The relation between radial-velocity range and light-range for RR Lyrae stars

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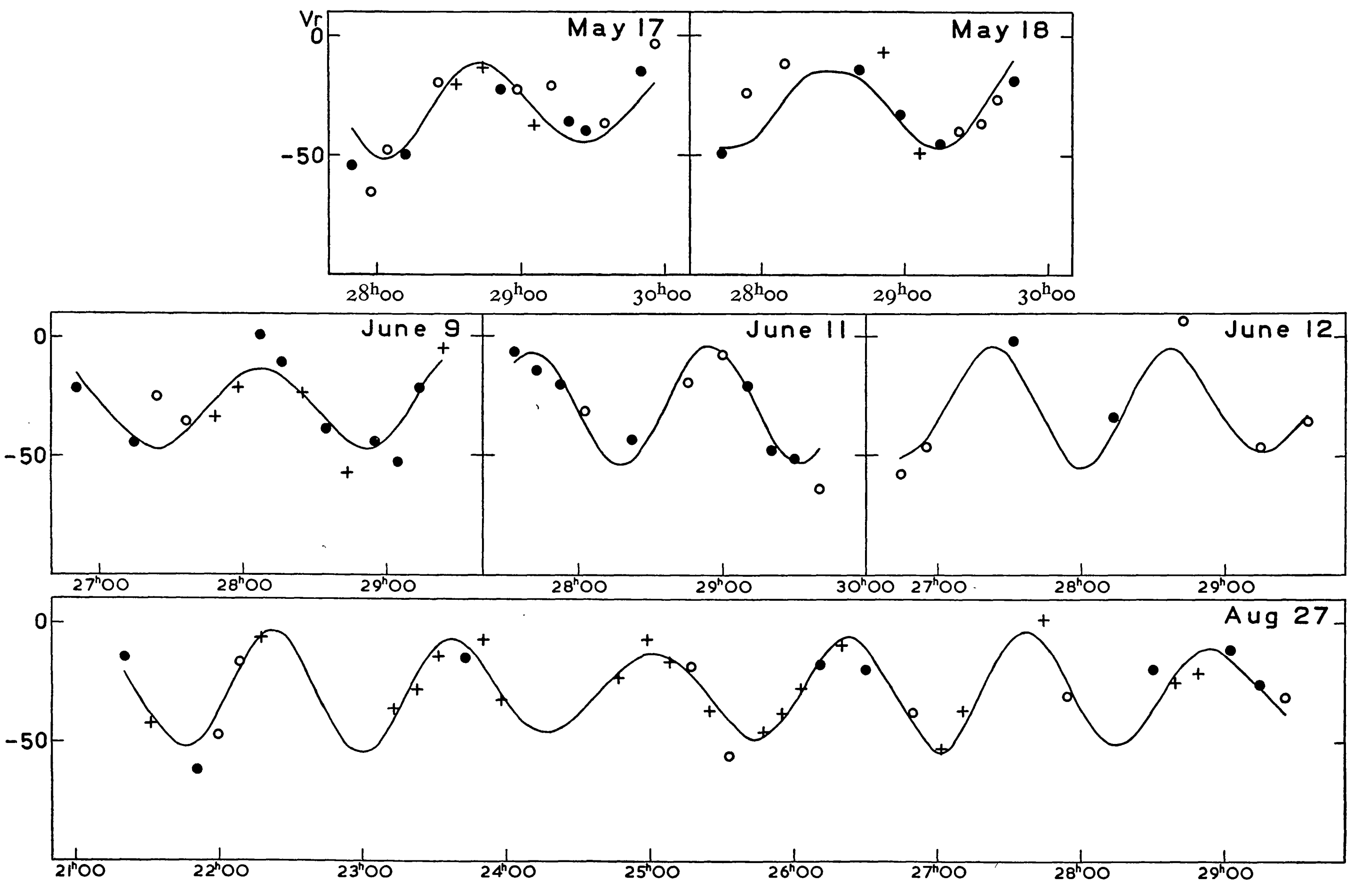
FIGURE 2



The radial velocity-curve of SX Phe. o weight unity, • weight two, + weight three.
The full-drawn curve is given by $v = -29.1 + 20.6 \times \sin 2\pi (\varphi_0 - 0.540) + 5.4 \sin 2\pi (\varphi_1 - 0.540)$.

