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THEORY OF ABSORPTION LINES IN THE ATMOSPHERE OF THE EARTH

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SUMMARY. — *In the present article theoretical profiles and equivalent widths of atmospheric absorption lines are calculated. Observational data concerning atmospheric oxygen are discussed in a separate article.*

Sections 2 and 3 contain general formulae, valid for any distribution of temperature and pressure. A general curve of growth does not exist, but a particular one might be constructed for each single line. Stress is laid on the two asymptotes of this curve. These define an effective damping constant, ranging from 0,48 γ_0 *to* 0,73 γ_0 *dependent on the initial level of the transition* (*section* 6).

The profile of a line in case of an isothermal atmosphere with exponentially decreasing pressure is computed in sections 4 *and* 5, *including the Doppler-effect. The profile nearly equals a pure damping profile with* $\gamma_{\text{eff}} = \frac{1}{2}\gamma_0$. *But if the Lenz formula of impact damping is valid, the line is shifted towards the red and has an asymmetrical core* (*figure* 2).

The effective height of the atmosphere, both for weak and strong lines, is computed in the last section. Effective temperatures, such as can be derived from band spectra, are found to differ markedly from the ground temperature.

1. INTRODUCTION.

Any luminous point, whose light has travelled over a certain distance through the air, shows in its spectrum atmospheric absorption lines and bands. These can best be seen in the solar spectrum, because of its strong continuous background and of the very long light path. The great fall of pressure, however, and the small drop of temperature in the atmosphere of the earth cause considerable complications. In the present article these complications are investigated theoretically.

The meteorological importance of our subject is manifest. Absorption of light by the air is a primary act in the physics of the atmosphere. The physicist is also interested in these absorption bands. Most of them are too weak for reproduction in the laboratory. But even an astronomer, who wishes to direct his attention to extra-terrestrial objects only, may not overlook these bands for three reasons.

1) The telluric lines are the only spectral lines having a *standard profile* and a *standard wave length* in the spectra of the sun and of the stars. If the theoretical profile is accurately known, we can determine the instrumental distortion from the observed profile, and then in turn improve the observed profile of the stellar lines. And the positions of the weak atmospheric lines, all of which are very sharp, may be used for reference in measuring radial velocities.

2) The absorption bands cause a *selective extinction* depending on the zenith distance for each band in a particular way.

3) The manner in which absorption lines are generated in the atmosphere of the earth, presents some *analogy* to the manner in stellar atmospheres where, to be sure, the complications are still greater. Our study of some effects in the simple atmospheres of the earth might be directive for studying similar effects in the atmospheres of the sun or of the stars.

2. The profile of a single line.

At a point of the light-path through a medium, the local absorbing power, belonging to a certain fixed transition, is given by :

$$S = KN_n \tag{1}$$

where N_n = number of molecules pro cm³ in the lower state and K = molecular absorption coefficient of the transition. The latter is related to Einstein's transition probability for absorption B_{nm} by

$$K = B_{nm}h\nu/c \tag{2}$$

and to the « oscillator strength » f_{nm} by

$$K = f_{nm}e^2/mc. \tag{3}$$

The local strength S is spread out by damping and by thermal Doppler broadening into the absorption coefficient s_ν, in such a way that

$$S = \int_{-\infty}^{+\infty} s_\nu d\nu. \tag{4}$$

If the light path has a total length L, the integrated absorption coefficient α_ν for frequency ν is equal to

$$\alpha_\nu = \int_0^L s_\nu dl. \tag{5}$$

Now all absorption in the earth's atmosphere is *consumptive* : no energy is reradiated into the absorption line. The intensity inside the spectral line is therefore :

$$I_\nu = I_0 e^{-\alpha_\nu}. \tag{6}$$

The spreading of σ_ν is mainly due to *impact-damping*, the theory of which is somewhat less conclusive than it is sometimes considered to be (cf. section 5). For the time being, however, the LORENTZ-WEISSKOPF theory will do, giving

$$s_\nu = S \frac{\gamma}{(2\pi\nu - 2\pi\nu_0)^2 + (\gamma/2)^2} \tag{7}$$

where γ = local damping constant of the line and ν_0 = frequency of the undisturbed transition. The obliteration by the *Doppler effect* transforms this expression into the well-known *Voigt integral.* We observe that in the far wings the integrated absorption coefficient

$$\alpha_\nu = (2\pi\nu - 2\pi\nu_0)^{-2} \int_0^L S\gamma \mathrm{d}l \tag{8}$$

is independent of Doppler broadening.

