

## ZODIACAL LIGHT IN THE SOLAR CORONA

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## ABSTRACT

The observations of Grotrian and Öhman show that the continuous light of the corona consists of two components: a component *K*, strongly polarized and showing no Fraunhofer lines, and a component *F*, unpolarized and having unbroadened Fraunhofer lines. The former component is believed to arise from electron scattering; the latter is the subject of this paper.

An analysis of the photometric data on the corona and on the zodiacal light indicates that the *F*-component dominates in the outer corona and suggests strongly that this component is just the extension of the zodiacal light. Other suggested explanations seem to be insufficient.

A theory of the scattering in interplanetary space is given. The scattering by a solid particle consists of diffraction and reflection; the first and the most important part has been neglected in earlier discussion. Curves of the surface brightness as a function of the elongation are obtained for particles of various sizes on the assumption of a constant space density. The assumption that there are no particles within 0.1 A.U. from the sun changes the results only slightly.

Theory and observations are then combined to give the distribution function of the radii, *a*. It has the form

$$n(a) = 3.5 \times 10^{-20} a^{-2.6}.$$

Particles with radii larger than 0.035 cm are less abundant. In addition, the calculation shows that the color of the *F*-corona should be slightly redder than the sun. Comparison with the brightness of the zodiacal light at 90° elongation shows either that the albedo of the particles is about 1 per cent or that the space density increases a little toward the sun.

The mean free path in the interplanetary medium is 10<sup>6</sup> A.U. The mean space density is 5 × 10<sup>-21</sup> gm/cm<sup>3</sup>. The largest particles found in this analysis are of the same size as telescopic meteors but about ten thousand times more abundant. The difference is probably real and indicates the existence of a quiescent cloud besides the rapidly moving meteors.

Some methods by which the theory outlined in this paper may be confirmed and by which further data on the real corona may be obtained are suggested. The *F*-component is eliminated from Baumbach's table of electron densities, and a revised table is given.

In 1934 W. Grotrian<sup>1</sup> discovered that the Fraunhofer lines in the corona are weaker, but not broader, than the corresponding lines in the solar spectrum. The light of the corona is therefore thought to consist of two superposed components, denoted by *F* and *K*. The *K*-component is characterized by the fact that it has no Fraunhofer lines; they are blurred out by Doppler effect. This component can be explained on the usual assumption that the light is scattered by an atmosphere of free electrons around the sun. On the other hand, the *F*-component has Fraunhofer lines which have the ordinary depth and width. Near the limb it has only a small fraction of the intensity of the *K*-component, but it decreases more slowly than does the *K*-component with increasing distance from the limb and exceeds the *K*-component at distances larger than 1 solar radius. Presumably, the *F*-component contributes most of the light of the outer corona.

The explanation suggested by Grotrian is that the *F*-component arises from scattering by solid particles, possibly related to the particles that cause the zodiacal light. It appears improbable, however, that such particles could exist in the hot region near the sun. But the observations themselves appear reliable. Spectrographic observations by J. H. Moore<sup>2</sup> have given similar results. Furthermore, the way in which the polarization of the corona changes with the distance from the limb strongly indicates the existence of an unpolarized component that dominates in the outer corona. This component may be

<sup>1</sup> *Zs. f. Ap.*, **8**, 124, 1934.

<sup>2</sup> *Pub. A.S.P.*, **46**, 298, 1934.

identified with the  $F$ -component. Recently, polarigraphic observations of the eclipse in 1945 by Y. Öhman<sup>3</sup> have established beyond doubt the existence of these two components. In particular, Öhman has obtained some spectrograms for the different directions of polarization which demonstrate that the polarization is due wholly to the  $K$ -component. The characteristics of the two components, as summarized by Öhman, are shown in Table 1.

The last difference in Table 1 is particularly significant. It shows that the  $F$ -component at the 1945 eclipse had no relation to the equatorial streamers<sup>4</sup> that mark the form of the corona at a minimum in solar activity. Further, it has also been noticed<sup>5</sup> that the isophotes at very large distances from the center are nearly circular. It is therefore quite likely that the  $F$ -component has a perfect circular symmetry both in its inner and in its outer parts. Consequently, it is possible that this component has no direct connection with the sun at all and is caused by a diffusing medium somewhere between the sun and the earth. This suggestion implies that—as far as *solar* physics is concerned—the corona outside one or two solar radii from the limb is, for the greater part, a spurious corona. In view of the present widespread interest in the physical state of the corona, a clarification of this problem would be useful.

TABLE 1  
CHARACTERISTICS OF THE TWO COMPONENTS OF THE CORONA

$K$ -Component	$F$ -Component
No spectral lines	Fraunhofer lines
Steep decrease of intensity outward	Slow decrease of intensity outward
High degree of polarization	Low polarization, or none
Same color as the sun	Same color as the sun
Much stronger near the sun's equator than near the sun's pole	Equally bright in polar and equatorial regions

#### PHOTOMETRIC DATA

Let  $\rho$  denote the angular distance from the center of the sun, measured in solar radii. Then, according to S. Baumbach,<sup>6</sup> the surface brightness of the corona in the range  $1.02 < \rho < 5$  can be represented by the formula

$$H(\rho) = \frac{0.053}{\rho^{2.5}} + \frac{1.425}{\rho^7} + \frac{2.565}{\rho^{17}},$$

where  $H(\rho)$  is expressed in the unit which is  $10^{-6}$  times the surface brightness at the center of the disk; we shall denote this unit by "unit<sub>B</sub>."

Grotrian<sup>1</sup> expresses the results of his measurements in an arbitrary unit, "unit<sub>G</sub>." By plotting the curves on a logarithmic scale, it was found that Grotrian's curve for the  $F$ - and  $K$ -components combined coincides approximately with Baumbach's curve if

$$\log \text{unit}_G - \log \text{unit}_B = -2.20.$$

Figure 1 shows the resulting position of these curves and also the curves for the separate  $F$ - and  $K$ -components. Since the steep decrease of the  $K$ -component continues beyond  $\rho = 2$ , we may assume that the light beyond  $\rho = 3$  is almost entirely due to the  $F$ -component. Thus we have fairly good curves for the  $F$ -component in the ranges  $1.2 < \rho < 2$  (Grotrian) and  $3 < \rho < 5$  (Baumbach). These curves are shown also by the heavy lines in the upper left corner of Figure 2, in which the  $\rho$ -scale is logarithmic. They can be con-

<sup>3</sup> *Pop. astr. Tidskr.*, 27, 133, 1946.

<sup>5</sup> Cf. D. K. Bailey, *Ap. J.*, 87, 74, 1938

<sup>4</sup> Photographs in this *Journal*, 102, 135, 1945.

