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TRACTARIAN EXPRESSIONS AND THEIR USE IN CONSTRUCTIVE MATHEMATICS

Göran Sundholm

My aim in the present paper is systematic rather than exegetic; I am more concerned to emphasize certain distinctions than to present an interpretation of the *Tractatus*. Accordingly, my exposition will proceed by contrasts and logicians other than Wittgenstein will be dealt with at least to the same extent as that accorded to the author of the *Tractatus*.

Wittgenstein's is not a name that modern analytical philosophy primarily associates with the notion of an expression. The household names are rather those of Quine and Tarski. We have all read *Mathematical Logic*, § 4, on *Use versus Mention* of expressions, and the ensuing views on quotation-mark names of expressions.¹ Tarski, however, held basically the same views a decade earlier in the main body of *Der Wahrheitsbegriff* (the research for which was carried out in 1929 and which was first presented in 1931).² There he deals, explicitly and in considerable detail, with the problem of how to refer, in natural and formalized language, to expressions of various types of language. He also provided an informal, philosophically non-technical, treatment in his elementary textbook *Introduction to Logic and to the Methodology of Deductive Sciences*, and some of his remarks there can serve as a starting-point for my discussion:

[I]t is well to make clear to oneself a very general and important principle upon which the useful employability of any language is dependent. According to this principle, whenever, in a sentence, we wish to say something about a certain thing, we have to use, in this sentence, not the thing itself but its name or designation.

...

This principle is, however, frequently violated if the thing talked about happens to be a word or a symbol. And yet the application of the principle

¹W. V. O. Quine, *Mathematical Logic*, Harvard U.P., 1940.

²'Der Wahrheitsbegriff in den formalisierten Sprachen', *Studia Philosophica* 1 (1936) [pamphlet offprints dated 1935], reprinted in Karel Berka and Lothar Kreiser (eds.), *Logik-Texte*, Akademie Verlag, Berlin, 1973, where the original pagination is given. English translation in *Logic, Semantics, Metamathematics* (ed. J. Woodger), Clarendon Press, Oxford, 1956. Reference will be made to the English translation, with the corresponding reference in the original German off-print version in square brackets, using the abbreviation WB. See footnote 1, p. 154 [6], for detailed information about the genesis of the work.

is indispensable also in this case; for otherwise, we would arrive at the whole which, though being a linguistic expression, would fail to express the thought intended by us, and very often might even be a meaningless aggregate of words. Let us consider, for example, the following two words:

well, *Mary.*

Clearly, the first consists of four letters, and the second is a proper name. But let us imagine that we would express these thoughts, which are quite correct, in the following manner:

(I) *well consists of four letters;*

(II) *Mary is a proper name;*

we would then, in talking about words, be using the words themselves and not their names. And if we examine the expressions (I) and (II) more closely, we must admit that the first is not a sentence at all since the subject can only be a noun and not an adverb, while the second might be considered a meaningful sentence, but, at any rate, a false one since no woman is a proper name.

In order to avoid these difficulties we might assume that the words '*well*' and '*Mary*' occur in such contexts as (I) and (II) in a meaning distinct from the usual one, and that they here function as their own names. In generalization of this viewpoint, we should have to admit that any word may, at times function as its own name; to use a terminology of medieval logic, we may say that in a case like this, the word is used in SUPPOSITIO MATERIALIS, as opposed to its use in SUPPOSITIO FORMALIS, that is, in its ordinary meaning. As a consequence, every word of common or scientific language would possess at least two different meanings, and one would not have to look far for examples of situations in which serious doubts might arise as to which meaning was intended. With this consequence we do not wish to reconcile ourselves, and therefore we will make it a rule that every expression should differ (at least in writing) from its name.

The problem arises as to how we can set about to form names of words and expressions. There are various devices to this effect. The simplest one among them is based on the convention of forming a name of an expression by placing it between quotation marks. On the basis of this agreement the thoughts expressed in (I) and (II) can now be stated correctly and without ambiguity, thus:

(I') '*well*' consists of four letters;

(II') '*Mary*' is a proper name.³

³Oxford U. P., 1941, *Galaxy* book 1965, pp. 59-60. I have altered Tarski's use of double quotes into single.

A quotation-mark name, such as ‘Mary’, according to Tarski

may be treated like single words of a language, and thus like syntactically simple expressions. The single constituents of these names — the quotation marks and the expressions standing between them — fulfil the same functions as the letters and complexes of successive letters in single words. Hence they can possess no independent meaning. Every quotation-mark name is then a constant individual name of a definite expression (the expression enclosed by the quotation-marks) and in fact a name of the same nature as the proper name of a man. For example, the name ‘p’ denotes one of the letters of the alphabet.⁴

Thus, for instance,

‘EG’

certainly is a purely accidental, non-significant part of the expression

‘FREGE’,

and, in the same fashion, the expression

‘FREGE’,

which is not a quotation-mark name of an expression, but a name of a person, is a purely accidental, non-significant part of the expression

