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Model Independent Signatures of New Physics in the Inflationary Power Spectrum

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We compute the universal generic corrections to the inflationary power spectrum due to unknown high-energy physics. We arrive at this result via a careful integrating out of massive fields in the “in-in” formalism yielding a consistent and predictive low-energy effective description in time-dependent backgrounds. We find that the power spectrum is universally modified at order H/M , where H is the scale of inflation. This is qualitatively different from the universal corrections in time-independent backgrounds, and it suggests that such effects may be present in upcoming cosmological observations.

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Introduction.—Inflationary theory has become a cornerstone of modern cosmology, elegantly solving many problems with standard big bang cosmology and predicting the primordial power spectrum whose evolution determines the temperature fluctuations in the cosmic microwave background. These are now matched to observation with spectacular precision [1]. Together with the realization that the inflationary energy scale may not be far from that of quantum gravity, the continuing advance in high precision observation may provide an opportunity to observe new fundamental physics near the Planck scale of quantum gravity in cosmological data [2–10]. In this Letter, we shall show and compute potentially observable *universal generic* corrections to the prediction of inflation that are *independent* of the precise details of the theory of quantum gravity or other unknown physics near the Planck scale.

Any new fundamental physics signals are small corrections to the existing measured characteristics and can therefore be seen only if the effects are large enough that they can be detected with upcoming precision experiments such as Planck [11] or CMBPol/Inflation Probe [12]. The primary measurement of interest is the primordial density (scalar) fluctuation power spectrum $P_s(k)$ itself. This has been observationally determined [1] to be very nearly scale-invariant:

$$P_s(k) \sim k^{n_s-1}, \quad n_s \approx 0.960 \pm 0.013. \quad (1)$$

Inflationary theories predict the amplitude and the momentum dependence, in particular, the value of n_s . The question of the observability of Planck scale corrections to $P_s(k)$ was actively pursued some time ago with the conclusion that in toy models [13–36] one can obtain measurable corrections of the order H/M , comparable to intrinsic cosmic variance, with $H \lesssim 10^{14}$ GeV the Hubble scale during inflation and M the energy scale of new physics. It is believed that such corrections linear in H/M encode fundamental physics effects on the initial state rather than the dynamics. Broadly put, all previously considered models fall into two classes: (a) new physics hypersurface (NPH) models, where initial conditions for each

momentum mode are set at the redshift where it equals the scale of new physics—all such initial conditions are *ad hoc* and lack a direct connection with the new physics—and (b) boundary effective field theory models, which have a manifest connection with the new physics. This framework is not universal, as the effects are controlled by the initial time of inflation rather than the redshift where new physics becomes relevant. To make a definitive statement, one needs the universal generic model independent corrections to the power spectrum in terms of an effective field theory (provided adiabaticity is maintained [37–39]). This long-standing question has been hampered by the obstacle of constructing low-energy effective theories in cosmological spacetimes where energy is not a conserved quantity.

Here we use our recent insight on how this obstacle can be overcome to compute the universal generic new physics corrections to the inflationary power spectrum. One can generate the universal low-energy effective action by integrating out a massive field in any particular new physics model. This is sensible in a cosmological setting, as long as one computes expectation values directly rather than transition amplitudes. The details behind the construction of low-energy effective actions in cosmological backgrounds will be given in a separate publication [40].

Universal corrections to the power spectrum.—To demonstrate how this procedure works in practice, let us consider an example of new physics. The simplest theories of inflation are a single scalar field ϕ coupled to gravity:

$$S_{\text{inf}}[\phi] = \int d^4x \sqrt{g} \left[\frac{1}{2} M_{\text{pl}}^2 R - \frac{1}{2} (\partial\phi)^2 - V(\phi) \right]. \quad (2)$$

The fluctuations $\varphi(t, \mathbf{x}) \equiv \delta\phi$ around a classical background solution $\phi_0(t)$ that inflates determine the spectrum (1). This power spectrum is computed through the equal-time two-point correlation function via the “in-in” formalism [41]:

$$P_\varphi(k) \equiv \lim_{t \rightarrow \infty} \frac{k^3}{2\pi^2} \langle \text{in}(t) | \varphi_{\mathbf{k}}(t) \varphi_{-\mathbf{k}}(t) | \text{in}(t) \rangle. \quad (3)$$

