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appendix II
Spatial analysis: a note[1]

After a short discussion of some writings on spatial analysis it is argued that of the three methods commonly used for this purpose (nearest neighbour, local density and correlation) the local density approach frequently suffers from the impossibility of establishing the domain relative to which the relative densities can be calculated. This problem is usually evaded by turning to nearest neighbour or correlation methods, which, however, require more computational efforts. Instead it is suggested that for the related case of contingency table analysis the domain is the sum of the areas covered by the individual distributions. An example is included based on data on bone and flint artefact distributions at Belvédère Site C.

1. Introduction

In chapter 4 the problem of tied or dependent distributions of two artefact classes was considered from an archaeological point of view. An attempt was also made to provide a statistical answer to this problem. The pertinent literature, however, did not give an easy model solution (e.g. Orton 1980: 150-154) and Roebroeks turned to nearest neighbour analysis.

The data as originally presented to me are shown in table 28; the problem was to find a statistical way to calculate the degree of association between the two artefact classes. Of course, nearest neighbour methods or correlation analysis can provide valid answers, but they involve much computational effort. Contingency table analysis is a more familiar method, which can be done by hand and is much easier. However, for such an analysis the number of 'empty' quadrats has to be known, and this is where difficulties appear, as will be shown below.

The situation illustrated in table 28 and the associated research problem are fairly common in archaeology, and therefore some attention should be paid to them.

In table 29 the figures of table 28 have been recalculated for the case that the two distributions A and B are independent of one another. The values in the table are proportional to marginal expectations, i.e. to the sum of the rows and columns.

Table 28: The distribution of flint artefacts (A) and bones (B) at Belvédère Site C. The figures indicate presence (+) and absence (-) of artefacts per square metre excavated.

		A		
		+	–	sum
B	+	8	18	26
	–	22	216	238
	sum	30	234	264

Table 29: Marginally expectable numbers for table 28.

		A		
		+	–	sum
B	+	3	23	26
	–	27	211	238
	sum	30	234	264

2. A specification of the problem.

From the figures in tables 28 and 29 a Chi-square value of 10.78 can be computed; for 1 degree of freedom the probability of non-association/dependence is only 0.001. That is, the chances are only 1 to 1000 that the distributions recorded in table 28 are not associated. Simply put, the observed number of quadrats with both classes of artefacts present is 8, whereas the expected figure is only 3, i.e. there is a much greater degree of association than can be explained by chance alone. Hence there is a statistically significant dependency between the two distributions and it can be said that the bones and tools bear some relation to one another.

However, in the present case the Chi-square value is largely determined by the contents of the 'not-A/not-B' cell, which outnumbers the other table values by a factor of 10. A further increase in the number of empty quadrats ('not-A/not-B,' is the same as 'empty'), would raise the figure of this cell, increase its proportion in the total number of quadrats, and thus soar the Chi-square. The interpretation would be that the two distributions are even more related

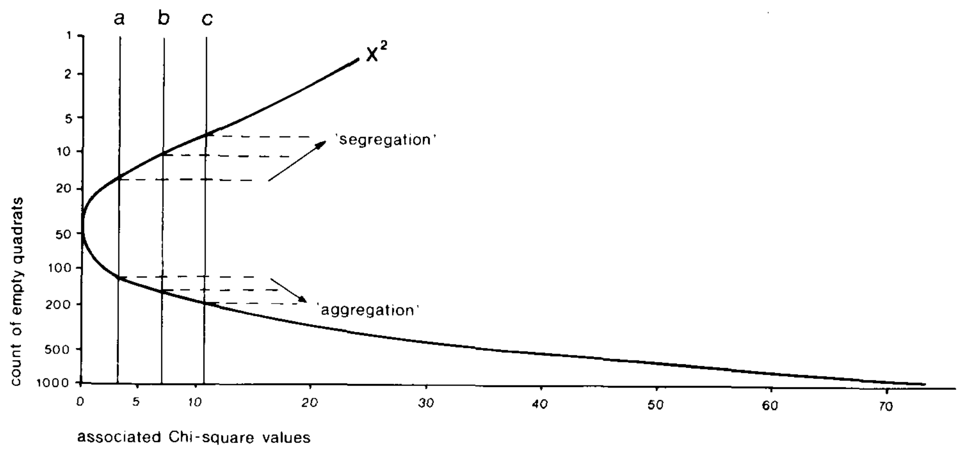

Fig. 152. When the number of empty quadrats in Table 28 is changed, the associated chi-square value changes too, and so does the interpretation. An illustration of a problem with contingency table analysis: a: p= .05 X^2= 3.84; b: p= .01 X^2= 6.63; c: p= .001 X^2= 10.83; all for df= 1.

than would appear from table 28. Conclusion: there is no apparent limit here.

