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On the Radius Determination of the Variable F-type Supergiant BL Tel(F)

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Summary. The radius of the variable F type supergiant in the eclipsing system BL Tel is derived with the aid of the Baade-Wesselink method, the visual surface brightness-colour index relation technique [which is related to the previous method] and with a third method based on a relation between pulsational amplitude and light amplitude. The three methods give similar results viz. $R = 200 \pm 40\, R_\odot$ in accordance with the value derived from the orbital light curve. An ambiguity arose since the value for the pulsational constant Q (0.10 ± 0.03) is twice as high as those for radial pulsators. This suggests a non-radial pulsation (cf. Maeder, 1980), while the techniques mentioned above, resulting in an acceptable result, are assumed to be only applicable to radial pulsators.

Key words: variable stars – supergiants – oscillations

1. Introduction

Arguments that the variable supergiants of spectral type A–F are pulsating are given by Abt (1957), Maeder and Rufener (1972), and Burki (1978). In the case of the G type supergiant FG Sge ($M_v = -4.3$, $P \sim 110^d$), Mayor and Acker (1980) presume that the regular velocity changes signify a radial pulsation.

The exact nature of the oscillatory mechanism is still open, and it could differ among the members of this group. Maeder (1980) for example, suggested that intermediate term variations (in visual < 0.1 mag) of B–G supergiants are the result of non-radial gravity oscillations, since the empirical pulsation constants Q are larger than the values for the fundamental mode of radial pulsations.

Since the visual amplitude of FG Sge is ~ 0.2 mag it does not belong to this group and an attempt by Mayor and Acker to determine its radius with the aid of the Baade-Wesselink method seemed to be then also successful.

In the present study the same method is applied to the variable F type supergiant in the eclipsing system BL Tel denoted by BL Tel (F). Its visual amplitude varies between 0.1–0.2 mag and consequently its oscillatory mechanism may be similar to FG Sge (thus a radial pulsator). It may be also a border line case. Earlier studies of BL Tel (F) have been made by Feast (1967), Walraven and Walraven (1970), van Genderen et al. (1974), and van Genderen (1977, 1980.)

A second method used is closely related to the classical Baade-Wesselink method and gives the distance first and then the radius: the so called visual surface brightness-colour index relation technique (cf. Barnes et al., 1976). A third method is based on a relation between pulsational amplitude and light amplitude. Mass, age, and the pulsational constant Q are derived with the aid of this radius and theoretical evolutionary models.

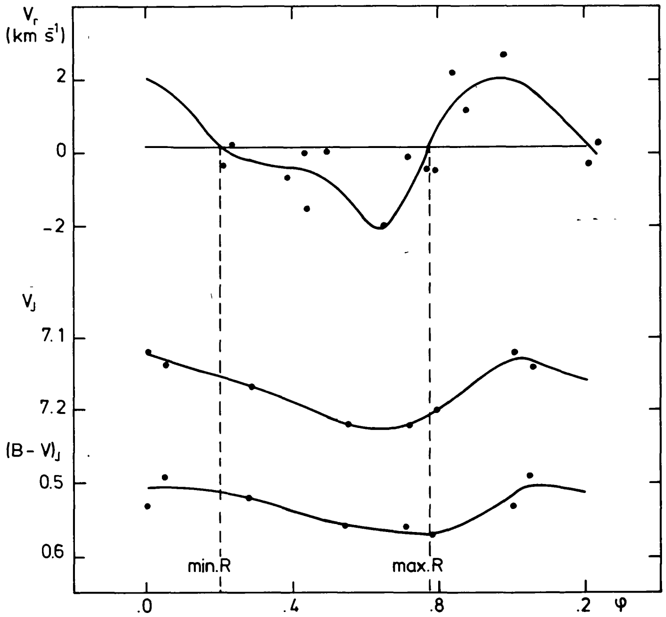

Fig. 1. The radial velocity-, light-, and colour curves of BL Tel (F) during more than one cycle in 1954, phased with $P = 65^d.1$