3. The equivalent width and its curve of growth.

Let the energy removed by the absorption line be equal to the energy contained in length W of the continuous spectrum. The *equivalent width* W, thus defined is

$$W = \int_{-\infty}^{+\infty} (1 - e^{-\alpha_\nu})\mathrm{d}\nu. \tag{9}$$

Its most important property is, that W is not altered by instrumental distortion.

If the temperature and pressure are constant along the light path, W can be derived from numerical tables [1] for any length of the path. But if the damping constant and the Doppler-broadening show local variations, no such tables are available. In two limiting cases, however, we have asymptotic laws of general validity at our disposal.

I. *Very weak lines.* — Let $\alpha_\nu \gg 1$ throughout the line ; then

$$W_{\mathrm{I}} = \int_0^L S\mathrm{d}l. \tag{I}$$

This formula will be referred to as law I, or *linear law.*

II. *Very strong lines.* — Let the intensity inside the line differ sensibly from zero only in the far wings ; then

$$W_{\mathrm{II}} = \sqrt{\frac{1}{\pi}\int_0^L S\gamma \mathrm{d}l}. \tag{II}$$

This formula will be referred to as law II, or *square root law.*

In a homogeneous atmosphere we can plot log W as a function of log SL ; this graph is called the *curve of growth.* It has the straight asymptotes

$$W_{\mathrm{I}} = SL, \qquad W_{\mathrm{II}} = (SL\gamma/\pi)^{1/2}$$

with slopes 1 and $^1/_2$, which intersect at the equivalent width

$$W_{\mathrm{I\,II}} = \gamma/\pi.$$

Astrophysicists have made extensive use of the curve of growth for determining the profile of the lines — in particular the values of the damping constant and the Doppler broadening — from measured equivalent widths. The « number of absorbing molecules » $N_n L$ may be varied in three ways.

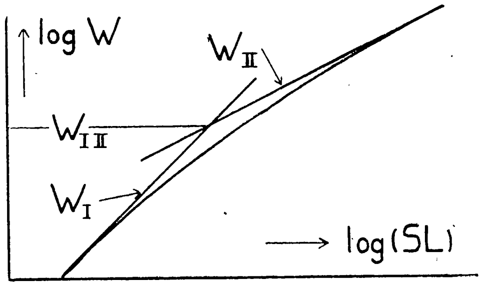

FIG. 1. — Schematic curve of growth.

1. Variation of the light path L (by variation of zenith distance or by measurements with artificial light sources).

2. Variation of the statistical weight in N_n (multiplets).

3. Variation of the Boltzmann factor in N_n by comparing lines originating from different levels of the molecule (Band spectra ; these yield also a determination of the temperature).

In a non-homogeneous atmosphere, however, these methods cannot be applied straight away. The curve of growth shows the increase of W, when the integrated absorption coefficient α_ν increases proportionally for all frequencies. Every form of α_ν yields a particular curve of growth and vice versa. We might construct, therefore, a particular curve of growth for each absorption line by varying the zenith distance. This would have two straight asymptotes I and II intersecting at $W_{\mathrm{I\,II}} = \gamma_{\mathrm{eff}}/\pi$, the effective damping constant being

$$\gamma_{\mathrm{eff}} = \int_0^L S\gamma dl \Big/ \int_0^L S dl. \tag{10}$$

This method is of little practical use, however, since sec z can only be varied up to a factor 8.

