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Some remarks on the fundamental systems of the General Catalogue and the Dritter Fundamentalkatalog, by *J. H. Oort.*

Summary.

In the first sections an explanation is sought for the striking differences existing between the systems of proper motion embodied in the FK3 and the *General Catalogue*. It is noted that, where the variation of systematic error with δ is concerned, the FK3 system is to be preferred for the motions in declination, while for the proper motions in right-ascension the GC has considerably greater weight than the FK3. For the, much smaller, but very important systematic terms depending on α the FK3 system is likely to be the more accurate of the two.

In the last two sections an attempt has been made to determine the true uncertainty of the fundamental systems. South of + 20° the mean error of the FK3 system of μ_δ is as high as $\pm$ $''^{a}\cdot0027$; north of + 20° it decreases to $\pm$ $''^{a}\cdot0015$. These values apply to the systematic errors depending upon δ; for the terms depending upon α the mean error is only $\pm$ $''^{a}\cdot00035$. From the motions of faint stars (*B.A.N.* No. 298, Table 3*b*) and of early-type stars (BLAAUW, unpublished) indications have been found, however, that, notwithstanding its large inherent uncertainty, the GC system of μ_δ happens to be nearly correct north of −20°, and that of the FK3 south of this limit; at least if we may rely on the assumptions concerning the apex on which these results are based.

For most of the sky the system of right-ascensions, at least that of the GC, is much more reliable than that of the declinations. If we leave out the uncertainty in the motion of the equinox, the mean error depending on δ is $\pm$ $''^{a}\cdot0007$ for the proper motions in arc of great circle at $\delta = \pm 30°$ and + 60°; only at − 60° it runs up to $\pm$ $''^{a}\cdot0033$. The uncertainty of the terms depending on α is again much less.

The mean errors of the systems of position for 1900 and 1950 have also been determined, and may be found in Tables 2, 4, 6 and the accompanying text.

Attention is drawn to the fact that the true weight of the equatorial system of declinations from catalogues corrected by observations of sun, moon and planets is about equal to that which can be derived from all good catalogues without such corrections. As these two determinations are systematically independent, a system having near the equator about twice the weight of the FK3 may be obtained by combining them.

General.

In many investigations in which proper motions are used, the choice between the two modern systems of proper motions, the *Albany General Catalogue* (GC) and the *Dritter Fundamentalkatalog des Berliner Astronomischen Jahrbuchs* (FK3), will be of importance. There are large systematic differences between the proper motions in the two fundamental catalogues, as may be seen from the comparison given in the GC (**1**, p. 330) or in that published by KOPFF [1]. Table 1 below has been condensed from Tables 4 and 7 of the latter publication.

TABLE 1.

Systematic differences FK3−GC [2]
(unit $''\cdot0001$ per year)

δ	$\mu_\alpha \cos\delta$	μ_δ	α	$\mu_\alpha \cos\delta$	μ_δ
			h		
+ 83° to + 90°	− 1	− 5	0·5	+ 15	+ 5
+ 68 „ + 82	− 32	+ 11	2·5	+ 1	− 6
+ 53 „ + 67	− 38	+ 9	4·5	− 6	− 9
+ 38 „ + 52	− 16	+ 26	6·5	− 13	− 14
+ 23 „ + 37	− 20	+ 23	8·5	− 14	− 19
+ 8 „ + 22	− 12	+ 33	10·5	− 3	− 15
− 7 „ + 7	− 5	+ 16	12·5	+ 2	+ 1
− 8 „ − 22	+ 3	+ 5	14·5	+ 7	+ 11
− 23 „ − 37	+ 20	+ 34	16·5	+ 4	+ 17
− 38 „ − 52	+ 10	+ 54	18·5	− 3	+ 9
− 53 „ − 67	+ 14	+ 68	20·5	− 3	+ 5
− 68 „ − 82	+ 21	+ 44	22·5	+ 11	+ 9
− 83 „ − 90	− 22	+ 20			

The differences FK3−GC depending on δ are particularly large. The two systems are based on the same observations; the differences must therefore be

[1] *A.N.* **269**, 160, 1939; *Mitt. Coppernicus-Inst.* **5**, No. 4.

[2] In the articles of this Bulletin we shall follow the practice usually adopted in statistical astronomy, and denote the proper motions in α by μ_α (or by $\mu_\alpha \cos\delta$ when reduced to arc of great circle), those in δ by μ_δ.