Traditionally, the in state $|\text{in}\rangle$ is taken to be the Bunch-Davies vacuum state, but this is not necessarily so. Expanding cosmological backgrounds allow for a more general class of vacua, which can be heuristically considered to be excited states of inflaton fluctuations. In the present context, we will find that integrating out high-energy physics generically results in boundary terms in the effective action, which represent such excited states. This gives a qualitative connection with the aforementioned toy models with potentially observable corrections.

To the inflationary action (2), we add a massive field χ with a simple interaction to the inflaton fluctuations:

$$S_{\text{new}}[\varphi, \chi] = - \int d^4x \sqrt{g} \left[\frac{1}{2} (\partial\chi)^2 + \frac{1}{2} M^2 \chi^2 + \frac{g}{2} \varphi^2 \chi \right].$$

We ignore self-interactions of φ , because we are interested only in scale-dependent corrections. There are no linear terms in the fluctuations, ensuring that a solution to the action S_{inf} is also a solution of the combined action $S \equiv S_{\text{inf}} + S_{\text{new}}$. If $V(\phi)$ has a minimum at a value ϕ_0 with $V(\phi_0) > 0$, the combined action S produces an inflationary phase de Sitter background metric

$$ds^2 = a(\tau)^2 (-d\tau^2 + d\mathbf{x}^2), \quad a(\tau) = -1/H\tau, \quad (4)$$

with a constant Hubble scale H , but contains new physics in the fluctuations parameterized by g and M . As de Sitter inflation is representative for all slow-roll models, we shall take (4) as the metric background.

For nonequilibrium systems such as a cosmological background, the fundamentally sound approach to computing expectation values such as the power spectrum (3) is the Schwinger-Keldysh approach. At some early time t_{in} , we impose the Bunch-Davies vacuum $|\mathbf{0}\rangle$ for φ and χ , evolve the system for the bra and ket state separately until some late time t , and then evaluate the two-point fluctuation correlation:

$$P_\varphi(k) = \lim_{t \rightarrow \infty} \frac{k^3}{2\pi^2} \times \langle \mathbf{0}(t_{\text{in}}) | e^{i \int_{t_{\text{in}}}^t dt \mathcal{H}(t)} | \varphi_{\mathbf{k}}(t) \rangle^2 e^{-i \int_{t_{\text{in}}}^t dt' \mathcal{H}(t')} | \mathbf{0}(t_{\text{in}}) \rangle. \quad (5)$$

Focusing now on the fluctuations in the action, if we denote the fields representing the “evolving” ket to be $\{\varphi_+, \chi_+\}$ and those for the “devolving” bra to be $\{\varphi_-, \chi_-\}$, the in-in expectation value (5) can be computed from a path integral with action

$$\mathcal{S} \equiv S[\varphi_+, \chi_+] - S[\varphi_-, \chi_-]$$

together with the constraint that $\varphi_-(t) = \varphi_+(t)$ and $\chi_-(t) = \chi_+(t)$. It is then helpful to transform into the Keldysh basis:

$$\begin{aligned} \bar{\varphi} &\equiv (\varphi_+ + \varphi_-)/2, & \Phi &\equiv \varphi_+ - \varphi_-, \\ \bar{\chi} &\equiv (\chi_+ + \chi_-)/2, & X &\equiv \chi_+ - \chi_-, \end{aligned}$$

where the action equals

$$\begin{aligned} \mathcal{S}[\bar{\varphi}, \Phi, \bar{\chi}, X] &= - \int d^4x \sqrt{g} \left[\partial\bar{\varphi}\partial\Phi + \partial\bar{\chi}\partial X + M^2 \bar{\chi}X \right. \\ &\quad \left. + g\bar{\chi}\bar{\varphi}\Phi + \frac{g}{2} X \left(\bar{\varphi}^2 + \frac{\Phi^2}{4} \right) \right]. \quad (6) \end{aligned}$$

