

Special relativistic jet collimation by inertial confinement

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Abstract. We present an explicit numerical method to handle problems in special relativistic hydrodynamics. This method is a special relativistic extension of a Roe solver method for nonrelativistic flow.

The method is used to simulate the collimation of an initially spherical outflow by a thickened disk (i.e. collimation by inertial confinement). Previous nonrelativistic results indicated that this is a mechanism to form jets. Here we show that this mechanism can also generate relativistic jets.

The highest Lorentz factor achieved in these simulations is 15. We discuss the differences between the relativistic and nonrelativistic flow.

Key words: relativity – hydrodynamics – methods: numerical – galaxies: jets

1. Introduction

The origin of jets is one of the unanswered questions in astrophysics. Jets, or more generally collimated outflows, are seen under various conditions at different scales. Active galaxies, X-ray binaries like SS433, proto-stars, and planetary nebulae all show more or less collimated outflows, in some cases extending over nine powers of ten in size. The characteristic velocities of the jets range from a few tens of kilometers per second to ultra-relativistic. Unfortunately the origin of these outflows remains, even in the case of nearby objects, beyond the possibilities of observations. Consequently a variety of models has been proposed to explain the emergence of jets. The purely hydrodynamical models can be divided into two basic mechanisms. The first one is the so called twin-exhaust mechanism by Blandford and Rees 1974 (see also Norman et al. 1981; Königl 1982; Smith et al. 1983). In this mechanism the collimation takes place through a nozzle structure, set up in a flattened cold gas distribution by hot gas emanating from the central source. The other mechanism uses the funnel of a thick accretion disk to collimate the

outflow (Lynden-Bell 1978; Paczyński & Wiita 1980). In this class of mechanisms the funnel is also used to collimate the radiative flux, thus supplying a means of accelerating the gas. This, however, appears not to be a very efficient mechanism (Abramowicz & Piran 1980; Abramowicz & Sharp 1983). Because both of these models appear to have trouble explaining real narrow jets, attention has focused on hydromagnetic models (see the reviews by Wiita 1990 and Padman et al. 1990).

It should be noted however that in the funnel type models detailed hydrodynamics was not taken into account. Authors tended to take the funnel to be a solid or slowly evaporating tunnel and the flow inside this structure is then analysed (see e.g. Calvani & Nobili 1983). In reality some sort of interaction between the outflow and the disk is expected to take place. The numerical work in Mellema et al. 1991 (henceforth MEI91), Icke et al. 1992 (henceforth IBF92), and Icke et al. 1992 (henceforth IMBEF92) revealed what might be happening. Studying the emergence of aspherical planetary nebulae, they found that a remarkable jet structure is formed in the interaction of a fast spherical outflow with a thickened disk. This mechanism has been dubbed ‘collimation by inertial confinement’. The interaction results in the formation of a dense chimney that serves to confine the flow into a jet. Velocity focusing by the barrel shaped inner shock helps in the collimation.

Since jets occur on a multitude of velocity scales (Begelman et al. 1984), it is important to study how the inertial confinement mechanism behaves in a relativistic setting. We realise that in the case of relativistic jets no observational data exist to support the assumption of a spherical outflow near the central object. However, given the absence of observational evidence for the nature of the flow in the innermost region, to obtain jet collimation from an initially spherical outflow represents the most extreme case.

The calculations in MEI91 were done using the classical limit of a general relativistic hydro code. In this paper calculations will be presented using a special relativistic version of this code. Since jets are one of the rare phenomena in which observations show that relativistic velocities are reached, we chose the inertial confinement mechanism to be the first application of the special relativistic Roe solver. The availability of nonrelativistic simulations with which to compare the relativistic results,

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and the experience with the nonrelativistic flow pattern made it a suitable problem to test the performance of our method. The choice of this specific problem of course also has its disadvantages. The initial conditions originate from the planetary nebulae (PN) setting. This means for instance that there is no source of gravitation in the centre, because the influence of the central star is negligible in the case of a planetary nebula. We did not apply this (relatively trivial) correction, since this would require the use of general relativistic hydrodynamics as well as inhibit the comparison to the nonrelativistic results of MEI91 and IBF 92. Gravity being absent we have taken the disk to be static.

Other explicit numerical methods for relativistic flow exist, see e.g. Van Riper (1979), Hawley et al. (1984a, 1984b), Van Putten (1992, 1993a, 1993b, 1993c). Most methods do not make use of the concept of characteristics and experience major problems with extremely relativistic velocities ($\gamma > 4$, see Hawley et al. 1984b). In Eulerink (1993) it is shown that our method overcomes these problems and is capable of handling these extremely relativistic flows.

Marti et al. (1991) and Marquina et al. (1992) describe a numerical method for relativistic flow which is also based on a Roe solver. Test results indicate that their code also overcomes the problems other methods experience.

Van Putten (1992, 1993a, 1993b, 1993c) constructed a numerical method for relativistic magnetohydrodynamics. He uses a spectral method which in shock tube tests appears to slightly more diffusive than our method. Note however that his method handles magnetohydrodynamics.

The purpose of this paper is twofold: to check whether the jet formation by inertial confinement works at relativistic velocities and to apply our numerical method for special relativistic flow to a realistic situation.

The paper is subdivided in four sections. Section 2 describes the numerical method and Sect. 3 discusses the application to relativistic jet collimation. Section 4 sums up our conclusions.

2. Special relativistic Roe solver

In MEI91 a nonrelativistic Roe solver for a two-dimensional spherical grid was presented. Although the actual expressions change when applying it to the special relativistic case, the formalism does not change. We therefore only introduce the special relativistic description of the flow and refer to MEI91 for the actual method. The full numerical background of the Roe solver method will appear in Eulerink (1993). In the following, we have chosen not to present the most compact algebraic form of the equations, but rather a formulation which is convenient for numerical implementation. See Taub (1978) for a review of the theory of relativistic flow.