<sup>6</sup> *A.N.*, 263, 121, 1937.

ned by a smooth curve with an initial slope of  $-5$  and a final slope of  $-2.5$ . We can accordingly represent this curve by the approximate expression,

$$H_F(\rho) = \frac{0.053}{\rho^{2.5}} + \frac{0.20}{\rho^5}.$$

We can now estimate the total light. For the solar disk we may assume a limb-darkening coefficient of 0.8. Integrating the surface brightness of the various components over the entire surface, we find

$$I_{\text{disk}} = 0.8 \times 10^6,$$

$$I_{\text{corona}} = 1.12,$$

$$I_{F\text{-corona}} = 0.34.$$

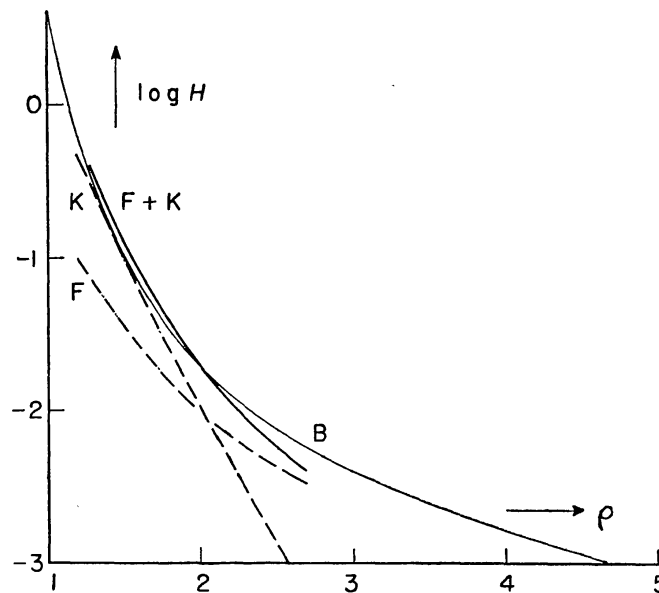


FIG. 1.—Surface brightness of the corona against distance from the center of the sun. *B*, Baumbach's curve; *F + K*, *F*, *K*, Grotrian's curves for the total light and the separate components.

Consequently, the *F*-component contributes 30 per cent of the total light of the corona. It should be noted, however, that the ratio of the two components is not constant. It varies considerably with the position angle and may also vary from eclipse to eclipse. Photometry is mostly made in the region of the coronal streamers. Since these streamers consist of electrons, they belong to the *K*-corona. It is possible, therefore, that an unbiased estimate of the light of the *F*-component would amount to even more than 30 per cent of the total light. On the whole, the accurate photometric separation of the *F*- and *K*-components deserves further attention.

Nothing reliable is known about the surface brightness between elongations  $1^{\circ}20'$  ( $\rho = 5$ ) and  $30^{\circ}$  from the sun. Observations in this region, both during an eclipse and after sunset, are hindered by scattered light in the earth's atmosphere. For greater elongations, however, we have measurements of the zodiacal light. During a solar eclipse it cannot be observed, but it is certainly present. Therefore, apart from the different circumstances of observation, there is no obvious distinction between the outer corona and the inner zodiacal light.

Accurate photoelectric photometry of the zodiacal light has been made by C. T. Elvey and F. E. Roach.<sup>7</sup> From their isophotal maps the intensities along the ecliptic were read. They show a seasonal variation of some 20 per cent around the average, but the mean curve is well defined in the entire range,  $40^\circ < \epsilon < 180^\circ$  ( $\epsilon =$  elongation). The unit of surface brightness, unit<sub>ER</sub>, corresponds to 1 star of the tenth magnitude per square degree. The sun has the photovisual magnitude<sup>8</sup> of  $-26.8$ , an area of 0.22 square degrees, and a mean surface brightness of  $0.73 \times 10^6$  Baumbach units. Combining these data, we find

$$\log \text{unit}_B = \log \text{unit}_{ER} + 9.50 .$$

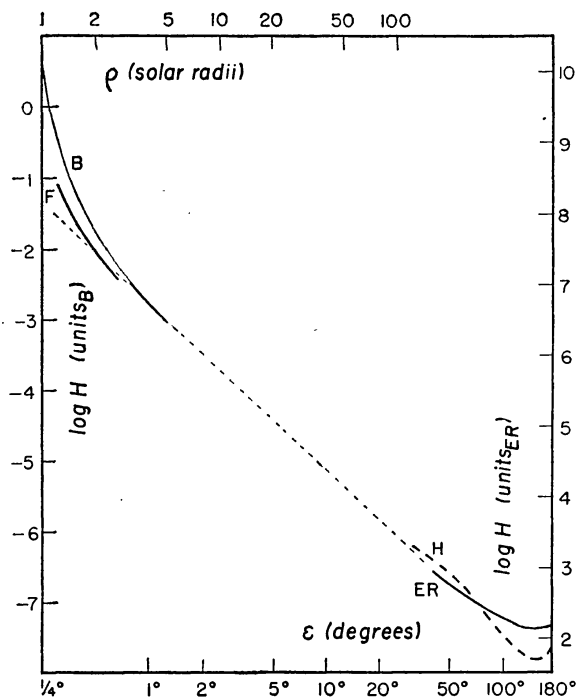


FIG. 2.—Surface brightness of the corona and of the zodiacal light on corresponding scales: Curves *F* and *B* are taken from Fig. 1; *ER*, Elvey and Roach; *H*, Hoffmeister. The two parts are tentatively connected by the dotted line, slope  $-2.4$ .

Further, there is an extensive set of visual observations by C. Hoffmeister<sup>9</sup> in the range  $30^\circ < \epsilon < 180^\circ$ . The results are given in a unit which corresponds to one star of magnitude 2.22 per square degree. A simple reduction gives

$$\log \text{unit}_H = \log \text{unit}_{ER} + 3.11 .$$

We can now plot the surface brightness of the zodiacal light in the same graph in which the surface brightness of the corona was plotted. The new curves are shown in the lower right corner of Figure 2. Though there is a wide gap between the elongations  $1.3$  and  $30^\circ$ , a smooth connection between both curves is strongly suggested by the slopes of the curves at either end of the gap. Tentatively connecting both ends by a straight line (the dotted line in Fig. 2), we find that the surface brightness decreases with the inverse 2.4th power of the elongation.

<sup>7</sup> *Ap. J.*, **85**, 231, 1937.

<sup>8</sup> G. P. Kuiper, *Ap. J.*, **88**, 429, 1938.

<sup>9</sup> *Veröff. Berlin-Babelsberg*, Vol. **8**, No. 2, 1930, and Vol. **10**, No. 1, 1932.

## SOME INSUFFICIENT EXPLANATIONS

We may first mention some suggested explanations of the  $F$ -component that would seem inadmissible.

*a)* First, there is scattering—atmospheric and instrumental—of the coronal light itself. Various effects of this type have been discussed by Baumbach<sup>10</sup> and by Y. Hagi-hara.<sup>11</sup> However, they cannot explain the presence of the Fraunhofer lines in the  $F$ -component. We must therefore look for some other mechanism by which *photospheric* light can be received without the aid of the free electrons.

*b)* Direct scattering in the atmosphere of the earth may be excluded for geometrical reasons. Let the observing station be in the center of the shadow cone. The line of sight directed to some point of the corona then passes into sunlit regions of space at several thousands of kilometers above the surface of the earth. At such altitudes there is no atmosphere that would cause any appreciable scattering.

*c)* Baumbach has also made calculation on multiple scattering in the atmosphere. This effect involves at least two scatterings under fairly large angles. It probably determines the general brightness of the sky during an eclipse.<sup>12</sup> However, the light thus scattered also covers the dark surface of the moon, and there is no reason why it should show any increase of intensity near the sun's limb. Moreover, it may be partially polarized. For all these reasons it cannot explain the  $F$ -component.

