BULLETIN OF THE ASTRONOMICAL INSTITUTES OF THE NETHERLANDS.

1939 March 15.

Volume VIII.

No. 317.

COMMUNICATION FROM THE OBSERVATORY AT LEIDEN.

Photovisual observations of V Puppis, by H. van Gent.

1. Introduction.

The variability of V Puppis (= Lacaille 3105 = 233 G Pup = HD 65818, sp. B1, B3, 07h55m4 - 48°58′, 1900), was discovered in 1885 by A. Stanley Williams 1) whilst on a voyage to Australia round the Cape by sailing vessel. The discovery of this fourth magnitude variable from aboard ship was followed by several measures of brightness by means of field glasses, using the star Lacaille 3069 as a comparison star. While the discoverer recognized the star at once as a variable of the eclipsing type, the period was still left indeterminate.

In 1895 E. C. Pickering ²) found the star to be a spectroscopic binary with double lines on objective prism plates taken by Solon I. Bailey at the Harvard Southern Station, Arequipa, Peru. The maximum separation of the lines is more than 600 km/sec. It is the fourth double line spectroscopic binary known, ζ UMa, β Aur and μ_1 Sco being discovered previous to it. Pickering thought the period to be 3^d·115.

In the meanwhile Alexander W. Roberts had started a long series of visual photometric observations with his $7\frac{1}{2}$ cm telescope at Lovedale, Cape of Good Hope, already as early as 1891, and continued these for many years. When discussing his 1899—1900 observations 3) he finds it impossible to satisfy them by Pickering's period and then derives a new one. This latter period, 1^{d} ·454476, even at present hardly needs any correction at all.

From 371 visual observations (not published in detail) Roberts derives the light curve of the variable and makes a first computation of the principal data of this system, assuming it to consist of two spherical bodies of equal size which present uniformly illuminated discs and which revolve in a circular orbit practically in contact with each

other. As the secondary minimum in the light curve is almost exactly in the middle between two consecutive primary minima, and very similar in shape, leaving only some difference in depth arising from difference in surface brightness, the assumption of a circular orbit appears to be very firmly founded.

After a general method for the derivation of the fundamental data in an Algol system from its light curve had been worked out by Russell and Shapley 1), the latter computed the system's elements 2) according to this rigorous method, taking into account the elongation of the bodies by tidal distortion and making separate solutions for uniform discs as well as for discs darkened towards the limb. The relative orbit is again considered to be circular; the two bodies are no longer supposed to be in contact with each other, but separated by a distance between their surfaces slightly over $^{1}/_{10}$ of the orbital radius. The computations are based on Roberts's 371 visual observations mentioned before.

Bailey's spectroscopic observations of the years 1893—1898, supplemented by two spectrograms taken in 1917, have been discussed by Miss A. Maury 3) who derives a spectroscopic orbit. The material consists of objective prism spectra showing double lines; no comparison spectra being present, only the distance between the lines can be measured and consequently only the relative orbit is derived, the mass ratio remaining unknown. Miss Maury adopts Roberts's period, which fully satisfies the Arequipa radial velocities. Whereas Roberts as well as Shapley from the photometric observations assume zero eccentricity for the orbit, the radial velocities yield the values: e = .08, $\omega = .72^{\circ}$. Taking Shapley's data for size and shape of the two components, this orbital eccentricity would make the system likely to show rotation of the line of apsides.

¹⁾ Mon. Not. 47, p. 91; A.N. No. 3410, p. 25.

²) *H.C.* No. 14.

³⁾ Ap. J. 13, p. 181.

¹⁾ Ap. J. 35, p. 315; 36, pp. 54, 239, 385.

²⁾ Contrib. Princeton Obs. 3, p. 82.

³) *H.A.* **84,** p. 172.

For this reason Luyten 1) has recomputed the spectroscopic orbit from the same material as used by Miss Maury by the Wilsing-Russell method, and finds: $e = .088 \pm .014$ m.e.; $\omega = 59^{\circ} \pm .10^{\circ}$ m.e. In the light curve such values should cause the time of secondary minimum to deviate $^{\text{P}} \cdot 030$ from the point midway between two consecutive primary minima, this deviation being proportional to $e \cos \omega$. In Roberts's light curve, however, no effects from this cause are visible. Therefore further photometric observations were thought to be very desirable and the star was put upon the observing program at Johannesburg.

2. Plate material.

The observational material consists of a trial plate of small weight on 1937 January 1 and a series of 85 plates from 1937 February 23 till April 28 on 13 nights, all taken with the Franklin Adams star camera (a = 25 cm, f = 112 cm) of the Union Observatory, Johannesburg, South Africa. A coarse grating was put in front of the objective in order to obtain the magnitude scale. The dimensions of this grating are d = l = .950 mm. The star Lacaille $3069 (= 216 \text{ G Pup} = \text{HD } 64740, \text{ sp. B3}, 07^{\text{h}}50^{\text{m}} \cdot 2$ $-49^{\circ}21'$, 1900) was used as a comparison star. This is the same star as used by A. Stanley Williams when discovering the variability of V Pup. Its brightness is intermediate between that of the variable at maximum and at minimum. As the spectrum of V Pup is Bip no serious errors from difference in colour between variable and comparison star are to be feared. All plates are of the brand Eisenberger Ultra Rapid hochfarbenempfindlich and have been exposed through a yellow screen in order to make the photometry in a limited region of the spectrum and to reduce the effect from darkening at the limb. By measuring the distance between the first order grating images the effective wavelength of the exposures on the variable as well as on the comparison star could be derived, the scale of the plate being known. For this purpose three plates have been selected, at maximum, minimum and intermediate brightness of the variable respectively. The result is $\lambda_{\text{eff}} = 5604 \text{ Å}$. This corresponds very well with the maximum in the spectral sensitivity curves for these plates as derived by Wesselink 2). As could be expected, no sensible difference in effective wavelength was found between variable and comparison star, nor between the variable at maximum and at minimum.

The plates are of size 9×12 cm. The telescope has been equipped with a plateholder mounted on a double slide. In front of this slide is a fixed screen with a square hole of 2.5×2.5 cm. This arrangement allows 12 exposures to be made on a 9×12 cm plate in three rows of four without adding sky fog to exposures not yet or already made. The device saves much time which otherwise is lost in changing plates. The exposure time was 55 seconds; 5 seconds were used to change from one exposure to the next one. In this way exposures were made at the rate of one a minute. In the last field exposed an extra exposure was made to prevent errors about the plate's orientation and about the order in which the exposures had been taken.