In computing the last two columns of the table systematic differences depending on δ had first been eliminated. The total differences FK3−GC in $\mu_\alpha \cos\delta$ are therefore obtained by adding together columns 2 and 5, and similarly for μ_δ.

due to the systematic corrections applied to the original data, and to the weights with which they were combined.

I have recently had to study the relative merits of the processes of formation of the two catalogues. As the experience obtained may possibly be of some help to others who have to make a choice, I am presenting in the following note some of the considerations to which I have been led.

The proper motions in declination.

The determination of absolute declinations has always been, and still is, a great stumbling block in positional astronomy, because it involves the measurement of large angles by means of meridian circles, which are influenced by unknown flexures and refractions. Even among good modern catalogues systematic differences up to 1″ are found near the equator.

The astronomers constructing the FK3 have tried to overcome these difficulties by determining the error of a catalogue in the vicinity of the equator from observations of the sun (and sometimes of moon and planets) according to the principle used by NEWCOMB in composing his fundamental catalogue. A new difficulty is introduced by this procedure, because the instrumental errors for the solar observations, made in mid-day, cannot be assumed to be the same as those for the night observations, while eventual personal errors in the observations of the sun may also enter. The difference between day and night observations can be determined, however, and has been eliminated from many series of observations. HEINEMANN [1]) has directed attention to the important fact that, for catalogues of the present century, the equator points determined by means of solar observations are more accordant than those from the stellar observations alone. For the equatorial zones of the 13 catalogues in this period which can be reduced with the aid of observations of sun and planets the average systematic deviation from the FK3 is ± ″·11 when they are reduced with the aid of the solar observations, while it is ± ″·47 for the catalogues as published. The advantage of the reduction with the aid of the sun seems striking; as the number of catalogues is still small, it will be of importance to see if it is confirmed by new series of observations. For the catalogues observed before 1900 there appears to be no such gain in accordance by using solar observations. As, moreover, the restriction to catalogues with solar observations diminishes materially the number which can be used, the accuracy of the system of proper motions in declination near the equator as determined from solar observations does not come out greater than that of the system determined from all good catalogues not corrected by positions of the sun (see below). In this respect, therefore, there seems to be no reason to prefer the FK3 system above that of the GC, which latter makes no use of the sun.

There is, however, a second aspect. In order to obtain a system which is the same for all stars, and which avoids jumps at certain declinations, it is necessary to apply empirical corrections to the various catalogues used. The exact determination of these empirical corrections for the FK3 system has been very complicated, but what it comes to is roughly this: The positions of equatorial stars in the catalogues corrected by means of solar observations were compared to those of a provisional system, for which AUWERS' *Neuer Fundamentalkatalog* (NFK) was adopted; from the systematic differences so obtained, average corrections were computed to the positions and proper motions of the NFK, which corrections defined the new FK3 system. Catalogues without accompanying solar observations played no role in fixing this new system near the equator.

The systematic corrections at the equator being determined, the errors at other declinations were derived by means of the working hypothesis that the systematic error varies linearly from the given value at the equator to zero values at the poles. This rather reasonable assumption is, on the whole, supported by the direct evidence.

Because of the rather intricate process of smoothing out the systematic differences between the individual catalogues used for the FK3, it is not easy to see, or to state briefly, with what relative weights these catalogues have entered into the equator points and other declinations of the final system. A priori weights, as used in the GC, are to be preferred, in my opinion; these have also the advantage of greater clarity.

In the *General Catalogue* no use has been made of observations of sun or planets; all available good catalogues have been combined for fixing the system. The systematic error of a northern catalogue was determined by comparing it with several southern catalogues of not too different epochs and by computing empirical corrections to the constants of refraction used, such as to bring northern and southern catalogues into agreement. The correction to be applied to the declinations of the northern catalogue was thus of the form $-(k \operatorname{ctg} z + k \operatorname{ctg} \varphi)$, in which z is the zenith distance to the south and φ the latitude [1]). Similar corrections had simultaneously to be determined for each southern catalogue. The average coefficients k are tabulated in Table 21 of the

[1]) *A.N.* **241**, 145, 1931; *Mitt. Astr. Rechen-Inst.* **2**, No. 14.