In this Keldysh basis, the propagators are the advanced and retarded Green’s functions $G^{A,R}$ and the Wightman function F :

$$\begin{aligned} F_{\mathbf{k}}(\tau_1, \tau_2) &\equiv \langle \bar{\varphi}_{\mathbf{k}}(\tau_1) \bar{\varphi}_{-\mathbf{k}}(\tau_2) \rangle = \text{Re}[U_{\mathbf{k}}(\tau_1) U_{\mathbf{k}}^*(\tau_2)], \\ G_{\mathbf{k}}^R(\tau_1, \tau_2) &\equiv i \langle \bar{\varphi}_{\mathbf{k}}(\tau_1) \Phi_{-\mathbf{k}}(\tau_2) \rangle \\ &= -2\theta(\tau_1 - \tau_2) \text{Im}[U_{\mathbf{k}}(\tau_1) U_{\mathbf{k}}^*(\tau_2)], \\ G_{\mathbf{k}}^A(\tau_1, \tau_2) &\equiv G_{\mathbf{k}}^R(\tau_2, \tau_1), \\ 0 &= \langle \Phi_{\mathbf{k}}(\tau_1) \Phi_{-\mathbf{k}}(\tau_2) \rangle, \quad (7) \end{aligned}$$

where

$$U_{\mathbf{k}}(\tau) = \frac{H}{\sqrt{2k^3}} (1 - ik\tau) e^{-ik\tau}$$

is a solution to the free massless φ equation of motion, chosen to obey Bunch-Davies boundary conditions at early times. For the massive χ field, one has $\mathcal{F}_{\mathbf{k}}(\tau_1, \tau_2) = \text{Re}[V_{\mathbf{k}}(\tau_1) V_{\mathbf{k}}^*(\tau_2)]$, etc., where the free field solution $V_{\mathbf{k}}(\tau)$ can be approximated by

$$V_{\mathbf{k}}(\tau) \approx - \frac{H\tau \exp[-i \int_{t_{\text{in}}}^{\tau} dt' \sqrt{k^2 + \frac{M^2}{H^2 t'^2}}]}{\sqrt{2}(k^2 + \frac{M^2}{H^2 \tau^2})^{1/4}}$$

in the WKB limit $|\dot{\omega}|/\omega^2 \ll 1$, which is always valid for $H/M \ll 1$.

In the decoupling limit $g \rightarrow 0$ or $M \rightarrow \infty$, the inflaton fluctuation power spectrum is simply

$$P_\varphi^{(0)} = \frac{k^3}{2\pi^2} F_{\mathbf{k}}(0, 0) = \left(\frac{H}{2\pi} \right)^2. \quad (8)$$

Corrections to this will come from the interactions (6), which contribute to all connected diagrams with two external solid lines (Fig. 1). We have assumed that all tadpoles (1-point diagrams) can be canceled via local counterterms—i.e., the cosmological background is quantum-mechanically stable; this issue is more thoroughly addressed in a forthcoming treatment of the details of renormalization in cosmological backgrounds [40].

Let us analyze the first diagram (recall that future infinity equals $\tau = 0^-$):

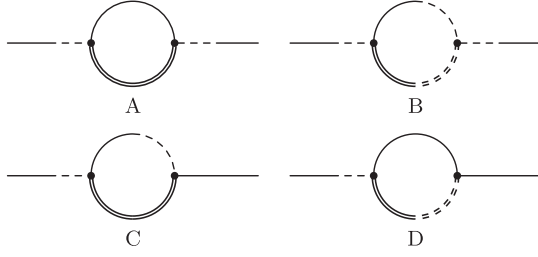


FIG. 1. Power-spectrum corrections mediated by the heavy field. Single solid lines indicate contractions of $\bar{\varphi}$, and dashed single lines indicate those of Φ , with analogous notation for double lines indicating the heavy field components $\{\bar{\chi}, X\}$.