The distance between the variable and the comparison star is °91903, or nearly two centimetres on the plate. At this distance the non-uniformity of the plate's sensitivity has already serious effects on the accuracy of the photometry. Therefore, the following procedure was carried out. After 13 exposures had been made in the manner already described, the plateholder was closed and taken off and the plate was turned 180° in the dark room. A new series of 12 exposures was then made on the same plate. In this way each field of 2.5 × 2.5 cm carries a set of 2 exposures in positions reversed with respect to each other. As the telescope was pointed midway between variable and comparison star the images of the variable at the exposure before reversal and the comparison star after reversal are very close together, and vice versa. Consequently each field of 2.5 \times 2.5 cm on the plate yields two magnitude differences of images which are in the mean about I mm from each other. Effects from changes in sensitivity across the plate are almost completely eliminated in this way.

A drawback is that each magnitude difference obtained results from images not taken simultaneously, so that errors from change in atmospheric conditions may come in. For the two magnitude differences from the two pairs of images in each square field these errors should be of opposite sign and consequently disappear by taking their mean. In this way simultaneity is to a certain extent reintroduced and the full advantages of small mutual distances are enjoyed as well. In two cases the images of two consecutive exposures were superposed, the plate by mistake not having been shifted between them. Nevertheless the mean of the magnitude differences of the set of images was good (errors of m·o4 and m·o5 only), showing how well also errors in exposure time are compensated by our procedure.

A choice had to be made between photographing

¹⁾ Minnesota Obs. Publ. 2, p. 41.

²) B.A.N. No. 294, p. 127.

the 12 fields on the plate after reversal in the same order as before reversal or in the opposite direction. The first manner has the advantage that the time elapsed between the two exposures is constant, the second that the time halfway between the exposures is the same for all fields.

It should be remembered that the mean of two brightnesses of a variable star at two different times is not equal to its brightness halfway between these times. The difference is in first approximation proportional to the square of the time interval and to the second derivative of the light curve. If it is necessary to correct for this effect it is convenient to have the time interval the same for all pairs of exposures. Therefore the first alternative was chosen. Consequently a normally exposed plate followed the scheme shown in Figure 1. As the time lost for plate reversal was 1 minute, the time between two exposures of a set to be compared was 14 minutes.

FIGURE 1.

13,12		11		10		9	
	25		24		23	2	2
5		6		7 .		8	
	18		19		20	2	1
4		3		2		I	
	17		16		15	1	4

3. Measures and reduction.

The plates have been measured in the old thermopile photometer 1) at the Leiden Observatory. The diaphragm used has a diameter of 4.15 mm, corresponding to a diameter of 113 mm for its projected bright image on the plate. At each set of measures the intensity of the lamp was so regulated by a resistance that a constant deflection of the galvanometer was obtained for the plate fog about midway between the two sets of three images to be compared, which is very convenient for the reduction of the measures. Then settings were made on the two sets of images. As each set consists of a central image and two first order grating spectra on opposite sides of it, the mean of the latter two was taken.

The four quantities obtained in this way for each set have been converted into provisional magnitudes by means of a table constructed by A. J. Wesselink²).

To these provisional magnitudes the formula $\frac{\Sigma_{\nu} - \Sigma_{c}}{\Delta_{\nu} + \Delta_{c}}$ was applied 1), Σ_{ν} and Σ_{c} being the sums of the provisional magnitudes of first order spectrum and central image for variable and comparison star respectively, and Δ_{ν} and Δ_{c} their differences. This formula is rigorous if the provisional magnitudes obtained by the table are supposed to be a quadratic function of the correct magnitudes.

The validity of this supposition was tested for a few plates by reducing them in three different ways:

- 1° by the procedure mentioned;
- 2° by applying the formula $\frac{\Sigma_{\nu} \Sigma_{c}}{\Delta_{\nu} + \Delta_{\nu}}$ directly to the galvanometer readings;
- 3° by drawing first a characteristic curve from the readings on a plate connecting magnitude with galvanometer reading and reducing all readings by means of this curve, the formula mentioned being then applied to the figures obtained.

The result was that the second procedure was likely to give slightly different values; for the other two they were in good accordance. As the third method involved considerably more labour, the first method was chosen.

For five plates, viz. Nos. 13421 till 13425, many of the first order grating spectra were very faint, giving galvanometer readings close to the value for the plate fog. In such a case a small error in the reading from irregular fog or inhomogeneous plate sensitivity causes a large error in the result. Therefore these five plates have been reduced by means of a characteristic curve derived for each plate separately as mentioned under 3°. In order to give smaller weight to the faint first order spectra the magnitude difference was then obtained by dividing the difference of the provisional magnitudes of the central images of variable and comparison star by their mean gradation, the difference in provisional magnitude between central image and first order spectrum being considered as the gradation. In this way the first order spectra contribute to the denominator only, whereas in the formula mentioned before they contribute to both denominator and numerator.

Some exposures have been rejected owing to defective images or disturbing insensitive spots on the plate. In a few fields the reversed image had by accident been put too close to the non-reversed one. In all these cases both exposures, reversed and non-reversed, have been sacrificed.

For all plates the means have been taken of the two magnitude differences resulting from the two

¹) For description see B.A.N. No. 60.

²⁾ A. J. Wesselink, Thesis for the doctor's degree, Leiden, 1937, p. 13. See also B.A.N. No. 318.

¹⁾ E. HERTZSPRUNG, A.N. No. 4543, p. 121.

exposures, reversed and non-reversed, in each of the twelve fields on a plate. For exposures 12 and 13 their average difference in brightness with exposure 25 was used. In this way each plate with 25 exposures gave 12 magnitude differences, expressed in the difference between first order spectrum and central image as unit. For a grating the bars of which are of the same width as the spaces between them this difference is theoretically m·981. Therefore, all magnitude differences have been multiplied by this quantity.

For each plate the mean of its 12 magnitude differences was taken. Again it should be remembered that the mean of these 12 brightnesses may differ

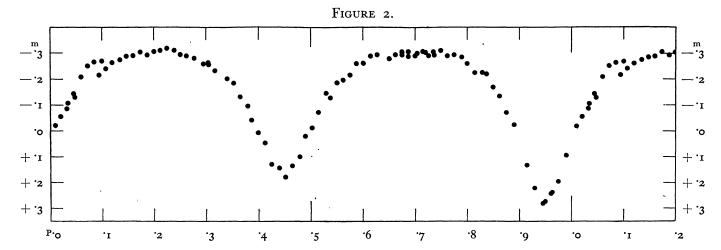
from the brightness at the mean of the exposure times. The correction for this effect is small; only in those parts of the light curve that show strong curvature it is worth being applied.