[1]) It was not implied that the systematic errors were wholly due to errors in the refraction; the above form was adopted as a plausible one for all errors depending upon z.

first volume of the GC. I feel two objections against this procedure. In the first place, the determination of the quantities k depends largely upon those regions where the coefficients of k are largest, i.e. at large zenith distances. Usually these extreme zenith distances will be particularly uncertain, and therefore ill suited for giving a good picture of the systematic errors of the whole catalogue. It is also questionable whether, if the systematic errors are rather due to flexure than to erroneous refractions, they may not vary with z in a somewhat different manner, so that forcing a tg z-correction would be dangerous. The second drawback of the GC system of declinations is that, in determining the systematic corrections, the total of the southern catalogues necessarily receives a weight equal to the total of the northern ones. Now there cannot be any doubt that, especially in homogeneity, but likewise in extent and quality, the series of southern catalogues is far inferior to that of northern ones. It is principally for this reason that, I think, preference should be given to the declination system of the FK3 [1]). For the period before 1900 this latter rests almost exclusively on equator points determined from northern catalogues; for the period after 1900, where in the FK3 the southern catalogues have received a high weight in determining the correction at the equator, there is no appreciable difference between the north and south catalogues used, so that the weights are irrelevant.

In the introduction to the GC an interesting attempt is shown, instigated, I believe, by KAPTEYN (c.f. H. RAYMOND, *A.J.* **36**, 136, 1925) to find the systematic errors in the proper motions in declination by determining them separately from three long series of observations made at the same observatory and, in two of the series, with the same instrument. As shown in Table 22 of that introduction, the agreement between the motions derived from these "three series" and those derived in the manner described above is quite satisfactory; much better than that between the GC and the FK3. In the north, the "series" may be considered to give a significant support to the GC system, but in the south – just where the largest differences between GC and FK3 occur – little significance can be attached to the solution from the "series", which in that part do not fulfill the necessary requirement of homogeneity, the Cape series in question resting on three different instruments.

As might have been expected, the variation of the systematic differences in the declination motions with right-ascension is less serious. The Albany astronomers have taken over the system derived by LEWIS BOSS for his *Preliminary General Catalogue*. The system of the FK3 has been newly determined with great care, and it is probable that the variation of systematic error with α has been better eliminated from this system than from the GC.

The proper motions in right-ascension.

The second important difference between the GC and the FK3 is that shown by the proper motions in right-ascension if these are arranged according to declination; as measured in arc of great circle there is a range of no less than $^{s}/_{a}\cdot 006$ in the systematic differences between the two catalogues. There is a considerable difference in the methods by which the systematic corrections of the proper motions were determined. Whereas in the GC they were derived in a straightforward manner from a combination with proper weights of all catalogues which might be expected to have a fundamental and reliable character as regards the determination of right-ascensions for different declinations, the proper motion corrections for the FK3 rest exclusively on series determined with the same instrument. Now it is, of course, an advantage – as KAHRSTEDT argues [1]) – that in this way instrumental errors are largely eliminated; but, in particular for earlier catalogues, there are also quite large personal errors, which are not eliminated, and by which most of the advantage of the restriction to the same instrument disappears again. The slight remaining gain appears to be greatly outweighed by the considerable increase of time interval which is obtained if the restriction to series of the same instrument is dropped. This argument applies with enhanced strength to the southern hemisphere, where three out of the four badly discordant series used by KAHRSTEDT extend over only 14 to 20 years, and one over 46 years. Plots of the residuals of various first-class catalogues show that, even in the north, the catalogues of one and the same series are not more accordant than all good catalogues together, so that there can be no real gain in the restriction to these series. The uncertainty of the individual series is also well illustrated by the systematic differences between them [2]). Even for the two best ones, Greenwich and Pulkovo, the difference runs up to $^{s}/_{a}\cdot 007$ at $\delta = +30°$, and $^{s}/_{a}\cdot 006$ at $\delta = +60°$, both measured in arc of great circle. For these reasons I believe that the GC system is to be preferred in this case, particularly in the south.

[1]) According to a private communication Mr BLAAUW at Groningen has recently derived rather convincing evidence that the southern μ_δ system of the FK3 is more nearly correct than that of the GC. This result, which is in line with the expectation formulated above, was found from a detailed study of the proper motions of O and B stars and δ Cephei variables. The investigation is soon to appear in the *B.A.N.*

[1]) *A.N.* **235**, 369, 1929; *Mitt. Astr. Rechen-Inst.* **2**, No. 5.