$$P_{\varphi}^{(A)}(k) = \frac{k^3}{2\pi^2} (-ig)^2 \int_{\tau_{\text{in}}}^0 d\tau_1 a(\tau_1)^4 \int_{\tau_{\text{in}}}^0 d\tau_2 a(\tau_2)^4 \\ \times \int \frac{d^3\mathbf{q}}{(2\pi)^3} [-iG_{\mathbf{k}}^R(0, \tau_1)] \mathcal{F}_{\mathbf{q}+\mathbf{k}}(\tau_1, \tau_2) \\ \times F_{\mathbf{q}}(\tau_1, \tau_2) [-iG_{\mathbf{k}}^A(\tau_2, 0)].$$

Writing out the Green and Wightman functions in terms of U 's and V 's, we see that there are three types of vertices. The first is

$$\mathcal{A}_1(\mathbf{k}_1, \mathbf{k}_2) \equiv \int_{\tau_{\text{in}}}^0 d\tau a(\tau)^4 U_{\mathbf{k}_1}(\tau) U_{\mathbf{k}_2}(\tau) V_{-(\mathbf{k}_1+\mathbf{k}_2)}^*(\tau) f(\tau) \\ = -\frac{1}{2\sqrt{2k_1^3 k_2^3} H} \int_{\tau_{\text{in}}}^0 \frac{d\tau}{\tau^3} \\ \times \frac{(1 - ik_1\tau)(1 - ik_2\tau)}{(|\mathbf{k}_1 + \mathbf{k}_2|^2 + \frac{M^2}{H^2\tau^2})^{1/4}} f(\tau) \\ \times \exp\left[-i(k_1 + k_2)\tau\right. \\ \left.+ i \int_{\tau_{\text{in}}}^{\tau} d\tau' \sqrt{|\mathbf{k}_1 + \mathbf{k}_2|^2 + \frac{M^2}{H^2\tau'^2}}\right],$$

where we have introduced a function $f(\tau)$ to account for any step functions. By introducing the rescaled time $u \equiv (H/M)\tau$, the vertex $\mathcal{A}_1(\mathbf{k}_1, \mathbf{k}_2)$ admits a stationary phase approximation at the energy-conservation moment

$$k_1 + k_2 = \sqrt{|\mathbf{k}_1 + \mathbf{k}_2|^2 + u_c^{-2}}. \quad (9)$$

The solution to this defines the NPH:

$$u_c^{-1} = -\sqrt{2k_1 k_2 (1 - \cos\theta)}, \quad \cos\theta = \frac{\mathbf{k}_1 \cdot \mathbf{k}_2}{k_1 k_2}.$$

Then to leading order in H/M the amplitude is

$$\mathcal{A}_1(\mathbf{k}_1, \mathbf{k}_2) \approx -\frac{\sqrt{\pi} i f(\tau_c) e^{-i(M/H)\sqrt{|\mathbf{k}_1 + \mathbf{k}_2|^2 u_m^2 + 1}}}{2\sqrt{k_1 k_2} [2k_1 k_2 (1 - \cos\theta)]^{1/4} \sqrt{HM}} \\ \times \left(\frac{k_1 + k_2 + \sqrt{2k_1 k_2 (1 - \cos\theta)}}{\sqrt{|\mathbf{k}_1 + \mathbf{k}_2|^2 + u_{\text{in}}^{-2}} + |u_{\text{in}}|^{-1}} \right)^{-i(M/H)}. \quad (10)$$

The physics of this is clear. This diagram accounts for the threshold production or decay of heavy particles at high redshift in the early Universe. Note that, in order to evaluate $f(\tau_c)$, one should use the step function appropriately “averaged” due to the Gaussian fluctuations:

$$\theta(\tau) = \begin{cases} 1 & \text{if } \tau > 0, \\ 1/2 & \text{if } \tau = 0, \\ 0 & \text{if } \tau < 0. \end{cases} \quad (11)$$

The second possible vertex is identical to \mathcal{A}_1 but with one U conjugated. This has only imaginary-time saddle-point solutions. Since our τ integral is confined to the real axis, we will never pass over this point in our integration, and so this amplitude will be suppressed as $\mathcal{A}_2 \sim \text{erf}(\frac{M}{H}) \sim \frac{H}{M} e^{-(M/H)^2}$, allowing us to neglect such interactions. Finally, we consider \mathcal{A}_3 , which has both U 's conjugated and so admits no saddle-point solutions and, thus, can also be neglected.