All lines of a molecular band, on the contrary, are not conducive to the construction of a single curve of growth : Lines absorbed by molecules in high states prefer high temperatures and accordingly low layers at the atmosphere, and this gives rise to a comparatively large effective damping constant. For that reason ALLEN'S method of analysing the atmospheric oxygen bands is inaccurate. In section 6 more accurate formulae are given.

So far all formulae are valid if the distribution of density, temperature and pressure along the light path is arbitrary. They may be applied in investigations concerning the *water vapour* lines. In all further sections we assume some law according to which the temperature and pressure decrease. The expressions there derived, will be applied to the *atmospheric oxygen* lines in the next paper.

4. LINE PROFILE IN AN ISOTHERMAL ATMOSPHERE.

Although we lay stress on the use of the two asymptotes of the curve of growth, it is necessary to have some idea of the form of its intermediate part. ALLEN [2] has computed a curve of growth, supposing :

a) The partial pressures of the oxygen molecules in each state decrease at the same rate as the total pressure ; *b*) This pressure decreases exponentially with increasing height in the atmosphere. We shall use this approximation to obtain the form of the intermediate part of the curve of growth. Two methods of computing the integrated absorption coefficient are available.

1° Direct method. — Put $x = \nu - \nu_0$, $b = \gamma/4\pi$; by (7) the local absorption coefficient, spread out by damping only, is :

$$s(x) = Sb/\pi(x^2 + b^2). \tag{11}$$

Both S and b are proportional to $u = \mathrm{e}^{-h/h_1}$, where h = altitude, measured from the observing station. On substituting $\mathrm{d}l = -h_1 \sec z \, \mathrm{d}u/u$, an elementary integration gives

$$\alpha(x) = \frac{S_0 h_1 \sec z}{2\pi b_0} \ln(1 + b_0^2/x^2), \tag{12}$$

where the subscript 0 denotes values at the level $h = 0$. Next the obliteration by the Doppler effect must be taken into account and this is only possible by numerical integration. Finally the corresponding curve of growth is obtained by graphical integration. Using this method Allen computed his curve of growth for the oxygen lines.

2° Fourier expansion. — Numerical computations can be avoided, when we approximate $\alpha(x)$ by a Voigt profile. The general principles of this method were outlined elsewhere [3]. The Fourier integral associated with $s(x)$ is

$$\int_{-\infty}^{+\infty} s(x) \cos xt \, \mathrm{d}x = S\mathrm{e}^{-bt}. \tag{13}$$

On substituting the exponential expressions for S and b, we obtain by integration over the height of the atmosphere :

$$\int_{-\infty}^{+\infty} \alpha(x) \cos xt \, \mathrm{d}x = S_0 h_1 \sec z(1 - \mathrm{e}^{-bt})/bt. \tag{14}$$

The obliteration by Doppler broadening is effected by simply adding a factor $\mathrm{e}^{-\delta^2 t^2/4}$ to this function, where $\delta = (2RT/\mu)^{1/2}\nu_0/c$ is the root mean square Doppler displacement. In order to approximate $\alpha(x)$ by a Voigt profile we then develop the logarithm of its associated function. The approximation is a good one, the third coefficient of

$$-\ln \frac{1 - \mathrm{e}^{-u}}{u} = \frac{1}{2}u - \frac{1}{24}u^2 + 0u^3 + \ldots \tag{15}$$

being zero. The first and second ones show the Voigt parameters to be

$$\beta_1 = 1/2\, b_0, \qquad \beta_2^2 = -\frac{1}{6} b_0^2 + \delta^2. \tag{16}$$

For example. inserting Allen's values $b_0 = 3{,}40$, $\delta = 1{,}28$ for the atmospheric oxygen lines near λ 6 900 observed at Pasadena, we find $\beta_1 = 1{,}70$, $\beta_2^2 = -1{,}92 + 1{,}64 = -0{,}28$, all values being expressed in micro-wavelengths.