THE RELATION BETWEEN RADIAL-VELOCITY RANGE AND LIGHT-RANGE FOR RR LYRAE STARS

BY L. WOLTJER

It is shown that there exists a relation between velocity range and light-amplitude for RR Lyrae variables. This relation is linear and given by $\Delta v = 64 \, \Delta m$, where Δv is the velocity range in km/sec and Δm the light-amplitude in photographic magnitudes. The average deviation from this relation for the individual stars is $0^m.1$, when one star is omitted. This result is compared with similar relations for other variables.

At the moment about ten more or less reliable radial-velocity curves for RR Lyrae variables are available and for some other stars of this class we can make some estimate about the velocity amplitude. It therefore seems of interest to investigate a possible relation between velocity amplitude and light-range. Such a relation is clearly present in the classical cepheids, as was demonstrated e.g. by O. J. EGGEN[1]). The relevant data for some RR Lyrae stars are given below. No distinction is made between the ordinary RR Lyrae variables and the RR Lyrae variables with very short periods, which form probably a separate group.

[1]) *Ap. J.* **113**, 367, 1951.

CY Aqr. From the photographic light-curve by A. J. WESSELINK[2]) $\Delta m_{pg} = 0^m.88$. Velocity amplitude 64 km/sec from 21 observations by O. STRUVE[3]).

UY Boo. From Harvard plates according to Mrs C. H. PAYNE GAPOSCHKIN[4]) $\Delta m_{pg} = 1^m.4$. Velocity amplitude 90 km/sec from 18 observations by Miss A. V. FARQUHAR[5]).

VZ Cnc. Photoelectric light-curves by W. S. FITCH[6]). A yellow filter was used in this investigation. A. H.

[2]) *B.A.N.* **9**, 217, 1941.
[3]) *A.J.* **54**, 137, 1949.
[4]) *H.A.* **115**, 265, 1952.
[5]) *Ap.J.* **107**, 276, 1948.
[6]) *Ap.J.* **121**, 690, 1955.

JOY and R. E. WILSON [1]) obtained a few velocity observations for this star, from which the amplitude seems to be about 52 km/sec. The amplitude in yellow light at the time the velocity observations were made, was $0^m.667$ according to FITCH. The spectral type varied between A7 and F2, as was seen from the spectrographic observations. The photographic amplitude is therefore about $0^m.18$ larger than the yellow one. The photographic amplitude at this time was thus $0^m.85$.

W CVn. Photographic light-curve by V. A. NACHAPKIN [2]) gives $\Delta m_{pg} = 1^m.0$. From 24 velocity observations A. H. JOY [3]) finds $\Delta v_R = 70$ km/sec.

XZ Cyg. Photoelectric light-curves by A. B. MULLER [4]). A blue filter was used, thus the response is almost photographic. The beat period causes a strong variation in the light-amplitude, which varies between $\Delta m_{pg} = 1^m.09$ and $\Delta m_{pg} = 1^m.66$. In the numerous velocities measured by O. STRUVE and A. VAN HOOF [5]) this variation is also present. The velocity amplitude varies between 53 km/sec and 64 km/sec.

DY Her. From a photographic light-curve by J. ASHBROOK [6]) $\Delta m_{pg} = 0^m.69$. It should be remarked that some visual observers found very strong irregularities in the light-curves. Also for this reason not too much weight should be given to the velocity amplitude, estimated by A. H. JOY [7]) from only 3 observations as 70 km/sec. Therefore the star is not plotted in Figure 1.

RR Lyr. Extensive series of photoelectric measurements were made by TH. WALRAVEN [8]). No filter was used, but in WALRAVEN's paper the colour sensitivity of the system is given. Using some six-colour observations published by J. STEBBINS [9]) we could derive an approximate correction to be applied to WALRAVEN's results to bring them on the photographic scale. We find that the photographic amplitude varied between $0^m.97$ and $1^m.15$ at the time when the velocity observations were made by A. BLAAUW and O. STRUVE [10]). From a very large number of observations these authors found the velocity amplitude to vary between 54 km/sec and 60 km/sec.

DY Peg. The photographic light-curve by C. H. D. STEINMETZ [11]) gives $\Delta m_{pg} = 0^m.70$. From 10 spectra W. P. BIDELMAN [12]) obtained $\Delta v_R = 55$ km/sec.