*d)* An apparently new suggestion is that we might observe photospheric light that is diffracted around the edge of the moon. Such light is certainly present; only by a quantitative computation can we show that its effect is negligible. If the sun were a point source and the moon were a screen with a straight edge, the intensity at the surface of the earth would have the regular Fresnel-type distribution,  $F(v)$ . Here  $v$  is the distance outward from the geometric edge of the shadow limit, divided by  $(b\lambda/2)^{1/2}$ , where  $b$  is the distance from the moon to the earth. Substituting  $\lambda = 5000 \text{ \AA}$  and  $b = 380,000 \text{ km}$ , we find that this unit is 10 meters. More than 100 meters outside the edge of the shadow ( $v > 10$ ) we have  $F(v) = 1$ , i.e., normal illumination. Near the edge of the shadow we have the typical fluctuations of the Fresnel distribution that can be observed during occultations of bright stars.<sup>13</sup> More than 100 meters inside the shadow edge ( $v < -10$ ) we have

$$F(v) = (2\pi^2 v^2)^{-1} = \frac{0.05}{v^2}.$$

It follows that at the center of a band of totality, of width 34 km, the illumination by a point on the solar limb is  $2 \times 10^{-8}$  times the normal and the illumination by a point near the center of the disk is  $2 \times 10^{-12}$  times the normal. Integration gives the illumination by light from the whole solar disk in that case to be  $4 \times 10^{-10}$  times the normal illumination. This means that the "lunar corona" has a total intensity 0.0003 times that of the observed corona, i.e., 0.001 times that of the  $F$ -component.

Though this effect is now ruled out as a possible explanation of the  $F$ -component, we may ask whether it would still be observable. The diffracted light should be observed as coming from a brilliant line along the edge of the moon. The theory<sup>14</sup> has not been worked out in sufficient detail to predict the intensity distribution across this brilliant line. In order for its surface brightness to exceed the surface brightness of the inner corona, it should extend over only  $0''.04$ . Since this distance cannot be resolved, the effect is unobservable.<sup>15</sup>

<sup>10</sup> *A.N.*, 267, 273, 1939.

<sup>11</sup> *Ann. Tokyo Astr. Obs.*, Vol. 1, Nos. 3 and 4, 1939.

<sup>12</sup> J. Q. Stewart and C. D. MacCracken, *Ap. J.*, 91, 51, 1940.

<sup>13</sup> A. E. Whitford, *Ap. J.*, 89, 472, 1939; *A. J.*, 52, 131, 1947.

<sup>14</sup> A. Sommerfeld, *Frank und Von Mises, Diff. Gl. der Physik*. (New York, 1943), 2, 845 ff.

<sup>15</sup> The effect might be stronger near the beginning and the end of totality. However, even the brightness gradient at the edge of the solar disk, measured by B. Lindblad and calculated by R. Wildt (*Ap. J.*, 105, 82, 1947), is not very steep. The corrections for diffraction may amount to 1 per cent at most.



## SCATTERING IN INTERPLANETARY SPACE

The only explanation left is Grottrian's original hypothesis that the  $F$ -component is due to scattering by solid particles. For convenience we assume that the particles are spherical. We define<sup>16</sup> the scattering function  $I(\theta)$  of a spherical particle as the amount of radiation scattered per unit solid angle in a direction inclined at an angle  $\theta$  with the direction of incidence, divided by the incident radiation that hits the geometrical cross-section of the particle.

Let  $J$  denote the intensity of the solar radiation at a distance  $R = 1$  A.U. from the sun.

Consider a column of height  $d\Delta$  and cross-section  $\sigma$ , seen at an elongation  $\epsilon$  from the sun (see Fig. 3). Let its distance from the sun be  $r$  and its distance from the earth be  $\Delta$ .

Let the column ( $\sigma$ ,  $d\Delta$ ) contain  $n$  particles per cubic centimeter with radii  $a$ . These particles receive the flux of solar radiation,

$$J \cdot (r/R)^{-2} \cdot \sigma d\Delta n \cdot \pi a^2,$$

of which the fraction  $I(\theta)\Delta^{-2}$  is scattered to 1 cm<sup>2</sup> of the surface of the earth. This flux is seen coming from a solid angle  $\sigma\Delta^{-2}$ , so that the surface brightness of the column is

$$H_{\text{col}} = JR^2 r^{-2} n \pi a^2 I(\theta) d\Delta.$$

The total surface brightness at elongation  $\epsilon$  from the sun is found by integrating  $H_{\text{col}}$  over the entire line of sight  $EQ$ . Substituting

$$r = R \frac{\sin \epsilon}{\sin \theta} \quad \text{and} \quad d\Delta = R \frac{\sin \epsilon}{\sin^2 \theta} d\theta$$

and assuming, further, that there are  $n(a)da$  particles per cubic centimeter that have radii in the interval  $da$ , we find for the surface brightness at elongation  $\epsilon$ :

$$H(\epsilon) = JR (\sin \epsilon)^{-1} \int_0^\infty \pi a^2 da \int_\epsilon^\pi n(a) I(\theta) d\theta. \quad (1)$$

All earlier theories of the zodiacal light emphasize Seeliger's statement that the curve of  $H(\epsilon)$  for  $\epsilon < 90^\circ$  does not depend very much on the exact form of  $I(\theta)$ . The main cause of the increasing brightness toward the sun must then be an increase of the "optical space density,"

$$f(r) = \int_0^\infty n(a) a^2 da,$$

with decreasing  $r$ . In particular, the observed law,  $H(\epsilon) \sim \epsilon^{-2.4}$ , would require  $f(r) \sim r^{-1.4}$ , and near the sun the optical density would have to be very high.

The error in this reasoning is that it *neglects the diffracted light*. The light diffracted by a large particle shows a rapid decrease of intensity with increasing  $\theta$ . Taking this fact into consideration, the importance of the two effects is completely reversed. The high gradient of the surface brightness is entirely due to the form of the scattering function,  $I(\theta)$ , and we need not assume an increase of the optical space density. This will become apparent from the following analysis.

The intensity of diffracted light may be calculated by means of the usual Fraunhofer diffraction theory. It can also be derived from G. Mie's<sup>17</sup> electromagnetic theory of the scattering by spheres of arbitrary size and refractive index. In a recent study<sup>16</sup> of this

<sup>16</sup> H. C. van de Hulst, "Optics of Spherical Particles" (Thesis, Utrecht, 1946); also published in *Rech. astr. de l'Obs. d'Utrecht*, Vol. 11, Part I, 1946.

<sup>17</sup> *Ann. d. phys.*, 25, 377, 1908.

theory and its asymptotic cases it was shown that ordinary diffraction, reflection, and refraction together constitute the scattering by a sphere whose radius is much larger than the wave length. Setting  $2\pi/\lambda = k$  and  $x = ka$ , we can write the scattering function for  $x \gg 1$  in the form

$$I(\theta) = I_r(\theta) + I_d(\theta).$$

The first part, due to reflection and refraction, depends on the material of the sphere but not on  $x$ . For instance, smooth, totally reflecting spheres give

$$I_r(\theta) = \frac{1}{4\pi}.$$

The second part, due to diffraction, depends on  $x$  but not on the material. It is expressed by the formula<sup>18</sup>

$$I_d(\theta) = \frac{1}{4\pi} x^2 \Phi(x\theta),$$

where

$$\Phi(z) = \{2z^{-1}J_1(z)\}^2.$$

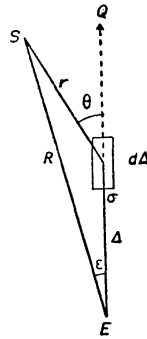


FIG. 3.