The coordinates are time ($x^0 = t$), and three spatial directions: radius ($x^1 = r$), polar angle ($x^2 = \theta$), and azimuthal angle ($x^3 = \phi$). The units are such that the speed of light (c) equals

one. The non-zero elements of the special relativistic covariant metric in spherical coordinates are

$$g_{00} = -1, \quad g_{11} = 1, \quad g_{22} = r^2, \quad g_{33} = r^2 \sin^2 \theta. \quad (1)$$

The determinant of the metric is consequently $g = -r^4 \sin^2 \theta$. The invariant interval is $-[d\tau]^2 = -[dx^0]^2 + g_{11}[dx^1]^2 + g_{22}[dx^2]^2 + g_{33}[dx^3]^2$. Hence, the four-velocity vector is¹

$$\left(\frac{dt}{d\tau}, \frac{dx^1}{d\tau}, \frac{dx^2}{d\tau}, \frac{dx^3}{d\tau} \right) \equiv (\gamma, u^1, u^2, u^3), \quad (2)$$

which is related to the three-velocity by

$$\frac{dx^1}{dt} = \frac{u^1}{\gamma}, \quad \frac{dx^2}{dt} = \frac{u^2}{\gamma}, \quad \frac{dx^3}{dt} = \frac{u^3}{\gamma}. \quad (3)$$

The physical components of the velocity are

$$v^1 = \sqrt{g_{11}} \frac{dx^1}{dt}, \quad v^2 = \sqrt{g_{22}} \frac{dx^2}{dt}, \quad v^3 = \sqrt{g_{33}} \frac{dx^3}{dt}. \quad (4)$$

Velocity normalisation may be written as

$$-1 = -\gamma^2 + g_{11}[u^1]^2 + g_{22}[u^2]^2 + g_{33}[u^3]^2 = \gamma^2(-1 + [v^1]^2 + [v^2]^2 + [v^3]^2). \quad (5)$$

The fluid at a particular event in spacetime can be described by the primitive variables ρ, v^1, v^2, v^3, p , where ρ is the density of the fluid and p the pressure. Let h denote the specific relativistic enthalpy of the flow. The Euler equations in spherical coordinates can then be written as

$$W_{,0} + F_{,1} + G_{,2} + H_{,3} = S, \quad (6)$$

where W is the state

$$W \equiv \sqrt{g}(\rho\gamma, \rho h\gamma^2 - p, \rho h\gamma u^1, \rho h\gamma u^2, \rho h\gamma u^3), \quad (7)$$

F is the flux in spatial direction r

$$F \equiv \sqrt{g}(\rho u^1, \rho h u^1 \gamma, \rho h u^1 u^1 + p g^{11}, \rho h u^1 u^2, \rho h u^1 u^3), \quad (8)$$

G is the flux in spatial direction θ

$$G \equiv \sqrt{g}(\rho u^2, \rho h u^2 \gamma, \rho h u^2 u^1, \rho h u^2 u^2 + p g^{22}, \rho h u^2 u^3), \quad (9)$$

H is the flux in spatial direction ϕ

$$H \equiv \sqrt{g}(\rho u^3, \rho h u^3 \gamma, \rho h u^3 u^1, \rho h u^3 u^2, \rho h u^3 u^3 + p g^{33}), \quad (10)$$

and S is the source term,

$$S \equiv r^2 \sin \theta (0, \quad 2p/r + \rho h u^2 u^2 r + \rho h u^3 u^3 r \sin^2 \theta, \quad p \cot \theta / r^2 - 2\rho h u^1 u^2 / r + \rho h u^3 u^3 \sin \theta \cos \theta, \quad -2\rho h u^1 u^3 / r - 2\rho h u^2 u^3 \cot \theta). \quad (11)$$

¹ All vectors in this paper are actually column vectors, but for ease of notation the transpose signs are omitted.

These formulae replace Eqs. (2–5) in MEI91. Assuming a perfect gas with adiabatic index Γ , the enthalpy density of the fluid is given by $\rho h \equiv \rho + \frac{\Gamma}{\Gamma-1}p$. The squared speed of sound is given by $s^2 = \Gamma p / (\rho h)$.

As in MEI91 we assume cylindrical symmetry without rotation, which means that $u^\phi = 0$. The appropriate split of the source terms (Eq. (11–12) of MEI91) then becomes

$$\mathbf{R} = r^2 \sin \theta (0, 2p/r + \rho h u^2 u^2 r, -2\rho h u^1 u^2 / r, 0), \quad (12)$$

and

$$\mathbf{T} = r^2 \sin \theta (0, 0, p \cot \theta / r^2, 0). \quad (13)$$

Hence the fourth component of Eq. 6 simplifies to an identity. This effectively reduces the number of spatial dimensions to two, namely r and θ .

The special relativistic expressions for $\mathbf{W}$, $\mathbf{F}$, $\mathbf{G}$, and $\mathbf{S}$ are now used in Eqs. (6–26) of MEI91 to obtain the special relativistic Roe solver in spherical coordinates. The expressions for the eigenvalues, eigenvectors and projection coefficients also change; these expressions can be found in the Appendix. To calculate these expressions one needs the ‘primitive variables’ (ρ, v^1, v^2, v^3, p). In the nonrelativistic case these can easily be calculated from the state vector $\mathbf{W}$. In relativistic flow one needs an iterative method to do this. We use a Newton-Kantorovich method (see Eulerink 1993).

We have one new aspect to add concerning the flux limiter. In MEI91 the limiter function was only applied to state differences (called a_k , see their Eqs. 19–22). Here we make use of the fact that one can also use it on the flux differences (b_k):

$$\mathbf{F}_{i+\frac{1}{2},j}^{n+\frac{1}{2}} = \frac{1}{2} (\mathbf{F}_L + \mathbf{F}_R) - \frac{1}{2} \sum_k \left[\sigma_k b_k + \psi(b_k, b_{ku})(v_k - \sigma_k) \right] \mathbf{e}_k. \quad (14)$$

Applying the limiter to the flux differences takes into account changes in the characteristic velocities. Therefore it prescribes a switch to the first order upwind scheme before the state difference limiter does, which makes the scheme more robust. However, near sonic points the flux difference limiter does not recognize the transition from a subsonic to a supersonic state because the subsonic and supersonic fluxes are nearly identical. This results in unphysical expansion shocks. To remedy this we change to the state difference limiter near sonic points.