SX Phe. From the photoelectric light-curve, obtained with a blue filter, by TH. WALRAVEN [13]) we find that Δm_{pg} varies between $0^m.38$ and $0^m.82$. In the previous paper we found that Δv_R varies between 30.4 km/sec and 52.0 km/sec.

δ Sct. From an inspection of unpublished photoelectric light-curves made by TH. WALRAVEN in 1952 we find that the photographic amplitude varies between $0^m.16$ and $0^m.27$. My thanks are due to Dr WALRAVEN for putting his data at my disposal. The amplitude of the velocity curve varies between 8.0 km/sec and 13.0 km/sec, as was found by G. F. PADDOCK and O. STRUVE [14]) from more than 200 observations. There has been some doubt about the class of variables to which δ Sct belongs, but from the relation between velocity-amplitude and light-amplitude, as given in Figure 1, it seems very probable that the star is an RR Lyrae-type variable.

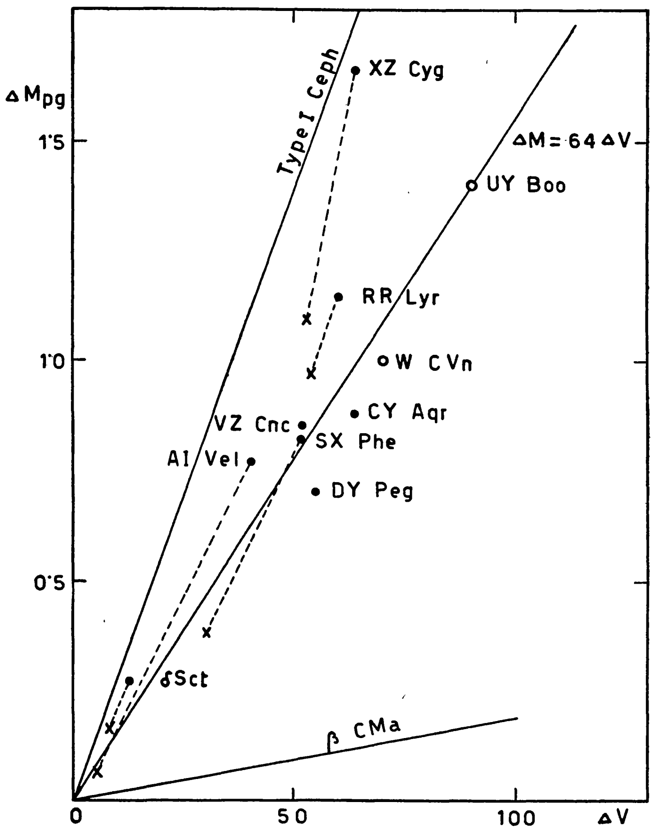
AI Vel. From the photoelectric light-curves obtained with a blue filter by TH. WALRAVEN [15]) it appears that the photographic range in 1951 varied between $0^m.07$ and $0^m.77$. In 1950 C. J. LAVAGNINO and L. GRATTON [16]) obtained some 150 spectra. The velocity amplitude varies between 5.6 km/sec and 40.4 km/sec, as was shown by L. GRATTON [17]).

AT Vir. Only a visual light-curve by A. V. SOLOVYEV [18]), with an amplitude of $1^m.3$ is available. The velocity amplitude was estimated by A. H. JOY [19]) from 4 spectra to be 85 km/sec. As the velocity amplitude is rather uncertain and as we do not know the correction from visual to photographic amplitude either, this star was not plotted in Figure 1.

The velocity amplitudes, for the ten stars for which reliable data are known, were plotted against their photographic ranges (Figure 1). For stars with secondary periods the largest and smallest values for the amplitudes are plotted and connected by dashed lines. It is seen that apart from XZ Cyg the data are represented rather well by a straight line, having a slope of 64 km/sec/magnitude. The average deviation from the line in magnitudes is about $0^m.1$ when XZ Cyg is omitted. This star has also an abnormally high velocity, as was pointed out to me by Professor OORT. The relation for classical cepheids as found by EGGEN [20]) and the relation for β CMa stars are also indicated in the figure. To obtain a significant result however it is necessary to convert all relations from photographic magnitudes to bolometric ones.

