Note on the luminosity of Cepheids and the discrepancies in the pulsation theory

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magnitudes of the observed maxima and minima, relative to the median magnitude, are plotted against ψ, computed with (3).

The ascending branches observed by EGGEN [1]) have arrived about 0.02 of a primary cycle earlier than the moments computed with (2).

This would suggest a correction for P_0, which is about three times the error expected from our observations. At the moment we cannot decide whether this is caused by a change in period or by inaccurate observations. Interesting is the comparison between the light-curves of HD 223065 and of AI Velorum [2]). The light-curves of HD 223065 can be derived from two interfering sine curves, by steepening and magnitude distortion, in very much the same way as for AI Velorum.

The ratio of the periods P_0 and P_1 is the same for both stars.

[1]) O. J. EGGEN, *P.A.S.P.* **64**, 305, 1952.
[2]) *B.A.N.* **11**, 421, 1952.

Star	P_1/P_0
HD 223065	0.778
AI Velorum	0.773

Very different for the two stars are the amplitudes of the overtone pulsations. Whereas in AI Velorum the amplitude of the oscillation P_1 is about three quarters of that of P_0, the corresponding ratio in HD 223065 is only one quarter.

Furthermore the oscillations with still shorter periods, which are so conspicuous in AI Velorum, seem to be absent in HD 223065, for not one of the light-curves obtained from this star shows a trace of a hump. This is confirmed by Figure 2, of which the points are much less scattered than in the corresponding figure for AI Velorum.

So the conclusion might be that HD 223065 is a star of the same type as AI Velorum but with less excitation of overtones.

NOTE ON THE LUMINOSITY OF CEPHEIDS AND THE DISCREPANCIES IN THE PULSATION THEORY

by M. SAVEDOFF *)

With a provisional absolute magnitude, the radius, mass and pulsation period for δ Cephei, η Aquilae, and RR Lyrae have been computed. Comparison with pulsation theory shows that, if the change in zero-point is $-1^{m}.4$, the computed and observed periods for δ Cephei and η Aquilae are consistent. It is suggested that RR Lyrae would have a period consistent with theory, if its mass were reduced 50%.

In recent discussions of the discrepancy between the computed and observed periods of δ Cephei variables, the effect of a systematic error in the luminosity has been neglected. W. BAADE [1]) has indicated that the absolute magnitude of δ Cephei variables has been underestimated by between $1^{m}.0$ and $1^{m}.5$ [2]). This change within present uncertainties removes the inconsistency between the period-density, mass-luminosity, and radius-temperature-luminosity relations.

A. BLAAUW has redetermined the zero-point of the period-luminosity law, using newly determined proper motions communicated by H. R. MORGAN. I am indebted to DR BLAAUW for permission to use in the following discussion his provisional result, $\Delta M = -1^{m}.4 \pm .3$ (m.e.) referred to SHAPLEY's period-luminosity curve.

In Table 1 are collected the pertinent data for the two galactic cepheids, η Aquilae and δ Cephe, and the cluster-type cepheid RR Lyrae. The effective temperatures quoted are based upon Six Color Photometry of Stars II [3]), VII [4]), and incomplete unpublished measures of RR Lyrae by STEBBINS. For the δ Cephei variables, minimum light was chosen in both cases, as STRUVE [5]) states that the spectrum is nearest normal at that phase. The effective temperature for δ Cephei at minimum light, obtained from the equivalent spectral type and KUIPER's relation [6]) between spectral type and effective temperature, is in good accord with the temperatures T_1 from V−I of Six Color Photometry III [7]). For η Aquilae at minimum light, the temperature on the T_1-scale is 4550° K, while the temperature for the equivalent spectral class according to KUIPER's table is 4850° K (provided that the supergiants have the same temperature as giants). SCHWARZSCHILD [8]) from the same data has obtained 4800° K. The increased distance

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[1]) *Trans. I.A.U.* vol. VIII, 1952, in press.
[2]) *Note added in proof.* THACKERAY and WESSELINK (*Nature* **171**, 693, 1953) find from a comparison of classical and cluster cepheids in both the Large and the Small Magellanic Clouds $\Delta M \sim -1^{m}.2$.

[3]) *Ap. J.* **101**, 47, 1945.
[4]) *Ap. J.* **115**, 292, 1952.
[5]) *Observatory* **65**, 257, 1944.
[6]) *Ap. J.* **88**, 429, 1938.
[7]) *Ap. J.* **102**, 318, 1945.
[8]) *Ap. J.* **108**, 207, 1948.

TABLE 1

	η Aql		δ Cep	RR Lyr
M_v min.	$-3.4 \pm .3$		$-3.0 \pm .3$	.00*
M_b med.	$-4.1 \pm .3$		$-3.7 \pm .3$	$-.04$
P (day)	7.176		5.366	.5668
$m \times 10^{-34}$ (gm)	$1.99 \pm .18$		$1.78 \pm .16$	.646
T_e (°K)	4550	4800	4950	6800*
$\overline{R} \times 10^{-12}$ (cm)	$5.67 \pm .8$	$4.89 \pm .7$	$3.82 \pm .4$	.457*
$\overline{R} \times 10^{-12}$ (cm) obs.	$4.7 \pm .2$**		$3.70 \pm .14$**	
$P\sqrt{\rho/\rho_\odot}$ (day)	.031 $\pm$ .006	.038 $\pm$ .008	.039 $\pm$ .008	.061

* for median descending branch
** m.e. from source quoted. Other mean errors in the table are computed from the mean error of the zero-point.

of the cepheids which follows from BLAAUW's zero-point implies that some interstellar absorption is to be expected. If $A_{pg} = + 0^m.1$, then T_1 is increased by about 80°K. No formal attempt has been made to obtain the actual value of A_{pg}, but there are some indications that η Aquilae is slightly reddened compared to δ Cephei. The author therefore prefers the higher value obtained by SCHWARZSCHILD.

