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## **The origins of the Korteweg-De Vries equation: Collaboration between Korteweg and De Vries**

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# Physics as a Calling, Science for Society

Studies in Honour of A.J. Kox

Edited by

Ad Maas and Henriëtte Schatz

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### 3 The origins of the Korteweg-De Vries equation: Collaboration between Korteweg and De Vries<sup>1</sup>

*Bastiaan Willink*

The Korteweg-De Vries equation that describes the behaviour of long wavelength waves in shallow water has generated an extensive body of literature since Zabusky and Kruskal rediscovered it in 1965, and today it is used in many physical and technological applications.<sup>2</sup> It is not necessary to add to the discussions about the contents of the equation or the genesis of the theory of non-linear partial differential equations, about which Eduard de Jager has recently written two papers that also serve as a general introduction to the topic.<sup>3</sup> Earlier, Robert Pego and others questioned the originality of the work by De Vries and Korteweg, especially in relation to the work done by Boussinesq (1842-1929).<sup>4</sup> Pego has pointed out that the Korteweg-De Vries equation is exactly equivalent to a pair of equations that appear in three of Boussinesq's publications. Moreover, in the third of these publications, the monumental (680 pages) *Essai sur les eaux courantes* (1877), the equation itself appears in a footnote.<sup>5</sup> De Jager, on the other hand, posits that it is plausible that, although the Korteweg-De Vries equation can be deduced from the work of Boussinesq by means of relatively simple substitutions, Korteweg and De Vries still arrived at new and important results via a different route than Boussinesq. Apart from the mathematical and hydrodynamical aspects of the discussion on priority and originality, there are other historical aspects to be considered. Anne Kox has previously described Korteweg, the senior author of the article, as the nexus between the Departments of Physics and Mathematics of the University of Amsterdam, and Ad Maas considers him a transitional figure in the mathematical and academic traditions.<sup>6</sup>

In this chapter, I will elaborate on the personal backgrounds of both Korteweg and De Vries to shed new light on the peculiar genesis of De Vries' dissertation (*proefschrift*) and the paper about the Korteweg-De Vries equation. Though Korteweg is known as a Dutch pioneer in the area of scientific bibliography, paradoxically enough something appears to have gone wrong with the review of the international literature by his doctoral student (*promovendus*) Gustav de Vries.

It seems strange that Korteweg, who has played such an important role in the professionalization of the exact sciences in the Netherlands, has somehow neglected to properly supervise his doctoral student, even though this was one of the essential aspects of his responsibilities as a professor. The biographies presented here will show that Korteweg and De Vries were people of flesh and blood, which makes the remarkable events surrounding De Vries' doctorate (*promotie*) in 1894 easier to understand.<sup>7</sup> I will make clear that, precisely in this period, Korteweg and De Vries were pressed for time, so in some respects they acted very hastily. I will also clarify what Korteweg and De Vries knew of the closely related work by Boussinesq. It was of particular importance to unearth these facts, because they have a tendency of disappearing without a trace if they are not discussed in detail. By presenting the situation in some detail, I will attempt to bring to life these two diverse personalities under stress, involved in a race against the clock.

I discovered the De Vries Archive, on which this chapter is based in part, as a result of my continued investigations following the publication of my book *De Tweede Gouden Eeuw* (The Second Golden Age) in 1998. Delving into Korteweg's archives was a straightforward task, as many of his documents have been preserved. The most important of these are in Korteweg's scientific archive, housed in the library of the University of Amsterdam.<sup>8</sup> Based on the Korteweg material in general, and in particular of the period around 1894, the picture can be made fairly complete. Eduard de Jager, making use of the newly discovered De Vries archives, shows convincingly that De Vries arrived at the results presented in his doctoral dissertation almost independently.<sup>9</sup> For this reason, De Vries deserves more credit than I have given him in *De Tweede Gouden Eeuw*, in which I concluded that Korteweg must have been the main author of the dissertation. Meanwhile, I have discovered other facts and documents that helped me find indirect evidence of De Vries' talent and made it easier to understand how he could arrive at this original work in the last year of his doctoral research. The interesting question is whether later in life he has done valuable scientific research as well. I will first present a more detailed impression of Korteweg as a person, so that his responses to the requirements of De Vries' doctoral dissertation in the period 1893-1895 can be better appreciated.

The Korteweg-de Vries (KdV) equation is a non-linear partial differential equation whose simplified form reads as follows<sup>10</sup>:

$$\frac{\partial u}{\partial t} - 6u \frac{\partial u}{\partial x} + \frac{\partial^3 u}{\partial x^3} = 0$$

Where  $u = u(x, t)$  denotes the wave amplitude at point  $x$  in space at time  $t$ . The non-linearity of this equation is due to the second term on the left-hand side of the equation



Fig. 1 – Diederik Korteweg (1848-1941) around 1898. Litho by Theo Molkenboer

## Life of Korteweg

Diederik Johannes Korteweg (1848-1941) was the eldest son in a family of six children, five boys and one girl.<sup>11</sup> His father, who was not a Catholic, was a judge in Den Bosch, the capital of the mainly Catholic province of Noord-Brabant in the South of the Netherlands. Not being Catholic, the family's social position was rather isolated. This is reminiscent of the situation of the painter Vincent van Gogh (1853-1890), whose father was a Protestant Minister in Nuenen, also in Noord-Brabant. The family's isolated position may have contributed to Korteweg's father becoming a Freemason, a move that was not unusual amongst progressive citizens of this period,<sup>12</sup> and it was within the Freemason milieu that

Korteweg's father made his real career. He achieved membership of the *Nederlands Hoofdbestuur* (Dutch Central Committee) under Prince Frederik (1797-1881), the brother of King Willem II, and in international arbitration commissions. He successfully passed on this independent and international outlook to his sons. His eldest son Died[erik] became a Freemason as well, but he left the institution at an early stage. The sons were all 'non-believers' and politically left-leaning or liberal. Died played a role in Amsterdam liberalism and he assisted the well-known activist and Dutch novelist Multatuli (Eduard Douwes Dekker, 1820-1887) financially.

