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Quasi-periodicity in deep redshift surveys

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SUMMARY

The recent result by Broadhurst *et al.*, showing a striking, nearly periodic, galaxy redshift distribution in a narrow pencil-beam survey, is explained within the Voronoi cellular model of clustering of galaxies.

Galaxies, whose luminosities are selected from a Schechter luminosity function, are placed randomly within the walls of this cellular model. Narrow and deep, magnitude-limited, pencil-beam surveys through these structures are simulated. Some 15 per cent of these beams show the observed regular pattern, with a spacing between the peaks of the order of 105 h^{-1}–150 h^{-1} Mpc, but most pencil-beams show peaks in the redshift distribution without periodicity, so we may conclude that, even within a cellular universe, periodicity is not a common phenomena.

1 INTRODUCTION

One way to study the clustering of galaxies on the largest scales is to determine the redshifts of all galaxies in a very small solid angle on the sky to a great depth. This is called a pencil-beam redshift survey, and is essentially a one-dimensional redshift survey. Combining several of these surveys around the North- and South Galactic Pole, Broadhurst *et al.* (1990) recently found a surprising clustering of galaxy redshifts in high peaks which were separated by regular spacings. This was confirmed by the pair-distribution and power-spectrum analysis. They found strong indications for the existence of a 128 h^{-1} Mpc period in the redshift distribution.

The positive news from their result is that we seem to have found an upper limit to the size of structures in the Universe, so that at last there is evidence for a scale of convergence. Another more striking point is the existence of structures of a size exceeding 100 h^{-1} Mpc, which are not expected in the known models of structure formation by gravitational instability. Most remarkable is the existence of a (quasi-)periodicity in the redshift distribution, which cannot be understood if the galaxy distribution behaves like that throughout the Universe, as was pointed out by Davis (1990).

Although at first sight the Broadhurst *et al.* result is puzzling, their results can be explained within the context of an elegant model for cellular structures in the distribution of galaxies, called Voronoi foam. We take $H_0 = 100$ km s^{-1} Mpc^{-1}.

2 THE VORONOI FOAM

The Voronoi foam is the simplest statistical model for a distribution of galaxies that is confined to sheets, filaments and clusters surrounding voids. It is physically motivated by the notion that structure formation under the influence of gravity can be described by looking to the evolution of underdense regions in the mass-density, which will expand with respect to the background and become more and more spherical (Icke 1984). Making the approximation that the excess expansion from each underdense region is nearly equal, one gets a geometrical skeleton of the ultimate mass distribution which is known mathematically as Voronoi tessellation. This tessellation is completely determined by the positions of the underdense region's 'expansion centres'. A Voronoi tessellation is the tiling of (comoving) space into cells, each cell consisting of that part of (comoving) space closer to its expansion centre than to any of the other expansion centres. Each cell is bounded by the bisecting planes between its expansion centre and its neighbouring expansion centres. These planes are identified with the 'walls' in the galaxy distribution. Each plane is bounded by edges, the lines where three planes meet. The density in these edges will be larger than in the planes because matter from three voids will gather in the filaments, while the planes are at the intersection of matter streaming from two voids. These edges are the filaments in the galaxy clustering. Finally, all matter will tend to stream towards the vertices of the tessellation, the highest density regions, where matter from four neighbouring voids will meet. These vertices can be identified with the Abell clusters found in the galaxy distribution.

Assuming the galaxies to populate the walls of the Voronoi cells, the Voronoi model qualitatively reproduces the observed pattern of cells and sheets in the observed galaxy distribution. Although the model cannot say much about the pattern of the galaxy distribution on smaller scales, within the sheets, it can be considered as a useful prescription for the spatial distribution of the sheets themselves. In the Voronoi model the 'Great Wall' discovered by Geller & Huchra (1989) will arise because adjacent Voronoi walls form large pleated sheets, and so is expected to be a rather common phenomenon.