From the values of β_1 and β_2^2 we infer two important facts. (β_1) The *effective damping constant* is just half the damping constant at ground level. (β_2^2) The core is broadened by the Doppler effect, but narrowed by the effect of the integrated damping profiles. The compensation is nearly complete, leaving only a slight narrowing of the core. Indeed, inspecting the curve of growth computed by ALLEN, we observe that it is hardly distinct from the well-known curve for lines *broadened by damping only* (cf. figure 1).

5. ASYMMETRY AND RED-SHIFT.

A spectral line is broadened by the influence of neighbouring molecules in various ways. Several writers gave comprehensive treatments of this subject [4] [5] [6]. The last reference also contains quite general formulae for the case of additive disturbances.

In the atmosphere of the earth, the pressure being small, the broadening can for the greater part be described by an " impact " theory and not by a " statistical " one. LORENTZ' theory is the prototype of an impact theory. Let the phase of radiation be disturbed totally if the centre of another molecule passes the centre of the radiating one at a distance of $r < \sigma$, and remain undisturbed by a more distant encounter. Then the absorption coefficient has the profile (7) with

$$b = \frac{\gamma}{4\pi} = \frac{1}{2} N\sigma^2\bar{v} = N\sigma^2 \left\{ \frac{2RT}{\pi}\left(\frac{1}{\mu_1} + \frac{1}{\mu_2}\right)\right\}^{1/2}, \tag{17}$$

where σ = collision diameter, N = number of disturbing molecules pro cm³, $\bar{v}$ = mean relative velocity of the colliding molecules, RT = gas constant $\times$ temperature, μ_1 and μ_2 are molecular weights.

WEISSKOPF modified this theory, supposing momentary frequency shifts equal to

$$x = -ar^{-p}, \tag{18}$$

where a and p are constants ; we suppose a to be positive, corresponding to a shift towards the red. The dimension of b is easily deduced. As to its absolute value WEISSKOPF only made an arbitrary assumption similar to the one in LORENTZ' theory. Afterwards LENZ [7] published the exact profile, which is written in a more convenient form [6] by the formula

$$s(x) = S\,\frac{b/\pi}{(x + b \operatorname{tg} \varphi)^2 + b^2}, \tag{19}$$

where

$$s = 2/(p-1), \qquad \varphi = s\pi/2,$$

$$b = \left\{\sqrt{\pi}\,\Gamma(1/s)/\Gamma(1/s + 1/2)\right\}^s \Gamma(1-s)\,\frac{1}{2}\,Na^s\overline{v^{1-s}} \cos\varphi.$$

For VAN DER WAALS disturbances :

$$p = 6, \qquad \varphi = 36^\circ, \qquad b = 0{,}68\, Na^{2/5}\overline{v}^{3/5}. \tag{20}$$

The most interesting feature of (19) is that the entire line has suffered a *displacement* towards the red proportional to its broadening. This can be proved to be the effect of encounters at distances of the order $1\frac{1}{2}\ \sigma$, which change the phase of radiation without disturbing it completely.

We now proceed to discuss the consequences of this red-shift in the atmosphere of the earth. The local red-shift is, like the local damping, proportional to the local pressure. For that reason we expect not only a red-shift of the total line, but also a considerable asymmetry. We again suppose an *ideal isothermal* atmosphere. If constant factors are avoided by dividing S by S_0, b and x by b_0, h by h_1, the integrated absorption coefficient is

$$\text{(21)} \qquad \pi\alpha(x) = \int_0^1 \frac{u du}{(x + u \operatorname{tg} \varphi)^2 + u^2}.$$

After an elementary integration and some minor reductions we find :

$$\text{(22)} \qquad \pi\alpha(x) = \cos^2 \varphi \, [1/2 \ln \{(1/x + \sin \varphi \cos \varphi)^2 + \cos^4 \varphi\} - \ln \cos \varphi] + \\ + \sin \varphi \cos \varphi \, [\operatorname{arc\,tg} (x + \operatorname{tg} \varphi) \pm \pi/2].$$