The Korteweg brothers were all relatively successful. The second son, Bas [tiaan], was also a mathematician, who owed his position as a lecturer at the *Koninklijke Militaire Academie* (Royal Military Academy) in Breda to Died, but he was unable to do his own independent research. Politically, Bas went further than Died and became a socialist. Like his brother, Bas came into direct contact with Multatuli, since he was married to the actress Elize Baart, who had acted in Multatuli's play 'Vorstenschool' (School for Monarchs) on its opening night. Died's third brother, Jo[han], became a professor of surgery, while his fourth brother, Piet[er], was an important malaria research scientist.<sup>13</sup> When they began to despair of the future, Bas and his wife committed suicide together in 1879, a tragedy that reached the national press and affected the family especially deeply. Dutch novelist Jeroen Brouwers has devoted a small book to this tragic event.<sup>14</sup> Although Brouwers considers it speculative, I believe that the strongly competitive atmosphere among the brothers may have contributed to the suicide. As students, the brothers used to give each other puzzles to solve, each from their own field of study, to force each other to study other subjects than their own.<sup>15</sup>

Having grown up in a family where children were expected to be high achievers, it is peculiar that Died Korteweg had a relatively sketchy education. He first received elementary education at the *Instituut Berman, Opleidingsschool voor Breda en Willemsoord* (Berman Training Institute for Breda and Willemsoord)<sup>16</sup> and later he took private lessons from School Inspector Ringeling. Not until September 1865 did he go to the *Polytechnische School* (Polytechnical School) in Delft. This later became the Delft Technical University, but according to the regulations of 1863 it was classified at that time as an institute of Secondary Education (*Middelbaar Onderwijs*). Korteweg was not happy with the curriculum in Delft: he found it too applied and too 'unscientific'.

After receiving his secondary school teacher's qualifications (*Middelbare Akte*) in 1869, at age 21, he became a teacher, first in Breda and in 1873 in Tilburg, at a *Hogere Burger School* (HBS), the newly developed school type preparing for (higher) technical and business professions (abolished in 1968). While teaching, he continued studying mathematics in his free time. It was not until 1876, at 28, that he took the university entrance examination in Utrecht. Remarkably enough, Diederik Korteweg's initially slow progress was followed by a rapid succession of career advancements. In April 1877, Died passed his *Kandidaatsexamen* (a University exam

at the bachelor level that has now been abolished), one month before his younger brother (by three years) Jo obtained his doctorate in medicine.<sup>17</sup> Since 1877 Diederik Korteweg had been assisting the physicist and future Nobel laureate [1910] Johannes Diderik van der Waals (1837-1923) in solving mathematical problems. Soon afterwards, on 31 January 1878, he passed his 'Doctoralexamen' (Master of Science) with honours at the University of Amsterdam. By this time, he must already have been quite far along with his doctoral research about the speed of the propagation of waves in elastic tubes, since he obtained his doctorate in the same year. On 12 July 1878 he was the first doctoral graduate of the University of Amsterdam. According to his own statement, he had received much assistance from Professor Van den Berg in Leiden.<sup>18</sup> Two weeks after receiving his doctoral degree, he married Bientje, Baronesse d'Aulnis de Bourouill, whose brother was a pioneer of mathematical economics in the Netherlands. At first glance, this union appears to be an instance of 'marrying upwards', but the d'Aulnis family was not very class-conscious. To their disappointment, the pair remained childless.

Not much later, in 1881, Korteweg was appointed professor at the University of Amsterdam.<sup>19</sup> In addition to his aspirations to become a university professor, the issue of the 'competing brothers' played a role in Korteweg's strong achievement drive, as was also the case with De Vries and later with Korteweg's most famous student Luitzen Egbertus Jan Brouwer (1881-1966), who specifically competed with his cousins. At the university, Korteweg lectured in mathematics, mechanics and astronomy. This he did meticulously. He could be very demanding of students, but in later years he certainly became a fatherly figure, too. In addition to teaching, he worked – encouraged by Van der Waals – on the mathematical description of plaits on surfaces and the application of this to Van der Waals' theory on equilibrium phases of binary mixtures. The extensive papers by Korteweg on this subject appeared in 1889 and 1891, and have been rediscovered and put in their historical context by J. Sengers-Levelt.<sup>20</sup> Apart from the work by Pieter Hendrik Schoute (1846-1923), these were the first mathematical papers of international significance during the 'Dutch Second Golden Age'. Ad Maas has pointed out that, in some respects, Korteweg can be considered a transitional figure between professors who did useful work involving applied mathematics around the middle of the nineteenth century, and genuine scientific investigators, such as Brouwer, after 1900.<sup>21</sup> Nonetheless, in his role as a mathematical investigator Korteweg appears to be more significant than most 'professionals' of the generation succeeding him.

What is remarkable about Korteweg is how he spent considerable energy working in many diverse fields, and how he almost always obtained important results. After finishing his mathematical career, he did most of the editorial work for the collected works of Christiaan Huygens (1629-1695) from 1905 to 1927, during which period he made discoveries about Willebrord Snellius' (1580-1626) influence on René Descartes (1596-1650) and that of Descartes on Huygens. At the

same time, he immersed himself in work for the Dutch regional office of the International Catalogue of Scientific Literature, where all scientific papers and books were registered and classified. In the doctoral dissertation of Paul Schneiders, dealing with the library and documentation movement from 1880 to 1914, Korteweg's Sisyphean task is discussed in detail.<sup>22</sup>

## Life of De Vries

Gustav de Vries (1866-1934) came from a family that, in many respects, resembles that of Korteweg, although his father, a bookseller in Amsterdam, had a different social status than Korteweg's father as a judge in Den Bosch. Yet, an equally competitive atmosphere prevailed in the equally respectable 'bourgeois' family of Korteweg's doctoral student. To gain a better perspective on De Vries' family background, a book from 1936 about the De Vries family was a good starting point for further research.<sup>23</sup> Further material on the De Vries family was found in the possession of two of Gustav's grandsons, living in Leiden, who represent the only remaining De Vries family line. These documents, originally belonging to their grandfather, offer some interesting perspectives into Gustav de Vries' personal and professional life.