However, with a decrease in the number of empty quadrats, Chi-square values drop at first to then rise steeply and become significant again when the sum of the other table values becomes larger than that under scrutiny. In that case, however, 'significance' would have to be interpreted as dissociation, instead of association. Figure 152 illustrates these changes in Chi-square values and significance with reference to the present case; the conclusion must be that these values are more dependent on the number of empty quadrats (that is, on the size of the excavation) than on properties of the distributions being compared: '...we can get almost any answer by a suitable choice of site boundary...' (Orton 1980: 145).

We thus encounter the following the problem: what is the relevant domain for the above distributions and how many empty quadrats (if any) should be assumed in the computations?

3. A review of the literature

In a general discussion of techniques for estimating association, Hietala and Stevens (1977:541-542, 549) note the problem of 'overly abundant' negative concordances (i.e. what have here been labelled 'empty quadrats') and the consequent inflation of the associated coefficients. For such cases they advocate the use of Kendall's tau-b (e.g. Siegel 1956: 213-223; Nie *et al.* 1975: 288-290). Basically, however, this is a kind of correlation coefficient, so they do not come to grips with the central problem, viz. the determination of the size of the relevant domain. They do in fact mention (but further ignore) the distinction between completely and incompletely excavated distributions.

Orton (Orton 1980: 150-154; see also Johnson 1984: 83-85) compares 'local densities' of artefact classes in each other's vicinity. 'Vicinity' is defined as a circular area of arbitrary size to be fixed by trial-and-error and statistical intuition. Comparison of the results obtained for different radii gives important clues regarding the relations between the distributions (for an illustration see Graham 1980 or Johnson 1984). At the time of Orton's writing, the significance of the coefficients had not been worked out. Graham (1980) and Johnson (1984) both continued in this direction. To Johnson, the originator of the technique, Local Density Analysis is mainly a descriptive method rather than a test of association (Johnson 1984).

Hodder and Orton (1976: 204) briefly discuss some coefficients which disregard the empty quadrats; quoting Pielou they conclude that 'one cannot judge whether the value of the coefficient departs significantly from expectation, on the null hypothesis of independence of the distributions, without taking ..[the count of empty quadrats].. into account'. Their remedies are nearest neighbour or correlation analytical methods.

Berry *et al.* (1984) discuss a method which is a generalized comparison of distances between artefacts of different classes; one obvious advantage over nearest neighbour analyses is the independence of area or density measures, and both approaches are characterized by the irrelevance of empty quadrats. It would seem however, that the shape of the distribution in the field has consequences for the results of the averaging process; a practical disadvantage is that the calculations are so complex as to require a computer.

No doubt I will have missed some discussions of this problem in the archaeological literature. But the basic problem has apparently not been solved (yet). The issue is side-stepped via bypasses to correlation analysis in standard statistical textbooks (e.g. Dixon/Massey 1956, or Hays 1973). Nevertheless, I hold to the opinion that because of the comparative ease with which Chi-squares can be computed, a straightforward solution is to be preferred (Thomas 1978).

4. Discussion: establishing a domain

For the study of the association of two artefact classes, Hietala and Stevens (1977) developed a scale ranging from uniform aggregation through indepencence to uniform segregation (cf. Orton 1982: 9). Different techniques are recommended for every interval on that scale; for instance, Chi-square analysis is appropriate for uniform distributions. Clear as their scale may be, it presents one difficulty in that the intervals are defined through 'theoretical probabilities', whereas in archaeological practice frequencies often have to be checked. The former relate to distributions known to their limits, the latter to parts of distributions (such as distributions not excavated to their limits, or not fully known, or not reliably estimated) so this is precisely the other side of the problem noted above: the domain is unknown. This is also visible in their use of indices ('for all i, j'), which are implicitly defined (p. 540-541) as spanning the whole excavation, which, in turn, is suggestive of the irrelevance of the domain of the distributions studied, or of the tacit equation of the excavated area with the theoretical domain. As noted above, Hietala and Stevens evade the problem by using Kendall's tau-b coefficient in the remainder of their article. Again, not everybody has unrestricted access to a mainframe computer; or the data may not stand up to this method because they were not gathered individually but per grave, feature or quadrat (cf. e.g. Graham 1980). It is for such situations that I am trying to find a way out.

The problem may be approached from another angle, as in the accompanying figures. In the case of a situation like that shown in figure 153a nobody would presumably be willing to deny uniform segregation of the two distributions. Neither would strong segregation be questioned in the case of excavation plans like those illustrated in figure 153b or figure 153c (the latter probably being fairly common in archaeology; e.g. Hietala/Stevens 1977: fig. 1, 57). In such cases no complicated computations are necessary: their interpretation is straightforward and statistically uninteresting.

Note that in figure 153c only frequencies can be calculated: both distributions (may) extend beyond the excavation's limits. Coefficients calculated for this type of situation are not representative of the relations between the *total* distributions. For situations like that illustrated in figure 153b the frequency counts can be converted into probabilities, for the limits of the distributions are well within the boundaries of the excavation. Below, I will not deal with analogues of figure 153c, but with completely excavated distributions only.