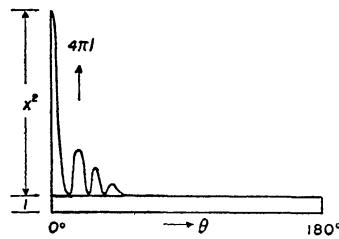


FIG. 4.

FIG. 3.—Position of scattering element in space;  $S = \text{sun}$ ,  $E = \text{earth}$   
 FIG. 4.—Schematic representation of the scattering function of a totally reflecting sphere

Figure 4 gives a schematic representation of the scattering function. The importance of the diffracted radiation is further illustrated by the fact that its total amount, found by integrating  $I_d(\theta)$  over all directions, is 1, i.e., just equal to the total amount of reflected and refracted radiation. For absorbing particles the latter amount is even less.

The separation of  $I(\theta)$  into the parts  $I_d(\theta)$  and  $I_r(\theta)$  is not possible when  $x$  is small. For transparent particles, e.g., water drops, the limit of correctness of the ordinary diffraction theory is at about  $x = 20$  (Van de Hulst, *op. cit.*, chaps. vii and viii). For totally reflecting particles it is at about  $x = 3$  (*ibid.*, chap. vi), and for metallic particles we may put the limit near  $x = 5$  (*ibid.*, chap. ix). We may assume that also for particles with a rough surface the ordinary diffraction theory is valid if  $x > 5$ . In this case we may choose for  $I_r(\theta)$  one of the classical phase functions.<sup>19</sup> On account of the present definition of  $I(\theta)$ , we have to multiply the usual expressions for these phase functions by  $\gamma/\pi$ , where  $\gamma$  is the albedo, and to replace the scattering angle by its supplement. For a sphere with Lambert's phase function we then have

$$I_r(\theta) = \frac{2\gamma}{3\pi^2} (\sin \theta - \theta \cos \theta).$$

<sup>18</sup> In textbooks one may find this formula with the argument  $x \sin \theta$  and with an additional factor  $\cos^4 \theta/2$ . It would seem that the difference has no meaning: for small  $\theta$  it vanishes and for large  $\theta$  the presence of the reflected and refracted light renders the distinction inessential (Van de Hulst, *op. cit.*, chap. v).

<sup>19</sup> E. Schoenberg, *Handb. d. Ap.*, 2, Part I, 159, 1929.

In our further analysis we assume that  $n(a)$  is the same function throughout the plane of the earth's orbit. Equation (1) may then be written in the form

$$H(\epsilon) = \frac{1}{4}JR\epsilon^{-1} \int_0^\infty n(a) a^2 F(\epsilon) da, \quad (2)$$

where

$$F(\epsilon) = \frac{\epsilon}{\sin \epsilon} \int_\epsilon^\pi 4\pi I(\theta) d\theta. \quad (3)$$

Apart from the factor  $\epsilon^{-1}$ , the function  $F(\epsilon)$  is proportional to the surface brightness due to particles of a given size. Our next step is therefore to calculate the functions  $F(\epsilon)$  for particles of various sizes and surface conditions. Again we may separate  $F(\epsilon)$  into the parts  $F_d(\epsilon)$  and  $F_r(\epsilon)$ .

TABLE 2  
FUNCTIONS CONNECTED WITH THE DIFFRACTED LIGHT

$z$	$\Phi(z)$	$\Psi(z)$	$z$	$\Phi(z)$	$\Psi(z)$
0.....	1.0000	1.698	5.....	0.0172	0.0301
1.....	0.7746	0.779	6.....	0.0085	0.0156
2.....	0.3336	0.225	7.....	0.0000	0.0127
3.....	0.0511	0.055	8.....	0.0035	0.0112
4.....	0.0011	0.040	9.....	0.0030	0.0074
			10.....	0.0001	0.0061

The function  $F_d(\epsilon)$  depends on  $x$ . Substituting for  $I_d(\theta)$ , we obtain

$$F_d(\epsilon) = x\Psi(x\epsilon),$$

with

$$\Psi(z) = \int_z^\infty \Phi(z') dz'.$$

Since the diffraction theory is accurate only for small values of  $\theta$ , we have omitted the factor  $\epsilon/\sin \epsilon$  and extended the range of integration to  $\infty$ . The functions  $\Phi(z)$  and  $\Psi(z)$  are tabulated in Table 2; the values of the latter function have been obtained by numerical integration. It may be mentioned<sup>20</sup> that  $\Psi(0) = 16/3\pi$  and that for large values of  $z$  the function  $\Phi(z)$  fluctuates around the values  $4/\pi z^3 = 1.27z^{-3}$ , while  $\Psi(z)$  fluctuates around  $2/\pi z^2 = 0.64z^{-2}$ . The surface brightness of the light diffracted by large particles is therefore proportional to  $\epsilon^{-3}$ , which is more than sufficient to allow for the observed intensity gradient of the corona and the zodiacal light.

The function  $F_r(\epsilon)$  depends on the composition and the surface condition of the spheres. For polished spheres we have

$$F_r(\epsilon) = \frac{\epsilon(\pi - \epsilon)}{\sin \epsilon};$$

this is approximately constant, varying between 2.45 at  $\epsilon = \pi/2$  and 3.14 at  $\epsilon = 0$  or  $\pi$ . If the spheres reflect the light according to Lambert's phase function, we obtain

$$F_r(\epsilon) = \gamma \frac{\epsilon}{\sin \epsilon} \frac{8}{3\pi} (2 + 2 \cos \epsilon + \epsilon \sin \epsilon).$$

<sup>20</sup> Cf. G. N. Watson, *Bessel Functions* (Cambridge, England: Cambridge University Press, 1944), p. 389.



Table 3 shows the values of this function, together with the values of  $I_r(\theta)$ ; the tabulated values are for  $\gamma = 1$ . Again we see that  $F_r(\epsilon)$  does not change its order of magnitude in the entire range of directions. Therefore, the surface brightness of the reflected light is approximately proportional to  $\epsilon^{-1}$ .

The values of  $F(\epsilon)$  thus calculated are shown together as the solid curves in Figure 5. The curve for polished totally reflecting spheres has only theoretical interest. For the

TABLE 3  
FUNCTIONS CONNECTED WITH LAMBERT'S PHASE FUNCTION

$\theta$	$I_r(\theta)$	$F_r(\theta)$	$\theta$	$I_r(\theta)$	$F_r(\theta)$
0.....	0.0000	3.40	105.....	0.097	5.25
15.....	.0004	3.44	120.....	.129	5.77
30.....	.0032	3.54	135.....	.160	6.35
45.....	.0102	3.74	150.....	.187	7.00
60.....	.0231	4.00	165.....	.205	7.71
75.....	.0424	4.36	180.....	0.212	8.39
90.....	0.0674	4.76			