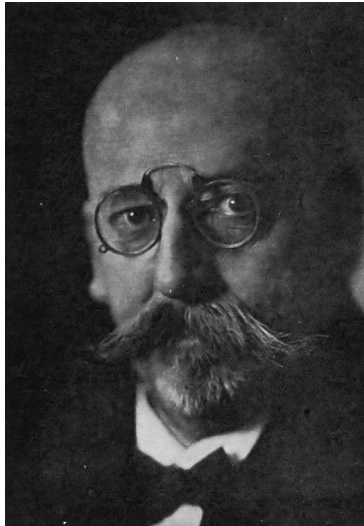


Fig. 2 – Gustav de Vries (1866-1934)

The available material clearly shows the resemblances between the two families. Two of Gustav's brothers made as much of a name for themselves as two of Korteweg's brothers did: Gustav's elder brother Jan was professor of mathematics at the University of Utrecht,<sup>24</sup> while a third brother, August, became Secretary-

General at the Ministry of Finance as well as *Staatsraad in Buitengewone Dienst* (State Councillor Extraordinary).<sup>25</sup> Died Korteweg and Jan de Vries – who were both eldest sons in their respective families – knew each other well, as they were both professors of mathematics and members of the *Koninklijke Akademie van Wetenschappen* (Royal Netherlands Academy of Arts and Sciences). They both helped their younger brothers, who were talented in mathematics, to obtain a position. As mentioned earlier, Died Korteweg managed to arrange a position at the Royal Military Academy in Breda for his brother Bastiaan – who later died tragically – and similarly, when he himself was appointed at the Delft Polytechnical School in 1894, Jan de Vries, was able to pass on to his brother Gustav his teaching position at the HBS in Haarlem. For a long time, Gustav was not quite comfortable in this position, possibly due to his frustration at his brothers' successes, although it is difficult to assess to what degree this has been the case.

Gustav de Vries reminds one of Bastiaan Korteweg also in other respects. Earlier, in 1892/93, he was a teacher at the Royal Military Academy in Breda, like Bastiaan Korteweg. In 1893/94, he became a teacher at the *Cadettenschool* (School for Cadets) in Alkmaar. Once he had taken over his brother's teaching position at the HBS in Haarlem on 4 April 1894, he remained there for the remainder of his career. During his first years of teaching all went well – he even substituted for a colleague who was ill –, but as time went on, the annual reports of the HBS for 1902, 1903 and 1904 repeatedly show passages like 'absent for a considerable period because of illness.'<sup>26</sup> In 1908, De Vries sent an open letter to Haarlem's *Wethouder van Onderwijs* (Alderman for Education) Thiel, which, ironically, resembles to some extent the pamphlet published by Bas Korteweg about his resignation from the Royal Military Academy. He explained that in 1902 he had spent five weeks in a sanatorium for mental patients, having had a nervous breakdown as a result of his disappointments due to failed job applications and lack of support from his Principal Brongersma.<sup>27</sup> Eventually, in 1909, he was dismissed from his post at the five-year HBS and appointed as a teacher at the less demanding three-year variant of the school.

## **De Vries' fight for recognition**

De Vries did not have an easy time having his academic papers published, even after he graduated from the university in December of 1894. Apart from his teaching, his family of four remaining children – after the death of his first child – demanded much of his attention, which he did not always give willingly. Still, he tried to continue studying and writing in his Haarlem home at Ripperdapark 45. In this way, he managed to finish two publications: a paper about cyclones in the Proceedings of the Royal Netherlands Academy in 1900 (which I have not looked into) and a textbook on arithmetic and algebra, *Beknopt leerboek der reken-en stellingen*, published by De Erven Bohn in 1907. A year later, he must have been un-

pleasantly surprised when a manuscript that he had submitted through Korteweg was returned to him by the journal *Nieuw Archief voor Wiskunde* (New Archive for Mathematics). Korteweg showed him the rejection letter from the journal's editor, professor of mathematics and algebraist at the University of Leiden, Kluyver, who wrote:

Amice (Dear Colleague),<sup>28</sup> I have perused the strange piece by Mr De Vries. It gives me the impression that the author has accidentally noticed a quite natural and unexceptional phenomenon, of whose true nature he makes no correct representation, and now more or less raises the status of what actually amounts to a commonplace thing to a miracle.

Kluyver then goes on to explain the weak aspects of the manuscript and calls De Vries an author of advanced age (*schrijver op leeftijd*). De Vries was 42 years old at the time!<sup>29</sup>

Kluyver's rejection letter dates from April 1908 and must have increased De Vries' frustration. From De Vries' long open letter to the Alderman, which appeared in print in August of the same year, it appears that he had many colleagues who found him socially inept, unable to discipline students, and incompetent as a teacher. Although De Vries appears to have done quite well teaching the lower grades, his demands on students in the higher grades may have been too high. Possibly, he may not have expected too much from the younger students, while in the older ones he may have looked for spiritual affinity. In his letter, De Vries cites the lack of support from his Principal and then continues, interestingly enough, that: '[this] added to the grief caused by the sudden death of a child'. The fact that he lists his professional disappointments first says something about the immensity of De Vries' frustrations. Clearly he was not happy in his teaching position, as he applied to all kinds of positions during the time he was required by the school's Principal to teach below his level of qualifications. Unfortunately these efforts were without success: he was never even put on the shortlist.