4. Light curve.

The time of each exposure has been converted into Julian Day Hel. M. Time Greenwich. Only the mean for each plate has been given in Table 1, 2nd column. For the construction of the light curve Prager's period r^d ·454488 has been used. The phases, in column 3, have been computed according to the formula: phase = $(J.D. - 2420000) \times d^{-1}$ ·6875278. The difference in brightness between the variable

TABLE I.

plate	J.D. Hel. M.T. Grw.	phase	observed brightness	hour angle correction	corrected brightness	n	sid. time Johbg.	plate	J.D. Hel. M.T. Grw.	phase	observed brightness	hour angle correction	corrected brightness	n	sid. time Johbg.
0-	d d		m	m	m		hm	0.	oc d	.00	m	m	m		hm
13083	8535.3711	3049	- 256	:004	- 260	3	05 ^h 27 ^m	13284	8623:3630	.8018	- 259	+ '020	- :239	12	11 ^h 04 ^m
13138	8588.2684	6733	- :293	'002	- 295	12	06 28	13285	.3819	8148	'226	+ '022	- 204	II	11 31
13139	2885	.6871	- 295	001	'296	12	06 58	13286	4023	.8288	:223	+ '025	198	II	12 00
13140	.3130	.7039	- :300	+ .001	- 299	12	07 33		8624.2148	3874	- :039	+ .001	038	10	07 34 08 01
13141	3373	7206	- :304	+ .003	- 301	12	08 08	13288	2335	4003	+ :007	+ .003	+ .010	12	08 28
13142	8590.2456	0327	083	003	:086	12	06 04	13289	2522	4131	+ :048	+ '005	+ .023	1 1	
13143	2660	.0467	- 128	- '002	130	1 1	06 33	13290	'2709	4260	+ .150	+ :007	+ .139	12	o8 55
13145	8591,4153	.8369	'211	+ '015	196	11	10 13	10001	.2899	.4201	±	+ .010	+ '154	11	00 22
13146	4344	.8500	-:171	+ '020	— ·116	12	10 40	13291			+ ·144 + ·178	+ '012	+ .190	II	09 49
13147	'4541	.8635	139	+ 020	- 110	12	11 08	13292	3080		+ 176	+ '015	+ 190	II	10 16
TAT 48	.4501	.8766	'071	٠٠٠٠ ا	'048	12	TT 06	13293	3269		+ 100	+ .018	+ 1118	12	10 43
13148	.4731 .4918	.8895	- '012	+ '023	+ .013	12	11 36 12 03	13294	3457 3642	4774	+ .018	+ .020	+ .038	II	11 10
13149	8593.2439	'0941	- 215	003	— ·218	12	06 13	13295	3042	.4901	010	+ .023	+ .013	12	11 37
13150	2626	.1060	'240	001	- 241	12	06 40	13296	4018	.2031	'071	+ .026	'045	12	12 04
13151 13152	2812	1197	- 262	,000	- '262	9	07 07	13297	4209		'146	+ .020	'117	12	12 37
13153	3043	11356	- 276	+ '002	- 274	9	07 40	13415	8648.2524	.0130	+ '132	+ '014	+ '146	12	10 04
	3229	1350	- 288	+ .003	- 285	6	08 07	13416	2711	9268	+ '222	+ .019	+ 238	12	10 31
13154	3229	1600	- 201	+ .009	— ·285	9	08 34	13410	2/11	9200	1 222	' 010	1 230	1~	10 31
13156	3603	1741	- '304	+ .008	'296	12	00 34	13417	.2906	.0402	+ '282	+ .019	+ .301	12	10 59
13157	3797	1874	- '293	+ .010	— ·283	10	09 28	13418	3237	.9630	+ '239	+ '024	+ '263	12	11 47
13137	3/9/	10/4	293	' 010	203	10	09 20	13421	8651.5126		+ '274	+ .010	+ .284	II	09 19
13158	.3980	2000	- 305	+ .013	'292	10	09 55	13422	2312		+ '241	+ '012	+ 253	II	09 46
13159	'4164		- 310	+ .019	- '294	12	10 22	13423	2499		+ '197	+ '015	+ '212	11	10 13
13160	4351	2255	-318	+ .018	300	12	10 49	13424	2745	'9917	+ '094	+ .018	+ '112	11	10 48
13161	4538		310	+ '021	289	12	11 16	13425	2977	.0077	- '022	+ '021	001	11	11 22
13162	4725	2514	- 293	+ '023	- 270	12	11 43	13426	3168		054	+ '024	030	11	11 49
13169	8595 3423	.5368	- 127	+ .006	- 121	12	08 43	13427	3358	.0339	107	+ '027	ogo	9	12 16
13170	3610		186	+ .000	177	12	09 10	13428	3543	.0466	- 143	+ .030	113	II	12 43
13171	3795	5624	- 195	+ .011	— ·184	9	09 37	١ ٠٠	00.0	•	1		1		
13172	3984	5754	'215	+ '014	- 201	12	10 04	13430	8652.2309	.6493	- '279	+ '012	'267	12	09 49
13173	'4171	5882	- 258	+ .019	'242	12	10 31	13431	.2498	.6623	- '295	+ '015	'280	II	10 16
		_		1				13432	'2685	.6751	303	+.018	- 285	II	10 43
13174	'4357	.6010	258	+ .010	'239	II	10 58	13433	.2870		306	+ .020	- 286	12	11 10
13175	4542	.6137	'288	+ '022	'266	10	11 25	13434	3057	.7007	- '292	+ .023	— . 269	12	11 37
13176	4729	.6266	'293	+ '024	- 269	11	11 52	13435	3246	.7137	306	+ .026	- 280	II	12 04
13179	8596.4017	.2651	'290	+ '015	'275	II	10 13	13436	'3431		'292	+ .028	- '264	12	12 31
13180	'4203	2779	'280	+ .012	- 263	II	10 40	13437	.3568	'7358	302	+ .030	- '275	4	12 51
13181	'4393	'2910	'261	+ '020	'241	12	11 07	13440	8653.2364	.3406	- 202	+ .013	189	12	10 01
13182			'262	+ .022	- 240	II	11 34	13441	'2551	3534	183	+ .019	162	12	10 28
13183	4767		533	+ '025	508	12	12 01	l	_						
13279			'294	+ .002	- '287	10	08 49	13442	2738		131	+ .010	- '112	12	10 55
13280	2885	.7506	310	+ .000	301	11	09 16	13443	'2922		002	+ '021	'074	II	II 24
-								13444	8654.5836		'210	+ '020	— .1 0 0	12	11 13
13281	.3066	, ,	1 5	1 '		10	/ 13	13445	.3053		225	+ .053	- 229	12	11 40
13282	3256		'296	+ '014		12	10 10	13446	'3210	-	'265	+ '026	— 239	12	12 07
13283	3443	7889	583	+ '017	- 266	12	10 37	13447	3397	.0991	- '272	+ '029	'243	12	12 34



and the comparison star has been given in column 4. When plotted against phase we find the light curve as shown in Figure 2.