3. Application to the problem of jet collimation

3.1. Nonrelativistic jet collimation

We will now describe briefly the kind of jet structures found at nonrelativistic velocities, so that a comparison can be made later.

We assume cylindrical symmetry and use a 2-D grid with spherical coordinates (r, θ) . The grid is one quarter of a plane of constant ϕ . In the tangential direction the grid boundaries lie at $\theta = 0$, the symmetry axis, and $\theta = \pi/2$, the equatorial plane.

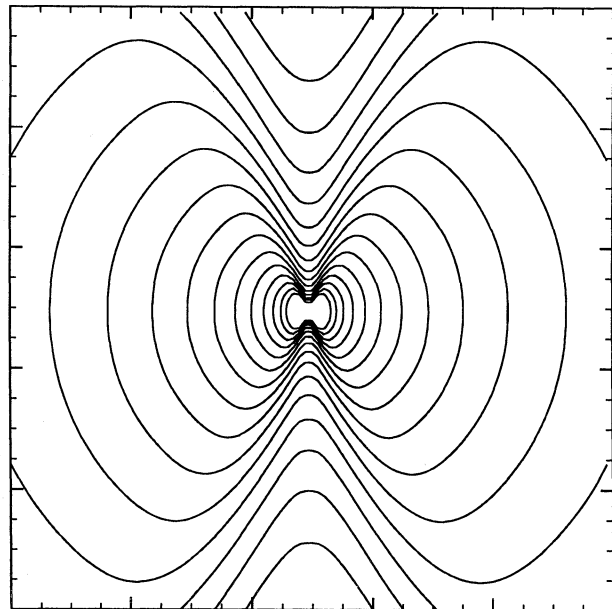


Fig. 1. The density profile of a typical thickened disk used in our simulations (logarithmic density contours). The parameters are $A = 0.9$ and $B = 8.0$ (see Eqs. 15–16)

We assume that $v_\theta = 0$ both at the symmetry axis and equatorial plane, and changes sign across it. In the radial direction the grid boundaries lie at $r_{\text{in}} = r_0 - dr$ and at $r_{\text{out}} = r_0 + (M - 1)dr$ (M being the total number of grid points in the radial direction). Since the initial condition of the disk is stationary (see below), we assume a stationary boundary condition at r_{out} . This is correct as long as the bubble does not cross r_{out} . When it does, the integration is stopped.

On this grid we let a high energy spherical outflow interact with a surrounding gas cloud the density distribution of which is given by

$$\rho(r, \theta) = \frac{\rho_0}{f(\theta)} \left(\frac{r_0}{r} \right)^2, \quad (15)$$

where

$$f(\theta) = 1 - A \left(\frac{1 - \exp(-2B \sin^2 \theta)}{1 - \exp(-2B)} \right). \quad (16)$$

This gas cloud is taken to be in global pressure equilibrium and has zero velocity. The density contours resulting from the above equations depend on the two parameters A and B . The first of these effectively determines the density contrast between pole and equator according to $\rho_p / \rho_e = 1 - A$, the second one controls the shape of the disk: the disk gets increasingly thicker with higher B . The opening angle of the funnel can be approximated by

$$\sin(2\theta_f) = \sqrt{1.5/B}. \quad (17)$$

Figure 1 shows the density profiles for the case $A = 0.9$, $B = 8.0$, which will be used in the remainder of this paper.

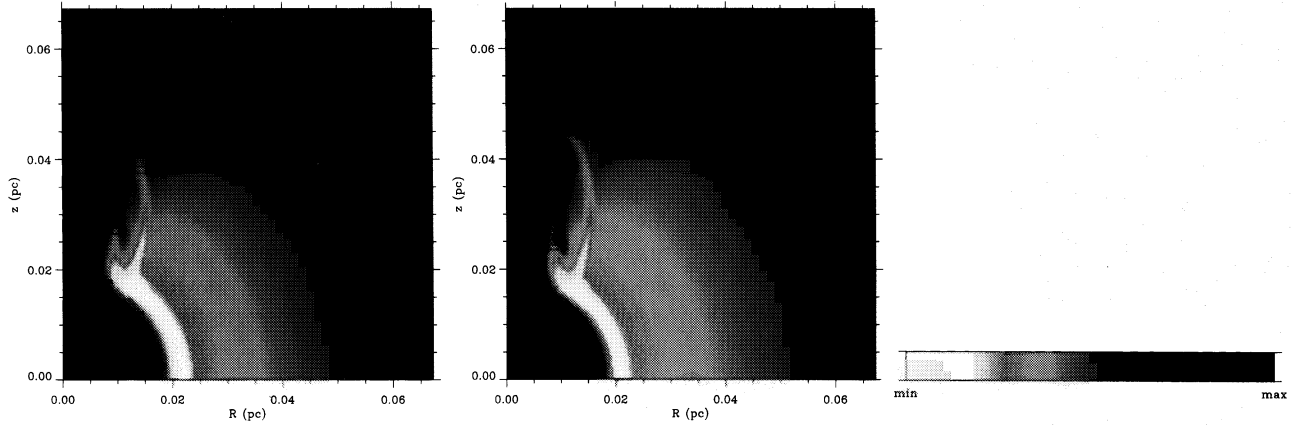


Fig. 2. Logarithmic density contour plots of jets formed by a nonrelativistic outflow (left) and a mildly relativistic outflow (right). The densities are shown at a time that the same morphology is reached ($t = 2.50 \cdot 10^{10}$ s on the left, $t = 1.60 \cdot 10^7$ s on the right). Densities run from $5.71 \cdot 10^{-19}$ to $3.40 \cdot 10^{-16}$ kg m^{-3} in the left plot and from $3.43 \cdot 10^{-19}$ to $3.64 \cdot 10^{-16}$ kg m^{-3} in the right plot. The colour bar shows the colour map from minimum to maximum density. Run parameters are given in the text. The velocities in the left plot a factor hundred lower than in the right plot. The left one was calculated using a nonrelativistic code