In spite of all these setbacks, De Vries published in 1912, through Korteweg, two papers about his own *calculus rationis* in the proceedings of the Royal Netherlands Academy of Arts and Sciences.<sup>30</sup> After 1912, his life became more fulfilling as he began to feel more at home in the new school where fewer demands were made on him. He taught mathematics for seven hours a week, in all the three grades, using his own textbook for the purpose – until 1911 –, as well as the one about planar geometry written by his brother and Janssen van Raaij. In addition, he taught accounting.

As De Vries was under less pressure, he had more time to focus on spiritual matters. In 1913, he was confirmed as a member of the Freemasons in the lodge 'Vicit vim virtus' (Virtue has overcome power), of which he became Master in 1916.<sup>31</sup> Together with some other members, he broke away from this lodge in

1916 and joined the new lodge of Kennemerland. For both lodges he organized long and elaborate discussions on a variety of philosophical subjects. Remarkable among the papers in the possession of De Vries' grandsons, in this respect, is a long analysis of Goethe's *Faust*, written in the footsteps of a French commentator.<sup>32</sup> As time went by, De Vries evolved in spiritual matters and eventually he became a spiritualist. This development may sound somewhat strange today, but, around 1900, many scientists, especially in Britain and the United States, carried out intensive 'psychical' research. In order to be able to explain the possibility of life after death, they theorized about the fourth and higher dimensions. Among them was the French Nobel laureate in medicine Charles Robert Richet (1850-1935) as well as the physicists Oliver Joseph Lodge (1851-1940), who proposed the term 'black hole', and Sir William Crookes (1832-1919), whose papers De Vries had studied in the course of his doctoral research. Over the years, he settled into a relatively uneventful existence of teaching and exercising his spiritual leanings. In December 1934, De Vries was knocked down by a car while returning from a séance in Haarlem-Noord.<sup>33</sup> He died not much later in the hospital, survived for three years by his ever-sickly wife.

At this point, there does not seem to be much about De Vries and his sad career that has not been discovered<sup>34</sup>. However, his writings are still an important topic of further study. Troelstra has suggested, and rightfully so, that an expert on mathematical-analysis should review and analyze De Vries' publications on the '*calculus ratiōnis*'. In addition, it would certainly be worthwhile to make a detailed comparison between De Vries' doctoral dissertation and the Korteweg-De Vries paper. In all, the remaining image of De Vries is that of a man with considerable communication problems.<sup>35</sup> He was a decent scientist and a good investigator, whose doctoral dissertation proved to be his most significant achievement. Directly or indirectly, this may have been due in part to the pressures put upon him by Korteweg and by his teaching obligations, which he clearly experienced as onerous.

## Korteweg's letter

In a footnote in my book *De Tweede Gouden Eeuw*, I published a letter from Korteweg, as dissertation advisor, to his doctoral student De Vries. From this letter I deduced that Korteweg was the main author of the dissertation. The content of the letter is surprising.<sup>36</sup> It dates from October 1893, one year before De Vries' dissertation defence, on 1 December 1894. It is striking for a dissertation advisor to criticize his doctoral student for having advanced very few ideas of his own so close to his dissertation defence. This appeared so peculiar to me that it induced me to start new investigations, which resulted in the discovery of the De Vries archive and Eduard de Jager's reappraisal of the contributions of De Vries.

Because of its historical interest, I am quoting the letter here:

Dear Sir,

To my regret I am unable to accept your dissertation in its present form. It contains too much translated material, where you follow Rayleigh and McCowan to the letter. The remarks and clarifications that you introduce now and then do not compensate for this shortcoming. The study of the literature concerning your subject matter must serve solely as a means for arriving at a more independent treatment [‘whereby you’ is put in by mistake, B.W.], expressed in your own words and in accordance with your own line of reasoning, prompted, possibly, by the literature, which should not be followed so literally. When you have mastered your subject matter to the extent that you can do this, then naturally you will also be confronted with the questions raised by Rayleigh and McCowan, which will provide you with the opportunity to display your strength.

In order to facilitate your progress, I send you the outline of a treatment of a single wave according to a slightly modified method due to Rayleigh<sup>37</sup> [to see whether I] could find a guiding principle to offer you for further elaboration. [...] Naturally, I cannot know whether I can succeed in this. [...] For an historical overview of the theory of waves, you should consult much more literature than you have done thus far, and this task will be difficult to carry out in Alkmaar. Your introduction consists too exclusively of issues that one can equally well find in handbooks (Lamb and Basset).

It is obviously a disappointment for you who must have deemed to have already almost completed your task, to discover that you have apparently only completed the preparatory work. In the meantime do not be downhearted. With pleasure I will do my best to help you mount the horse [...]<sup>38</sup>

## The doctorate of De Vries

De Vries must have been under considerable pressure in the period after he had received this letter. It appears, though, that this was also the case for Korteweg. The first directly relevant event that must have put Korteweg under quite a strain during the period surrounding the dissertation defence by De Vries was the death of Nicolaas de Roever (1850-1893), Archivist of the City of Amsterdam. As De Roever’s wife had already died earlier, Korteweg and his wife, though in their forties and childless, adopted the three De Roever children, two girls and one boy, in 1893 or 1894. The death in 1896, at age eleven, of their adopted son Arend, must surely have been a devastating blow to the family, too. It is not clear whether Arend had been ill long before his untimely death, but if this was the case, it may also have added considerable stress.<sup>39</sup>