2. The Observations

During 1953–1955 numerous UBV observations by Cousins (1966) and radial velocity measurements by Wing (1963) were made of BL Tel, a long period eclipsing system. The residuals of the orbital velocity curve outside the eclipse showed the presence of an oscillation, which was due to the F type primary. The influence of the M type secondary could be ignored. The characteristic period showed to be $65^d.1$ (van Genderen, 1977 hereafter called Paper I). Radial velocity and photometric measurements were made simultaneously only during slightly more than one cycle in 1954. This is of importance for the determination of a Wesselink radius, since the oscillations differ from cycle to cycle.

Figure 1 depicts the observations of $V_{\rm rad}$ (km s^{-1}), V and B–V in mag (with subscript J to indicate the UBV system). Phases are computed with the formula given in Paper I. A unique phenomenon is that radial velocity curve and light curve are *in phase* with each other, opposite to Cepheids.

Since the number of observations in this interval is low, some knowledge of the precision is important. From the light – and colour curves of Cousins (see Paper I), a precision of ± 0.01 mag is estimated for V_J and $(B-V)_J$. For $V_{\rm rad}$ Wing quotes probable errors of 1.5–2.1 km s^{-1}. However in view of the scatter around the smooth curve of $V_{\rm rad}$ against date, these values seem to be too large. They rather reflect the scatter when several cycles are phased with a period of $65^{\rm d}.1$. An error of ± 1 km s^{-1} is likely to be more realistic.

3. The Radius Determination

3.1. The Baade-Wesselink Technique

According to Wesselink's (1946) method pairs of equal colour, thus equal $T_{\rm eff}$ (assuming black bodies), were chosen and the corresponding differences in mag (ΔV_J) interpreted as differences in radius (ΔR). The time variation ΔR of the stellar radius R is obtained by integrating the radial velocity curve. However in applying this method one makes the primary assumptions that the ratio of the radii of the line forming and continuum forming regions is constant with phase and that the colour in uniquely related to $T_{\rm eff}$. Such assumptions and others, which are usually not completely true for pulsating stars and the trials to find still reliable radii are discussed by Abt (1959), Oke and Bonsack (1960), Karp (1975), and Breger (1975).

The assumption on the colour – $T_{\rm eff}$ relationship for example is violated in many pulsating stars by the fact that they describe a loop in the two-colour diagrams. An ambiguity then arises since the method does not specify which colour should be used. Since the observations of BL Tel (F) in the *VBLUW* system of Walraven describe very narrow loops in the two-colour diagrams (Walraven and Walraven, 1970; and still unpublished *VBLUW* photometry made in 1966–1968), it is assumed that a colour-$T_{\rm eff}$ relationship exists for this star. Especially since the 1954 curve of Fig. 1 shows the same asymmetry as those cycles for which *VBLUW* photometry has been made. The same shape may indicate similar physical circumstances during an oscillation. More symmetric curves are also possible like those in 1955 and 1957, while sometimes, as in 1969 (Paper I) the star shows practically no oscillation.

For conversion from limb darkened projected radial velocity to pulsational velocity a factor 1.31 has been used (Parsons, 1972; Karp. 1975). The zero velocity in Fig. 1 amounts to $V_0 = 0.2$ km s^{-1}. Nine pairs of equal colour were chosen at regular phase intervals. The approximate radius follows then from:

$$R = 2.17\,\Delta R/\Delta V_J \tag{1}$$

(Rosseland, 1949). The result for the smooth curves drawn in Fig. 1 is as follows:

$$R_F = 195\,R_\odot \pm 40\,R_\odot \text{ s.d. (=standard deviation)}$$

$$\pm 15\,R_\odot \text{ m.e. (=mean error).}$$

This is in surprising agreement with the radius derived from the orbital light curve $R_F = 195\,R_\odot$, using the mass ratio $M_F/M_M = 2.8$ (Feast, 1967) in the formula:

$$R_F = 0.36\ 10^8\ (1 + M_F/M_M) \text{ km}$$

(van Genderen et al., 1974). The small standard deviation is an indication that the curves of $V_{\rm rad}$, V_J, and $(B-V)_J$ have been drawn in reasonable consistency with each other. Yet the good agreement must be considered a coincidence. The reason is the too low number of observations used and the changing shape of the light curve from cycle to cycle. However if the radius amounts to, say, $100\,R_\odot$, it would not fit the semi-period-luminosity diagram for variable supergiants of Burki (1978) (see also Maeder, 1980; Fig. 6) at all.