[2]) C.f. the graphs by KAHRSTEDT, *A.N.* **235**, 369, 1929 (Tafel 1) and *A.N.* **240**, 385, 1930 (Tafel 2).

The variation of the errors in μ_α with α is of less importance. Here the GC has again copied the PGC system, while in the FK3 new systematic corrections were applied. The FK3 may be considered as perhaps slightly preferable for these terms.

The uncertainty of the fundamental system in declination.

Because of the empirical systematic corrections applied to the catalogues it is impossible for the user of the GC or the FK3 to form a judgment on the real uncertainty of the fundamental system. In order to obtain an impression of this we have made a new determination of the system directly from the catalogues used in the construction of the FK3, uncorrected for systematic errors. For the declinations this solution was based on the systematic differences between the individual catalogues used in the FK3 and the FK3 itself. These differences were formed from a publication by HEINEMANN and Frl. NOWACKI [1]) by taking the differences of the numbers given in their Table 1 minus those in Table 22 [2]); in order to restrict the number of solutions to be made, the differences were averaged over 10° intervals of declination. From the numbers so obtained corrections to the FK3 declinations at mean epoch, and to the FK3 proper motions were calculated, together with their mean errors. These latter were computed from the residuals left by the various uncorrected catalogues. The weights of the catalogues were taken in accordance with those adopted in Table 21 of the first volume of the GC, except that their range was somewhat reduced by assigning a weight 5 to all catalogues for which weights 5 or larger were given in the table mentioned; in the interval $-12°{\cdot}5$ to $-20°$ the normal weights of the northern catalogues were halved, while they were still more reduced for catalogues not extending over the whole interval. In the part of the sky north of $+20°$, in which the declinations are better than in the equatorial and southern regions, solutions for only one zone, from $+40°$ to $+47°{\cdot}5$, were carried out.

The results are given in Table 2, where under δ_{1950} are shown the corrections to the FK3 for 1950 as indicated by the present solutions, and under μ_δ the corrections to the FK3 motions in declination; n is the number of catalogues employed. Discussion of these corrections themselves is deferred to the following article. For our present purpose we are interested in the mean errors, which should give an idea of the true uncertainty of our fundamental systems. For the 1950 declinations south of $+20°$ these are about $\pm\,''{\cdot}15$; the declinations for 1900 (which date practically co-incides with the mean epoch) are more accurate, having an average mean error of $\pm\,''{\cdot}06$ in the same region. The mean error of the system of proper motions amounts to no less than $\pm\,^{s}{\cdot}003$ in the equatorial belt, and is therefore rather disquieting. The seriousness of this uncertainty may be further illustrated by comparing a solution based on all northern catalogues with one made from all southern catalogues. The resulting corrections to the FK3 system of proper motions are shown in Table 3.

TABLE 2.

Corrections to the FK3 and mean errors of the system of declinations and of proper motions

δ	n	δ_{1950}	m.e.	μ_δ	m.e.
		$''$	$''$	^{s}	^{s}
$+40°$ to $+47°{\cdot}5$	30	$+\,''{\cdot}14$	$\pm\,''{\cdot}08$	$+\,^{s}{\cdot}0038$	$\pm\,^{s}{\cdot}0015$
$+10$ „ $+17°{\cdot}5$	49	$+\,''{\cdot}04$	$\pm\,''{\cdot}15$	$+\,^{s}{\cdot}0012$	$\pm\,^{s}{\cdot}0027$
0 „ $+7°{\cdot}5$	49	$+\,''{\cdot}03$	$\pm\,''{\cdot}15$	$+\,^{s}{\cdot}0016$	$\pm\,^{s}{\cdot}0028$
-10 „ $-2°{\cdot}5$	49	$+\,''{\cdot}11$	$\pm\,''{\cdot}16$	$+\,^{s}{\cdot}0026$	$\pm\,^{s}{\cdot}0029$
-20 „ $-12°{\cdot}5$	46	$+\,''{\cdot}13$	$\pm\,''{\cdot}18$	$+\,^{s}{\cdot}0020$	$\pm\,^{s}{\cdot}0033$
-30 „ $-22°{\cdot}5$	14	$-\,''{\cdot}43$	$\pm\,''{\cdot}16$	$-\,^{s}{\cdot}0060$	$\pm\,^{s}{\cdot}0029$
-40 „ $-32°{\cdot}5$	13	$-\,''{\cdot}32$	$\pm\,''{\cdot}12$	$-\,^{s}{\cdot}0036$	$\pm\,^{s}{\cdot}0022$
-50 „ $-42°{\cdot}5$	13	$-\,''{\cdot}29$	$\pm\,''{\cdot}16$	$-\,^{s}{\cdot}0042$	$\pm\,^{s}{\cdot}0029$
-60 „ $-52°{\cdot}5$	13	$-\,''{\cdot}25$	$\pm\,''{\cdot}12$	$-\,^{s}{\cdot}0039$	$\pm\,^{s}{\cdot}0023$
-70 „ $-62°{\cdot}5$	13	$-\,''{\cdot}17$	$\pm\,''{\cdot}14$	$-\,^{s}{\cdot}0030$	$\pm\,^{s}{\cdot}0027$
-80 „ $-72°{\cdot}5$	13	$-\,''{\cdot}07$	$\pm\,''{\cdot}13$	$-\,^{s}{\cdot}0026$	$\pm\,^{s}{\cdot}0023$