In the last term + holds for $x < 0$, — for $x > 0$. The profile in the wings can be developed into :

$$\text{(23)} \qquad \pi\alpha(x) = \frac{1}{2} y^{-2} - \frac{1}{4}\left(1 - \frac{1}{3} \operatorname{tg}^2 \varphi\right) y^{-4} + \dots, \qquad y = x + \frac{2}{3} \operatorname{tg} \varphi.$$

Putting $\varphi = 0$ we get back Allen's profile deduced in the preceding section. Substituting the value $\varphi = 36^\circ$, we find :

$$\text{(24)} \qquad \pi\alpha(x) = 0{,}750 \log \{(1/x + 0{,}476)^2 + 0{,}428\} + 0{,}138 + \\ + 0{,}0083 \operatorname{arc\,tg} (x + 0{,}726) \pm 0{,}747,$$

where log denotes the 10–logarithm and the arc tg is expressed in degrees. In the far wings :

$$\text{(25)} \qquad \pi\alpha(x) = 0{,}50\, y^{-2} - 0{,}206\, y^{-4} + \dots, \qquad y = x + 0{,}484.$$

We computed this profile numerically and obliterated it by the Doppler effect. The resulting profiles, both for $\varphi = 0$ and $\varphi = 36^\circ$, are shown in figure 2. The median curve of the latter profile marks the asymmetry. Its upper part determines the red-shift of weak absorption lines : $x = -0{,}16\ b_0$; its lower part determines the red-shift of strong absorption lines : $x = -0{,}48\ b_0$, in accordance with (25).

Substituting the numerical values already given in the preceding section, we find :
effective broadening = 1,70 micro wave lengths,
displacement of very weak lines = 0,54 » ,
displacement of very strong lines = 1,64 » .

Displacements of this order might be detected easily, as 1 micro wave length corresponds to an apparent radial velocity of 300 m/sec. Several consequences lend themselves to observational tests.

1. The wave length of a line should present considerable differences when measured : *a*) in the solar spectrum of a mountain observatory ; *b*) in the solar spectrum at sea level ; *c*) in the spectrum of a terrestrial light source.

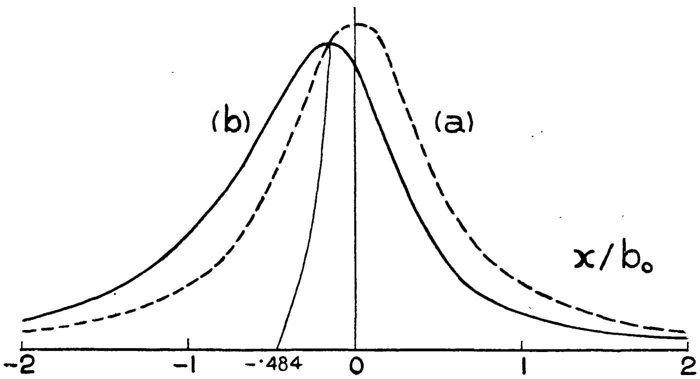

FIG. 2. — Integrated absorption coefficients : (a) according to WEISSKOPF'S damping formula; (b) according to the LENZ' damping formula. DOPPLER effect has been taken into account in both profiles.

2. The profile of a weak line should be asymmetrical. This effect, however, will be hardly visible, because of instrumental distortion.

3. The red-shift of a line should depend slightly on sec z.

These predictions are based on one single physical assumption. The momentary frequency of radiation is supposed to be reduced by an amount proportional to r^{-6} by the effect of a neighbouring molecule at a distance r between ca 4 and 8 Å. (The additivity supposed in deriving formula (19) is not essential.) If *polarisation effects* were the main cause of disturbance, these predictions would hold at least qualitatively. But extensive observations made at Mt. Wilson [8] [9] show conclusively that the atmospheric *oxygen* lines present *no* such asymmetry effects. We infer that some other damping effect, for example *hampering of the rotation* is the main cause of broadening. This question requires further investigation.