The total range of the radius variation during the cycle of Fig. 1 $\Delta R_t \sim 5.7\,R_\odot$. Thus $\Delta R_t/R \sim 3\%$. Often the amplitudes in $V_{\rm rad}$ and V_J of other cycles are twice as large, thus in those cases $\Delta R_t \sim 11\,R_\odot$ and $\Delta R_t/R \sim 6\%$.

The fact that the method followed resulted in an acceptable radius might indicate that the oscillation of Fig. 1 is a result of a pulsation of volume just like Cepheids. The same type of radius determination applied by Mayor and Acker (1980) to the supergiant FG Sge of which the $V_{\rm rad}$ and V_J curves are nearly in phase, also resulted in an apparently reliable radius ($R \sim 140 \pm 30\,R_\odot$) consistent with other parameters. A difference with BL Tel (F) is that FG Sge is situated at the red side of the Cepheid instability strip and evolved within 20 yr from spectral type B4–G2 (Whitney, 1978), while BL Tel (F) situated at the blue side of the strip (van Genderen, 1980) does not show any sign of such a variation. This is in agreement with theoretical evolution times in this part of the HR diagram (de Loore et al., 1978), assuming an initial mass of $\sim 20\,M_\odot$ (Sect. 4) [the mass of FG Sge is estimated to be $\sim 1\,M_\odot$ only].

3.2. The Visual Surface Brightness-colour Index Relation Technique

We also applied the visual surface brightness (F_v)-colour index relation technique of Barnes et al. (1976). Their method is to compare linear diameter variations to angular diameter variations (the latter is dependent of the variations in the visual surface brightness) thereby obtaining the distance. The radius can then be determined. This method is an extension and a revision of the original relation between F_v and colour discovered by Wesselink (1969).

The best results have been obtained with the colour index *V–R* (Johnson system). Since BL Tel (F) has been observed in *UBV* only, we shall use the colour index $(B-V)_J$. Three equations are of importance:

$$F_V = \text{constant } -0.21\ (B-V)_{J_0}. \tag{2}$$

[The subscript 0 means corrected for interstellar extinction.]

$$F_V = 4.2207 - 0.1\,V_{J_0} - 0.5\log\phi \tag{3}$$

in which ϕ is the stellar angular diameter – in milli arc s,

$$F_V = \log T_{\rm eff} + 0.1\ \text{B.C.} \tag{4}$$

Equation (2) is according to Thompson's (1975) result for Cepheids. The approximate constant for F type supergiants can be derived by substituting in Eq. (4) for BL Tel (F) $T_{\rm eff} = 6700$ K (Feast, 1967; van Genderen, 1980) and B.C. = 0.18 mag (Lub and Pel, 1977). Thus $F_V = 3.841$. Further $(B-V)_{J_0} = 0.37$, $[(B-V)_J = 0.49$ and $E_{(B-V)_J} = 0.12$ (van Genderen et al., 1974; van Genderen, 1980)] and the constant turns out to be 3.919. This is in excellent agreement with 3.916 found by Whitney (1978) using two other F type supergiants, in his discussion of FG Sge.