The clustering of the vertices (van de Weygaert & Icke 1989) has a power-law correlation function ξ_{vv} with slope -2.0 and a scale of 32 h^{-1} Mpc where $\xi=1$ (when normalized with the mean distance between Abell clusters with $R \geqslant 1$ of 55 h^{-1} Mpc). This value is just within the bounds of the cluster–cluster correlation function determined by Bahcall & Soneira (1983). At the same time this normalizes the (comoving) distances within our cellular model. Note that, because a Voronoi foam consisting of 1000 cells gives about 6750 vertices, this means the mean distance between cell centres is about 104 h^{-1} Mpc, which is far larger than the voids seen in the slices of the CfA group (de Lapparent, Geller & Huchra 1986; Geller & Huchra 1989). These structures on the scale of the cluster distribution do not, however, exclude the existence of structure on smaller scales, like smaller voids within gigantic ones, although the word void should be used carefully. The best description of a void in this context may be a volume bounded by dense sheet-like regions, which can be nested hierarchically in exactly the same way as hierarchical galaxy clustering.

Since we are only looking out to $z \simeq 0.5$, this description is probably reasonable all over the simulation volume. The reason for this is that the curvature term is not yet dominant, and that structure on those scales is 'advanced linear', meaning that the imprint of structure formation is visible already, but not in a non-linear way.

3 THE SIMULATION

Our geometrical model is not restricted by the resolution or number of particles in the simulation. Therefore a cellular structure can be generated over a part of space (in three dimensions!) beyond the reach of any N-body experiment. This makes it possible to look at the results of galaxy clustering in very deep surveys through these cellular structures.

We set a Voronoi tessellation consisting of 2500 cells (periodic boundary conditions) generated by 2500 Poisson-distributed expansion centres, in a box with side ratios of 2:1:1. Normalization of the distances in the box with respect to the intervertex separation ($\approx 16\,875$ vertices) means the box has comoving sides of 2239 $h^{-1} \times 1120$ $h^{-1} \times 1120$ h^{-1} Mpc^3. To simulate the Broadhurst *et al.* results, two opposite beams are taken through this cellular structure, both with a survey redshift limit of $z=0.5$, which in an $\Omega_0=1$ universe corresponds to a comoving distance of 1101 h^{-1} Mpc. In order to prevent artefacts due to the periodic boundary conditions, the beams were oriented within 25° of the central axis. The code is written in such a way that the position of the observer can be chosen anywhere within the box. Another parameter is the opening angle of the observation cones, taken to be 20 arcmin in the simulations presented here, which is approximately the size of the observation cones of Broadhurst *et al.*, although they use a compilation of several neighbouring beams.

The first step in the simulation procedure is purely geometrical, and uses the advantage of the geometrical Voronoi code. Because all matter is assumed to be situated within the walls, the code starts by determining in a completely geometrical way which Voronoi walls can be found within the observation cone. Note that this is not trivial, because there are many walls (depending on the opening angle and extent of the cone) which have a part of their area inside the cone, although the central axis of the cone does not pass through them. This part of the code is by far the most complicated one.

The next step is the filling-in of the walls by galaxies, bringing up the issues of their spatial and luminosity distribution. First we have to specify the spatial distribution of the galaxies. The simplest assumption is that of a Poisson distribution of galaxies within the walls. Although observations show this not to be the case, in order to control the simulation it is better to use as few parameters as possible. In a later stage more complex distributions will be tested. The distribution perpendicular to the plane is assumed to be Gaussian with $\sigma=2.5$ h^{-1} Mpc, the value of σ being motivated by the fact that the 'Great Wall' seems to have a width of 5 h^{-1} Mpc (Geller & Huchra 1989).