Apart from Korteweg’s sudden fatherhood, the other relevant event in the period around De Vries’ doctorate was the fact that Korteweg became *Rector Magnificus* (Chancellor) of the University of Amsterdam (1893-1894) and had to work on,

among other things, the text of his acceptance address entitled 'Het Bloeitijdperk der Wiskundige Wetenschappen in Nederland' (The Golden Age of mathematics in the Netherlands). In this address, delivered in January 1894, he discussed the seventeenth-century heyday of mathematics, thus anticipating his historical research during the latter part of his life as a scientist. All these time-consuming responsibilities, in combination with his teaching duties, make it clear why Korteweg was rather blunt in his judgement when he received the first draft of De Vries' dissertation. As a doctoral advisor, Korteweg was a meticulous, level-headed, and efficient worker of great versatility, but he had apparently reached the limits of his capabilities. When reading the dissertation draft, he must have looked back on his own efficient doctoral research and must have realized that De Vries had not really made much progress. In the past few years he had been working on the history of the Dutch mathematicians of an earlier age and on the theory of analytic surfaces, and presumably he had not had a chance to closely follow the literature on hydrodynamics. Thus, while he did see that De Vries had studied John Scott Russell (1808-1882), George Biddell Airy (1801-1892), John William Strutt, 3rd Baron Rayleigh (1842-1919), John McCowan (1863-1900), Sir Alfred George Greenhill (1847-1927), and Boussinesq, he did not notice that important publications by Boussinesq were missing.

### **Pressure and carelessness**

In view of De Vries' circumstances in 1893-1894, it is interesting to elaborate on the interaction between him and Korteweg and to find out more about their knowledge of the work of Boussinesq. As I have shown earlier, De Vries had submitted the first draft of his doctoral thesis to Korteweg, just before his elder brother Jan arranged for his new position in Haarlem. Korteweg's rebuke of De Vries made him shift into higher gear, but meanwhile the young HBS teacher had already become very busy teaching. In fact, after his doctorate, De Vries complained that he no longer had any time for writing or research because of the large number of tests he had to grade, though he must already have been obliged to do this kind of work before. Yet, while the paper for the Royal Society was being prepared by 'my young friend and myself', as Korteweg writes in his submission letter to the editor of *Philosophical Magazine*<sup>40</sup>, Korteweg insisted on much more scholarship, as is also seen in the letter quoted earlier. Thus, at the beginning of 1894, both dissertation advisor and doctoral student were under considerable pressure to deliver and had little time to do so. They must have thought: it is now or never.

Since De Vries did, unfortunately, not include a bibliography in his doctoral dissertation, we cannot be sure, but it appears that the pressure he was under at the time affected De Vries' literature review. He left behind some neat summaries of the literature he studied, which included papers by Boussinesq that were pub-

lished in 1870 and 1871<sup>41</sup>. Thus, we are able to follow him closely in his readings. He began with the English literature on his subject of interest and must have encountered Boussinesq's name in Rayleigh's paper 'On Waves' that appeared in April 1876 in *Philosophical Magazine*. As Rayleigh<sup>42</sup> writes:

I have lately seen a memoir by Mr Boussinesq (1871, *Comptes Rendus*, Vol. LXXII.), in which is contained a theory of the solitary wave very similar to that of this paper. So far as our results are common, the credit of the priority belongs of course to Mr Boussinesq.

Apart from Boussinesq's paper of 1870 and his first paper of 1871, De Vries later also studied some other French publications, including papers by Adhémar Jean Claude Barré de Saint-Venant (1797-1886) from 1885. One of these 1885 Saint-Venant papers, published in *Comptes Rendus*, Vol. CI, makes a clear reference to Boussinesq's *Essai sur la théorie des eaux courantes* (Treatise on the theory of running water), his 680-page treatise published in 1877. As Pego has pointed out, the Korteweg-De Vries equation is used by Boussinesq in a somewhat different form in earlier articles, but in the same form in a footnote of the *essay*. Later, De Vries explicitly refers to Boussinesq's *Essai* on pages 38 and 40 of his summary of Saint-Venant's paper and he even writes at the bottom of page 37: 'Essai sur les eaux courantes', followed by an exclamation mark '!' (See Fig. 3).<sup>43</sup>

De Jager believes that Korteweg and De Vries might also have cited other work by Boussinesq if only they had had more access to international sources, as he writes in his extensive paper:<sup>44</sup>

It is somewhat surprising that Korteweg and De Vries refer in their paper only to Boussinesq's short communication in the *Comptes Rendus* of 1871 and not to the extensive article in the *J. Math. Pures et Appl.* and the *Essai sur la théorie des eaux courantes* of 1872, respectively 1877. However, we should realize that the international exchange of scientific achievements in those days was not at the level as it is today.

I believe, though, that De Jager is jumping to conclusions here. In spite of Korteweg's exhortations to De Vries to study more, there are no indications that De Vries was aware of Boussinesq's second 1871 paper, or of his 1872 paper. There is also nothing to suggest that De Vries read the 1877 treatise, which is especially odd, considering that he put an exclamation mark after its title in his literature summary. Very probably a copy of the work was already available in

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tusschen  $\xi$  en  $H + \zeta$ , en wordt  $\frac{d^2 \zeta}{dx^2} (2H\xi + \zeta^2)$  aange-

geleken als  $\frac{d^2 \zeta}{dx^2}$  verandert in  $\frac{d^2 \zeta}{dx^2}$ .

1827 (7)  $\frac{d^2 \zeta}{dx^2} = g(H + \zeta - x) + \frac{d^2 \zeta}{dx^2} \cdot \frac{H - \zeta}{2H}$

elliptische maar  $x$  en in (8) gebracht, konint en

(8)  $u' = -g \frac{d^2 \zeta}{dx^2} - \frac{d^2 \zeta}{dx^2} \frac{H - \zeta}{2H}$

id. beide vermengen met  $\frac{dx}{dx}$  en  $\frac{d^2 \zeta}{dx^2}$

(9)  $\frac{1}{H + \zeta} \int_0^{H + \zeta} u' dz = -g \frac{d^2 \zeta}{dx^2} - \frac{H}{3} \frac{d^2 \zeta}{dx^2}$

Hydrogely (2) geldt  $u' = \frac{du}{dt} + u \frac{du}{dx}$ , en die de laatste term als

product van kleinste grootheden te verwaarloogen,

de eerste term heeft ongeveer als gereduceerde  $\frac{dV}{dt}$  en gereduceerde

over  $u'$  in te verwaarloogen het  $\frac{dV}{dt} + u \frac{dV}{dx}$ . In geldt derhalve

(10)  $\frac{dV}{dt} + u \frac{dV}{dx} + g \frac{d^2 \zeta}{dx^2} + \frac{H}{3} \frac{d^2 \zeta}{dx^2} = 0$  nuunt de Contin.