F_V is computed by means of Eq. (2) at regular phase intervals along the colour curve and substituted in Eq. (3) yielding ϕ. The stellar angular diameter variation Δd (in arc s) has been computed by taking the minimum radius at $\phi = 0.2$ as the zeropoint. These

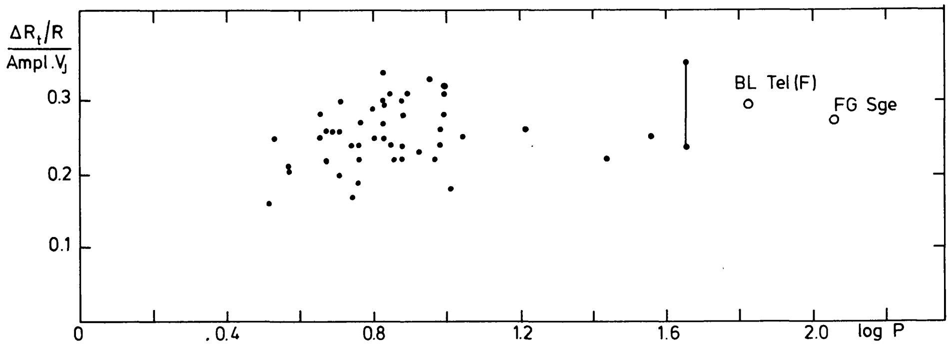

Fig. 2. The ratio between $\Delta R_t/R$ (the fraction of the total radius variation on the average radius) and the visual light curve amplitude (in mag) as a function of log P for a number of Cepheids of population I and two variable supergiants BL Tel (F) and FG Sge. The necessary data of δ Cep are taken from Barnes et al. (1974) and of the long-period Cepheids ζ Gem (log $P=1.005$), χ Cyg (log $P=1.215$), and T Mon (log $P=1.432$) from Barnes et al. (1977) [ΔR_t is computed with the aid of Eq. (5)]. Further we took for l Car (log $P=1.552$) the value of R from Parsons (1972), ΔR_t from Dawe (1969) and the amplitude from Madore (1975); for SV Vul (log $P=1.654$): two possibilities for R, one by Parsons (1972) and one by Sanford (1956), ΔR_t from Sanford (1956) and the amplitude from *VBLUW* photometry [still unpublished]; FG Sge a non-Cepheid (log $P=2.05$) the necessary data from Mayor and Acker (1980)

angular diameter variations were then correlated with ΔR (the linear diameter variations) by integrating the $V_{\rm rad}$ curve and using the conversion factor 1.31. The distance r has been computed at each of the chosen phases by means of:

$$r=2\Delta R/\Delta d \qquad (5)$$

(r in pc, ΔR in A.E., Δd in arc s). The result is:

$\bar{r}=6100\pm3300$ pc (s.d.)

$\pm$ 750 pc (m.e.)

The largest diameter is 0.″0002640 and the smallest 0.″0002554. Thus the total variation $\Delta d_t=0.''0000086$. With $2R=\Delta d_t\cdot r$ [the same units as in Eq. (5)],

$\bar{R}=170\pm90\,R_\odot$ (s.d.)

$\pm20\,R_\odot$ (m.e.)

and

$\Delta R_t=5\,R_\odot$.

Apart from the large standard deviation, these values are in agreement with those obtained in Sect. 3.1, which is not surprising since both methods are related to each other.

3.3. The Radius According to the Pulsational Amplitude – Light Amplitude Ratio

For radially pulsating stars a rough relation exists between the ratio $\Delta R_t/R$ and the amplitude of the visual light curve (Rosseland, 1949). Figure 2 shows a diagram in which the ratio

$$\frac{\Delta R_t/R}{\text{Ampl. } V_J}$$

is plotted against log P. Most of the short period variables are Cepheids (pop. I) taken from Pel (1978, Table 5a), Ampl. V_J (in mag) have been determined by eye estimation of the light curves (Pel, 1976). The intrinsic scatter in the vertical direction of Fig. 2 is much larger than the error in Ampl. V_J. The ratio $\Delta R_t/R$ (in %) is tabulated in Pel's Table 5a and converted into linear dimensions (for example in $R_\odot$). Also BL Tel (F) is plotted in Fig. 2.