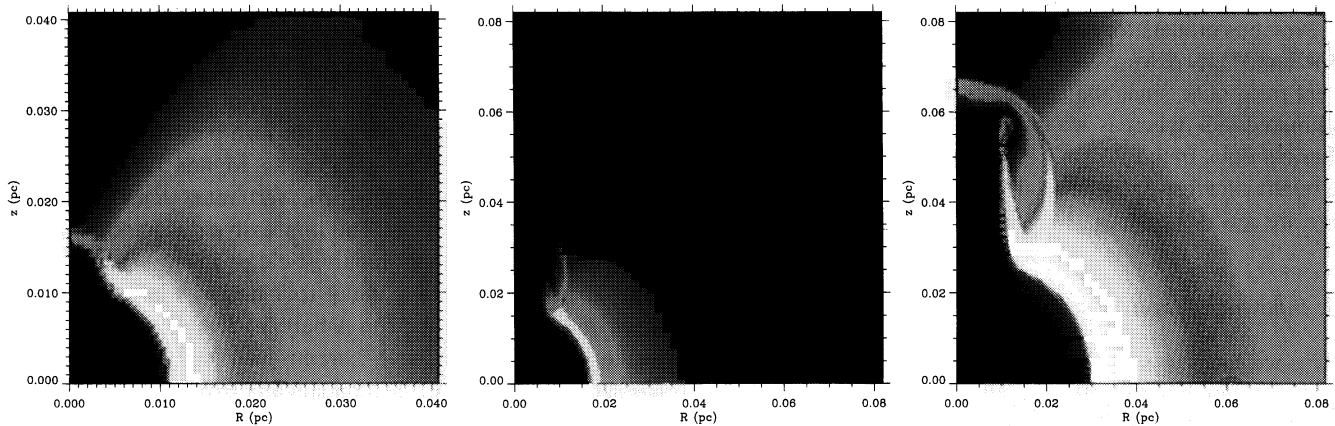


Fig. 3. Logarithmic density contour plots of jets formed by relativistic outflows of total energy densities corresponding to $\gamma = 1.34$ (left), $\gamma = 4.00$ (middle), and $\gamma = 12.3$ (right). All other parameters are the same. The densities are shown at the same time, $t = 8.5 \cdot 10^6$ s after the start of the interaction. The density ranges in kg m^{-3} are $1.27 \cdot 10^{-19}$ to $4.44 \cdot 10^{-16}$ (left), $2.73 \cdot 10^{-19}$ to $4.23 \cdot 10^{-16}$ (middle), $1.52 \cdot 10^{-19}$ to $1.69 \cdot 10^{-16}$ (right). The colour map is the same as in Fig. 2. Run parameters are given in the text. Note the smaller length scale in the left figure

Typical examples of the jets found in the nonrelativistic case can be found in MEI 91, IBF 92, and IMBEF 92. These structures are set up in the following way. The interaction between the outflow and the thick disk results in the well known bubble structure (Pikel'ner 1968; Dyson & de Vries 1972): there is an outer shock followed by a denser shell of swept up disk material, bounded on the inside by a contact discontinuity. Inside of that there is hot shocked outflow material filling the bubble. Because of the aspherical distribution of the surrounding material, the bubble becomes aspherical. The part near the equator still shows the simple bubble structure, because the thick disk is almost spherical in that region (see Fig. 1); the part of the bubble near the polar axis protrudes. The transition from the spherical part (which forms a barrel-shaped equatorial belt) to the protruding part is rather sudden and one can discern a clear cusp in the outer shock. It is at this position at the inside of

the bubble that the jet formation takes place. If one follows the development of the bubble one can see that the faster expanding protruding part bulges out beyond the barrel-shaped equatorial part, and curls around its edge. This leaves the rim of the barrel sticking into the bubble. The strong shearing flows consequently elongate this rim into a 'chimney', which serves as the jet confinement. Note that the effective opening angle of the jet thus formed, is much smaller than the opening angle of the funnel, in fact it is close to 0° . Turbulent hot gas (deflected from the head of the bubble) streams back on the outside of the chimney. The collimation is caused by three mechanisms: the inertial confinement by the chimney, the pressure of the back flowing gas outside it, and the refraction of fast gas by the aspherical inner shock (see IMBEF 92).

3.2. Relativistic flow

The objective is to see if relativistic jets are produced by the inertial confinement mechanism, and to compare them with their nonrelativistic counterparts. Before we present the results it is useful to consider what differences we expect. Looking at the equations describing relativistic flow (Eqs. 6–11), we see two major differences as compared to nonrelativistic flow. The first one is the appearance of Lorentz factors in all terms, due to Lorentz contraction in the continuity equation and due to Lorentz contraction and kinetic energy in the momentum and energy equations. The other one is the appearance of an enthalpy term in the equation of motion. This is due to the different way systematic and random velocities are handled relativistically.

The Lorentz factors change the time and length scales as compared to the nonrelativistic case. This is not simply a proportional scaling, because there are both γ and γ^2 terms coming in. The differences are consequently expected to be of order γ . There is of course the limit that the shock cannot travel faster than $c(=1)$. The effect of the enthalpy term in the equation of motion is less easily evaluated.

Previous models for relativistic jets can be found in Wilson (1987) and Daly & Marscher (1988). These models are very different from ours, since the authors consider stationary, already collimated jets, whereas we consider time dependent, initially spherical flow. In the Wilson (1987) paper the author compares the relativistic results to nonrelativistic results and explains why there are few differences.

3.3. Inflow conditions

At r_{in} we have to supply a boundary condition which at the same time models the driving outflow. In the nonrelativistic case we put in a stationary shock (see MEI 91), but for extremely relativistic velocities this proved to be unstable. Instead we have made an equipartition between the thermal and kinetic energy of the outflow. The total relativistic energy density is given by

$$\rho e = \left(\rho + \frac{\Gamma}{\Gamma - 1} p\right) \gamma^2 - p \quad (18)$$

(see eg. Mihalas & Mihalas 1984, Eq. [43.33]). Below we will indicate the total energy density of the driving outflow by its equivalent gamma value. So, an outflow with $\gamma = 4$ means that $e = 16\rho$, of which half is kinetic energy and half is internal energy. The quantities describing the inner boundary condition will have subscript c, those describing the disk subscript 0.