Another cause of asymmetry should be noted. The " statistical " profile falls off proportional to $x^{-3/2}$, and predominates in the extreme red wing at any pressure. In the atmospheric oxygen lines the exponent might change from — 2 into — 3/2 at a distance 1,85 Å from the centre of the line. This would be just discernable in the strong lines of the A band. These details, however, are rather hypothetical, since for encounters at small distances the supposition made is no longer valid. The spectrum then merges into the spectrum of an O_4-molecule [10] [11] with quite different properties.

6. Effective temperatures and damping constants.

We now compute the positions of the two *asymptotes* of the curve of growth. They depend on the energy E of the molecular state from which the transition starts and on the distribution of the temperature and pressure in the atmosphere.

We fix the following meaning to indices : I and II denote values according to the two asymptotic laws, a subscript 0 denotes values on the level of the observatory $h = 0$, and a superscript 0 denotes values for a transition from the ground state $E = 0$ of the molecule. Let the temperature be T and the number of air molecules pro cm³ be N. We anticipate from the next article that the local strength of a transition is

(26) $$S = 2iCK\mathrm{e}^{-E/KT}N/T.$$

By (17) the local damping constant is

(27) $$\gamma = \gamma_0 N T^{1/2}/N_0 T_0^{1/2}$$

and the element of the light path is $\mathrm{d}l = \mathrm{d}h \sec z$. Substituting these in (I) and (II), we obtain :

(28) $$W_{\rm I} = 2iCKN_0 \sec z/T_0 \times I_{\rm I},$$
$$W_{\rm II}^2 = 2iCKN_0 \sec z/T_0 \times I_{\rm II} \times \gamma_0/\pi.$$

with

(29) $$I_{\rm I} = \int_0^H \frac{NT^{-1}}{N_0 T_0^{-1}} \mathrm{e}^{-E/kT}\mathrm{d}h,$$
$$I_{\rm I} = \int_0^H \frac{N^2 T^{-1/2}}{N_0^2 T_0^{-1/2}} \mathrm{e}^{-E/kT}\mathrm{d}h.$$

The dependence of the equivalent widths on the energy of the initial level is not strictly exponential, so that no effective temperature of general validity exists. Yet such temperatures $T_{\rm I}$ and $T_{\rm II}$ may be defined, as are tangently valid near $E = 0$, so that :

(30) $$I_{\rm I} = I_{\rm I}^0 \mathrm{e}^{-E/kT_{\rm I}} f_{\rm I},$$
$$I_{\rm II} = I_{\rm II}^0 \mathrm{e}^{-E/kT_{\rm II}} f_{\rm II},$$

where the correction factors $f_{\rm I}$ and $f_{\rm II}$ are tangent to 1 for $E = 0$. They deviate sensibly from 1 only for lines starting from high levels.

The integrals, the effective temperatures and the correction factors must now be computed after substituting the atmospheric distribution of density and temperature. We proceed with a *model atmosphere* introduced by Childs :

(31) $$T = T_0(1 - \varepsilon x), \qquad N = N_0(1 - \varepsilon x)^q, \qquad H = 1/\varepsilon.$$

This model represents the real distributions quite well, if adequate values of ε

and q are chosen. From tables by HUMPHREYS [12] we computed $1/\varepsilon = 55$ km and $q = 6{,}0$ as mean values for summer and winter. (The bad agreement in the stratosphere does not affect the results.)

