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Line absorption and absorption-coefficient inside a star, by *J. Woltjer Jr.*

About a year ago S. ROSSELAND [*] showed that the mean absorption-coefficient x involved in the theory of the radiative equilibrium of a star's interior is equal to a combination of the absorption-coefficients x_ν for the frequency ν, expressed by the formula:

$$(1) \qquad \frac{1}{x} = \frac{\displaystyle\int_0^\infty \frac{1}{x_\nu} \frac{\partial B_\nu}{\partial T} \, d\nu}{\displaystyle\int_0^\infty \frac{\partial B_\nu}{\partial T} \, d\nu} \; ;$$

B_ν is the intensity of black body radiation of frequency ν and temperature T, a known function of ν and T.

Since then the idea has been prevalent that line absorption does not contribute to the value of x, the argument being that some values nearly zero of the integrand in (1) during a small interval $\Delta\nu$ would not make out anything of importance for the value of the integral.

However the value of x, computed from (1) starting from physical considerations about the nature of the absorption inside a star, only accounted for about $1/12$ of the value deduced from astronomical data [**], thus making the discrepancy already present in EDDINGTON'S [***] researches on the absorption-coefficient still worse.

The purpose of this note is to reconsider the question of line absorption and to show how, under the circumstances existing inside a star, its influence on the value of x may become predominating and may easily account for the large value of x deduced from astronomical data.

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[*] *M. N.* 84, 7.
[**] EDDINGTON, *M. N.* 85, p. 409.
[***] *M. N.* 84, 3.

1. The first thing to consider is the range of values of the frequency ν, in which line absorption occurs.

The atoms inside a star are strongly ionised; the degree of ionisation will depend on conditions of temperature and pressure and on the ionisation potentials of the atoms. Energy levels with low ionisation potential will have lost all electrons, those with extremely high ionisation potentials will retain their electrons. Following EDDINGTON [*] we introduce a critical ionisation potential defined by the condition that for this value one half of the atoms shall be ionised as far as the corresponding energy-level, and the other half unionised. This critical value is useful to estimate the condition of the atoms inside the star.

Indeed, $h\nu_1$ being the value of the ionisation potential, we may consider ν_1 as the limit of the absorption line spectrum of the atom.

The value of ν_1 in the centre of Capella results from the equation [**]:

$$(2) \qquad \frac{h\nu_1}{kT} = 7.5 \; ;$$

h and k are the usual constants, T the absolute temperature.

The maximum of the weight-factor $\dfrac{\partial B_\nu}{\partial T}$ in (1) is determined by the value of ν satisfying the equation:

$$(3) \qquad \frac{2 \dfrac{h\nu}{kT}}{4 + \dfrac{h\nu}{kT}} = 1 - \exp\left\{-\frac{h\nu}{kT}\right\}.$$

The resulting value is:

$$(4) \qquad \frac{h\nu}{kT} = 3.8$$

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[*] *M. N.* 84, p. 108. For an exact definition the reader may be referred to this publication.
[**] EDDINGTON, *M. N.* 84, p. 110.

Comparing (2) and (4) we conclude that in the centre of Capella the absorption lines belong to a frequency range affected with relatively large weight in the mean value (1).

2. The idea of a diffusion of the stationary states of an atom under the influence of electrical fields is not new.

The consideration of this effect on the width of the absorption lines inside a star is necessary for the evaluation of the mean value (1).

Indeed, if the absorption lines correspond to large frequency intervals (contrary to laboratory conditions), then the argument of the inefficiency of these lines in contributing to the integral in (1) fails.

An estimation of the intensity of the electrical field due to the radiation compared with the electrostatic force exercised on the photo-electron by the nucleus and the surrounding electrons, may be performed in the following way.

The energy-density of the radiation inside a star corresponding to a temperature of, say, 10^7 degrees is equal to

(5) $\quad 7.64 \times 10^{-15} \times 10^{28} = 7.64 \times 10^{13}$ erg per cm³.