[1]) *P.A.S.P.* **62**, 58, 1950.
[2]) *Tadzhik Circ.* No. 44, 1940.
[3]) *Ap.J.* **88**, 408, 1938.
[4]) *B.A.N.* **12**, 11, (No. 443), 1953.
[5]) *Ap.J.* **109**, 215, 1949.
[6]) *A.J.* **59**, 6, 1954.
[7]) *P.A.S.P.* **62**, 60, 1950.
[8]) *B.A.N.* **11**, 17, (No. 403), 1949.
[9]) *P.A.S.P.* **65**, 118, 1953.
[10]) *Ap.J.* **108**, 60, 1948.
[11]) *B.A.N.* **10**, 391, (No. 393), 1948.
[12]) *Ap.J.* **106**, 135, 1947.
[13]) *B.A.N.* **12**, 242, (No. 459), 1955.
[14]) *Ap.J.* **119**, 346, 1953.
[15]) *B.A.N.* **12**, 223, 1955.
[16]) *Zs.f. Ap.* **32**, 69, 1953.
[17]) *B.A.N.* **12**, 31, 1953.
[18]) *Tadzhik Ann.* **1**, 3, 1941.
[19]) *P.A.S.P.* **62**, 60, 1950.
[20]) *L.c.*

FIGURE 1



The relation between photographic amplitude and radial-velocity amplitude for a number of RR Lyrae variables. The relations for the classical cepheids and the β Canis Majoris stars are indicated for comparison.

For some RR Lyrae stars the spectral classification depends on the group of lines selected for this purpose. Therefore it seems a safer procedure to derive the bolometric corrections from the colours of the stars. Reliable colour indices are only known for three of the stars used in this investigation. From six-colour photometry of RR Lyrae J. STEBBINS[1]) found the spectral type varying between A7 and F5. Using KUIPER's[2]) bolometric corrections, it follows that the bolometric amplitude for RR Lyrae is $0^m.08$ smaller than the photographic one. O. J. EGGEN[3]) measured colours for SX Phe. The spectral type from these colours varies between A2 and F0 and we find the bolometric amplitude to be $0^m.12$ larger than the photographic one. For XZ Cyg the difference between bolometric and photographic amplitudes comes out as $+ 0^m.15$, using colours by A. B. MULLER[4]), which correspond to spectral variations between A2 and F3. It thus seems that the differences between bolometric and photographic amplitudes for RR Lyrae stars are small on the average.

Therefore the relation between photographic amplitude and velocity amplitude can approximately also be used for bolometric amplitudes.

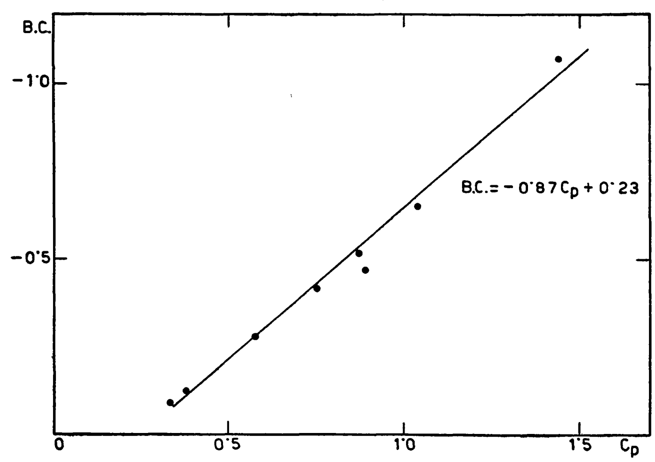
For the classical cepheids the bolometric corrections are much larger and less reliable. It was shown by EGGEN[5]) that the following relation between photographic amplitude and velocity range is accurately fulfilled for all types of classical cepheids:

$$\Delta v_R = + 3.6 + 31.7\, \Delta m_{pg} . \qquad (1)$$

EGGEN also found linear relations between the photographic amplitude and the period, and between the range in colour index Δc_p and the period.