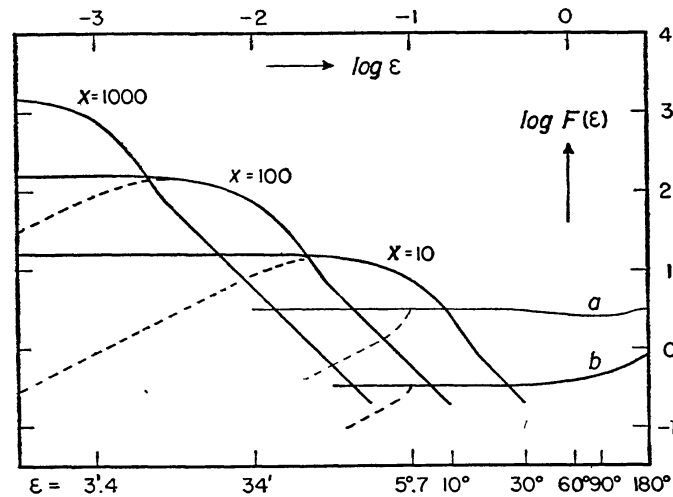


FIG. 5.—Theoretical scattering curves for particles of various sizes. Both scales are logarithmic;  $F(\epsilon)$  is proportional to  $\epsilon$ , the surface brightness. The solid curves are derived on the assumption of a homogeneous density throughout space, the dashed curves on the assumption that there are no particles within 0.1 A.U. from the sun. *Left*, the diffraction part for various values of  $x$ ; *right*, the reflection part for polished spheres (*a*) and for Lambert spheres with albedo 0.1 (*b*).

actual particles we assume that Lambert's phase function is valid and that the albedo is 0.10; this value of  $\gamma$  may be a suitable estimate for metallic or stony particles. The complete curve of  $F(\epsilon)$  for particles of a given size consists now of three distinct parts. For  $\epsilon < \epsilon_1$ ,  $F(\epsilon)$  has the constant value  $1.70x$ ; for  $\epsilon_1 < \epsilon < \epsilon_2$  it decreases proportionally to  $\epsilon^{-2}$ ; and, finally, for  $\epsilon > \epsilon_2$  reflection is the main source of scattered light and  $F(\epsilon)$  is again approximately constant, having a value between 0.4 and 0.8. The surface brightness in these three regions is proportional to  $\epsilon^{-1}$ ,  $\epsilon^{-3}$ , and  $\epsilon^{-1}$ , respectively. Table 4 shows the values of  $\epsilon_1$  and  $\epsilon_2$ ; they have been computed on the assumption that  $\lambda = 4400 \text{ \AA}$ ,  $k = 1.4 \times 10^{-5} \text{ cm}^{-1}$ .

One serious objection may be raised against the foregoing calculations. The factor  $(\sin \epsilon)^{-1}$  in formula (1) arises from the fact that the particles are more strongly illuminated, the nearer they are to the sun. Obviously, this relation must break down for very small values of  $\epsilon$ , because solid particles cannot exist in the regions close to the sun. By fairly complex calculations<sup>21</sup> one can estimate the distances in which particles of various sizes and thermal properties can approach the sun. However, we shall make here the simple assumption that there is a region of radius 0.1 A.U. around the sun which is void of particles and that outside this region the density is uniform as before. A black body at 0.1 A.U. from the sun would have a temperature of 870° K.

The effect on  $F(\epsilon)$  is easily calculated; we denote the revised values by  $F'(\epsilon)$ . For  $\epsilon > 5^\circ.7$  we have  $F'(\epsilon) = F(\epsilon)$ , because the line of sight does not pass through the empty region. For  $\epsilon < 5^\circ.7$  the range of integration in formula (3) has to be changed. It now consists of two parts, one extending from  $\epsilon$  to  $\theta_1$ , and the other extending from  $\theta_2$  to  $\pi$ . Here  $\theta_1$  and  $\theta_2$  are the sharp and obtuse angle, respectively, defined by

$$\sin \theta_1 = \sin \theta_2 = 10 \sin \epsilon .$$

TABLE 4  
ANGLES BETWEEN WHICH THE SURFACE BRIGHTNESS CHANGES  
PROPORTIONALLY TO  $\epsilon^{-3}$

$a$	$x$	$\epsilon_1$	$\epsilon_2$	$a$	$x$	$\epsilon_1$	$\epsilon_2$
$7 \times 10^{-5}$ cm. . . . .	10	3.5	25°	$7 \times 10^{-3}$ cm. . . . .	1,000	2.1	2.5
$7 \times 10^{-4}$ . . . . .	100	21'	8°	$7 \times 10^{-2}$ . . . . .	10,000	0.2	47'

Obviously, the correction is largest if  $\epsilon$  is very small. In that case we have  $\theta_1 = 10\epsilon$  and  $\theta_2 = \pi - 10\epsilon$ . The revised expression for the diffracted light is then

$$F'_d(\epsilon) = x \{ \Psi(x\epsilon) - \Psi(10x\epsilon) \} .$$

Only for  $x\epsilon < 0.1$  the second term has a serious effect; the expression then reduces to

$$F'_d(\epsilon) = 9x^2\epsilon .$$

For moderate values of  $x\epsilon$  the correction can be derived from Table 2. For large values of  $x\epsilon$  the second term is just 1 per cent of the first one, so that the correction is negligible.

This result reveals an interesting fact: As long as we deal with the outer portions of the diffraction pattern, *the brightness observed at small elongations from the sun is due to particles that are not relatively close to the sun*. The explanation is apparent from equation (2). The effect that the particles near the sun are more strongly illuminated is overcompensated by the effect that the particles closer to the earth diffract the light more effectively because of the smaller angle.

Conversely, the reflected light is contributed mostly by that part of the line of sight that is nearest to the sun. The exclusion of the empty region has therefore a serious effect on  $F_r(\epsilon)$ . It is sufficient to write down the revised formulae for the case where  $\epsilon$  and  $\theta_1$  are small. These formulae are

$$\text{Polished spheres : } F'_r(\epsilon) = \pi - \theta_2 + \theta_1 - \epsilon = 19\epsilon ,$$

$$\text{Lambert spheres : } F'_r(\epsilon) = \frac{8}{3}\gamma(\pi - \theta_2) = 26.7\gamma\epsilon .$$

<sup>21</sup>H. N. Russell, *Ap. J.*, 69, 49, 1929.

The values of  $F'(\epsilon)$  are shown by the dashed curves in Figure 5. It is seen that for very small angles the  $F'(\epsilon)$  of each component is proportional to  $\epsilon$ , so that the corresponding surface brightness is a constant. For the reflected light this occurs as soon as  $\epsilon < 5^\circ 7$  but for the diffracted light it occurs only when  $\epsilon < \epsilon_1/4$ . The surprising result emerges that those parts of the curves which are most effective in determining the scattering by a mixture of particles of various sizes are just the parts that are not affected by the absence of particles in the hot region around the sun.

#### THE DISTRIBUTION FUNCTION OF THE RADII

We shall now investigate what information about the interplanetary particles can be gained from a comparison of the photometric data and the theoretical scattering curves. First, we assume that the photometric data are exactly represented by the straight line in Figure 2. The equation of this line is

$$H(\epsilon) = 1.2 \times 10^{-7} \epsilon^{-2.4}, \quad (4)$$

where  $H$  is expressed in units<sub>B</sub> and  $\epsilon$  in radians. We may anticipate that the distribution function of the radii has the form

$$n(a) da = C a^{-p} da \quad (a_1 < a < a_2). \quad (5)$$

We confine ourselves to elongations  $< 30^\circ$ , for which the reflected light may be neglected. Equation (2) then becomes

$$H(\epsilon) = \frac{1}{4} JR \epsilon^{-1} \int_{a_1}^{a_2} C a^{-p+2} k a \Psi(k a \epsilon) da.$$

After replacing the limits of integration by 0 and  $\infty$ , we obtain

$$H(\epsilon) = \frac{1}{4} JR C C_p k^{p-3} \epsilon^{p-5}, \quad (6)$$

where

$$C_p = \int_0^\infty z^{-p+3} \Psi(z) dz.$$

Comparison of equations (4) and (6) gives  $p = 2.6$ . By numerical integration it was found that  $C_{2.6} = 2.1$ . If  $\Psi$  is replaced by  $\Psi'$  (empty region around the sun), the value is only 4 per cent less.