On the other hand the energy-density of the electromagnetic field is equal to:

(6) $\quad \dfrac{1}{8\pi}\left\{\overline{E_x^2} + \overline{E_y^2} + \overline{E_z^2} + \overline{H_x^2} + \overline{H_y^2} + \overline{H_z^2}\right\} = \dfrac{\overline{E^2}}{4\pi};$

E is the intensity of the electric field, H the intensity of the magnetic field. Equating (5) and (6) we get the value of $\overline{E^2}$:

(7) $\qquad \overline{E^2} = 9.6 \times 10^{14}$

$\qquad \sqrt{(\overline{E^2})} = 3.1 \times 10^7$ electrostatic units.

Hence we may consider the amplitude of the oscillating electric vector equal to:

(8) $\qquad \sqrt{(2\overline{E^2})} = 4.4 \times 10^7$ electrostatic units.

Generally expressed (5) is equal to:

(5a) $\qquad\qquad a T^4,$

a being the energy-density of black body radiation of temperature 1°; thus (8) is equal to:

(8a) $\qquad\qquad T^2 \sqrt{\left\lvert\, 8\,\pi a \,\right\rvert}.$

Suppose that the photo-electron describes a circular orbit under the attraction of the nucleus and the surrounding electrons, together aequivalent to a single charge Ze (e is the electronic charge); let the quantum number of this orbit be n. Then the attraction per unit charge exercised on the photo-electron by the aequivalent charge Ze is equal to:

(9) $\qquad \dfrac{16\,\pi^4\,e^5\,Z^3\,m_e{}^2}{n^4\,h^4},$

m_e being the electronic mass; the numerical value of (9) is equal to:

(10) $\qquad 1.7 \times 10^7 \times \dfrac{Z^3}{n^4}$ electrostatic units.

Hence the ratio of the electric forces exercised on the photo-electron by the radiation and by the nucleus and surrounding electrons respectively is equal to:

(11) $\qquad 2.6 \times 10^{-14}\,\dfrac{n^4\,T^2}{Z^3}.$

Substituting again $T = 10^7$ we get:

(12) $\qquad 2.6\,\dfrac{n^4}{Z^3}.$

Generally n will have small values; Z will be rather large, of the order of magnitude of say, 25. Thus[*] the influence of the radiation field on the photo-electron, changing each moment in direction and magnitude, will in the mean be a relatively large fraction of the influence of the nucleus and surrounding electrons. Hence the width of the absorption lines will be a large fraction of the frequency and the absorption lines may fill up the whole frequency range efficient in the integral from (1) and line absorption determines the value of the mean absorption coefficient. A special circumstance making the filling up still more efficient is the fact that we are concerned not with one element but with a mixture of a great number of elements; for their absorption spectra generally will not exactly coincide.

3. As we have seen that line absorption preponderates in the determination of the mean absorption coefficient we shall try to estimate the magnitude of line absorption.

We start from EINSTEIN's relation, connecting the probability of transition of the photo-electron from the lower energy level to the higher one under the influence of radiation, with the probalility of spontaneous emission accompanied by a transition of the electron in the inverse direction. Let τ be the mean duration of the life of the atom in the excited state. Then this relation may be expressed by the formula[**]).

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[*] The action of the radiation field on the nucleus is negligible on account of the heavy nuclear mass.

Strictly speaking we should consider the difference of the action of the field on the two stationary orbits connected with an absorption line. The change of (12) with n however is rapid enough to allow us to forego a discussion of this effect.

The magnetic field has only a small influence on account of the small value of the ratio of electron velocity to light velocity.

[**] E. A. MILNE, *M. N.* **85**, p. 119.

$$(13) \qquad x_\nu \, m_A \Delta\nu = \frac{1}{8\pi} \frac{\lambda^2}{\tau};$$

m_A is the mass of the atom, $\Delta\nu$ the frequency interval corresponding to the width of the line, λ the wavelength.