3.4. Mildly relativistic velocities

We first show the comparison between a mildly relativistic and a nonrelativistic simulation. For the nonrelativistic calculations we take as initial conditions $\rho_0 = 3 \cdot 10^{-17} \text{ kg m}^{-3}$, $T_0 = 10^4 \text{ K}$, $\rho_c = 1.23 \cdot 10^{-18} \text{ kg m}^{-3}$, $v_c = 6.667 \cdot 10^{-1} c$, $p_c = 5.086 \cdot 10^{-10} \text{ J m}^{-3}$. The mildly relativistic simulation has identical initial conditions except for $\rho_c = 9.1 \cdot 10^{-19} \text{ kg m}^{-3}$, $v_c = 0.553c$, and

$p_c = 2.51 \cdot 10^{-2} \text{ J m}^{-3}$. This corresponds to a total energy density equivalent to $\gamma = 1.34$ (see Sect. 3.3). The grid starts at $r_0 = 9.7 \cdot 10^{-3} \text{ pc}$ and extends to $6.7 \cdot 10^{-2} \text{ pc}$. It made up of 100 radial and 100 tangential cells. Figure 2 shows the contour plots of the (observed) mass density $\rho\gamma$ for both cases at the same dynamical time. The relativistic calculation has a maximum velocity of $\gamma = 1.5$ in the jet. It is evident that for these low values of γ the differences between nonrelativistic and relativistic flow are small. In fact the values of the densities and velocities agree to within a few percent with what one gets classically. This also shows that the code performs correctly in the limit of nonrelativistic velocities.

3.5. Extremely relativistic velocities

To make the comparison with the extremely relativistic case we have performed three runs with identical initial conditions ($\rho_0 = 3 \cdot 10^{-17} \text{ kg m}^{-3}$, $p_0 = 2.48 \cdot 10^{-9} \text{ J m}^{-3}$, $\rho_c = 1.00 \cdot 10^{-19} \text{ kg m}^{-3}$) but with different driving energy densities (see Sect. 3.3) in the outflow: run A has $v_c = 0.55c$, $p_c = 1.25 \cdot 10^{-3} \text{ J m}^{-3}$ (a total energy density equivalent to $\gamma = 1.34$), run B has $v_c = 0.942c$, $p_c = 2.74 \cdot 10^{-3} \text{ J m}^{-3}$ (a total energy density equivalent to $\gamma = 4.00$), and run C has $v_c = 0.994c$, $p_c = 2.92 \cdot 10^{-3} \text{ J m}^{-3}$ (a total energy density equivalent to $\gamma = 12.3$). The grid runs from $r_0 = 9.7 \cdot 10^{-3} \text{ pc}$ to $8.1 \cdot 10^{-2} \text{ pc}$ and has 125 by 125 cells. Figure 3 shows a logarithmic grey scale plot of the (observed) mass density for these three runs at $t = 8.5 \cdot 10^6 \text{ s}$. The impressive difference between the mildly and extremely relativistic cases is immediately visible. The run C bubble is about four times as large and much more developed than the run A bubble. The velocities reached in the jet at this time have Lorentz factors 3.2 in case B and 6.9 in case C. In case C the adiabatically expanding central outflow reaches a Lorentz factor of 15. To show the difference in scale between the three runs, we plot the position of the equatorial shock radius as a function of time in Fig. 4. The expansion velocity of the equatorial belt of the bubble is approximately constant in time for all three cases, but its value increases dramatically from 0.03 (case A) to 0.12 (case B) to 0.35 (case C), so it scales approximately proportional to γ . This is to be expected since the apparent density of the driving outflow increases as γ . The nonrelativistic description of the expansion of a bubble is given by Icke (1988). He shows that the expansion is exponential, but the characteristic time scale is very large in this case. This means that in the time regime of our simulations the expansion appears almost linear. The plot shows that even in the relativistic regime the expansion into a $1/r^2$ atmosphere has an initially constant velocity, but unfortunately we are not aware of an analytical solution in this case. The theoretical limit to the expansion velocity of course lies at $v = 1$ (also indicated in Fig. 4).

So the expected difference in scaling is indeed present. The difference in morphology is less obvious. Judging from Fig. 3 one would say that the general form of the flow pattern does not change very much, even in the ultra-relativistic case. Comparing Fig. 3c with Fig. 2, we see more or less the same features:

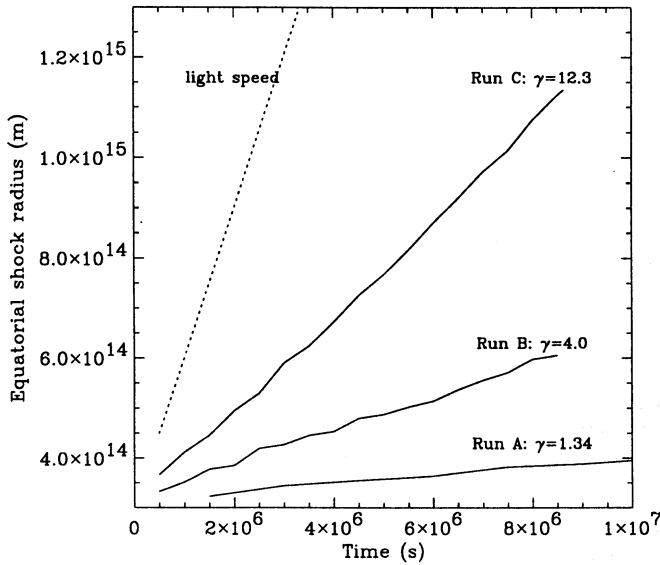


Fig. 4. Graph showing the position of the outer shock position at the equator against time for the runs A, B, and C. The increase in expansion velocity when going to extremely relativistic velocities is clear. The line on the left indicates the velocity of light

equatorial belt, chimney, terminal shock, and back flow. Only the time scale has drastically changed. The similarity of the relativistic and nonrelativistic cases is in line with Wilson (1987).