The intensity  $J$  may be expressed in the present units by writing it as the product of the mean surface brightness of the sun and the solid angle subtended by the solar disk. These factors are  $0.8 \times 10^6$  units<sub>B</sub> and  $0.64 \times 10^{-4}$  steradians, respectively, so that  $J = 51$ . By substituting further the values  $R = 1.5 \times 10^{13}$  and  $k = 1.4 \times 10^5$  ( $\lambda = 4400 \text{ \AA}$ ), equation (6) is reduced to

$$H(\epsilon) = 3.5 \times 10^{12} C \epsilon^{-2.4},$$

so that equation (4) is satisfied by  $C = 3.5 \times 10^{-20}$

The factor  $k$  ( $= 2\pi/\lambda$ ) appears in equation (6) with the exponent  $-0.4$ . This means that the  $F$ -corona and the zodiacal light at small elongations should be a little redder than the sun. This result may seem surprising at first sight. It should be noted, however, that the total light diffracted by a large particle is independent of wave length (and equal to the light falling on its cross-section<sup>22</sup>). The outer parts of the diffraction pattern are relatively red, while the inner part is relatively blue; the intermediate parts are the most effective parts in the present solution.

The predicted color difference between the corona and the sun is even smaller than the difference between the center and the limb of the sun. Therefore, it may easily have

<sup>22</sup> Van de Hulst, *op. cit.*, chaps. ii and v.

escaped detection. Accurate photometry at two widely different wave lengths would seem desirable for a check. Radiometric observations by V. B. Nikonov<sup>23</sup> have indeed indicated an infrared excess of the corona; however, the observation is very uncertain.

We shall next discuss the surface brightness in the immediate neighborhood of the sun. The curve marked  $F$  in Figure 2 shows an increase over the dotted line, which represents the exponential relation (4). An obvious explanation for such an increase is given by the sun's size. In order to confirm this explanation quantitatively, we may formulate the following problem: Given the surface brightness of diffuse light at an angular distance  $\rho$  from a point source of intensity 1 equal to  $\rho^{-q}$ , what will be the surface brightness at the angular distance  $\rho$  from the center of a luminous disk with radius 1 and total intensity 1? The answer to this problem is expressed by the integral

$$f(\rho) = \frac{1}{\pi} \int_0^{2\pi} d\varphi \int_0^1 (\rho^2 + r^2 - 2r\rho \cos \varphi)^{-q/2} r dr.$$

It should be noted that we are allowed to consider the sun as a luminous disk instead of a luminous sphere because it was stated earlier that the effect of particles close to the sun is negligible. For the case  $q = 2$ , analytical integration gives

$$f(\rho) = -\ln(1 - \rho^{-2}).$$

TABLE 5  
INCREASE OF LOG  $H(\rho)$  DUE TO THE SUN'S SIZE

$\rho$	Calc. $q=2$	Calc. $q=2.5$	Obs. $q=2.4$	$\rho$	Calc. $q=2$	Calc. $q=2.5$	Obs. $q=2.4$
1.2.....	0.23	0.32	0.41	1.6.....	0.10	0.18	0.18
1.4.....	0.14	0.23	0.27	2.0.....	0.06	0.10	0.01

Further, a graphical integration for the case  $q = 2.5$  was made. The disk was divided into about six parts, separated by concentric circles around a point at a distance  $\rho$  from its center; the areas of these part were measured and multiplied by the appropriate factor  $\rho^{-2.5}$ , and the products were added. Table 5 gives the resulting values of  $\log f(\rho) - \log(\rho^{-q})$ , i.e., the increase of  $\log H(\epsilon)$  over the exponential relation. The corresponding increase shown by the photometric data is given in the last column. The observed increase seems a little steeper than the calculated ones, and this difference would be enhanced if the limb darkening were taken into account. Yet the measurements of the  $F$ -component near the sun are not accurate enough to consider the differences as real.

We may conclude that the exponential relation (5) holds, down to the smallest angle for which observations were made. This angle—now measured from the limb—is about 0.2 solar radii, i.e., 0.001 radian. The definite integral  $C_{2.6}$  decreases by 20 per cent if the upper limit of integration is taken as 5 instead of as  $\infty$ . If 20 per cent is taken for the observational tolerance, this means that the approximation which we made is correct if  $ka_2\epsilon > 5$ . It is correct down to  $\epsilon = 0.001$  if  $ka_2 > 5000$ , i.e.,  $a_2 > 0.035$  cm.

The surface brightness at large angles requires a quite different discussion. For  $\epsilon > 30^\circ$  the diffracted light comes mainly from very small particles, for which the diffraction formula is not accurately valid. Further, the reflection comes in as an additional cause of scattering. However, we may assume that for  $\epsilon > 60^\circ$ , reflection is the only effect to

<sup>23</sup> *Bull. Abastumani Obs.*, 7, 33, 1943.

be taken into account. Substituting the value  $F_r(\epsilon) = 4.4$ , valid for Lambert spheres and  $\epsilon$  near  $75^\circ$ , and assuming again the distribution function of the radii given by equation (5), we find from equation (2) that

$$H(\epsilon) = \frac{1}{4}JR\epsilon^{-1}4.4\gamma C \frac{(a_2^{3-p} - a_1^{3-p})}{(3-p)}.$$

Since  $3 - p = 0.4$ , we may replace  $a_1$  by 0. The photometric curve in the range  $60^\circ < \epsilon < 90^\circ$  is approximately represented by the equation

$$H(\epsilon) = 1.0 \times 10^{-7} \epsilon^{-1}.$$

Equating these expressions and using the values of the constants derived earlier, we find

$$\gamma = 0.0013 a_2^{-0.4},$$

which, in combination with the lower limit of  $a_2$ , gives

$$\gamma < 0.005.$$

This result is a little lower than we had expected. Since meteorites and meteoric dust are often black,<sup>24</sup> however, a fairly low albedo appears not impossible. In addition, there are several possibilities to widen the margin. First, the phase function of the moon would make the reflected light about three times weaker than it would be on account of Lambert's phase function. Furthermore, we may expect that an irregular form of the particles would enhance the diffracted light without causing a proportional increase of the reflected light. Finally, almost any change in the surface brightness may be effected by assuming that the density varies through the plane of the ecliptic.

For example, we may consider as an extreme case the one in which the distribution of the radii has the form (4) within 0.5 A.U. from the sun, and space is empty outside this region. The surface brightness for  $\epsilon > 30^\circ$  is then zero. For  $\epsilon < 30^\circ$  it is found by adding a factor  $2^{p-4} = 0.38$  to the right-hand member of equation (5). The resulting value of  $C$  is then 2.6 times the previous value. This model would therefore explain the surface brightness of the corona without leading to too high a value for the surface brightness of the zodiacal light.

Summarizing the results, we may say that the photometric data are well represented by theory if we assume: (a) that the albedo is about 0.1; (b) that the distribution of radii has the form

$$n(a) = Ca^{-2.6} \quad (5a)$$

but that particles with dimensions  $> 1$  mm are less abundant than would follow from this formula; and (c) that  $C$  is about  $10^{-20}$  near the orbit of the earth,  $5 \times 10^{-20}$  at 0.5 A.U. from the sun and 0 within 0.1 A.U. from the sun.