Secondly, the number of galaxies within each particular wall has to be determined. Here it is assumed that all matter within the pyramid spanned by the expansion centre and the wall will ultimately stream towards the wall, due to the void expansion. The volume of this cone has to be multiplied by two because the matter streaming from the opposite side has also to be taken into account. Multiplying that volume by the average number density of galaxies gives the expected number of galaxies within the wall. The number of galaxies within the wall is then a random integer number drawn from a Poisson distribution with this expected number as its mean. These galaxies are then distributed within the wall according to the prescription discussed above. Notice that the number of galaxies in the wall is determined solely by the surface area of the wall and the distance of the expansion centre from the wall. This can be important, because in this way one expects that small walls will be missed from the redshift survey, as are large walls at a small distance from their expansion centre (their density contrast is just too small).

Finally we have to prescribe the kind of galaxies, in particular their luminosities. Only galaxies with absolute magnitudes between $M_{B_T}=-17$ and -22 are taken; fainter galaxies are only seen at small distances, and brighter ones are relatively rare. Their luminosity distribution is taken to be prescribed by the Schechter luminosity function for field galaxies as determined by Efstathiou, Ellis & Peterson (1988): $\alpha=-1.07$, $M^*_{B_T}=-19.68$ and $\phi^*=1.56\times10^{-2}$ h^3 Mpc^{-3}. The number density of galaxies used is then taken to be the number density of galaxies brighter than $M_{B_T}=-17$. The luminosity of each galaxy in the cone is taken by randomly sampling from the specified luminosity function.

Having decided upon both the spatial and luminosity distribution, and having the list of walls within the cone, the cone is filled up with galaxies. The comoving distance is translated to the redshift z, assuming $\Omega=1$, while the apparent luminosity is determined by the absolute luminosity and the redshift of the galaxy (we assume a K-correction of the form κz, with $\kappa=3$; Peebles 1980). The galaxy is in the redshift survey when its apparent magnitude is smaller than the magnitude limit of the survey. Our simulations have a limiting magnitude of $m_{B_T}=21.5$, as was taken in one part of the pencil beams by Broadhurst *et al.* (1990).

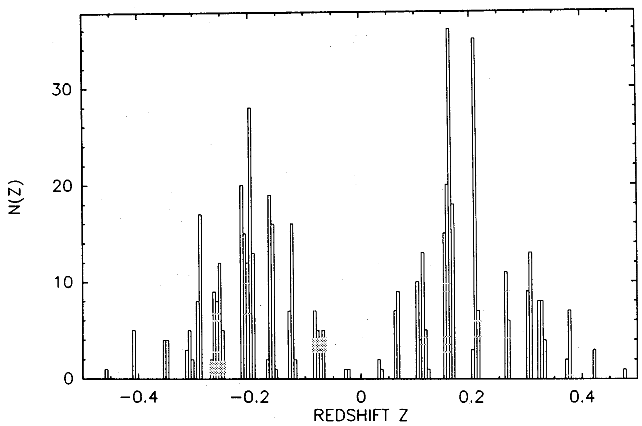

Figure 1. The redshift distribution for pencil-beam surveys taken in opposite directions through a three-dimensional Voronoi tessellation. For further description see text.

4 RESULTS

The final result, a distribution of number of galaxies against their redshift z is shown in Fig. 1. Positive redshift means the galaxy is in the cone pointing in one direction, while negative redshift means the galaxy is situated in the opposite pencil beam. The sample redshift distribution indeed shows a striking regularity. This regularity is confirmed by the corresponding pair number count shown in Fig. 2 and the power-spectrum analysis shown in Fig. 3.

Fig. 2 shows 13 peaks in the pair distribution out to a comoving separation of 1500 h^{-1} Mpc, with a very regular spacing in the order of 107 h^{-1} Mpc. This result thus confirms the possibility of getting a 'quasi-periodic' redshift distribution in cones taken through cellular structures. The wiggly 'large-scale' behaviour of the pair-counts in Fig. 2 is due to the geometry of the sample volume, two opposite cones of equal depth. This probably explains the qualitative difference with fig. 2(a) in Broadhurst *et al.* This geometrical effect can be taken out by normalizing the counts with respect to counts of Poissonian distributed galaxies inside the observation cone, using the same selection criteria. This leads to the correlation function of Fig. 2(b), which shows that the peaks all have roughly the same size.