We present the changes in morphology in two ways. One manner to represent the form of the bubble is to evaluate the polar to equatorial shock radius (R_p/R_e) as a function of time. In the nonrelativistic case this is expected to increase exponentially, until the fastest waves have outrun all others (at that stage the protruding part has swallowed up the entire equatorial belt), after which R_p/R_e remains constant (Icke 1988). This last phase has not been reached by far in our simulations, so we expect R_p/R_e to increase. Figure 5 shows R_p/R_e against time for our three cases. This plot shows that whereas the extreme case C initially has the steepest rise in asphericity, this increase levels off, and indeed, at a given time beyond 5.5×10^6 seconds, the case B bubble is more aspherical (although at a smaller size, see Fig. 4). As we have seen, the expansion velocity at the equator is more or less constant and the increase in R_p/R_e is caused by an acceleration of the bubble in the polar direction. So, the stagnation in case C means that this acceleration is diminishing. This is reasonable, since acceleration gets increasingly hard in the ultra-relativistic case.

Another way to look at it is to compare the form of the bubble at a time when all three cases have reached the same value of R_p/R_e (Fig. 6). This figure shows that the opening angle of the chimney remains close to zero in all cases. We see no signs that the collimation mechanism becomes less efficient in case C: the fierceness of the outflow produces a very nice jet, whereas in cases A and B the chimney is less well developed. It takes quite some time for a reasonable jet structure to develop in case B (see Fig. 6b), and in case A we found no clear jet up to the point where the integration was stopped (Fig. 6a). This last fact

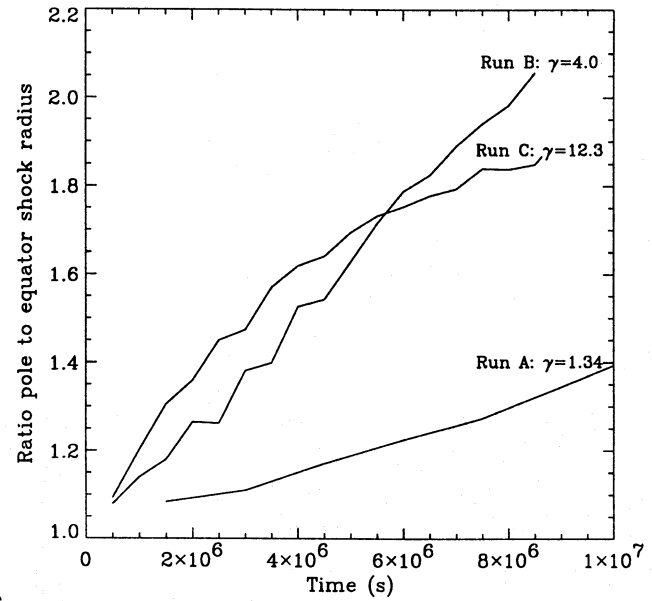


Fig. 5. Graph showing the ratio of outer shock position at pole and equator against time for the runs A, B, and C. The asphericity of the ultra-relativistic case C is initially highest, but gradually falls below that of the relativistic case B

indicates that it requires more than just a thickened disk to get a jet structure: some conditions have to be met by the outflow. A more massive outflow does the trick (as shown by Fig. 2b, which has identical parameters to case A, but the outflow is a factor 9.1 more massive), but other factors might come in too.

A new feature that is found only in the extremely relativistic case C is an area of very low density along the chimney. Clearly, the gas being deflected from the equatorial region cannot change its velocity too drastically and flows into the jet, leaving an area along the chimney almost evacuated. This is yet another consequence of the increased inertia of relativistically moving mass.

3.6. Code behaviour

Because we are using a new numerical method we briefly discuss its behaviour in these simulations. The simulations were done on a Convex C210, taking up some 10 CPU hours per run. Most runs were stopped when the flow pattern reached the edge of the grid.

As is also noted in Eulderink (1993), the Roe solver produces very good approximations to the actual flow in a great number of cases, but for some interactions it clearly makes mistakes. Because of the low internal diffusion, small errors are not smoothed out, in some cases leading to unphysical solutions with negative pressures or densities. These errors are due to the fact that one is working with averages over a cell, instead of with the true, continuously varying quantities. The introduction of source terms due to the grid only makes it worse. This annoying property of the method already becomes clear in the nonrelativistic case, and experience shows that it does not get any better in the relativistic case. In practice this means that