Deze Solitaire  $\zeta$  en  $V$  veranderen hierbij niet (wijze

uitz.) als  $t$  met  $dt$  ontrent  $x$  met  $cdt$  ongeveer

gegeven deze functies van  $x$  en  $t$ . In (10) kan men

$\frac{d^2 \zeta}{dx^2}$  door  $-\omega \frac{d^2 \zeta}{dx^2}$  vervangen. En konint dan

(11)  $\left\{ \begin{aligned} \frac{d}{dt} [-\omega V + \frac{1}{2} V^2 + g \zeta + \frac{H \omega^2}{3} \frac{d^2 \zeta}{dx^2}] &= 0 \\ \frac{d}{dx} [-\omega \zeta + H V + V \zeta] &= 0 \end{aligned} \right.$

en gereduceerde  $\frac{d^2 \zeta}{dx^2}$  bes. Cond., alle termen zijn ongeveer

des ongeveer  $\omega$ . Men kan hieruit (approx.) af

(12)  $\omega^2 = g H (1 + \frac{3}{2} \frac{\zeta}{H} + \frac{H}{3} \frac{d^2 \zeta}{dx^2})$

F. Inai tusschen deze omtrent!

Fig. 3 – De Vries mentioning Boussinesq’s Essai at the bottom of page 37 of his summary of Saint-Venant, 1885.

the Netherlands at that time, as it had been given an award by the French Académie des Sciences.<sup>45</sup> In the library of the Delft Technical University, for example, I found a copy of the work that must have arrived there not long after its publication in 1877. It is also possible that De Vries got lost in the voluminous and difficult book. In that case De Vries should have discussed his difficulties with Korteweg, but there is no trace of this to be found, either in the dissertation, or in the correspondence and the De Vries archives. However, not only De Vries appears to have been remiss in not paying attention to Boussinesq’s work: it is equally strange that Korteweg appears not to have been informed about the award-winning treatise through other channels.



Fig. 4 – Joseph Valentin Boussinesq (1842-1929)

In any case, under pressure, De Vries has, consciously or unconsciously, ignored something of significance. Notwithstanding Korteweg's considerable attention to bibliographically responsible research, which is amply evident from the letter quoted earlier, in which he explicitly asks for additional literature study, and in spite of De Vries' carefully prepared summaries, it appears that no study of Boussinesq's works of 1872 and 1877 has followed De Vries' exclamation mark. In the case of Rayleigh, it was already peculiar that in 1876 he had not yet investigated the 1872 literature, but then Boussinesq's major work of 1877 and Saint-Venant's citations of 1885 had not been published. However, by 1894, De Vries' omission, in my opinion, qualifies as gross negligence, even though at that time he and Korteweg may have already been following a different line of theoretical inquiry.

It is equally strange that Boussinesq, who died only in 1929, never made any complaints to Korteweg or to the Editor of *Philosophical Magazine*. Saint-Venant did not live long enough to experience this 'comedy of errors'. In 1885 he was already 88 years old and he died the following year. Earlier, he had played a significant role in Boussinesq's life and had helped him with his appointment to his first professorial position in Lille, the place where Louis Pasteur (1822-1895) had also begun his career. Boussinesq had looked forward to collaborating with the energetic old man when he moved to the Sorbonne in 1886, but unfortunately that was not to be. Later, especially after becoming Dean of the Department of Natural Sciences of the *Académie des Sciences*, Boussinesq certainly must have had an international network. It would be worth investigating whether there was direct or indirect contact between Boussinesq and Korteweg.<sup>46</sup> Korteweg has, for instance, exchanged some correspondence with Paul Emile Appell (1855-1930), who must have known Boussinesq well.

To explain the neglect by Rayleigh of the 1872 Boussinesq paper and his lack of response to the Korteweg-De Vries paper of 1896, we should take into account that, at that time, in large countries the national scientific cultures were dominant. Rayleigh read in particular the works by British scientists and the editors of *Philosophical Magazine* did not know the international literature sufficiently well either. Boussinesq, on the other hand, read mostly works by French authors. Occasionally they looked across their national borders, but in those cases they turned chiefly to Germany. There the first two volumes of the Poggendorff bibliography, the 'Biographisch-Literarisches Handwörterbuch zur Geschichte der Exakten Wissenschaften' (Biographical Literary Hand Dictionary for the History of the Exact Sciences) <sup>47</sup> that were, above all, of an historical nature, had already been published. The Catalogue of Scientific Papers of the Royal Society also already existed, but evidently not every professional investigator consulted it.

Korteweg and De Vries, living in a relatively small country, were clearly more internationally oriented than Rayleigh and Boussinesq, and Korteweg was also a good bibliographer. Yet, under the pressure of time they failed to be as meticulous as they should have been. De Vries stopped studying Boussinesq after having flagged Boussinesq's work with an exclamation mark in his notes to indicate its significance. It is conceivable that he did not inform Korteweg about the existence of this extensive treatise, especially if Korteweg had already decided that all the rewriting had been more than enough. Apparently even Korteweg, in spite of his insistence by the end of 1893 on much more study of the literature, made a mistake because of being too busy. The positive side of the affair, though, is that the pressure by Korteweg, as well as De Vries' own circumstances caused him to surpass his own expectations. Of course, in the final analysis we should be happy with the doctoral dissertation by De Vries and the Korteweg-De Vries paper, especially since it is certainly conceivable that De Vries would not have obtained his doctorate at all if he had studied Boussinesq's publications more deeply.