A few words may be added about the zodiacal light at elongations  $> 90^\circ$ . Part of the variation of brightness in this region may be due to real density variations. This explanation is likely, since there are also seasonal variations in the position of the axis and of the Gegenschein<sup>25</sup> that seem to have some relation with the orbit of Jupiter.<sup>26</sup> In addition, the scattering angles are different from the angles that are effective in the region of small and medium elongations. Consequently, the zodiacal light at large elongations offers a problem which has only little relation to the problem treated above.

<sup>24</sup> I found no values of the albedo mentioned.

<sup>25</sup> C. T. Elvey, *Ap. J.*, **77**, 56, 1933.

<sup>26</sup> Hoffmeister, *op. cit.*



## DENSITY OF THE INTERPLANETARY MEDIUM AND RELATION TO METEORS

We return to the original solution, in which the density was supposed to be homogeneous. Using the values  $p = 2.6$ ,  $C = 3.5 \times 10^{-20}$ , and  $a_2 = 0.035$  cm, we find that the mean free path among the interplanetary particles is

$$\left( \frac{\pi C a_2^{0.4}}{0.4} \right)^{-1} = 10^6 \text{ A.U.}$$

Roughly the same value follows directly from the ratio of the total intensity of the corona to the total intensity of the sun. It shows that the interplanetary dust cloud is extremely transparent and justifies our assumption that interaction between the scattering particles may be neglected.

The space density is

$$\frac{\frac{4}{3} \pi s C a_2^{1.4}}{1.4} = 5 \times 10^{-21} \text{ gm/cm}^3,$$

where  $s$ , the density of the particles, is taken to be 5. P. van Rhyn<sup>27</sup> estimated a density of  $5 \times 10^{-18}$  gm/cm<sup>3</sup> near the earth. The difference of a factor of 1,000 is mainly due to the fact that Van Rhyn assumes arbitrarily that the zodiacal light is caused by rocks with radii of about 50 cm, which is more than 1000 times larger than the radii following from the present investigation.

The thickness of the dust cloud perpendicular to the plane of the ecliptic may be about 0.1 A.U. The total mass of the particles within the orbit of the earth then is  $5 \times 10^{18}$  gm, i.e.,  $10^{-9}$  times the mass of the earth. This mass is far too small to produce observable effects on the motion of the planets.

Perhaps the most interesting problem is the relation of these particles to the meteors. The following discussion is based on F. Watson's<sup>28</sup> analysis of meteor frequencies. Watson's formula (1') gives the relation between mass, velocity, and magnitude of the meteor, based on calculations by E. Öpik.<sup>29</sup> If  $M$  = mass in grams,  $m_z$  = magnitude reduced to the zenith, and the geocentric velocity is 56 km/sec, this relation becomes

$$\log M = -1.1 - 0.4 m_z. \quad (7)$$

For instance, a second-magnitude meteor has a mass of 0.012 gm. Assuming a density of 5, we have, further,

$$\log M = 1.3 + 3 \log a,$$

so that

$$\log a = -0.8 - 0.133 m_z.$$

It follows that the naked-eye meteors ( $-1.5 < m_z < 5$ ) are caused by particles with radii in the range  $0.25 > a > 0.03$ , while the range for telescopic meteors ( $5 < m_z < 9$ ) is  $0.03 > a > 0.01$ . The particles that cause the  $F$ -component of the corona, according to the preceding analysis, have radii up to 0.03 cm. The largest particles among them correspond, therefore, to the telescopic meteors.

Let  $N$  be the number of particles per cubic centimeter and let the number in a given interval of mass, radius, or magnitude be denoted by  $dN$ . Watson's Figure 2 gives the number of meteors of a given magnitude that daily enter the entire atmosphere of the earth. Assuming again a geocentric velocity of 56 km/sec, we find that this number is

<sup>27</sup> *Pub. Astr. Lab., Groningen*, Vol. 31, 1921.

<sup>28</sup> *Harvard Ann.*, 105, 623, 1937.

<sup>29</sup> *Tartu Pub.*, Vol. 29, No. 5, 1937.



$10^{29.8}$  times the number per cubic centimeter. The straight line in this figure is therefore represented by

$$10^{29.8} \frac{dN}{dm_z} = 10^{4.4+0.60m_z}.$$

Since  $dm_z = 10^{0.5} a^{-1} da$ , this relation can be reduced to

$$\frac{dN}{da} = 10^{-28.5} a^{-5.5}. \quad (8a)$$

A similar result is found from Watson's Figure 4: the total number per cubic parsec in a given interval of  $\log M$  is

$$10^{55.5} \frac{dN}{d(\log M)} = 10^{29.0-1.40 \log M},$$

which is reduced by means of  $d(\log M) = 10^{0.1} a^{-1} da$  to

$$\frac{dN}{da} = 10^{-28.2} a^{-5.2}. \quad (8b)$$

TABLE 6  
NUMBER OF PARTICLES IN INTERPLANETARY SPACE

$a$ (Cm)	$m_z$	LOG $dN/da$			DIFFERENCE (5) - (8)
		(5)	(8a)	(8b)	
0.10.....	1.5	< -16.9	-23.0	-23.0	< 6.1
.03.....	5.2	-15.6	-20.3	-20.4	4.8
0.01.....	9.0	-14.3	-17.5	-17.8	3.4

These results may be compared with the result of the present paper:

$$\frac{dN}{da} = 10^{-19.5} a^{-2.6}. \quad (5)$$

Table 6 shows the values of  $\log dN/da$  for some values of  $a$  in the region where both determinations are valid. The differences shown in the last column are considerable: *the number of particles with sizes between 1 mm and 0.1 mm in interplanetary space is about 10,000 times larger than can be inferred from the number of telescopic meteors.*

Before drawing further conclusions we shall estimate just how reliable this result is. The constants in equation (5) do not depend on any arbitrary estimates; the resultant numbers may be accurate within a factor two. As we have seen earlier, it is also very unlikely that the  $F$ -corona can be explained in any different way. The meteor data are more flexible; in particular, the question of the meteor velocities is still a matter for controversy. Fortunately, it does not matter for the present estimate whether the particles come from interstellar space or belong to the solar system: in either case they must be present in interplanetary space. Therefore, even drastic corrections of the velocities cannot change the order of magnitude of the estimated density. Further, Watson's counts refer to sporadic meteors; the number of shower meteors should still be added. However, there are fewer shower meteors than there are sporadic meteors, for the telescopic magnitudes in particular. Therefore, no appreciable change results from this

correction. Independent counts cited by Watson and a later paper by J. D. Williams<sup>30</sup> confirm the general order of magnitude. The total uncertainty would amount to about a factor of 10. The only way to change appreciably the estimates derived from equation (8) is to assume that the physical theory of meteors<sup>29</sup> leads to masses which are much too small. At present this possibility seems unlikely.