Matters become ambiguous only when situations like that

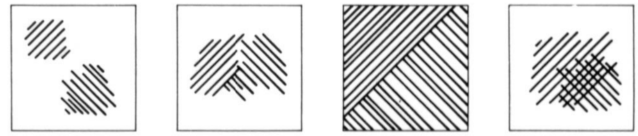

Fig. 153. See the text for an explication.

shown in figure 153d are encountered, i.e. when there is some overlap of the distributions; only then questions about the degree of association become meaningful (such a situation was also found in the excavation of Belvédère Site C). It seems therefore that a situation as in figure 153c constitutes the limit *beyond* which a statistical measure of association is not very illuminating, and within which such a coefficient could be useful. This suggests using the sum of the areas of separate distributions as the domain relative to which coefficients of association may be computed. In other words, the size of the domain is dependent only upon properties of the distributions involved, and not upon such extraneous factors as the size of the excavated area.

If a domain is established in this way, the number of jointly occupied quadrats is exactly balanced by that of empty ones; that is, their size or weight are neutralized. They contribute to the Chi-square value only in relation to the distributions of which they are part; the marginal values are reflective only of the respective joint/single dichotomies, and not of occupied/empty ones. And this is precisely the solution we were looking for.

It may be objected that the sum of maximum distributions per artefact class could also be used as a baseline: n artefacts may be distributed over at most n quadrats. The ratio of the observed and the theoretical maximum dispersion (cf. the section on densities, below) is indicative of the density of the distribution of the artefact class. By simultaneously introducing this density measure into the computation, the significance of the Chi-square becomes opaque. However, it seems best to take the *densities* as given and to study them separately. After all, the problem was the association of the artefact classes, not the densities.

If this reasoning has some ground, then (the outcome of) a Chi-square test along these lines should yield results that are similar to those obtained in nearest neighbour analysis. I will compare the outcomes obtained for the Belvédère Site C data below.

5. A test of association: an adjusted Chi-square computation

It has been suggested that in establishing coefficients of association between two artefact distributions the relevant domain is the maximum space that can be occupied by them (given their densities). In the case of Belvédère Site C the flint tools occupied 30 square metres or quadrats and the bones 26 square metres (table 28). Together, the two distributions could conceivably occupy 30 + 26 = 56 squares at the most, given their observed densities -and this is to be the sum of their counts, the domain. Tables 30 and 31 have been calculated accordingly.

The Chi-square coefficient equals 10.39 and for one degree of freedom the probability that the null hypothesis of independent distributions is true is approximately 0.001 (cf. fig. 152). In this case, however, the observed frequency of combined occurrence (8) is much *less* than that of a randomized or marginal expectation (14); there is evidence of 'strong segregation' on the Hietala and Stevens scale.

Table 30: As table 28, though adjusted for domain/number of empty quadrats as suggested in the text.

		A		
		+	–	sum
B	+	8	18	26
	–	22	8	30
	sum	30	26	56

Table 31: Expected frequencies for table 30 in the case that A and B are independent.

		A		
		+	–	sum
B	+	14	12	26
	–	16	14	30
	sum	30	26	56

On local densities

It is not difficult to compute relative local density figures for the individual distributions. In principle, n artefacts can occupy n quadrats at the most; when the n artefacts are distributed randomly over these n quadrats, the resultant expectation (binomial) for empty quadrats is:

$$p(0)= (1/n)^0.(1-1/n)^{n-0}.\binom{n}{0} = 0.364 \text{ (for n=34)}$$

Accordingly, the probability of a quadrat being occupied by at least one artefact is 1 – p(0) = 0.636 (Corresponding figures for 41 artefacts in 41 quadrats are 0.363 and 0.637). This means that if the artefacts are randomly distributed, 0.636 x 43 = 27 (26, respectively) quadrats should be occupied; compare this with the observed value of 30 (26 respectively) quadrats. Probabilities could then also be assigned. However, this kind of excercise does not lead to any meaningful results, for what if an archaeological distribution is described as 'clustered' or 'dispersed', or even random (and preferably significantly so) (cf. Johnson 1984: 80).

Note that these densities are properties of the individual distributions, and not measures of association between distributions, as implied in the Local Density Analytical techniques described by Orton (1980), Graham (1980) and Johnson (1984).

6. Conclusions: a comparison of outcomes

In previous sections it was said that if the idea has some ground that the joint domain of two archaeological distributions is the sum of the individual distributions, then the outcome of tests based on that idea should square with the results of nearest neighbour analysis, which does not use empty quadrats. In chapter 4 nearest neighbour analysis resulted in a Chi-square value of 15.49, which is significant at the level of 0.001 (df= 1), which also indicates segregation. With a value of S= 0.429, Pielou's coefficient of segregation is between full segregation at + 1.00 and random occurrence at 0.00 (see Hodder/Orton 1976: 205).

Thus, the results obtained with the different techniques lead to the same conclusion. It may also be inferred that in this case at least the proposed solution to the delimitation of a domain yields an outcome comparable with those of other methods involving more computational effort.

notes

[1] Thanks are due to Wim van Zanten for his criticism and comments.

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