## Notes

1. I thank Dr Behnam Farid for his translation of an earlier version of this chapter, published in Dutch in *Nieuw Archief voor Wiskunde* 5/7 (September 2006), pp. 179-185; the translation was published on 27 October 2007, on the website of arXiv, section History and Philosophy of Physics, Cornell University: arXiv:0710.5227. I also thank him for his insightful comments concerning the Korteweg-De Vries equation, the Boussinesq equations, and De Vries' reference to Boussinesq's *Essai* in the footnote on page 37 of De Vries' excerpt, shown in Fig. 3. Thanks are also due to Dr. Sengers-Levelt for commenting on an earlier draft of this paper based on a lecture I delivered in September 2003 at the symposium dedicated to Korteweg and De Vries at the University of Amsterdam.
2. See e.g., the entry on the *Korteweg-de Vries Equation* in Wikipedia.

3. De Jager (2004); De Jager (2006) For further relevant bibliography see the references in these publications.
4. Joseph Valentin Boussinesq was born near Béziers in 1842. After being awarded the Poncelet prize in 1872, he became a professor of mathematics at the University of Lille. In 1886, he moved to the Faculty of Science at the University of Paris. He was a member of the *Académie des Sciences*. Boussinesq was a modest man, but a prolific author. He died aged 86 in 1929. There is a biography of Boussinesq in the *MacTutor history of mathematics archive*: Joseph Valentin Boussinesq.
5. Pego (1997). The relevant publications by Boussinesq are: Boussinesq (1871), equations 5a and 7a; Boussinesq (1872), equations 29 and 34; Boussinesq (1877), equations 283 and 291. In this 1877 book the Korteweg-De Vries equation itself appears for the first time in a footnote on page 360.
6. Kox (2000); Maas (2005).
7. The key publications are De Vries (1894); Gustav de Vries' dissertation; and Korteweg & De Vries (1895).
8. After Korteweg's death, his pupil Gerrit Mannoury arranged Korteweg's correspondence in alphabetical order of the correspondents' names. He did so in the Korteweg house – built by Pierre Cuypers and featuring a two-floor conservatory – on the corner of Vondelstraat opposite the *Heilige Hartkerk* (the Church of the Holy Heart), where he was taught for a long time by Korteweg in preparation for his university education. Unfortunately, the archivist at the Amsterdam University Library has rearranged the entire archive, this time ordered chronologically, so that I had to spend many hours searching for the Korteweg-De Vries correspondence. The archive also contains 'the Mountain of Brouwer', the many letters exchanged between Korteweg and L.E.J. Brouwer around 1906 (fortunately Korteweg preserved his own drafts), which Dirk van Dalen has used in his books about Brouwer. Also, Dr. Sengers-Levelt has searched through the Korteweg Archive for her book on thermodynamics in the Netherlands around 1900 (Sengers-Levelt (2002)).
9. Communicated to me in writing by Eduard de Jager.
10. De Jager, *op.cit.* (2006), 1
11. This year I am hoping to finalize a book about the Korteweg family in which Died Korteweg, his parents, brothers and their nuclear families are discussed in detail. In this book, I shift my emphasis from their instilling 'competitiveness' in the children to more general educational factors in the family. Their strict education at home may especially explain many apparently strange events. Around 1900, there was an educational climate among the Dutch bourgeoisie that was comparable to that defended by the Chinese-American Yale Professor Amy Chua in her book *The Battle Hymn of the Tiger Mother* (2011), New York: Penguin Press.
12. No academically rigorous book on the history of Freemasonry exists in the Netherlands, although recently a Chair at the University of Leiden has been established to deal with this history. It is well known, for example, that Multatuli as well as the poet and novelist Jacob van Lennep (1802-1868) were members. The Masonic lodges, under the patronage of Prince Frederik, were only open to men and were meeting places particularly for members of the higher social classes.

13. Johan Korteweg was my great-grandfather. I received a great deal of Korteweg-material through my grandmother, Ms W.M. van Steeden-Korteweg, and her only son, my uncle Jaap van Steeden. Johan's eldest son Adri was a medical doctor who was an internist at the Alkmaar hospital. The Died's fourth brother, the malaria specialist Piet, had a son named Remmert, who later became a pioneer in cancer research. The fifth Korteweg brother, Willem, died while studying philosophy at the University of Leiden. In addition to the brothers, there was a much younger sister, Ms Rand-Korteweg, of whom little is known.
14. Brouwers (1993).
15. Mentioned in autobiographical notes by my grandmother.
16. *Gedenkboek* (1932): entry about his brother Johan Korteweg.
17. Johan did not have a gymnasium diploma either; he too had received private education.
18. *Onze Hoogleraren* (1898), p. 256 (brother Johan follows on page 267).
19. Brother Johan was appointed in 1887.
20. Sengers-Levelt (2002).
21. Maas (2005).
22. Schneiders (1982).
23. *Een Friesch geslacht* (1936). The research into De Vries' background was a much more time-consuming task than the search into Korteweg's. There were more mathematicians named De Vries, but of course they were not all related to Gustav. Soon it became evident that Gustav de Vries was not related to Korteweg's junior colleague Hendrik de Vries. The 'golden' hint about this came from De Vries himself. In his doctoral dissertation he was required to put down his place of birth: Amsterdam. When searching the ten-year indexes of the Amsterdam Municipal Registry, his birthplace and his unusual first name led to his parents and his date of birth: 1866. At the Central Bureau of Genealogy in The Hague, a number of other relatives were found through the register of persons. After a few weeks of investigations, two elderly women from the Veluwe area in the Province of Gelderland turned out to know a great deal about Gustav de Vries. They were Mrs De Vries-Helmert from Ermelo, who had been married to David de Vries and was 87 years old in 1999, and Ms Rie Bosman from Harderwijk, 73 years old in 1999 and the partner of the late Rubertus Jan de Vries (Ruurd). These women explained that the family had counted amongst its members both right-wing individuals with anti-Semitic leanings (Gustav appears to have been one of them) and left-wing members of the resistance who were fighting the German occupation of the Netherlands during World War Two.
24. The Jan de Vries line of the family died out, when his grandson Jan, who was born in 1920 to his son Dr. Jan de Vries – a mathematics teacher at the Amersfoorts Gymnasium – was executed on 2 March 1945 in retaliation for an assassination of four German military men by the occupying German army at Varsseveld in the Province of Gelderland, together with 49 other resistance fighters. (Source: *Nederlands Instituut voor Oorlogsdocumentatie*, The Netherlands Institute for War Documentation).
25. At a meeting with the grandsons of Gustav de Vries, in a house on a small canal in Leiden, I sensed some degree of distrust. Here was a relative of Korteweg who, according to De Vries family lore, had run away with their grandfather's ideas! I had