Taking, then, the italicized result as significant, we may inquire into its meaning. An obvious explanation is that only a very small fraction of the interplanetary particles have velocities high enough to cause visible meteors. The majority of the particles might move in approximately circular orbits in the same way that the planets and the asteroids do. The dynamical relation of this system to the meteor streams and to the sporadic meteors is not clear. We may, however, mention that a particle moving in some cometary orbit would hit a particle of the quiescent cloud once in  $10^6$  or  $10^7$  years. One may therefore conclude from the existing motions that there is some supply from outside the solar system, even if the direct evidence for interstellar meteors is considered doubtful. Furthermore, the existence of such an interplanetary medium may have implications for the motion of comets and for the cosmogony of the solar system.

#### THE REAL CORONA: SOME SUGGESTIONS

After this discussion of the "spurious" corona, we may ask how the best information about the "real" corona can be obtained. First, we shall have to make quite certain that the *F*-component has indeed no direct connection with the sun. This might be confirmed by observations that would narrow the gap shown by Figure 2. Measurements of the zodiacal light might be extended to  $10^\circ$  from the sun if proper precautions were taken to eliminate twilight. The brightness of the corona, on the other hand, may be measured to considerably larger distances from the sun than has been done so far. The most serious disturbance is the light scattered from outside the eclipse cone. From the measurements by J. Stebbins and A. E. Whitford<sup>31</sup> with a potassium cell, it would follow that the surface brightness of the sky during an eclipse is about one four-hundredth that of the full moon, i.e.,  $10^{-2.4}$  units<sub>B</sub>. Since the skylight has the regular blue color, it might be reduced by a factor 25 by observing in the infrared about  $1\mu$ ; a further factor of 2 or more may be gained by observing from an aircraft. The sky brightness would then come down to  $10^{-4.1}$  units<sub>B</sub>, so that, according to Figure 2, it would equal the brightness of the corona at  $4^\circ$  from the sun.

One unexplained observation should be mentioned. On two occasions Moore<sup>32</sup> found that the Fraunhofer lines in the spectrum of the corona showed a red shift of about 20 km/sec. This observation conflicts with the present explanation of the *F*-component. Even if the diffracting particles had large radial motions, the Doppler effect would not influence the diffracted light. However, the free electrons probably are ejected from the sun with a speed<sup>33</sup> of the order of 1 A.U. per 4 days, i.e., 400 km/sec. Consequently, the Fraunhofer lines of the *K*-corona should have a red shift of this amount. These lines are broadened into very shallow, nearly invisible depressions. Grotrian<sup>34</sup> finds that their broadening between  $1'$  and  $5'$  from the limb corresponds to a mean random velocity of 4000 km/sec. At the levels of  $10'$  and  $20'$  from the limb, where Moore's observations were made, the broadening might be considerably less, though still large enough to make the lines very shallow. Superposition of the displaced shallow lines over the undisplaced narrow lines of the *F*-corona might then cause an apparent shift of the latter, especially if the dispersion is low.

<sup>30</sup> *A. p. J.*, **92**, 424, 1940.

<sup>31</sup> *A. p. J.*, **87**, 225, 1938.

<sup>32</sup> *Pub. A.S.P.*, **35**, 333, 1923, and **45**, 147, 1933.

<sup>33</sup> K. O. Kiepenheuer, *A. p. J.*, **105**, 408, 1947.

<sup>34</sup> *Zs. f. Ap.*, **3**, 199, 1931, and **8**, 124, 1934.

In several papers on the physical state of the corona, reference is made to the electron densities derived by Baumbach.<sup>6</sup> They have been computed on the assumption that the entire brightness of the corona is due to electron scattering. However, on our present ideas the  $F$ -component should be excluded. We may obtain the corrected values for the electron density in the following way.

With little change in the numerical values we can write the brightness of the  $F$ -component in the form

$$H_F(\rho) = \frac{0.053}{\rho^{2.5}} + \frac{0.30}{\rho^7}.$$

This formula contains the same powers of  $\rho$  as Baumbach's formula for the total brightness, viz.,  $\rho^{-2.5}$ ,  $\rho^{-7}$ , and  $\rho^{-17}$ ; the coefficients are the fractions 1.00, 0.21, and 0.00 of the coefficients for the total light. The corrected formula is therefore obtained by multiplying these coefficients by the factors 0.00, 0.79, and 1.00, respectively. Baumbach's analysis

TABLE 7  
CORRECTED ELECTRON DENSITIES

$r$	$f$	$10^{-6} N(r)$		$r$	$f$	$10^{-6} N(r)$	
		Old	New			Old	New
1.....	0.94	458	430	2.2.....	0.47	2.50	1.2
1.03.....	.92	311	290	2.4.....	.39	1.79	0.7
1.06.....	.90	229	210	2.6.....	.31	1.35	0.42
1.10.....	.88	156	137	2.8.....	.26	1.10	0.29
1.2.....	.83	70.4	58	3.0.....	.21	0.91	0.19
1.3.....	.79	38.4	30	3.5.....	(.11)	0.63	(0.08)
1.4.....	.74	23.8	18	4.0.....	(.07)	0.51	(0.04)
1.6.....	.67	11.1	7.5	5.0.....	(0.03)	0.38	(0.01)
1.8.....	.61	6.1	3.8	6.0.....	.....	0.25	.....
2.0.....	0.54	3.7	2.0	8.0.....	.....	0.16	.....

shows that the same factors should be applied to the formula for the source function,  $F(r)$  (Baumbach's formula [6]). The new values of  $F(r)$  are computed and found to be a fraction,  $f$ , of the original values. Since the geometrical conditions are not changed,  $f$  is also the correction factor for the electron densities,  $N(r)$ . Table 7 shows the results. Near the limb the correction is small; beyond  $r = 3$  the correction is so large that the electron density becomes very uncertain.

The  $F$ -component is only roughly eliminated in this way. The resulting picture of an electron density which is constant in spherical shells around the sun is certainly much too simple. One of the reasons why the data compiled by Baumbach fit so nicely to a single curve is undoubtedly just the presence of the constant  $F$ -component. The  $K$ -corona alone probably shows much stronger fluctuations. We may also expect that the coronal streamers, especially their outer parts, actually show more distinct features than are seen on the usual photographs. Taking one calibration spectrum (e.g., along the polar axis) and assuming that the  $F$ -component has circular symmetry, one might correct the isophotes of the total light so as to obtain the isophotes of the  $K$ -component. A more reliable method would be the use of various calibration spectra along different lines. It would be still better to exclude the  $F$ -component directly from the observations. This might be effected by observing the corona, or a part of it, through a filter transmitting a wave-length interval of about 5 Å around the center of the H- or K-line. The

*F*-corona would then be cut down by a factor of 6, while the *K*-corona would show in approximately full strength.

*Addendum, March 31.*—In a paper just received, C. W. Allen<sup>35</sup> gives the same explanation of the *F*-corona as that proposed in the present paper. Allen's paper is particularly important, since it gives new data, obtained at the eclipse of October 1, 1940. A few further points that lend support to the present explanation may now be noted: (*a*) The *F*-corona seems slightly reddish according to Allen's measurements, in agreement with our formula (6); (*b*) the discrepancy suggested by our Table 5 vanishes when Allen's data are used instead of Grotrian's; and (*c*) Allen finds no confirmation of the unexplained red shift.

The ratio of the *F*-component to the total intensity found by Allen is smaller than the ratio found by Grotrian at the same distance from the limb. We may suggest that the difference is due to the *K*-component; this component may have been weak in 1923 (minimum of solar activity) and strong in 1940 (3 years after maximum). Future eclipse expeditions should consider the separation of the two components as a major point of their photometric program.

<sup>35</sup> *M.N.*, **106**, 137, 1947.