For RR Lyrae the phase of the median light on the descending branch was chosen as less liable to errors caused by the fluctuating amplitude. The color index $V-I = -0^m.70$ corresponds to $T_1 = 6800$°K. The equivalent color class F2.5 is close to the mean spectral type determined by STRUVE [1]) from the appearance of the Balmer series for this phase. Examination of 10 A/mm Mount Wilson Coudé plates at this phase, obtained by R. SANFORD and the author, indicates that the spectral class defined by the excitation temperature may be later than that indicated by the MKK criteria for metallic lines. The uncertainty in T_1 is probably larger in this case than for the δ Cephei variables because of the peculiarities encountered in Population II stars.

The masses of these stars were computed through the use of KUIPER's empirical mass-luminosity law and empirical bolometric corrections [2]). The radii were obtained from HERTZSPRUNG's empirical relation between M_v, T_e, and R [3]).

The last entry in the table is $P\sqrt{\rho/\rho_\odot}$, which is the observed value of the constant in the period-density relation. The observed radii, $\overline{R}$, are obtained from an application of BAADE's criterion: that the temperature, radius, and luminosity variations be consistent. The value for δ Cephei is that of WESSELINK [4]) based on the measures of Six Color Photometry II [5]), while the value for η Aquilae is that of STEBBINS *et al.* [6]) in Six Color Photometry VII. Intermediate limb darkening, $x = 0.6$, has been assumed.

I. EPSTEIN [7]) has investigated the radial-pulsation periods for a giant model with chemical inhomogeneity and also other hypothetical giant models. He concludes that the period is largely determined by conditions in the outer envelope at a distance of about one fourth the radius from the surface, and he states that $P\sqrt{\rho/\rho_\odot} = 0^d.041$ is the most probable value of the constant in the period-density relation for a giant model. One may expect some changes in this quantity resulting from effects of changes in L, M, and R on the opacity and on the ratio of gas to light pressure. A comparison of this theoretical value with the "observed" values in the last line of Table 1 shows that, for the δ Cephei variables, the observed periods agree with the computed periods without any abuse of the observations. Further, the independently determined radii of WESSELINK and of STEBBINS *et al.* are also consistent with these data (although earlier results of WESSELINK suggest smaller radii).

PANNEKOEK [8]) has noted a discrepancy in excess of a factor of 10 between the gravitational acceleration computed from M and $\overline{R}$ and that obtained from the spectra of δ Cephei. SCHWARZSCHILD [9]) has discussed this discrepancy for η Aquilae and found that if the observed line profiles are produced by large-scale turbulence, and not by rotation, then the discrepancy for η Aquilae is such that the spectra are consistent with $g = 32$ cm/sec^2. The data of Table 1 lead to $g = 55$ cm/sec^2, a factor of 2 larger, which is no longer serious, in view of the uncertainties in spectrophotometry. If the line profiles result from rotation, the spectra are consistent with $g = 8$ cm/sec^2. For δ Cephei, PANNEKOEK's estimate of the gravity does

[1]) *Ap. J.* **108**, 60, 1948.
[2]) *Ap. J.* **88**, 429, 472, 1938.
[3]) *Ap. J.* **108**, 207, 1948.
[4]) *B.A.N.* **10**, 256, 1947.
[5]) *Ap. J.* **101**, 47, 1945.
[6]) *L.c.*
[7]) *Ap. J.* **112**, 6, 1950.
[8]) *Physica* XII, 761, 1946.
[9]) *L.c.*

not include measurement of the kinematical support. In this case, therefore, the larger discrepancy may probably be removed in a comparable manner.

For the Population II variable, RR Lyrae, there is a discrepancy between the observed and computed periods. In this case, the relation between effective temperatures and the colour index most certainly needs improvement. (No account has been taken of the difference between $\overline{R}$ and the listed R_{med}, although the total range of $r - \overline{R} \backsim 6.6 \times 10^{10}$ cm.)

The application of the empirical mass-luminosity law to Population II objects is doubtful, since the writer knows of no Population II objects of known mass. For an assumed mass as low as 2.95×10^{33} gm or an effective temperature as low as 6000° K, the computed period is consistent with the luminosity. The investigations of SANDAGE, ARP, and BAUM [1]) on globular clusters indicate that the absolute magnitude of the RR Lyrae variables does not need any correction. It is interesting to note that the suggested mass of RR Lyrae is about 1.49 $M_{\odot}$ and comparable to the mass attributed by SANDAGE and SCHWARZSCHILD [2]) to stars of M3 and M92 near the junction of the main sequence and the prolonged giant branch.

The data upon which this note is based are all liable to change. An increase of $0^m.3$ in the absolute magnitude decreases $P\sqrt{\rho/\rho_{\odot}}$, for example, by 20%. It is expected, however, that the improvement of the zero-point of the period-luminosity law will make future theoretical discussion of δ Cephei variable periods more dependent upon precise effective temperatures. The need for normal colours of cepheids cannot be exaggerated. Extension to RR Lyrae and other Population II cepheids of the analysis previously carried out on η Aquilae and δ Cephei seems most necessary, in spite of the difficulties imposed by secondary variations. For these, simultaneous wide-base two-colour photometric and spectrographic observations are essential.

I wish to thank Dr A. BLAAUW and Prof. J. H. OORT for their most valued discussions.

[1]) *A. J.* **57**, 4, 1952.

[2]) *Ap. J.* **116**, 463, 1952.