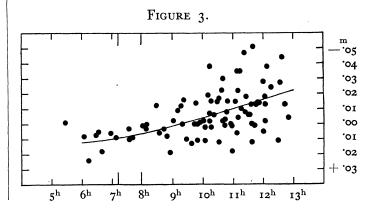
Inspection of the light curve shows that the difference in phase between primary minimum and secondary minimum is very nearly half a period. Consequently the value for $e\cos\omega$ can only be small. The shape of the minima is very symmetrical. There are, however, some discrepancies which occur especially at those places in the light curve where the observations jump from one night to another. The observations of a same night are mostly lying in a row without jump. Further inspection shows that at the end of a series of plates from one night the variable appears to be too bright and at the beginning of a series of another night which follows in phase it mostly appears too faint. This suggests a systematic error depending on the hour angle at which the exposures have been taken. Therefore, the sidereal time of the plates has been given in column 8 of Table 1. The error is far too great to be explained by the ordinary differential atmospheric extinction, although its effect is in the same direction. A bad thing is that, for a long row of observations covering a minimum like the series of twelve plates covering the secondary minimum, the effect tends to produce a spurious asymmetry in the light curve. The complete light curve has been covered by observations only once, so that in the same phase only observations from one hour angle are available. Consequently it is difficult to draw a mean light curve. In the following way this difficulty was more or less overcome. By applying a correction of double the differential extinction at sea level the light curve has been corrected for at least part of the hour angle effect, so that the minima have become more symmetrical. By means of the method exposed in B.A.N. No. 147, p.179, the phases of primary and secondary minimum have been determined, the result being:

primary minimum P .94996 \pm P .0004 secondary minimum 2 .45003 \pm .0005

The difference in phase between secondary minimum and the point midway between two consecutive primary minima, which is equal to $\frac{2e\cos\omega}{\pi}$

becomes: $^{P}\cdot 00007 \pm ^{P}\cdot 0006$. As the quantity concerned is much smaller than its mean error, the value for $e\cos\omega$ may be taken as zero. The light curve may be regarded as fully symmetrical and was now reflected about the line of symmetry at phase '4500. Still the observations taken at great western hour angles showed large deviations. In order not to be disturbed by them when drawing the light curve the observations taken at more than 3^h hour angle were temporarily omitted. Then the differences between the uncorrected observations and this light curve have been formed and plotted against the sidereal time as shown in Figure 3.

The diagram fully confirms the suspicion that the error depends on the hour angle. A second fact now shows its presence: the dispersion of the observations increases with the hour angle. Computation shows the mean error for plates up to three hours out of the meridian to be \pm m·oog, and for greater hour

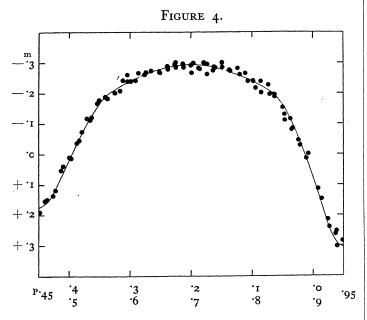


angles ± m·015, corresponding to a ratio in their weights of approximately three to one.

Means for each hour of sidereal time are shown in the following table:

sidereal time	mean deviation from light curve
h	, m
$6\frac{1}{2}$	+.011
$7\frac{1}{2}$	+.008
$8\frac{1}{2}$	+:004
$9\frac{1}{2}$	002
IO $rac{1}{2}$	007
I I $\frac{1}{2}$	—·o15
$12\frac{1}{2}$	—·o16

A curve has been drawn through the corresponding points in Figure 3. As many more observations have been made after the meridian than before it the effect is not zero at the meridian but $+^{m}$:006. If the effect is of the nature of a differential



extinction it should be zero when comparison star and variable are at equal altitudes above the horizon. This happens at 7^h12^m sidereal time. Therefore all observations have been reduced to 7^h12^m sidereal time by means of the curve mentioned. The resulting light curve is shown in Figure 4. Hour angle correction and corrected brightness are shown in columns 5 and 6 of Table 1.

Although nothing definite can be said about the cause of this hour angle effect the similarity with differential extinction suggests that it may be due to abnormally great extinction by dust in the atmosphere. At the Union Observatory, Johannesburg, for declination -49° at great western hour angles

the telescope is pointed across the town, the centre of which is at a distance of about $3\frac{1}{2}$ km, and across the range of gold mines. Both are a source of smoke and dust in the air so that this explanation is not unacceptable.

For accurate photometry the neighbourhood of a big industrial centre seems to be a serious handicap.

5. Mean errors and weight.

The mean error of a single plate as found from the symmetrical light curve (Figure 4) is \pm m·o116. The weight of the light curve computed from this value is $86/(\text{m·o116})^2 = 640000 \text{ m.}^{-2}$

This result will be flattered if part of the error is of a systematical nature. Therefore different sources of error have been considered, viz. night error, plate error, measuring error and the error in the image.

The mean error ε of the individual measures with respect to the average of a plate was found to be \pm m·043. It is composed of image error and measuring error. By measuring a plate twice the second error was found to be \pm m·014; consequently an error of ± m·041 is inherent to the image. By studying the systematic deviation of plates of the same night from the light curve, an idea was obtained about the night error and the plate error. The values found are \pm ***.0058 and \pm ***.0041 respectively. The number of nights being small, not too much importance should be attached to these last two figures. However, a comparison may be given of the values found here and those determined by Wesselink 1) in his discussion of 12479 exposures on Σ 485 with the Leiden 34 cm refractor:

VAN GENT, V Puppis Wesselink, Σ 485

	m	m
$\varepsilon_{ ext{night}}$	\pm :005 8	土:0053
$\varepsilon_{ m plate}$.0041	.0023
$arepsilon_{ ext{measure}}$.014	.013
$\varepsilon_{\mathrm{image}}$. 041	.019

The great difference between the values for the accuracy of the image may be satisfactorily explained by the difference between the instruments with which they have been taken. The exposures on V Pup have been obtained with a focal distance of 113 cm through yellow filter on colour sensitive plates, whereas Wesselink's Σ 485 exposures have been made with a focal distance of 520 cm without filter on ordinary blue sensitive plates. The character of the images is consequently entirely different, those of V Pup being of much smaller diameter and with a steeper gradation.