TABLE 3.

Differences between systems of proper motion in declination from northern and southern catalogues

δ	north catalogues	south catalogues
	^{s}	^{s}
$+10°$ to $+17°{\cdot}5$	$+\,^{s}{\cdot}0037$	$-\,^{s}{\cdot}0083$
0 „ $+7°{\cdot}5$	$+\,^{s}{\cdot}0041$	$-\,^{s}{\cdot}0082$
-10 „ $-2°{\cdot}5$	$+\,^{s}{\cdot}0054$	$-\,^{s}{\cdot}0080$
-20 „ $-12°{\cdot}5$	$+\,^{s}{\cdot}0098$	$-\,^{s}{\cdot}0077$

The differences are so enormous that one might well question whether the present system of proper motions can be of any use for determining such minute quantities as the rotation of the galaxy. That it is nevertheless possible to obtain a fair estimate of the constant B (which amounts to only $^{s}{\cdot}002$) is due to the circumstance that only rather special types of systematic errors can affect the determination of this constant (see p. 426). Moreover, the FK3 system rests almost entirely on the north catalogues alone, and these are considerably more certain than the southern ones; also, the system is supported by the solar observations used in the FK3, as well as by a careful discussion of equator points from sun, moon and

[1]) *A.N.* **252**, 353, 1934; *Mitt. Astr. Rechen-Inst.* **3**, No. 19.

[2]) The "definitive errors" so obtained were considered to represent the differences catalogue minus FK3. It would have been more satisfactory to have derived these differences from direct comparisons with the FK3, but this would have increased the labour beyond what seemed appropriate in view of the schematic character of the present note. However, the most essential part of the results has been completely and independently checked by a solution based on the systematic corrections to various catalogues as given in Volume I, Appendix 3, of the *General Catalogue* (see Table 5).

planets, given by MORGAN [1]), which indicates a correction of only about $-$ ″/ᵃ·001 to the declination motions of the FK3 near the equator.

Nevertheless, it remains evident that large systematic errors are still to be feared even in the best system of declinations which can at present be constructed, and that therefore such undertakings as the Numerov-Brouwer plan of minor planet observations [2]), and the observation of azimuths from an equatorial observatory [3]), both aiming at an improvement of the absolute declinations, are urgent.

A solution similar to that given in Table 2 has been made from the catalogues of which the errors near the equator have been determined with the aid of observations of the sun or planets. The individual errors of the various catalogues were taken from HEINEMANN's discussions [4]), without any smoothing. These errors have been subtracted from the differences "catalogue minus FK3" used above and new solutions have been made for the systematic corrections to the FK3 from the corrected differences so obtained. As the corrections apply only to the equatorial belt the solutions were confined to the zones between $\pm$ 20° declination. The weights were left unchanged, but, following HEINEMANN, Odessa 1910 was excluded. The results are given in Table 4; n is again the number of catalogues.

TABLE 4.