We put $y = E/kT_0$, $p_{\mathrm{I}} = q - 1 = 5{,}0$, $p_{\mathrm{II}} = 2q - \frac{1}{2} = 11{,}5$ and temporarily drop the suffices I and II. Then both integrals obtain the form :

$$I = \int_0^{1/\varepsilon} (1 - \varepsilon x)^p \, \mathrm{e}^{-y/(1-\varepsilon x)} \mathrm{d}x, \tag{32}$$

which on substituting $1 - \varepsilon x = 1/t$ and $t = \mathrm{e}^z$ is transformed into :

$$I = \frac{1}{\varepsilon} \int_1^{\infty} t^{-(p+2)} \, \mathrm{e}^{yt} \mathrm{d}t = \frac{1}{\varepsilon} \int_0^{\infty} \mathrm{e}^{-[(p-1)z + y\mathrm{e}^z]} \mathrm{d}z.$$

The integral can be expressed in known functions only for $E = 0$:

$$I^0 = 1/\varepsilon(p + 1), \text{ giving } \quad I_{\mathrm{I}}^0 = 9{,}3 \text{ km}, \qquad I_{\mathrm{II}}^0 = 4{,}4 \text{ km}. \tag{33}$$

These values are the *effective heights* of the atmosphere for weak lines and strong lines respectively.

The *effective temperatures* are found by differentiating I with regard to y :

$$-\left(\frac{\mathrm{d} \ln I}{\mathrm{d}y}\right)_{y=0} = T_0/T_{eff} = 1 + \frac{1}{p}, \tag{34}$$

from which $T_0/T_{\mathrm{I}} = 1{,}20$ and $T_0/T_{\mathrm{II}} = 1{,}09$.

An observational proof of these formulae will be given in the next article. CHILD's conclusion [13] that the effective temperature is equal to the ground tempe rature is found to be incorrect.

The correction factors f may be neglected up to $E = 400$ cm^{-1}. For higher E a more accurate value of the integral was found by enclosing it between a smaller and a larger value. Using :

$$(p + 1)z + y(1 + z) < (p + 1)z + y\mathrm{e}^z < -z + (p + 2)(\mathrm{e}^z - 1) + y\mathrm{e}^z,$$

we obtain :

$$\mathrm{e}^{-y}/\varepsilon(p + 1 + y) < I < \mathrm{e}^{-y}/\varepsilon(p + 2 + y). \tag{35}$$

From a closer examination of the formulae we found that the difference between the smaller and larger values is cut by I into two parts, roughly proportional to $p + 2$ and y respectively. The factors thus computed are given in table 1. Collecting (28), (30), (33) and (34) we find the equivalent width of very weak lines to be :

$$W_{\mathrm{I}} = 2iCKN_0/T_0 \sec z \, f_{\mathrm{I}} \, 9{,}3 \text{ km } \mathrm{e}^{-1{,}20E/kT_0},$$

and of very strong lines :

$$W_{\mathrm{II}} = \{2iCKN_0/T_0 \sec z \, f_{\mathrm{II}} \, 4{,}4 \text{ km } \mathrm{e}^{1{,}09E/kT_0} \gamma_0/\pi\}^{1/2}.$$

Finally by (10) the *effective damping constant* is :

$$\gamma_{eff} = \gamma_0 I_{\mathrm{II}}/I_{\mathrm{I}}. \tag{36}$$

TABLE I

Correction factors and effective damping constants.

y	E (for $T_0 = 288^o$)	10 logarithms of f_I	f_{II}	γ_{eff}/γ_0
—	—		—	
0	0 cm^{-1}	0,00	0,00	— 0,32
1	200	0,01	0,00	— 0,28
2	400	0,04	0,00	— 0,25
3	600	0,07	0,01	— 0,23
4	800	0,11	0,03	— 0,21
5	1 000	0,15	0,04	— 0,19
6	1 200	0,20	0,05	— 0,18
7	1 400	0,26	0,06	— 0,17
8	1 600	0,31	0,08	— 0,16
9	1 800	0,37	0,10	— 0,15
10	2 000	0,43	0,11	— 0,14

Its values range, as can be seen from table 1, from $\gamma_{eff} = 0{,}48\ \gamma_0$ for lines starting from low levels to $\gamma_{eff} = 0{,}73\ \gamma_0$ for the highest oxygen lines observed, so that ALLEN's theory, giving the constant value $\gamma_{eff} = 0{,}50\ \gamma_0$, is seriously in error.

In the next article values of the effective temperatures and damping constants will be derived from observational data.

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