The integrals over τ_1 and τ_2 yield (10) and its conjugate, removing any phase. The integral over the loop momentum should then be traded for an integral over the NPH. While one cannot truly assign an energy to a field in an expanding background, we can define an energy in the practical sense. Multiply the stationary phase definition (9) by $H|\tau_c|$ to yield the physical energy conservation at the NPH moment

$$E_q + E_k = E_{\chi},$$

where E_q is the energy of the virtual φ field, E_k is the energy of the external φ field, and E_{χ} is the energy of the χ field at the moment of interaction. To evaluate the integral over \mathbf{q} , we perform the coordinate transformation given by

$$\tau^{-1} \equiv -\frac{H}{M} \sqrt{2kq(1 - \cos\theta)}, \\ E_q \equiv Hq|\tau| = M \sqrt{\frac{q}{2k(1 - \cos\theta)}}. \quad (12)$$

The \mathbf{q} integral then transforms as

$$\int \frac{d^3\mathbf{q}}{(2\pi)^3} |\mathcal{A}_1(\mathbf{k}, \mathbf{q})|^2 = \frac{1}{16\sqrt{2}\pi k^{3/2} M} \int \frac{\sqrt{q} dq d(1 - \cos\theta)}{H\sqrt{1 - \cos\theta}} \\ \rightarrow -\frac{1}{16\pi k^2 H^4} \int \frac{d\tau dE_q}{\tau^3}. \quad (13)$$

Imposing the UV constraint $E_{\chi} \leq \Lambda$ and the geometrical constraint $1 - \cos\theta \leq 2$, one finds

$$\frac{M^2}{4Hk|\tau|} \leq E_q \leq -k|\tau|H + \Lambda,$$

$$-\frac{\Lambda + \sqrt{\Lambda^2 - M^2}}{2Hk} \leq \tau \leq -\frac{\Lambda - \sqrt{\Lambda^2 - M^2}}{2Hk}.$$

By performing the integrals, the leading $\Lambda \gg M$ correction for this diagram is then the scale-invariant result

$$\Delta P_\varphi^{(A)} \approx \frac{g^2 H \Lambda^3}{96 \pi^3 M^4}. \quad (14)$$

The second correction, represented by diagram *B*, can be evaluated in a similar manner but contains a minus sign relative to diagram *A*. There is also a subtlety in that there is now a Heaviside function $\theta(\tau_1 - \tau_2)$ producing a factor of $1/2$ via Eq. (11):

$$\Delta P_\varphi^{(B)} \approx -\frac{g^2 H \Lambda^3}{192 \pi^3 M^4}. \quad (15)$$

Finally, it is easy to see that diagrams *C* and *D* cancel against each other to leading order: By ignoring the external lines, they can be seen to be corrections to the Green's

function rather than initial state effects. Thus to leading order in H/M ,

$$\Delta P_\varphi \approx \frac{g^2 H \Lambda^3}{192 \pi^3 M^4} \quad (16)$$

is the complete power-spectrum correction due to high-energy physics. It is a simple overall enhancement of the amplitude without any characteristic momentum-dependent features. In a slow-roll background, H will acquire a weak dependence on k ; the full details of the slow-roll expansion are presented in a separate publication [42].

Universal effective action, vacuum choice, observability, and conclusion.—Intuitively, there exists a low-energy effective action in terms of only the inflaton fluctuations φ which reproduces these corrections. First Fourier expand as

$$\bar{\varphi}_{\mathbf{q}_i}(\tau) = \frac{1}{a(\tau)} \int \frac{d\omega_i}{2\pi} \tilde{\varphi}_{\mathbf{q}_i, \omega_i} e^{-i\omega_i \tau}$$

and similarly for Φ . By integrating out χ , the leading term in H/M is [42]