Taking into account the systematic errors, we

¹⁾ L.c., p. 31.

find for the total weight of the observations on V Pup: $\frac{14}{\left[(\text{m} \cdot \text{oo}58)^2 + \frac{(\text{m} \cdot \text{oo}41)^2}{6 \cdot 5} + \frac{(\text{m} \cdot \text{o}41)^2 + (\text{m} \cdot \text{o}14)^2}{6 \cdot 5 \times 25} \right] }{\text{about 250000 m}^{-2}}$

6. Period.

Roberts's observations 1) have been reduced with the reciprocal period already mentioned, viz. d⁻¹·6875278, and in the resulting light curve the phase of minimum has been determined in the same way as already described for the 1937 photovisual observations. For both series of observations the corresponding epochs of minimum have been computed. An estimate of the accuracy of these minima was obtained from the accuracy of the points on the respective light curves, viz. \pm m·o19 for Roberts's and \pm m·0116 for the 1937 Johannesburg curve. Considering the slopes of the descending and ascending branches near the minima, which are the main factor determining the accuracy of these epochs, and considering the number of points upon these branches, the following figures result:

Min. Roberts J.D. Hel. M.T. Grw.

 $2415021^{\text{d}} \cdot 2186 \pm {}^{\text{d}} \cdot 0015 \text{ m.e.}$

Min. van Gent J.D. Hel. M.T. Grw.

 $2428648^{d} \cdot 3048 \pm {}^{d} \cdot 0007$ m.e.

The number of periods elapsed between is 9369. Consequently the period is ²):

$$1^{d}$$
.4544867 \pm d·0000002 m.e.

The uncertainty of the period is of the order of '02 seconds.

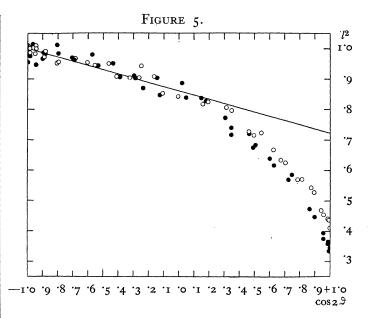
7. Determination of the orbital elements.

As the light curve has considerable weight it was made the basis of a new determination of the fundamental quantities in this eclipsing system. A solution for uniform discs was made, assuming two similar three-axial ellipsoids with their longest axes in a line, as is customary in such cases. The orbit was considered to be circular.

The intensity of light l is connected with the eccentricity ε of the equatorial section in the following way:

$$l^2 = 1 - \varepsilon^2 \sin^2 i \cos^2 \vartheta,$$

where i denotes the orbital inclination and \mathfrak{I} the system's anomaly, counted from principal minimum. From Table 1 l^2 was computed and plotted against $\cos 2\mathfrak{I} = 2 \cos^2 \mathfrak{I} - 1$. The result is shown in Table 2, column 4, and in Figure 5, open circles



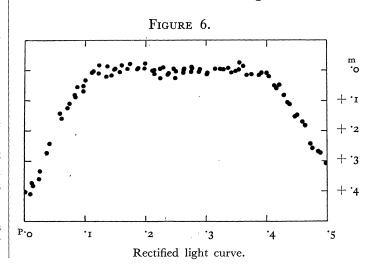
denoting secondary minimum, dots principal minimum. The relation between the coordinates should be linear outside the eclipses. From inspection of the diagram the eclipse was considered not to start before $\cos 2\beta = + \cdot 2$ and a least squares solution was made for a straight line through the points to the left. Its result is:

$$l^2 = +.8614 -.1383 \cos 2 \ \%$$

 $\pm.0073 \text{ m.e.}$

Accordingly: $\varepsilon^2 \sin^2 i = +2766 \pm 0146$ m.e. The light curve was now rectified as shown in column 6 of Table 2 and in Figure 6.

The next quantity to be derived is the ratio of the surface brightnesses $\frac{J_b}{J_t}$. For this purpose the ordinates of the eclipse in Figure 6 have been converted from magnitudes into intensities as shown in column 7 of Table 2. The area in the rectified light curve en-



¹⁾ Ap. J. 13, p. 189.

²⁾ Although the old observations by STANLEY WILLIAMS actually hit three minima, they could not be used for improving the period, their number being too small.

TABLE 2.