Although the number of available long-period variables is low, it seems that for log $P>0.8$ $(\Delta R_t/R)$/Ampl. V_J is constant. The intrinsic scatter is rather large. The scatter is of the same order, if the amplitudes of the bolometric light curves are used (tabulated in Table 5a of Pel). An approximate radius can be computed with:

$R=\Delta R_t/0.27$. Ampl. V_J

±0.05 e.e. (=estimated error).

For BL Tel we then find assuming that it is a radial pulsator:

$R=211\pm40\,R_\odot$ (e.e.)

which is close to the values previously obtained. It is obvious that the same assumptions as for the Baade-Wesselink method have been made, for example that the ratio of the radii of line forming and continuum forming regions is constant with phase.

4. Mass, Age, the Pulsational Constant *Q* and Discussion

According to the previous radius determinations, an approximate radius of $R=200\pm40\,R_\odot$ (e.e.) is adopted. We have the following additional parameters: $E_{(B-V)_J}=0.12$, thus $V_{J_0}=6.77$, and $(B-V)_{J_0}=0.37$ (Table 5 in van Genderen, 1980). Consequently $M_{\rm bol}=7.38\pm0.45$ and log $L/L_\odot=4.852\pm0.180$. With $T_{\rm eff}=6700\pm100$ K, log $T_{\rm eff}=3.826\pm0.006$, the distance $r=7.3\pm1.5$ kpc and the height above the galactic plane $z=2.9\pm0.6$ kpc. All errors are estimated.

A mass and an age determination has been made by using the evolutionary computations for massive stars of de Loore et al. (1978) with $X=0.7$, $Y=0.27$, and $Z=0.03$. Adopting the mass loss parameters $N=100$ (moderate mass loss) and 300 (high mass loss) respectively, we find:

$M=20\pm2.5\,M_\odot$

$t=6.2$ (±0.5) 10^6 yr

and

$M=15.8\pm1.6\,M_\odot$

$t=6.5$ (±0.5) 10^6 yr.

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All errors are mainly influenced by the error in the radius. The error in the temperature is of minor importance. The choice of N has practically no influence on the age. According to de Loore et al. (1977) and Lamers et al. (1980) $N=100$ is the best value.

The computation of the pulsational constant Q has been made according to the equation:

$$\log Q = \log P + 0.5 \log M/M_\odot + 0.3 M_{\rm bol} + 3 \log T_{\rm eff} - 12.71 \quad (6)$$

(cf. Burki, 1978). The results are: $N=100$: $Q=0.10$ with the extreme limits of 0.14 [for $R=160 R_\odot$, $M_{\rm bol}=-6.90$, $M=17.5 M_\odot$, $T_{\rm eff}=6800$] and 0.08 [for $R=240$, $M_{\rm bol}=-7.78$, $M=22.5 M_\odot$, $T_{\rm eff}=6600$]. $N=300$: $Q=0.09$ with the extreme limits of 0.13 and 0.08 [with the same parameters]. Thus for different mass loss parameters, Q has practically the same value.

Burki (1978) and Maeder (1980) computed for BL Tel (F) $Q=0.06$ and 0.05 respectively, since they used different physical parameters like $R=300 R_\odot$ according to Feast's (1967) initial radius determination and $T_{\rm eff}=6300$ K, which is appreciably lower than the value used in this paper based on spectroscopic and photometric considerations.

The Q values computed above (0.10) and that for FG Sge [0.07 after substitution in Eq. (6) of the physical data found by Mayor and Acker] agree with the general phenomenon found by Burki (1978), Maeder (1980), and Sterken (1977), that the empirical Q values of variable supergiants are larger than those for the fundamental mode of radial pulsations, sometimes by more than a factor two. It should be noticed that these studies are based on supergiants with smaller light amplitudes than BL Tel (F) and FG Sge viz. visual amplitude <0.1 mag. Maeder's (1980) conclusion is that these supergiants may be subject to non-radial gravity pulsations. An obvious conclusion would then be, that the same rule may apply to larger amplitude supergiants, were it not that the success of the Baade-Wesselink method for the two stars suggests a (temporary?) radial pulsation.

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