648	+ ·1235
3·5	·3212	+ ·3309	+ ·1251	— ·1961
4·0	— ·6988	— ·3576	— ·1352	— ·8340
r_0	partic. sol. of (C)	sol. of (B)	C_e × sol. of (B)	final sol. of (C)
3·5	— 1·373	— ·02570	+ 1·177	— ·196
4·0	1·877	·02279	1·043	·834
4·5	2·479	— ·00919	+ ·421	2·058
5·0	3·130	+ ·02640	— 1·209	4·339
5·5	3·680	·10501	4·807	8·487
6·0	3·774	·26472	12·12	15·89
6·5	— 2·666	·5733	26·25	28·92
R	·000	+ 1·0000	— 45·78	— 45·78

For an observed boundary amplitude of the free oscillation equal to a_1, instead of the value 3·149, which has been used here as corresponding to unit central amplitude, the amplitudes of the forced vibration become:

$$\frac{a_1^2}{9{\cdot}916} C_i = + 0{\cdot}03812\, a_1^2 \text{ and } \frac{a_1^2}{9{\cdot}916} C_e = - 4{\cdot}617\, a_1^2.$$

5. The displacement can be represented by

$$r_0\,(a_1 \cos nt - a_2 \cos 2nt),$$

where a_1 and a_2 are positive.

The radial velocity is then given by the quantity

$$n\, r_0\,(a_1 \sin nt - 2\, a_2 \sin 2nt),$$

averaged over the stellar disc.

Values of $2\,a_2 : a_1$ for different values of a_1 are shown in Table 9.

TABLE 9.

a_1	$2a_2 : a_1$
·15	+ 1·3850
·12	1·1080
·10	·9234
·08	·7387
·07	·6464
·06	·5540
·05	·4617
·04	·3693
·03	·2770
·02	+ ·1847

It is seen that the effect in the radial velocity is quite considerable. For the equivalent spectroscopic orbit the distance from node to periastron is equal to 90°, which means that the rise to maximum is less rapid than the decline; the excentricity for a star like δ Cephei, with $a_1 = {\cdot}06$, is ·26.

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