plate	phase from min.	cos 2 🎝	$l^2_{ m obs.}$	$l^2_{ m ell}$.	m obs. minus m ell.	obs. loss of light due to eclipse	brightness maxim obs.		O-C	plate	phase from min.	cos 2 J	$l^2_{ m obs}.$	l ² ell.	m obs. minus m ell.	obs. loss of light due to eclipse	brightne maxin		O-C
					m		m	m	m						m		m	m	m
13421	.0008	+ .9999	'3448	.7232	+ '402	.3094	+ 578	.598	020	13140	.2461	— ·9988	1,0003	.9996			1	.000	
13417	.0098	'9924	'3342	7242	+ 420	3208	+ 595	584	+.011	13434	.2493	1,0000	'9550	9997	+ '025		+ 025	.000	+.022
13422	0120	9887	'3651	7247	+:372	2901	+ '547	578	031			1,0000			+ '002		+ '002	.000	+.005
13418	.0130	.9867		7250	+.382	2966	+ 557	574	012	13433		.9883	'9854	9981	+ .002		+ .008	.001	+:007
13416	.0232	.9578	3753	.7290	+:360	2822	+ .532	.230	+ 002	13159	.2627	.9873	1,0000	.9980	001		.000	.001	001
13423	.0248	.9218		.7298	+:335	2655	+ .506	521	-015			•9869	1.0032	9979	003		002	1001	—·003
13415	.0361	.8989			+ '274					13432	2749	.9514	.9836	.0930	+.002		+ .000	004	+.002
13424	.0412	.8659	'4734	7417	+ 244	.5013	+ '406	420	014										
13425	.0577	'7485			+'142							'9491	1,0111	9927	—. 010		006	004	010
13149	.0602	7247	.2681	7612	+:159	.1365	+:307	308	001	13138			1,0018	9920	- .006		001	004	•005
										13431	.2877	.8899		9845	+.006		+ '014	. 008	+.006
13426	0708	6296	6149	7744	+ 125	.1088	+ 264	253	+.011	13161	2884	·88 ₅ 8			'004		+ .002	.000	'004
13148	'0734	.6039			+.110	0964	+ 246	239	+:007	13430	3007	.8039	'9515	9726	+ '012				+ 012
13142		.2068		7913	180.+	.0218	+ 208	197	+.011	13162	.3012	.7963			+.008				+.008
13427		.4938		7932	+ .088	.0228	+ 214	1192	+ 022			.6837			—.ooè				—·005
13147		'4652		7971	+ .022	·0494	+ 178	.181	003	13176	3234	.6039			006				— •∞5
13428	1	'3494		.8131					+.038						'004				—·oo3
13143		'3482							+.021		.3363	'4673	'9497	.0261	'014		+ .056	042	 ⁺014
13146			7727	8187	+ 032	0290	+ 140	132	+ 008	_									
13444	1								+.001						+ .002				+:007
13145	.1131	1490	8348	8408	+ '004	0037	1+.008	097	+.001				9030	.9058	+ '002				+,001
06		0	.0	.00			1		l	13182					'004				-:004
13286	t .	.0478	8379	8548	+ .011				+.011				9393	8900	:026				—·026
	1234				018				017				9007	·8843	-:015		+ 052	007	:015
	1352				+ :020				+:020				8535	0759	+ '014		+ '000	072	+:014
	1363	1 -	1 2/		- 014		1 :		'014		1	ı	0420	.0021	+ '012		+ 093	000	+:013
	1441		1 - 3		+ .016	1	1		+.019				8047	0390	+ :015	0137	+ 110	098	+ 012
	1482		, ,		- '002	1			001						+ .002				
	1491			1 -	- 004				:005 -:005		4004	.3138	8001	8180	+ .008	00/3	T 117	12/	010
	1569			9154	+ .002		+ .038			13441	.4024	.2404	.7014	.8.2.	+ '014	.0128	1.125		008
	1611			. -	004				—·oo4				7914	.2026	+ '050	0120	127	1.164	7.000
13132	1097	3320	9420	9331	004	1	032	(030	004		4163		77152	7970	+ .026	.0404	+ . 782	175	+ '007
13282	1720	.5766	.0782	.0413	021		+ .013	.022	—·021						+ 046				
	1856			9412	004				004				.6668	.7748	+ 082	.0727	+ .220	.220	.000
~ ~~	1870			1	006				006						+.109				
-	1984		1	9717		- 1			007				.6240		+.110				
	1994		1,0130			1	007			13296					+ .123				
•	2109			9834		1	+ .000			13288			5712	7402	+ 147	.1566	+ .304	1.310	006
	2128			9849			+ .007	008	—.oo1	13205	4500				+ .169				
13437					+ '012				+.011	3-,3	1399	-,3/	37-3	, 1-3	•9	'	33-	332	
	2236	, ,		-	+ .026	.			+ 026	13280	4631	.8944	. 5277	1.7378	+ 182	1543	+ .34	367	020
	2241		1 0037	1	1 .	- 1	- 002			13294			4682		+ 242				
3-30		1173	37	']		4760								+.010
13141	2294	. 9667	1,0130	9951	010	o	007	002	009		1			7254			+ 443		
	2363				+ .013				+.013						+ 273				
	2374				+ .010				+.010				4100		+ 308				
5 57		, , , ,			• •											,		/	

closed between the intensity at maximum and the eclipse curve was determined for both minima. The ratio between these two areas is the quantity wanted. Its value is:

$$\frac{J_b}{J_f}$$
 = 1.36 ± .03, equivalent to -m.335.

It is appropriate to compare this value with the one derived from the spectral types of the two components which are classified by Miss Maury as Br and B3 respectively. The ratio of the radiation in a certain wavelength per unit area for the two components is:

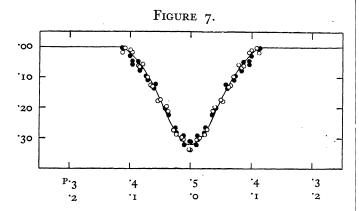
$$\frac{\frac{K}{\lambda T_{b}} - 1}{\frac{K}{\lambda T_{f}} - 1}.$$

Taking $\lambda = 5604$ Å, K = 6240 and interpolating in the temperature scale of Russell, Dugan and Stewart 1) we find: $\frac{J_b}{J_f} = 1.34$, in even better agreement with the value derived from the light curve than might be expected.

¹⁾ Astronomy, p. 734.

The remaining quantities to be determined from the light curve are $\frac{r_1}{a}$, $\frac{r_2}{a}$ and i.

For this purpose the two minima have been united into a single curve by multiplying the figures for the loss of light due to secondary eclipse from Table 2, column 7, with the value found for $\frac{J_b}{J_f}$. The values so obtained together with the values for the loss of light due to primary eclipse have been plotted against phase, counted either from mid-primary or midsecondary minimum. The result is shown in Figure 7,



dots denoting principal, open circles secondary eclipse. It is seen that there are hardly any systematic differences between the two eclipses, now that the effects of difference in surface brightness have been eliminated.

Theoretically three points on a light curve are sufficient to find the values of the three remaining unknowns. For these points the following set has been selected:

phase	loss of light
.0000	.328
.0412	·200
.0810	.075

and with a number of values for $k=\frac{r_{\rm B_3}}{r_{\rm B_1}}$ corresponding values for a and i have been computed satisfying the first two points. For the third point the values $\frac{\delta}{s_1}$ (projected distance of the centres of the two components in terms of the semi major axis of the equatorial section of the bigger one) have been computed for phase '0810 and compared with the values derived from the observed loss of light '075. The following table gives an account of these trial solutions:

It was hoped that by varying k the value for $\left(\frac{\delta}{s_1}\right)_{\text{obs.}} - \left(\frac{\delta}{s_1}\right)_{\text{comp.}}$. would pass through zero. As shown by the table, this hope was not fulfilled although the values for α_o , the fraction of the smaller disc eclipsed by the bigger one at mid-eclipse, practically cover the whole possible interval from 1 oo through a minimum to 1 oo. It appears that no set of values k, a and i can be found satisfying the three selected points on the light curve. Moreover, a big change in k involves only a small change in $\left(\frac{\delta}{s_1}\right)_{\text{obs}}$

 $\left(\frac{\delta}{s_1}\right)_{\text{comp.}}$ This raises the suspicion that this type of light curve is not capable of yielding a sharp determination of k, a and i anyhow.

Therefore k was determined in another way. The ratio of the strengths of the brighter B_I spectrum to the fainter B₃ spectrum is given by Miss Maury as $\frac{10}{7}$. This should be equal to $\frac{1}{k^2} \frac{J_{B_1}}{J_{B_3}}$. With the value already found for $\frac{J_{B_1}}{J_{B_3}}$ the value for k becomes:

already found for $\frac{J_{\rm B_1}}{J_{\rm B_3}}$ the value for k becomes: k=.98.

