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## **A method for determining the orbital elements of a spectroscopic binary, with application to $\gamma$ Geminorum**

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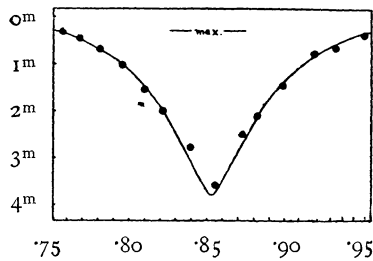
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shows mean phases and magnitudes while Figure 1 is a graphical representation of the primary minimum.

FIGURE 1.



The lightcurve is in some respects remarkable:

- 1° Because of the great range, viz.  $3^m.4$ ,
- 2° Because of the relatively long duration of the eclipse, viz.  $\cdot 2$  of the period, or about twice as long as is usual for

systems of the Algol type.

With the device explained in *B. A. N.* No. 147, p. 179, the phase of the primary minimum was found to be  $\cdot 8534$ . Accordingly the elements are

$$J. D. \text{ Hel M. Astr. T. Grw. } 2425850^d \cdot 9434 + 2^d \cdot 192306 E.$$

There is a doubtful depression at the phase of the

secondary minimum,  $\cdot 353 = \cdot 853 - \cdot 500$ , the eccentricity of the orbit being assumed to be zero.

A determination of the geometrical elements of the system was now made by means of a method (unpublished as yet) kindly put at my disposal by P. P. BRUNA.

It appeared possible to indicate relatively narrow limits for these elements. The principal minimum corresponds to the eclipse with the larger star in front. The limits found for the maximum eclipsed area of the smaller star in terms of its total area are  $\alpha_0 = \cdot 96$  and  $\alpha_0 = 1.00$ . In Table 3 the radii of the stars in terms of the radius of the orbit and the inclination  $i$  are given for three values of  $\alpha_0$ , viz.  $\alpha_0 = \cdot 96, \cdot 98$  and  $1.00$ . The full drawn line in Figure 1 corresponds to the elements for  $\alpha_0 = \cdot 98$ .

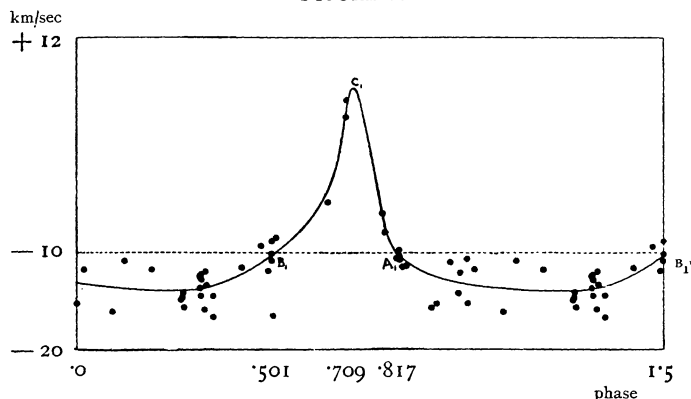
TABLE 3.

$\alpha_0$	$\cdot 96$	$\cdot 98$	$1.00$
$r_1$	$\cdot 43$	$\cdot 42$	$\cdot 41$
$r_2$	$\cdot 24$	$\cdot 25$	$\cdot 26$
$i$	$76^\circ 8$	$78^\circ 8$	$80^\circ 8$

## A method for determining the orbital elements of a spectroscopic binary, with application to $\gamma$ Geminorum, by *A. J. Wesselink*.

Suppose that a sufficient number of observations of radial velocities are available for constructing the velocity curve over one complete period. The velocity of the centre of gravity of the system is readily found in the usual way; this velocity corresponds to the horizontal line in Figure 1. The present method makes use of the *phases* of three definite points of the velocity

FIGURE 1.

Velocity curve of the spectroscopic binary  $\gamma$  Geminorum.

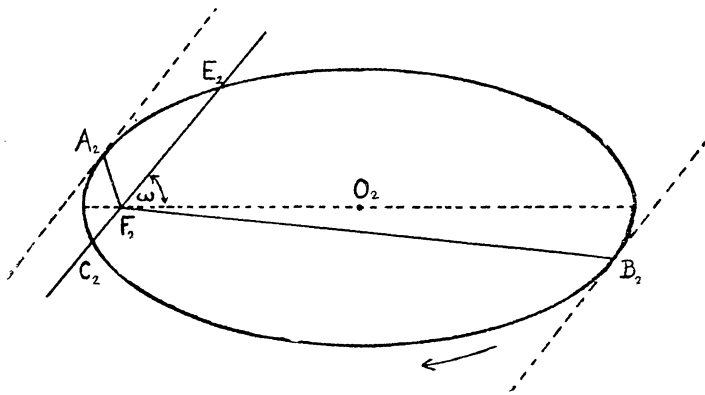
curve. These are (Fig. 1) the points of intersection  $A_1$  and  $B_1$  of the horizontal line with the velocity curve and  $C_1$  which is either the maximum or the minimum value, but always the most sharply defined of the two; only in the case of zero eccentricity or when  $\omega = 90^\circ$  or  $270^\circ$  there is no such a preferential value. The phases of  $A_1$  and  $B_1$  are immediately read from the diagram. The evaluation of the phase of  $C_1$  may be effected in the same way as the phase of maximum brightness is determined in the case of variable stars of the cluster- or  $\delta$  Cephei type. For example, we may draw a set of equidistant horizontal lines, from each of which a segment is cut off by the velocity curve. The curve connecting the centres of these segments intersects the velocity curve in  $C_1$ .