$$\mathcal{S}_{\text{int},4}[\bar{\varphi}, \Phi] = - \int \prod_i \frac{d\omega_i d^3 \mathbf{q}_i}{(2\pi)^4} (2\pi)^3 \delta^3\left(\sum_i \mathbf{q}_i\right) \frac{g^2}{2!} \left[2\tilde{\varphi}_1 \tilde{\Phi}_2 \theta(\tau_{1c} - \tau_{2c}) \text{Im}[\mathcal{B}^*(\omega_1, \omega_2, \mathbf{q}_1 + \mathbf{q}_2)] \right. \\ \left. \times \mathcal{B}(\omega_3, \omega_4, \mathbf{q}_3 + \mathbf{q}_4) \right] (\tilde{\varphi}_3 \tilde{\varphi}_4 + \frac{1}{4} \tilde{\Phi}_3 \tilde{\Phi}_4) + i \tilde{\varphi}_1 \tilde{\Phi}_2 \text{Re}[\mathcal{B}^*(\omega_1, \omega_2, \mathbf{q}_1 + \mathbf{q}_2)] \mathcal{B}(\omega_3, \omega_4, \mathbf{q}_3 + \mathbf{q}_4) \tilde{\varphi}_3 \tilde{\Phi}_4 \left. \right],$$

where

$$\mathcal{B}(\omega_1, \omega_2, \mathbf{q}) = \int_{\tau_{\text{in}}}^0 d\tau a(\tau)^2 e^{-i(\omega_1 + \omega_2)\tau} V_{\mathbf{q}}^*(\tau) \\ = \frac{1}{\sqrt{2}} \int_{\tau_{\text{in}}}^0 \frac{d\tau}{H\tau} e^{-i(\omega_1 + \omega_2)\tau + i \int_{\tau_{\text{in}}}^{\tau} d\tau' \sqrt{q^2 + \frac{M^2}{H^2 \tau'^2}}},$$

which can be evaluated by using a stationary phase approximation, and

$$\tau_{1c}^{-1} = -\frac{H}{M} \sqrt{|\omega_1 + \omega_2|^2 - |\mathbf{q}_1 + \mathbf{q}_2|^2}, \\ \tau_{2c}^{-1} = -\frac{H}{M} \sqrt{|\omega_3 + \omega_4|^2 - |\mathbf{q}_3 + \mathbf{q}_4|^2}.$$

We see that this effective action has specifically localized interactions on the NPH. It therefore has the virtues of both the NPH and boundary effective field theory models without either of their vices. As a “generalized” boundary effective action, it can be connected to microscopic physics, but it is controlled by the NPH.

Our computation reveals that the leading universal generic contribution to the inflationary power spectrum is indeed unambiguously of linear order in H/M . On the other hand, its profile, a flat enhancement, is qualitatively different from what was surmised. Both initial state NPH

and boundary effective field theory approaches indicated a characteristic oscillatory signal in the generic correction due to initial states [8]. With the fully consistent approach to compute the universal generic correction pioneered, we now can trace the origin of this oscillatory behavior. If one would chose a comoving cutoff instead of a physical cutoff as in Eq. (13), one cannot make tadpoles vanish consistently. The remaining terms yield the oscillatory signal. A first draft of this Letter showed this explicitly. Since the presence or absence of oscillatory features depends on the cutoff used, it cannot be a physical effect, and it should be absent in a properly renormalized theory when the cutoff is removed after the introduction of counterterms [40]. This does not mean that one can never have oscillatory features in the inflationary power spectrum. It is just that they are not a generic prediction of unknown high-energy physics but rather of some nongeneric phenomenon, e.g., [43–46].

In summary, we have developed a technique to explicitly calculate universal generic corrections to the inflaton power spectrum from fundamental high-energy physics. While the contribution from each loop momentum is localized to a unique new physics hypersurface [21], the integral over such loop momenta yields a correction to which is widely distributed in time. The result is a scale-invariant overall enhancement at order H/M to the power spectrum. This allows us to effectively represent

microscopic models through a generalized boundary effective field theory as in Ref. [33]. This effective action includes a spreading of the initial state density matrix, producing a loss of quantum coherence. Most importantly, these corrections are potentially observable; a definitive statement on this requires further detailed study.

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