Now that a value for k has been found without using the light curve, only two points upon the latter suffice for determining the two remaining quantities a and i. The two following points have been chosen:

It will be noted that the value for the loss of light at mid-minimum is less than the one adopted before from a free hand curve through the observed points. A theoretical curve drawn near the middle of the minimum showed it to be much blunter than the free hand curve, with consequent decrease of the depth at mid-minimum.

We thus find:
$$k = .98$$

 $a = .2.6196$
 $i = .76^{\circ}.89$

With this set of values a theoretical light curve has been computed, which has been compared with the observed points. An account of this comparison is given in Table 2, columns 8, 9 and 10, and in Figures 4 and 7.

The mean error of a single plate with respect to this curve is \pm m·o13.

8. Orbital eccentricity and rotation of periastron.

In the same way as described under 4 the difference in phase between secondary minimum and the point midway between two consecutive primary minima was determined for Roberts's 1) light curve of the year 1900. Together with the spectroscopic orbit derived from the Harvard-Arequipa radial velocities by Luyten 2) we now have the following information regarding the position of the periastron, adopting the spectroscopically determined eccentricity throughout:

1896 Harvard-Arequipa radial velocities . . .

$$e = .088 \pm .014; \omega = 59^{\circ} \pm 10^{\circ},$$

1900 Roberts's visual light curve...

$$e\cos\omega = + \cdot 0037 \pm \cdot 0033; \omega = 88^{\circ} \pm 2^{\circ},$$

1937 the author's photovisual light curve . . .

$$e \cos \omega = + .0002 \pm .0010; \omega = 90^{\circ} \pm .7^{\circ}.$$

From these data an advance of the periastron at the rate of 9° . 7 annually seems possible. It is, however, a suspicious fact that for both photometric determinations ω becomes very nearly 90° with almost exactly one complete rotation of the periastron in between.

As is well known, orbital eccentricity also produces a difference in width between primary and secondary minimum proportional to $e\sin\omega^3$). Therefore the magnitude of this effect was computed for the 1937 light curve with Luyten's value for e and the values for k, a and i as found in the previous section. The result was that primary minimum should last P.034 longer than secondary minimum. The light curve in Figure 7 certainly excludes so great a difference, although there is an indication that primary minimum might last longer than secondary minimum by an amount of not more than Post, corresponding to an eccentricity of not more than '026. It should be borne in mind that determination of $e\sin\omega$ from the difference between the widths of the two minima is far inferior to the determination of $e\cos\omega$ from the times of mid-minimum.

The author considers it very probable that the value found spectroscopically for e is too high and mainly due to systematic errors in the radial velo-

cities. Miss Maury ¹) states that the lines in the spectrum of V Pup are wide and hazy and sometimes asymmetrical and liable to cause systematic errors in the measures. Effects of this nature are since long held responsible for the clustering of the values for ω in spectroscopic orbits with moderate, eccentricity round $\omega = 90^{\circ 2}$). Therefore in the following section the orbit has been considered to be circular and the question of rotation of periastron was accordingly dropped.

9. Differential improvement of orbit by least squares.

As the differences between the observed and the computed light curve show something of a systematic nature, an attempt was made to improve the solution for k, a and i by the method of least squares. The set of values already found for these three unknowns was made the starting point for differential corrections dk, da, di, which are connected with the difference in intensity observed minus computed by the following equations of condition:

$$d\alpha = \frac{\partial \alpha}{\partial k} \cdot dk + \frac{\partial \alpha}{\partial \delta} \frac{\sqrt{\cos^2 i + \sin^2 i \sin^2 \beta}}{\sqrt{E}} \cdot d\alpha$$
$$-\frac{\partial \alpha}{\partial \delta} \frac{a \sin 2i \cos^2 \beta}{2 \sqrt{E} \sqrt{\cos^2 i + \sin^2 i \sin^2 \beta}} \cdot di$$

In this equation α denotes the fraction of the smaller disc obscured by the bigger one, δ the projected distance between the centres of the two components, and $E = 1 - \varepsilon^2 \sin^2 i \cos^2 \vartheta = 1 - 2766 \cos^2 \vartheta$.

The 39 plates nearest in phase to principal and secondary minimum have been made the basis of the least squares solution. For each plate the value $d\alpha$ was computed. The values for $\frac{\partial \alpha}{\partial k}$ and $\frac{\partial \alpha}{\partial \delta}$ have been taken from the table $\alpha = f(k, \delta)$ computed by M. Wend 3), which is very convenient for this purpose. The normal equations resulting from the 39 equations of condition are:

$$+ \text{ i.8022} \quad dk - \text{ i.0209} \quad da - \text{ 4.9597} \quad di$$
 $= + \text{ i.058214}$
 $- \text{ i.0209} \quad dk + \text{ i.657818} \quad da - \text{ 6.28456} \quad di$
 $= - \text{ i.016812}$
 $- \text{ 4.9597} \quad dk - \text{ 6.28456} \quad da + \text{ 39.9091} \quad di$
 $= - \text{ i.116812}$

¹⁾ L.c.

²) L.c.

³⁾ Cf. UITTERDIJK, B.A.N. No. 237.

¹⁾ L.c.

²⁾ Cf. Hellerich, A.N. **216**, p. 277; Struve and Pogo, A.N. **234**, p. 207.

³⁾ Eine Tafel zur Theorie der Bedeckungsveränderlichen, Dissertation Leipzig, 1931.

From these equations the following corrections to the elements were found:

$$dk = + .297 \pm .162$$

 $da = + .349 \pm .215$
 $di = + .089 \pm .054 = + 5^{\circ}.10 \pm 3^{\circ}.09$

The very great mean errors of these corrections fully confirm the view that a type of light curve as that of V Pup does not permit a sharp determination of k. A new light curve computed with the corrected elements indeed gave a better representation of the observations, but the improvement is small, the mean error decreasing by only 9 percent. After application of the correction dk the new value for k would become: k = 1.28. With the ratio of the surface brightnesses $\frac{J_{B_1}}{J_{B_3}} = 1.3616$ already found, this new value for k would yield the ratio of the intensities of the two components: $\frac{I_{B_3}}{I_{B_1}} = \frac{k^2}{1.3616} = 1.20$, in complete disagreement with the value '70 found by Miss Maury from estimates of the relative strengths of the two components, even reversing the position of stronger and weaker component. Such a great deviation from Miss Maury's value is impossible and nothing seems to be left but admitting that the set of values found under 6 is about the best set of elements satisfying all the observations, including Miss Maury's observations of the relative strengths of the two spectra.