Moreover, normalization of the scales of the cellular structure on the cluster distribution, as we did, gives a typical peak-to-peak distance in the order of 107 h^{-1} Mpc. This compares well with the observed regularity scale of 128 h^{-1} Mpc. The fact that the 107 h^{-1} Mpc scale seems to be a fundamental mode of the redshift data in Fig. 1 is confirmed by the power-spectrum analysis in Fig. 3. The resemblance of Fig. 3 to fig. 2(b) in Broadhurst *et al.* (1990) is remarkably good, except for the power at high frequencies, which is due to the fact that in our model we used a rather uniform distribution of galaxies on small scales.

There are several comments to be made. Of course, a result in accordance with the Broadhurst *et al.* result is shown here, but many more realizations were simulated. All cases show a spiky redshift distribution but only a small fraction show signs of regularity. This is not surprising in a cellular galaxy distribution such as the one we started from. On the basis of some 30 realizations, it is estimated that approximately 1 in every 6 to 7 show regularity. In the other cases, no periodicity is observed. This agrees roughly with the analytical estimate by Coles (1990). Note that the presence of regularity is independent of the normalization of the model. A more quantitative and better defined estimate, based on far more lines-of-sight, will be presented in a future paper, together with a study of the influence of the expansion centre distribution, as well as of the distribution of galaxies within the walls. The presence of regularity is influenced by the initial distribution of the expansion centres. First results from a tessellation based on correlated expansion centres showed that the chance of having regularity is substantially decreased.

The suitability of the model can be tested by determining the fraction of regular redshift distributions in pencil-beam surveys along far more, randomly chosen, directions. Better defined observational selection criteria, like the depth and geometry of the pencil beams, will facilitate a quantitative comparison.

Another point is that the spikes in Fig. 1 seem to be broader than the ones shown in Broadhurst *et al.* (1990). The width of these peaks depends on the angle between the wall and the central axis of the cone, narrow peaks resulting from a nearly face-on hit, while very broad peaks are seen when the central axis is oriented along the wall. Another important factor is the distribution of the galaxies within the walls. Numerical simulations show that matter will tend to move towards the edges (filaments) of the walls, thus causing the walls themselves to be more sparsely populated. This will cause smaller and narrower peaks. The effects will be investigated in a future paper.

5 CONCLUSIONS

We tested a full three-dimensional Monte Carlo study of clustering in a cone taken through a Voronoi foam. We may conclude that such a cellular distribution of galaxies indeed seems to be able to explain the puzzling observation of a quasi-periodic redshift distribution with a characteristic scale of the order of 128 h^{-1} Mpc. The model has only one free length-scale, the mean distance between clusters. So we know for certain that 107 h^{-1} Mpc is a scale intrinsic to the clustering model and not just a conspiracy of multiple parameter choices.

The advantage of such a Monte Carlo approach as opposed to studies based on lines-of-sight, as carried out by Coles (1990) and Kurki-Suonio, Mathews & Fuller (1990), is the possibility of taking into account the geometry of the cone. Effects due to the orientation of walls inside the cone with respect to the line-of-sight can be properly evaluated. Moreover, the study presented allows a direct visual comparison between the calculations and the observations, since the model correctly simulates the observational procedures.

An alternative approach is to use an N-body or adhesion-model approach to the clustering (e.g. Weinberg & Gunn 1990). This would have the advantage that the structure was dynamically self-consistent. However, only two-dimensional studies have the required length-scale range. The study of quasi-periodicity in two-dimensional pancake models by Buchert & Mo (1990) is an example of this. It might be