Corrections to the FK3 and mean errors of the system as defined by observations of sun, moon, and planets

δ	n	δ_{1950}	m.e.	μ_δ	m.e.
		″	″	″/ᵃ	″/ᵃ
+ 10° to + 17°5	28	+ ·11	± ·14	+ ·0015	± ·0026
0 „ + 7°5	28	+ ·12	± ·13	+ 19	± 24
− 10 „ − 2°5	28	+ ·20	± ·14	+ 30	± 25
− 20 „ − 12°5	26	+ ·33	± ·18	+ 40	± 31

It appears that the mean errors come out sensibly the same as in Table 2; the diminution of the number of catalogues is just counterbalanced by the gain in systematic accuracy resulting from the use of the solar observations. The systems defined by the two solutions are entirely independent. A fundamental system of approximately twice the weight of that of the FK3 in the equatorial zone may therefore be obtained if the average of the systems with and without solar corrections is taken, instead of only that with the solar corrections as in the FK3. This matter will be reconsidered in the next article.

The systematic difference between the solution of Table 4 and the FK3 is somewhat puzzling, as both rest on the same principle and the same material. I regret that, not disposing of all data necessary for following each step of the very complicated way in which the final FK3 system has been derived, I have not been able to trace exactly the cause of the difference. So far as I can judge, however, about half of it may be explained by differences in weighing.

In order to ascertain that the results derived for the corrections to the GC and FK3 systems are essentially correct, an independent check was made, based on the systematic corrections $\Delta\delta_\delta$ for δ = 0° given in Vol. I of the *General Catalogue*. The solution was confined to the same 49 catalogues that were used in the construction of the FK3, except Kasan 1926, for which the GC gives no systematic corrections; for Heidelberg 1926 and Nikolajew 1926, also omitted in the GC, the corrections were determined through the intermediary of the FK3 as + ″·60 and − 1″·35 respectively. From the catalogues with solar observations a second solution was made, corresponding to that given in Table 4 for the FK3; the systematic reductions to the sun were taken from HEINEMANN.

In Table 5 the results of these solutions are compared to those found in Tables 2 and 4 for the two zones nearest to the equator, after application of the reductions from FK3 to GC as given in Table 1.

TABLE 5.

Corrections to the GC for δ = 0°
(comparison of results directly from the GC, with those derived from Tables 2 and 4)

	From GC		From Tab. 2 and 4	
	δ_{1900}	μ_δ	δ_{1900}	μ_δ
	″	″/ᵃ	″	″/ᵃ
49 catalogues	+ ·05	+ ·0036	+ ·03	+ ·0037
28 cat.'s with sun's obs.'s	+ ·13	+ ·0036	+ ·10	+ ·0040

The agreement is entirely satisfactory.

Another control was made by adopting for the systematic differences FK3—NFK instead of the numbers of Table 22, *A.N.* **252**, 353, used above, the smoothed difference given by Frl. NOWACKI [1]). A solution for δ = 0° from the catalogues corrected by means of solar observations gave results identical with those of Table 4 for δ_{1900}, while for μ_δ the corrections came out ″/ᵃ·0007 higher.

The mean errors of the system as computed above refer to errors of the type $\Delta\delta_\delta$ and $\Delta(\mu_\delta)_\delta$ [2]). The errors depending upon α, $\Delta\delta_\alpha$ and $\Delta(\mu_\delta)_\alpha$ [2]), are of a much

[1]) *A.J.* **42**, 149, 1933.
[2]) C.f. *A.J.*, **44**, 57, 1935.
[3]) HINS and VAN HERK, *Leiden Annals* **18**, Pt 1, 1938.
[4]) *A.N.* **241**, 165, 1931 (Tab. 19) and **249**, 301, 1933 (Tab. 4); *Mitt. Astr. Rechen-Inst.* **2**, No. 14 and **3**, No. 11.

[1]) *A.N.* **255**, 301, 1935; *Mitt. Astr. Rechen-Inst.* **3**, No. 21, Tab. 10.
[2]) Systematic errors or differences found from an arrangement according to δ will be denoted by $\Delta\delta_\delta$ and $\Delta(\mu_\delta)_\delta$. The variation with α of the errors remaining after the part depending on δ has been eliminated will be designated as $\Delta\delta_\alpha$ and $\Delta(\mu_\delta)_\alpha$. Similar notations will be used for the errors in the α co-ordinates.

smaller order. As shown on p. 426 of this Bulletin, the coefficient of either $\sin\alpha$ or $\cos\alpha$ in the proper motions has a "mean systematic error" of only $\pm$ $^{s/a}$·00036. We can thus trust the system of proper motions in declination nearly ten times better for terms depending only upon α than for those depending upon δ. This is of considerable practical consequence, in as much as for this reason the constant B of galactic rotation, and the constant Δn connected with the precession, can be found with much greater precision than, for instance, the secular parallaxes (see p. 426).