Although in this case the least squares solution has apparently not brought us any further as to orbit improvement, one thing is gained by it: a quantitative determination of the uncertainty in the elements.

10. Reflection and darkening towards the limb.

If we apply Eddington's formula 1) to both components, the effect of reflection on the luminosity of an eclipsing variable may be expressed by

$$(b_1 - b_2) \cos \psi + \frac{16}{9\pi^2} (b_1 + b_2) \cos 2 \psi,$$

in which formula the phase angle ψ is connected to the anomaly ϑ by: $\cos \psi = \sin i \cos \vartheta$.

The effect of the first term is that the maxima are made asymmetrical, except when $b_1 = b_2$. The second term does not disturb the maximum's symmetry but makes it flatter, in this way working in opposite direction to the effect from tidal elongation of the components. Consequently the value for $\varepsilon^2 \sin^2 i$ determined by the usual procedure under 5 will need a positive correction. The amount of this correction is '36 $(b_1 + b_2)$. Inspection of the light curves in Figures 4 and 6 shows the maxima to have little or no asymmetry, so that not much reflection effect can be expected. With the values found for k, a, i and $\frac{J_1}{J_2}$ Eddington's formula would predict a value $b_1 - b_2 = +$ 0078. The residuals of column 10, Table 2, outside eclipse, have been analysed for presence of a term $(b_1 - b_2) \cos \psi$. The value found was:

$$b_1 - b_2 = + .0005 \pm .0030.$$

As $\frac{b_1}{b_2} = \frac{J_1}{J_2} = 1.3616$, the coefficient of the term with $\cos 2 \psi$ becomes: $+ \cdot 0012 \pm \cdot 007$. This correction is too small and too unreliable to consider it for improvement of the value found for $\varepsilon^2 \sin^2 i$. Consequently the reflection has been completely disregarded.

Nor is the case of V Pup favorable for showing effects from darkening at the limb. According to MILNE 1) the darkening is a function of λT . Taking the temperature of V Pup as 20000° and that of the sun as 5600°, we find that the darkening in V Pup for the wavelength used, viz. 5604 Å, should be the same as in the sun for wavelength 20000 \mathring{A} . This is already outside the range of wavelengths that has been chosen by Moll, Burger and van der Bilt²) for measuring the intensity of radiation across the sun's disc. The effect of darkening towards the limb upon the computed brightnesses found from the same elements by extrapolation from the figures obtained by Moll, Burger and van der Bilt is only small. The brightness during eclipse is slightly increased near the beginning and end, and decreased near mid-eclipse as compared with the non-darkened curve.

For the inverse problem, i.e. to find the law of darkening, c.q. the coefficient of darkening if the cosine law is accepted, the present light curve is insufficient. As has been remarked by PANNEKOEK and Miss van Dien 3) the major part of the effect of darkening upon the light curve is taken up by small adjustments of k, a and i. In order to find the darkening much more accurate observations are required, preferably simultaneously in different wavelengths 4) so that the effect may be found differentially. Even so V Pup will not be a favorable case; a variable like Castor C with about equal components of small size with respect to the orbit and of low temperature is much better suited to show effects from darkening.

¹⁾ Mon. Not. 86, p. 322.

Phil. Trans. A. 223, p. 201.

²⁾ B.A.N. No. 91.
3) B.A.N. No. 297.
4) Cf. Krat, Zs. f. Ap. 11, p. 71; Rosenberg, Ap. J. 83, p. 67; Hellerich, Bergedorf Mitt. 7, p. 179.

11. Dynamical parallax and absolute dimensions.

As is well known the mass-luminosity relation permits the calculation of the absolute magnitude of an eclipsing binary from its light curve and the spectral type of one component 1). The procedure consists in computing the surface brightnesses of both components with the aid of the spectral type of one of them and the ratio of the surface brightnesses as derived from the light curve, and then selecting the dimensions of the system in such a way that the absolute luminosities resulting from size and surface brightness, and the total mass calculated by Kepler's third law from orbital radius and period fit the mass-luminosity relation. Comparison between the absolute magnitudes thus found for both components and their apparent magnitudes will yield the parallax.

Neglecting the absorption, we find in this way for V Puppis:

$$p = + \text{".oo28},$$

Epoch of principal minimum Period Brightness at maximum Brightness at principal minimum Brightness at secondary minimum Ratio of surface brightnesses $\frac{\int_{B_1}}{r}$ Ellipticity constant $\varepsilon^2 \sin^2 i$ Ratio of radii k Light of brighter component Inclination of orbit i Oblateness of equatorial section $\sqrt{1-\epsilon^2}$ Oblateness of meridional section Orbital radius a Longest radius of B1 component Longest radius of B₃ component Mass of B1 component Mass of B₃ component Density of B₁ component Density of B₃ component Mean absolute magnitude of B1 component Mean absolute magnitude of B3 component Dynamical parallax

corresponding to:

With the aid of the spectroscopic observations a direct and much more reliable determination of orbital radius and masses was made, its result being:

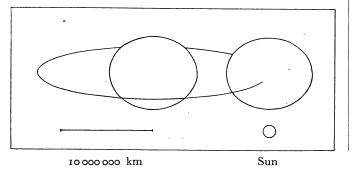
a = 0.0830 astronomical units

Mass B₁ component = 21.18 \odot Mass B₃ component = 14.91 \odot

The corresponding absolute magnitudes can now be derived either by means of the mass-luminosity relation ²) or by means of the relation between spectral type and surface brightness, the result for the brighter component being — 3^M·3^I and — 2^M·9^I respectively or ^M·64 and ^M·24 brighter than as determined before.

The data obtained for V Puppis may now be summed up in the following list. For the absolute magnitudes and the parallax the mean was taken of the two sets of results obtained with the use of the spectroscopic data.

FIGURE 8.



A drawing of the system is given in Figure 8.

I am indebted to the late Mr. A. STANLEY WILLIAMS for information about the discovery of variability and early observations of V Pup; to Prof. R. PRAGER for kindly sending me a résumé of the literature on V Pup in advance of publication in the next volume of Geschichte und Literatur des Lichtwechsels der veränderlichen Sterne and to Mr. F. DE HAAS for preparing the drawings for the diagrams in this paper.

Cf. S. Gaposchkin, *Harv. Repr.* No. 151.
 Cf. G. Durand, *Bull. Astr.* 11, p. 137.