Let us consider the points in the orbit which correspond to  $A_1$ ,  $B_1$  and  $C_1$ . At  $A_1$  and  $B_1$  the two components have the same radial velocity. The orbital motion is at right angles to the line of sight. Since the inclination of the orbit cannot be zero  $A_1$  and  $B_1$  are the instants in which the star moves parallel to the line of the nodes. From the equation of the radial velocity curve,  $R. V. = k \{ e \cos \omega + \cos (v + \omega) \} + \gamma$ ,

$e$  eccentricity,  $\omega$  longitude of periastron,  $\gamma$  velocity of the system,  $v$  true anomaly,  $2k$  the velocity range, we see that the extreme velocities correspond to the instants at which the star passes the nodes.  $C_1$  as defined above belongs to the node which is nearest to the periastron.

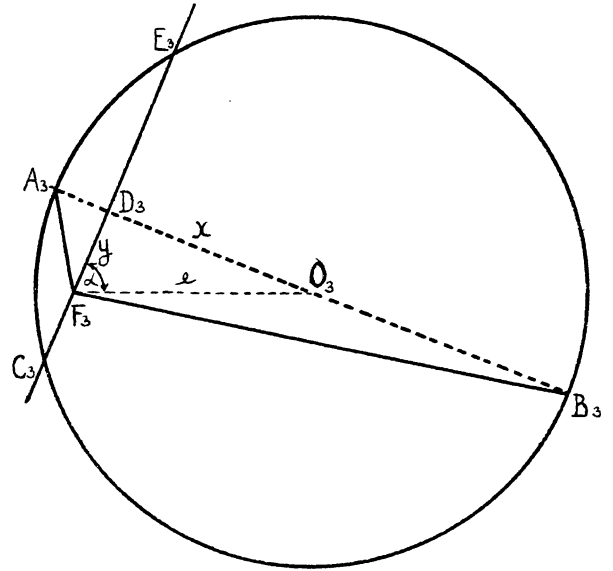
Figure 2 shows the true orbit, the nodal line, the points  $A_2, B_2$  and  $C_2$ , corresponding to  $A_1, B_1$  and  $C_1$ ,

FIGURE 2.



of Figure 1.  $F_2$  is the centre of gravity. The direction of the corresponding orbital motion is indicated by an arrow. Consider the true orbit as the orthogonal projection of a circle which is shown in Figure 3. Every point  $P_2$  in Figure 2 is the projection of a

FIGURE 3.



point  $P_3$  in Figure 3. Denoting the phase differences  $B_1 C_1, C_1 A_1, A_1 B_1^*$  as determined above, by  $p, q$  and  $r$  respectively, we have the following proportionality for the sectors in Figures 2 and 3:

$$\begin{aligned} F_2 B_2 C_2 : F_2 C_2 A_2 : F_2 A_2 B_2 &= \\ F_3 B_3 C_3 : F_3 C_3 A_3 : F_3 A_3 B_3 &= p : q : r \\ p + q + r &= 1. \end{aligned}$$

In the following we take the area of the circle as unit of area, the radius as unit of length. We define quantities  $x$  and  $y$  as the lengths of  $O_3 D_3$  and  $F_3 D_3$  respectively.  $p$  and  $q$  being known we calculate  $y$  from  $\frac{y}{\pi} = \frac{1}{2} - (p + q)$ , each side of the equation being equal to the area of the triangle  $A_3 B_3 F_3$ .

The area of the segment  $C_3 A_3 E_3$  is  $2q + \frac{y}{\pi} (1 - x)$ .

The area can also be derived directly from  $O_3 D_3$  and is thus a definite function of  $x$ , say  $z(x)$ , which is tabulated in Table I.  $x$  is found by solving the equation

$2q + \frac{y}{\pi} (1 - x) = z(x)$ . The solution is most easily made graphically. We find  $x$  as the abscissa of the point of intersection of the line representing the function  $2q + \frac{y}{\pi} (1 - x)$  with the graph representing  $z(x)$ .

$e$  and  $\omega$  are calculated from

$$\begin{aligned} e &= \sqrt{x^2 + y^2} \\ tg \omega &= tg \alpha \cdot \sqrt{1 - e^2} = \frac{x}{y} \cdot \sqrt{1 - e^2} \end{aligned}$$

TABLE I.