The effective wavelength for the yellow measurements is 5450 Å and thus very close to the effective wavelength to which KUIPER's bolometric corrections apply. Combining EGGEN's relations between colour and spectral type (Figure 40 of EGGEN's paper) with the bolometric corrections given by KUIPER for supergiants of different spectral classes, we obtain a relation between colour and bolometric corrections,

FIGURE 2



The relation between bolometric corrections and EGGEN's colours.

which is shown in Figure 2. It is seen that over the interval of cepheid colours these corrections can be represented quite well by the linear relation

$$B.C. = 0.23 - 0.87\, C_p . \qquad (2)$$

The bolometric amplitude is given by

$$\Delta m_B = \Delta m_{pg} - \Delta C_p + \Delta B.C. = \Delta m_{pg} - 1.87\, \Delta C_p . \qquad (3)$$

Using EGGEN's relations between period and photographic amplitude and colour amplitude we have

$$\Delta m_B = 0.303\, \Delta m_{pg} + \varkappa . \qquad (4)$$

1) *L.c.*
2) *Ap.J.* **88**, 429, 1938.
3) *P.A.S.P.* **64**, 305, 1952.
4) *L.c.*
5) *Ap.J.* **113**, 367, 1951.

The values found for $\varkappa$ for cepheids of respectively type A, B and C are $+0.09$, $+0.03$ and $+0.05$, thus on the average $+0.06$. The smallness of this quantity demonstrates the consistency of the data. It should be remarked that the type C cepheids scatter rather widely around a linear relation. For the determination of the constants of the relation between period and photographic amplitude for type C cepheids we used the average of the two relations indicated in Figure 36 of EGGEN's paper. From (1) and (4) we obtain

$$\Delta v_R = -2.7 + 105\,\Delta m_B\,. \qquad (5)$$

The relation between velocity range and bolometric amplitude is thus seen to be also linear.

For δ Cep and η Aql the bolometric amplitudes may be estimated also from the six-colour photometry by J. STEBBINS [1]) and J. STEBBINS, G. E. KRON and J. L. SMYTH [2]). Interpolating the amplitudes for an effective wavelength of 5500 Å and applying the bolometric corrections we find for the bolometric amplitudes of these stars resp. $0^m.49$ and $0^m.48$. From EGGEN's data these values are $0^m.45$ and $0^m.41$. The values observed by PETTIT and NICHOLSON [3]) are somewhat higher. These authors found from radiometric measurements for δ Cep and η Aql bolometric amplitudes of $0^m.58$ and $0^m.55$. Because of the uncertainties in the bolometric corrections we consider these radiometric data as the most reliable. Assuming a linear relation without a constant term between velocity range and bolometric amplitude, which seems permissible in view of the smallness of the constant term in eq. (5), we find, averaging the radiometric results for δ Cep and η Aql,

$$\Delta v_R = 70\,\Delta m_B\,. \qquad (6)$$

[1]) *Ap.J.* **101**, 47, 1945.
[2]) *Ap.J.* **115**, 292, 1952.
[3]) *Ap.J.* **78**, 320, 1933.

It thus seems probable that the classical cepheids satisfy the same relation between velocity range and bolometric amplitude as the RR Lyrae stars.

The bolometric amplitude of the type II cepheid W Virginis as determined by H. A. ABT [4]) from colours and KUIPER's bolometric corrections has the value $0^m.96$. The velocity range of this star is 55 km/sec. This star, which probably is representative for the type II cepheids, thus fits well into the relation for RR Lyrae stars.

The long-period variables do not satisfy the relation found for the cepheids and RR Lyrae stars. The star o Ceti for example has a bolometric range of about one magnitude, as follows from the radiometric measurements by PETTIT and NICHOLSON [5]). The velocity range is about 15 km/sec. The star is therefore situated far above the relation for cepheids in the diagram.

From the point of view of the relation between velocity range and light range we thus have three distinct groups of variables. The cepheids and RR Lyrae stars all satisfy more or less the same relation over a large interval of periods, luminosities and spectra. The β Canis Majoris stars and the long-period variables on the other hand behave quite differently. It is interesting that also the phase relations between radial-velocity curve and light-curve for these stars differ from those observed in cepheids and RR Lyrae stars.

It did not seem worth while to separate the luminosity variations due to changes in the surface area of the variables from those due to the changes in surface brightness. The available data are not sufficiently accurate for such a detailed discussion. More photoelectric and radiometric data are urgently needed.

[4]) *Ap.J.* Supplement No. 1, 63, 1954.
[5]) *L.c.*