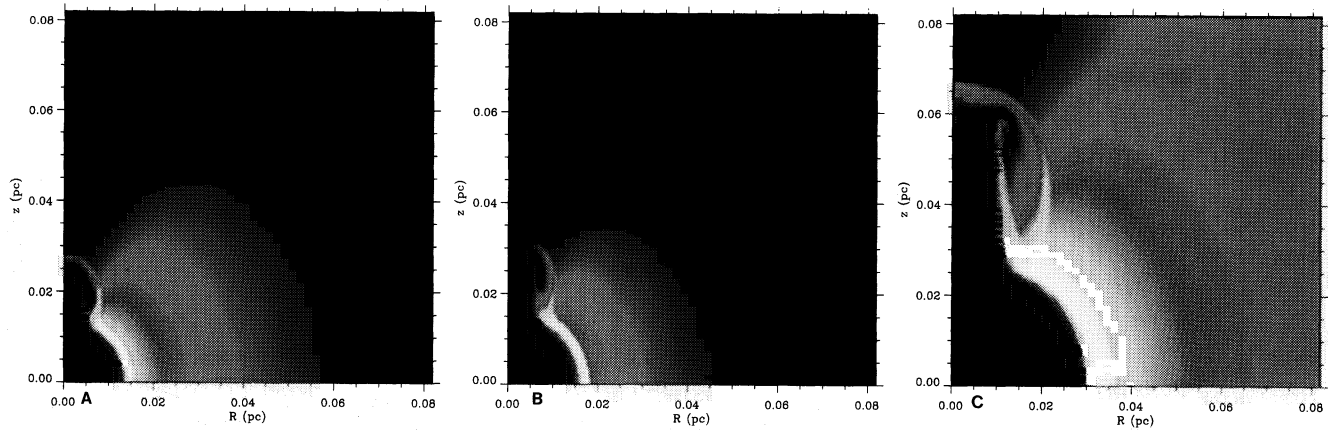


Fig. 6. Logarithmic density contour plots of jets formed by relativistic outflows of runs A (left), B (middle), C (right). The densities are shown at the time that the same R_p/R_e is reached; $t_A = 19.5 \cdot 10^6$ s, $t_B = 7.5 \cdot 10^6$ s, $t_C = 8.5 \cdot 10^6$ s. Note the different length scales and the fact that $t_B < t_C$. Density ranges in kg m^{-3} are $1.27 \cdot 10^{-19}$ to $4.74 \cdot 10^{-16}$ (left), $3.16 \cdot 10^{-19}$ to $4.44 \cdot 10^{-16}$ (middle) $1.52 \cdot 10^{-19}$ to $1.69 \cdot 10^{-16}$ (right). The colour map is the same as in Fig. 2

there are very strict limitations to get out results. To go all the way up to $\gamma = 15$ we have to limit *flux* differences in the radial integration and *state* differences in the tangential integration. Furthermore we had to apply a correction for negative pressure. This amounted to locally changing to more diffusive, first order fluxes whenever a negative pressure was produced, and if that did not help, actually borrowing internal energy from the neighbouring cells to keep the pressure positive. Such under pressures can be suppressed by taking smaller cell sizes, but this is impractical because of CPU time considerations. As was outlined in Sect. 3.3 we had to fix the inner boundary condition such that an equipartition between thermal and kinetic energy was imposed.

So, what does this mean for the applicability of the method? The problems requiring the above measures occur at a very specific position in the flow, namely at the inner boundary. The majority of points on the grid behave well, even at the most extreme velocities. This means that it might well be possible to work around some of these difficulties. Furthermore, the problem chosen is rather hard, even in the nonrelativistic case. At the inner edge of the grid a continuous inflow has to be supplied, and this is done by not updating these inner points. In reality an inner shock runs into the fast outflow, stalls and travels outward again. This last situation does indeed occur, but since we are not changing the state of the inner boundary, the inner shock has a hard time detaching itself from the boundary. The fact that the inner shock is not exactly spherical and hence not precisely parallel to grid lines of constant radius is a further complication (see Van Leer 1989)

It has to be kept in mind that generating unphysical solutions is largely inherent to numerical methods in general. To begin with, we do not know the exact state at every point, we only know the average over a cell. And if one starts to change these averages using any type of approximation which is not entirely physical (like Lax-Wendroff, FCT, PPM, or Roe solver), there is no absolute guarantee that the new state will be physical! Some methods may be less sensitive to errors than others

because of a very high diffusion (which effectively smoothes extreme gradients, so also the non-physical ones), or because the errors just are not fatal. For instance, the FCT method does not automatically crash in case of a negative pressure, because it only uses pressure gradients. A negative pressure kills the Roe solver because it needs its square root for the velocity of sound. So, robustness might often be just a sign of insensitivity.

To conclude, there is certainly room for improvements on the special relativistic Roe solver presented here, but as it stands it is a working method and is, as far as we know, the only explicit scheme capable of tackling relativistic velocities up to at least $\gamma = 15$.

4. Conclusions

Our conclusions can be summarized as follows:

1. The special relativistic Roe solver, as presented in this article, is capable of treating extremely relativistic flow patterns up to at least a Lorentz factor of 15.
2. For the case of jet formation by inertial confinement, we find no major differences between the nonrelativistic and mildly relativistic case.
3. At extremely relativistic velocities, the time and length scales involved change drastically, but the general flow pattern of the jet formation mechanism stays the same.
4. There are no signs that the collimation mechanism becomes less efficient at extremely relativistic velocities.
5. At extremely relativistic velocities some areas in the flow pattern of the jet formation mechanism become effectively devoid of gas, because of the inability of relativistically flowing gas to adjust to very sudden changes.

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Appendix : explicit expressions

The explicit expressions for the eigenvectors, the coefficients and characteristic velocities at a cell interface are given here. All flow variables are averaged between the value left and right of the interface using the square root of enthalpy density (K):

$$K = \sqrt{\sqrt{-g}\rho h}, \quad (1)$$

So for example the Lorentz factor at the cell interface becomes

$$\gamma \equiv \frac{[K\gamma]_L + [K\gamma]_R}{[K]_L + [K]_R}, \quad (2)$$

where L and R stand for left and right of the interface. All flow variables that appear below have been averaged like this.

As was discussed in MEI 91 the actual integrations are done separately in the radial and in the polar direction. First the expressions are given for the integration in the radial direction and then for the integration in the polar direction. The radius r corresponds to x^1 and the polar angle θ corresponds to x^2 .

At an interface at (r, θ) the characteristic speeds for the integration in the radial direction are given by

$$\lambda_1 = \frac{(1 - s^2) \gamma u^r - \bar{s} y^r}{(1 - s^2) \gamma^2 + \bar{s}^2}$$

$$\lambda_2 = \frac{(1 - s^2) \gamma u^r + \bar{s} y^r}{(1 - s^2) \gamma^2 + \bar{s}^2}$$

$$\lambda_3 = \frac{u^r}{\gamma}$$

$$\lambda_4 = \frac{u^r}{\gamma},$$

where and s is the sound speed

$$s^2 = \frac{\Gamma p}{\rho h},$$

and $\bar{s}^2$ and y^r are given by

$$\bar{s}^2 \equiv \frac{1}{2} s^2 \left(1 + \gamma^2 - [u^r]^2 - r^2 [u^\theta]^2 \right) - \frac{1}{2} (\Gamma - 1) \left(1 - \gamma^2 + [u^r]^2 + r^2 [u^\theta]^2 \right),$$

and

$$y^r \equiv \sqrt{(1 - s^2) (\gamma^2 - [u^r]^2) + \bar{s}^2}.$$

The eigenvectors are

$$e_1 = \left(c_-, \gamma - \frac{\bar{s}}{y^r} u^r, u^r - \frac{\bar{s}}{y^r} \gamma, u^\theta \right)$$