The uncertainty of the fundamental system in right-ascension.

The mean errors of the system of right-ascensions were determined with the aid of the systematic differences $\Delta\alpha_\delta$ between the individual catalogues and the PGC, as published in Table 7 of the first volume of the *General Catalogue*. From these differences corrections to the PGC for 1950 and to the PGC proper motions were solved for five declinations. The results are given in Table 6 below. They are valid for the GC system; the errors of the FK3 will be rather higher. The mean errors were again determined from the systematic residuals of the various catalogues. Weights were taken from Table 7 of the GC, except that those greater than 5 were again reduced to 5; values marked with colons were given half weight. Separate solutions were made from the "primary" catalogues in Table 7A alone, and from all catalogues of Tables 7A, B and C combined. The numbers of catalogues used are shown under n. For comparison, the systematic corrections to the proper motions as derived in the GC (Table 8) are also given (in the columns marked GC); the differences with the values derived here are probably due to the reduction of the weights of the best catalogues in the present solution.

TABLE 6.

Corrections to the PGC system of right-ascensions, and mean errors of the GC system

δ	Primary catalogues only							All catalogues						
	n	α_{1950}	m.e.	μ_α	m.e.	GC		n	α_{1950}	m.e.	μ_α	m.e.	GC	
		s	s	s/a	s/a	s/a			s	s	s/a	s/a	s/a	
+ 60°	28	− ·035	± ·010	− ·000 43	± ·000 15	− ·000 46		64	− ·033	± ·008	− ·000 35	± ·000 11	− ·000 39	
+ 30°	41	− 13	± 4	− 16	± 6	+ 2		88	− 10	± 3	− 10	± 4	+ 2	
0°	42	+ 1	± 2	+ 2	± 3	+ 4		89	+ 2	± 1	+ 1	± 2	+ 2	
− 30°	30	+ 2	± 5	+ 1	± 10	0		58	+ 5	± 4	+ 9	± 7	+ 9	
− 60°	12	+ 45	± 12	+ 44	± 24	+ 64		19	+ 51	± 13	+ 63	± 22	+ 72	

It should be noted that the effect of an error in the equinox is not included in the above determination of the mean errors of the system. For a discussion of the uncertainty of the motion of the equinox we may refer to several previous investigations [1]). Abstracting from this uncertainty the mean errors are satisfactorily small [2]) except below − 30°. The mean errors of the positions at the average epoch (about 1895), which are not given in the table, are about 2/5 of those at 1950. For the zones at ± 30° and + 60° the mean errors of the system of proper motions, reduced to arc of great circle, average only ± $^{s/a}$·0007. This is considerably smaller than the systematic differences between FK3 and GC, so that the application of a correction to the FK3 proper motions in right-ascension (c.f. p. 419) would seem to improve this system considerably. The systematic differences required may be taken from the second column of Table 1.

As in the case of the declinations, the systematic errors depending upon α are much smaller than those depending upon δ, which have been considered above. No numerical determination concerning the terms depending on α has been made.

The circumstance that the mean error of the system of μ_α is between two and four times smaller than that of μ_δ might make it desirable in special cases to base determinations of apices and parallaxes of distant objects principally on μ_α, in conjunction with the projection of the sun's velocity on the equator for transforming them into ordinary mean parallaxes. In the case of objects concentrated to the galaxy errors of the type $\Delta(\mu_\alpha)_\delta$ enter into such a solution, which should therefore be based on the GC system. For stars evenly distributed over the sky errors of this type are eliminated. In this case the FK3 system is to be preferred; the remaining errors of the μ_α system are then probably no more than one tenth of those in the μ_δ system, so that, as far as systematic errors are concerned, the weight of the determination from μ_α becomes overwhelmingly greater.

[1]) C.f. for instance *B.A.N.* **8**, 149, 1937; R. E. WILSON and H. RAYMOND, *A.J.* **47**, 49, 1938; GLIESE, *A.N.* **270**, 127, 1940, *Mitt. Coppernicus-Inst.* **5**, No. 9.

[2]) It should be noted that the smallness of the mean errors near the equator is largely artificial, and due to the reductions to NEWCOMB's equinox which have generally been applied to the observations before they were published.