The quantity τ has been determined in several cases, e. g. by MILNE from astrophysical data *) for the H- and K-lines of Ca^+. Substituting this value, viz.:

$$(14) \qquad \tau = 0.6 \times 10^{-8} \text{ sec.,}$$

the value of $x_\nu \Delta\nu$ for the H- and K-lines of Ca^+ appears to be equal to:

$$(15) \qquad 1.6 \times 10^{20} \text{ C. G. S.}$$

As we are concerned with the value of x_ν, or, as we may say, with the value of τ for large values of Z, we must make an estimate of the change of τ caused by a change in Z. Tentatively we use the well known hypothesis that τ is of the same order of magnitude as the time needed by an electron to emit the quantum $h\nu$ classically. A change of the aequivalent nuclear charge from 1 to Z effects:

a multiplication of the acceleration of the electron by Z^3 (see (9));
» 　　» 　　　» 　» classical rate of radiation » Z^6;
» 　　» 　　　» 　» quantum $h\nu$ 　　　　　　» Z^2;
» 　　» 　　　» 　» interval τ 　　　　　　» $Z^2/Z^6 = Z^{-4}$;
» 　　» 　　　» 　» $x_\nu\, m_A \Delta\nu$ (see (13)) 　» $Z^4/Z^4 = 1$.

As the change in m_A for the different elements (we are only concerned with the heavier elements, as the lighter ones are only bare nuclei) is small, we adopt for $x_\nu \Delta\nu$ a value of the order of magnitude of (15):

$$(16) \qquad 10^{20} \text{ C. G. S.}$$

4. Tentatively we may now make an estimate of the order of magnitude of the mean absorption-coefficient x.

If in (12) we consider an absorption line corresponding to the transition $n = 1$ to $n = 2$, and we take $Z = 25$ we are able to estimate the deformation of the energy levels, hence the width $\Delta\nu$.

Neglecting the relatively insignificant influence on the lower level and putting $\Delta\nu/\nu$ equal to twice the value of (12) for the higher level we get:

*) $M. N.$ **84**, 5.

$$(17) \qquad \frac{\Delta\nu}{\nu} = 0.5 \times 10^{-2}.$$

Taking ν equal to $3/4$ of ν_1 (see formula (2)) we get for the centre of Capella ($T = 10^7$ degrees):

$$(18) \qquad \nu = 1.2 \times 10^{18}.$$

Hence from (17) and (16):

$$(19) \qquad \begin{aligned} \Delta\nu &= 0.6 \times 10^{16} \\ x_\nu &= 2 \times 10^4 \text{ c. g. s.} \end{aligned}$$

Proceeding to the evaluation of the mean value of x_ν according to (1) we must consider the following circumstances:

Firstly: as the mean limit of absorption spectrum ν, determined by (2) lies rather outside of the maximum weight-factor determined by (4), only part of the elements will contribute with their large x_ν values to the lowering of the integrand in (1).

Hence x will have a smaller value than would be the case if x_ν were always equal to (19).

Secondly: if the line absorption spectrum, especially at the lower frequencies, does not sufficiently fill up the range of values of ν effective in formula (1), then we also will get a lowering of x as the value of the absorption coefficient x_ν corresponding to the free-free transitions of electrons is very small *).

Hence we conclude that line absorption is able to reconcile the value of the absorption coefficient x derived from physical considerations with the value [$x = 47.5$ at the centre of Capella **] deduced from astronomical data.

5. Accepting the considerations of the preceeding sections we evade the discrepancy between the absolute value of x from physical and from astronomical data, but at the same time a new difficulty arises: the variation of x with density and temperature involved in EDDINGTON's mass-luminosity function could previously be derived from physical arguments, but now must be established *de novo*. Whether this is possible, and in which way, may, however, at present be left aside as an object of further investigation.

*) EDDINGTON, $M .N.$ **84**, p. 107.
) 　　» 　　$M. N.$ **84, p. 109.