$$e_2 = \left(c_-, \gamma + \frac{\bar{s}}{y^r} u^r, u^r + \frac{\bar{s}}{y^r} \gamma, u^\theta \right)$$

$$e_3 = \left(c_- + \frac{\bar{s}^2}{\Gamma - 1}, \gamma, u^r, u^\theta \right)$$

$$e_4 = (-c_+ r^2 u^\theta, 0, 0, 1),$$

where

$$c_- = 1 - \frac{s^2}{\Gamma - 1} \quad (8)$$

$$c_+ = 1 + \frac{s^2}{\Gamma - 1}.$$

The projection coefficients of a flux difference $F_R - F_L \equiv (\delta, \Delta^0, \Delta^1, \Delta^2)$ in the radial direction are:

$$b_1 = - \frac{1}{2 (\gamma^2 - [u^r]^2) \bar{s}^2} \left(\bar{s}^2 k^r + \bar{s} y^r (\gamma \Delta^1 - u^r \Delta^0) + (\Gamma - 1) (\gamma^2 - [u^r]^2) (\delta + c_+ (-\gamma \Delta^0 + u^r \Delta^1 + r^2 u^\theta \Delta^2)) \right)$$

$$b_2 = - \frac{1}{2 (\gamma^2 - [u^r]^2) \bar{s}^2} \left(\bar{s}^2 k^r - \bar{s} y^r (\gamma \Delta^1 - u^r \Delta^0) + (\Gamma - 1) (\gamma^2 - [u^r]^2) (\delta + c_+ (-\gamma \Delta^0 + u^r \Delta^1 + r^2 u^\theta \Delta^2)) \right) \quad (9)$$

$$b_3 = - \frac{1}{(\gamma^2 - [u^r]^2) \bar{s}^2} \left(2 \bar{s}^2 k^r + (\Gamma - 1) (\gamma^2 - [u^r]^2) (\delta + c_+ (-\gamma \Delta^0 + u^r \Delta^1 + r^2 u^\theta \Delta^2)) \right)$$

$$b_4 = \Delta^2 - \frac{k^r u^\theta}{(\gamma^2 - [u^r]^2)},$$

where

$$k^r \equiv \gamma \Delta^0 - u^r \Delta^1. \quad (10)$$

In the polar direction the values of the characteristic speeds are given by

$$\lambda_1 = \frac{(1 - s^2) \gamma u^\theta - \bar{s} y^\theta}{(1 - s^2) \gamma^2 + \bar{s}^2} \quad (5)$$

$$\lambda_2 = \frac{(1 - s^2) \gamma u^\theta + \bar{s} y^\theta}{(1 - s^2) \gamma^2 + \bar{s}^2} \quad (6)$$

$$\lambda_3 = \frac{u^\theta}{\gamma}$$

$$\lambda_4 = \frac{u^\theta}{\gamma}, \quad (11)$$

where $\bar{s}$ as above and

$$y^\theta \equiv \sqrt{(1 - s^2) \left((\gamma/r)^2 - [u^\theta]^2 \right) + (\bar{s}^2/r^2)}. \quad (12)$$

The eigenvectors are

$$\begin{aligned} e_1 &= \left(c_-, \gamma - \frac{\bar{s}}{y^\theta} u^\theta, u^r, u^\theta - \frac{\bar{s}}{y^\theta} \frac{1}{r^2} \gamma \right) \\ e_2 &= \left(c_-, \gamma + \frac{\bar{s}}{y^\theta} u^\theta, u^r, u^\theta - \frac{\bar{s}}{y^\theta} \frac{1}{r^2} \gamma \right) \\ e_3 &= \left(c_- + \frac{\bar{s}^2}{\Gamma - 1}, \gamma, u^r, u^\theta \right) \\ e_4 &= (-c_+ u^r, 0, 1, 0), \end{aligned} \quad (13)$$

and the projection coefficients of a flux difference $F_R - F_L \equiv (\delta, \Delta^0, \Delta^1, \Delta^2)$ in the polar direction are

$$\begin{aligned} b_1 &= -\frac{1}{2 \left((\gamma/r)^2 - [u^\theta]^2 \right) \bar{s}^2} \left(\bar{s}^2 k^\theta + \right. \\ &\quad \left. \bar{s} y^\theta (\gamma \Delta^2 - u^\theta \Delta^0) + (\Gamma - 1) \left((\gamma/r)^2 - [u^\theta]^2 \right) \right. \\ &\quad \left. (\delta + c_+ (-\gamma \Delta^0 + u^r \Delta^1 + r^2 u^\theta \Delta^2)) \right) \\ b_2 &= -\frac{1}{2 \left((\gamma/r)^2 - [u^\theta]^2 \right) \bar{s}^2} \left(\bar{s}^2 k^\theta - \right. \\ &\quad \left. \bar{s} y^\theta (\gamma \Delta^2 - u^\theta \Delta^0) + (\Gamma - 1) \left((\gamma/r)^2 - [u^\theta]^2 \right) \right. \\ &\quad \left. (\delta + c_+ (-\gamma \Delta^0 + u^r \Delta^1 + r^2 u^\theta \Delta^2)) \right) \\ b_3 &= -\frac{1}{\left((\gamma/r)^2 - [u^\theta]^2 \right) \bar{s}^2} \left(2 \bar{s}^2 k^\theta + \right. \\ &\quad \left. (\Gamma - 1) \left((\gamma/r)^2 - [u^\theta]^2 \right) \right. \\ &\quad \left. (\delta + c_+ (-\gamma \Delta^0 + u^r \Delta^1 + r^2 u^\theta \Delta^2)) \right) \\ b_4 &= \Delta^1 - \frac{k^\theta u^r}{\left((\gamma/r)^2 - [u^\theta]^2 \right)}, \end{aligned} \quad (14)$$

where

$$k^\theta \equiv \frac{1}{r^2} \gamma \Delta^0 - u^\theta \Delta^2. \quad (15)$$

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