- just put behind me a prolonged, self-imposed ‘mission impossible’ after completing my book, to check once and for all if I had not done an injustice to De Vries, so I felt very uncomfortable. Fortunately, some gaps were filled in a little later by Gustav’s brother August. It transpired that August – who had bought an additional name and was later called August Laman de Vries – was an uncle of my grandfather Willink through the Roelvink family. In this way, at least one traceable genealogical connection between Korteweg and De Vries turned out to exist, and I was able to point out that my family loyalty was not in question.
26. Municipal Archive of Haarlem (*Gemeente Archief Haarlem*), since merged with the Provincial Archive of North Holland (*Noordhollands Archief*); Archives of Predecessors of the Spaarne Schools in Haarlem (*Archieven Rechtsvoorgangers Spaarne Scholengemeenschap te Haarlem*); Archives of the HBS-B, 1864-1958, inv. nrs. 35 and 36; Annual Reports 1889-1906 and 1907-1920.
  27. This letter was part of the documents held by De Vries’ grandsons.
  28. The word *amice* is a form of the Latin *amicus* (friend).
  29. The letter originates from the Korteweg Archive at the Library of the University of Amsterdam. Lourens van den Brom, with whom I became acquainted through his archival research pertaining to Korteweg’s academic career, has examined the arguments put forward by Kluyver and has shown them to be plausible.
  30. Professor Anne Troelstra has reviewed these papers and has also found that they did not go deeply into research done by others. According to him, they have nothing to do with fundamental research: ‘Perhaps someone with analytical expertise can find something more in these. My impression is that Korteweg has encouraged De Vries to build on the works of others in his doctoral research, but that De Vries has subsequently been working in too isolated an environment’. [Translated from Dutch, B.W.].
  31. It looks as though he had become a freemason earlier. As early as 1908 he used the word ‘brother’ in a letter, but I have not been able to find out why he did so. The data concerning his freemasonry originate from the Cultural Masonic Centre (*Cultureel Maconniek Centrum*) Prins Frederik in The Hague.
  32. This document is also with the brothers De Vries in Leiden.
  33. Source: Mrs de Vries-Helmert.
  34. Troelstra is right in suggesting that an expert on mathematical analysis should be consulted to review the publications on De Vries’ ‘*calculus ratiōis*’. In addition, it is certainly worthwhile to make a detailed comparison between De Vries’ doctoral dissertation and the Korteweg-De Vries paper.
  35. Specialists would nowadays perhaps speak of Asperger’s syndrome.
  36. Willink, (1998) p. 247. For a summary of my findings regarding the educational causes of the second Dutch golden age in science, see also: Willink (1991). In my book and in this chapter the important general social context of the educational reform, which is the topic of my dissertation Willink (1988), is discussed only marginally. For readers of books and articles in Dutch: it is also the central topic of the debate between Maas and myself in *Tijdschrift voor geschiedenis*: Maas (2001) and Willink (2001).
  37. According to the submission letter, this very point is central to the final version of the Korteweg-De Vries paper.

38. Willink (1998), p. 247. The letter has been preserved in the Korteweg Archive at the University of Amsterdam (see footnote 8). I have not been able to find Korteweg's own outline to which he refers, either in the Korteweg Archive or in the De Vries Archive in Leiden that are maintained by De Vries' grandsons.
39. Diederik Korteweg and Bientje d'Aulnis are buried with him in Zorgvlied, a cemetery in Amsterdam.
40. In the Korteweg Archive at the library of the University of Amsterdam.
41. See his summaries in the De Vries Archive maintained by his grandsons in Leiden.
42. From the way Rayleigh refers to Boussinesq as Mr, it appears that he either did not know that Boussinesq was already a professor at a university or that he did not find this worth noting.
43. The symbol 'F' in the footnote, followed by 'Essai...' refers to the expression on line 12 from the top. The translator of the second version of this paper, Dr Behnam Farid, suggested that the expression concerns the total derivative of  $v \equiv v(x, t)$ , where  $x \equiv x(t)$ , with respect to  $t$ ; De Vries uses the symbols  $dv/dt$  and  $dv/dx$ , rather than  $\partial v/\partial t$  and  $\partial v/\partial x$ , respectively, for partial derivatives. The passage on which page 37 of De Vries' summary (shown in Fig. 3) has bearing, is taken directly from Saint-Venant's paper, p. 1105. De Vries' exclamation mark in the footnote seems to indicate that he may have considered the derivation significant.
44. De Jager (2006), p 21.
45. Grattan-Guinness (2000), p 601.
46. I have not been able to find anything about this in the Korteweg Archive; however, as in the case of Brouwer, a great deal has yet to be found.
47. Johann Christian Pogendorf (1796-1877).

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