$x$	$z$	$x$	$z$	$x$	$z$
.00	.500	.36	.276	.72	.085
.02	.488	.38	.264	.74	.076
.04	.475	.40	.252	.76	.068
.06	.462	.42	.241	.78	.060
.08	.449	.44	.229	.80	.052
.10	.436	.46	.218	.82	.044
.12	.424	.48	.207	.84	.037
.14	.411	.50	.196	.86	.031
.16	.398	.52	.184	.88	.024
.18	.386	.54	.173	.90	.019
.20	.373	.56	.163	.92	.013
.22	.361	.58	.152	.94	.009
.24	.349	.60	.142	.96	.005
.26	.336	.62	.132	.98	.001
.28	.324	.64	.122	1.00	.000
.30	.312	.66	.113		
.32	.300	.68	.103		
.34	.288	.70	.094		

The remaining elements  $a \sin i$  and the time of periastron passage are calculated in the usual way.

Example:

We shall apply the method to the spectroscopic binary  $\gamma$  Geminorum,  $6^h 31^m 9 + 16^s 29'$  (1900). My purpose is not to give the most accurate orbit for  $\gamma$  Geminorum which can be derived at present from all the measures made on the star, but to give an illustration of the present method and to draw attention

to the fact that the star becomes of special interest during the next years.  $\gamma$  Geminorum is important in being one of the longest period spectroscopic binaries known. The velocities near maximum recession (the time of which is much more sharply defined than that of the maximum velocity of approach) have been observed very incompletely. Consequently the elements of the orbit, which largely depend on it, are still uncertain. The star is therefore recommended for further observation near the time of maximum velocity of recession which, adopting HARPER's period of 2160<sup>d</sup> (*R. A. S. Canada*, VI, p. 183) occurs on January 27, 1934.

For the construction of the velocity curve the observations given in the following publications were used:

<i>Lick Publ.</i> , 16, p. 94	(11)
<i>Yerkes Publ.</i> , VII, part 1, p. 29	(5)
<i>Cape Annals</i> , X, part 8, p. 159	(3)
<i>A. N.</i> , Bd. 192, p. 447	(6)
<i>R. A. S. Canada</i> , VI, 182	(16)

The phases were computed with the formula

$$\text{phase} = \frac{1}{2160} \times (\text{J. D. Hel. M. Astr. T. Grw} - 2400000).$$

The systematic corrections given in *Lick Publ.*, 16, p. XXXI were applied.

In Figure 1 the separate observations are represented by dots.

The full drawn line is the adopted velocity curve.

Phase of $A_1$	·817
"    " $B_1$	·508
"    " $C_1$	·709

and using the same notation as before

$$\begin{aligned} p &= \cdot 202 \\ q &= \cdot 108 \\ p + q &= \cdot 310 \end{aligned}$$

$$\frac{y}{\pi} = \cdot 500 - \cdot 310 = \cdot 190$$

$$y = \cdot 598$$

$$2q + \frac{y}{\pi} (1-x) = \cdot 406 - \cdot 190x = z(x)$$

With the aid of the graph we find  $x = \cdot 212$ ,

$$e = \sqrt{\cdot 212^2 + \cdot 598^2} = \cdot 634$$

$$tg\omega = tg\alpha \cdot \sqrt{1-e^2} = \frac{x}{y} \cdot \sqrt{1-e^2} = \frac{\cdot 212}{\cdot 598} \times \cdot 773 = \cdot 274$$

$$\omega = 15^\circ \cdot 3$$

The velocity of the system  $\gamma = -10\cdot 1$  km/sec. The mass function is  $\cdot 12 \odot$ . The phase of the periastron passage is  $\cdot 718$ . The radial velocity at periastron is  $+6\cdot 4$  km/sec. From the total velocity range of 21 km/sec we find  $a \sin i = 241 \times 10^6$  km  $= 1\cdot 6$  A. U. ( $a$  representing the semi major axis of the orbit relative to the centre of gravity).

If it is assumed that we look edgewise at the orbit ( $i = 90^\circ$ ) we find for its dimension 3 A. U. Since in reality  $i$  will be different from  $90^\circ$  this is a minimum value, but probably not far from the true dimension of the projected orbit. Abstracting from the proper motion the displacements measured on plates taken for parallax will thus be due to parallax as well as to orbital motion. In the time covered by the parallax observations (some years) these two effects will be of the same order of magnitude. It is, therefore, of interest for parallax observers to know that it is inadmissible to derive the parallax discarding the orbital motion. On the other hand it seems to be possible to find both the orbit and the parallax from a sufficient number of observations.

The parallax from SCHLESINGER's catalogue (1924) is  $+''\cdot 043 \pm ''\cdot 009$  (m. e).

I want to thank Mr. G. P. KUIPER for drawing my attention to  $\gamma$  Geminorum and for giving the sources of the available observations.