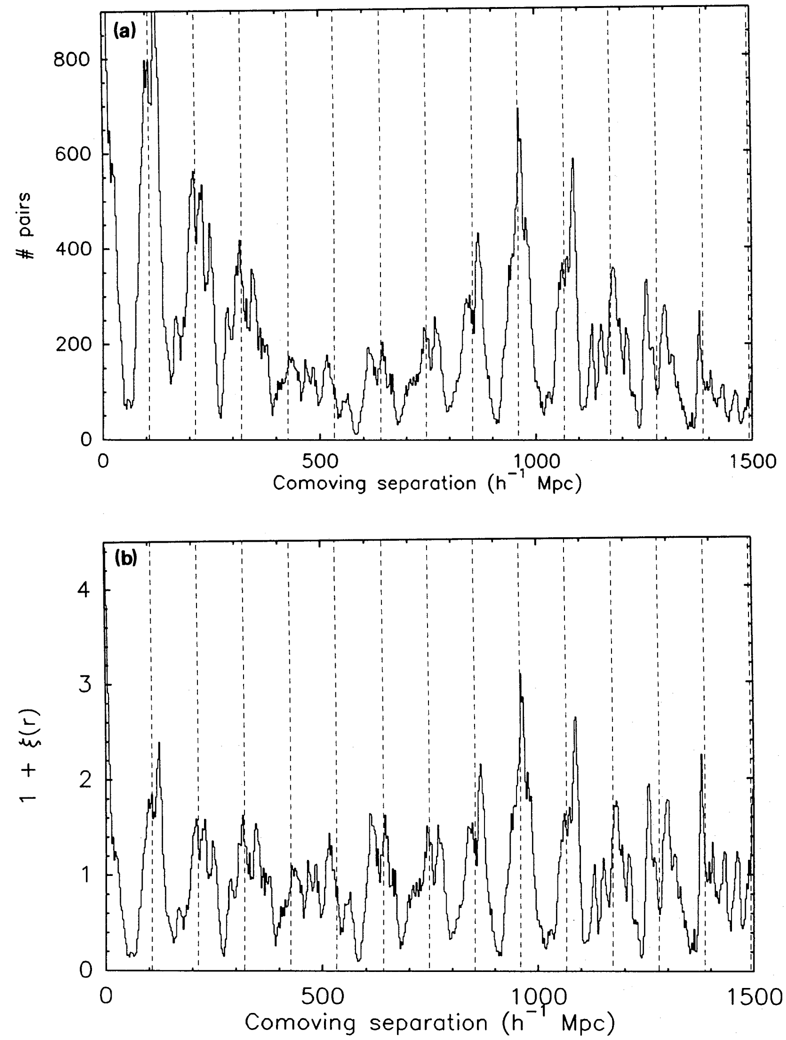

Figure 2. (a) The pair-count correlation diagram for all redshift data in Fig. 1. The dashed lines indicate scales as multiples of 107 h^{-1} Mpc. (b) The same as Fig. 2, but then normalized with respect to a sample of Poissonian-distributed galaxies within the same observation cone using the same selection criteria.

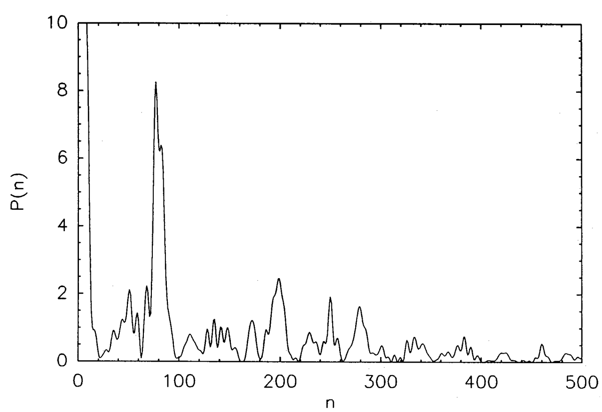

Figure 3. Power-spectrum analysis of the redshift data in Fig. 1, with $r(n) = 8000/n\ h^{-1}$ Mpc.

difficult to extrapolate conclusions from this two-dimensional approach to three dimensions, because we do not understand the phase-space restrictions arising out of the two-dimensionality of the flow.

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