‘‘FREGE’’’,

which, on the other hand, is a quotation-mark name of an expression, and, therefore, not of a person. One should here recall Miss Anscombe’s “paradox” concerning the impossibility of being told the name of a person.⁵ When asking for the name of the author of the essay *Funktion und Begriff* the reply: ‘Frege’ would be incorrect, since this is a name of the person in question and we need a *name of the name* to refer to the name; thus only ‘‘Frege’’ will do, but ‘Frege’ is not a significant part of ‘‘Frege’’’. In fact, one could, and perhaps better should, on this view, use, for instance, ‘Kurt’, or another conventionally introduced name of the name, to refer to ‘Frege’ (but not to Frege, where we would still have to use ‘Gottlob’ or ‘Frege’), so as to sever the connection between a name naming an object and another name naming a certain other object, namely the name of the first object in question. The important point is that both the name and the person are considered objects and that there is no route from the name of the name to the name of the person.⁶ Among the fourteen solutions offered all but one were disqualified by Miss Anscombe. The winning solution, offered by Al. Tajtelbaum, New York, resolves the problem by noting that

we have a (tacit) convention that a name and its name are denoted by the same word, and so the name of a name “tells” us the name.

⁴WB, pp.160-161 [13].

⁵Report on Analysis ‘Problem’ No 10, *Analysis* 17 (1957), with the winning entry appended, pp. 49-53.

⁶See also Miss Anscombe’s *Introduction to Wittgenstein’s Tractatus*, Hutchinson, London, 1959, pp. 82-85

This, of course, uses the names autonomously, much in the same way as they are used in the allegedly improper theory of material supposition. In his youth Tarski published under the names 'Alfred Tajtelbaum' and 'Alfred Tajtelbaum-Tarski'. I have not been able to determine whether the winning entry is by his hand. The pithiness of its style — especially in its final paragraph and the opinions there expressed — certainly lend support to the idea. It is well-known that Tarski was extremely reticent about committing his philosophical views to paper. Possibly we have here yet another example of this phenomenon. Fairness bids me to report that Tarski sometimes also exhibited a less hostile attitude towards the theory of supposition. Concerning the role of semantical concepts, he stated that

they give expression to certain relations between the expressions of language and the objects about which these expressions speak We could also say (making use of the *suppositio materialis*) that these concepts serve to set up the correlation between the names of expressions and the expressions themselves.⁷

In the main body of *Der Wahrheitsbegriff* Tarski professes explicit allegiance to the Husserl/Leśniewski Doctrine of Semantical Categories.⁸ Two expressions are of the same semantical category if they are substitutable *salva congruitate* with respect to some context. The semantical categories themselves are then obtained from this equivalence relation by means of abstraction. Thus the semantical categories are linguistic counterparts to Russell's types: the latter are, after all, the argument ranges of propositional functions.⁹ Tarski's adherence to this theory provides a neat example falling under the range of phenomena and views, collected by Jaakko Hintikka under the heading of Language as the Universal Medium.¹⁰ The example in question concerns the impossibility of a truth definition for a language of infinite order:¹¹ the definition of truth for a

⁷WB, p.252 [116].

⁸Only in the main body of the text, WB, pp.215-221 [74-81].

⁹Woodger translates the German *Aussagefunktion* as 'sentential function', but 'propositional function' might be better. In either case the problem concerning the status of the function remains: normally one would say that the arguments of a propositional function are, for instance, objects and not the expressions that serve to name them. Quine has charged Russell with confusion on this point, cf. his Introduction to Russell's 'Mathematical Logic as Based on the Theory of Types' in J. van Heijenoort (ed.): *From Frege to Gödel*, Harvard U. P. 1967. Accordingly I prefer to use 'context' for the place where the intersubstitutability has to hold. The important point here is that semantical equi-categoricity is an equivalence-relation between expressions. As Michael Dummett has remarked in his *Frege. Philosophy of Language*, Duckworth, London, 1973, Ch. 16, 'Identity', the corresponding relevant notion of type, or *ontological category*, is given, not just by means of (i) *application criteria*, that serve to lay down what an object of the category is, but also by (ii) *identity criteria*, that serve to lay down when two objects of the category are equal objects of that category. See also below concerning the Type Theory of Martin-Löf, *op. cit.*, footnote 16.

¹⁰*Investigating Wittgenstein*, Blackwell, Oxford, 1989, Ch. 1 and, 'On the Development of the Model-theoretic Viewpoint in Logical Theory', *Synthese* 77 (1988), pp. 1-36.

¹¹WB, § 5, in particular Theorem I, p. 247 [110], and the discussion preceding it.

language of a certain order will always have to be at least one higher, but for a language of infinite order the doctrine of semantical categories rules out that there could be a higher order and hence truth will be indefinable with respect to this language. There is no logical space left where it could be defined. This, of course, is a very clear instance of the rejection of a meta-perspective. Everything which can be said, can be said in language. If it cannot be said using the infinitely many orders we have got in our language, "the only language that I understand", it cannot be said at all.¹² "Whereof one cannot speak in categorical language, thereof one must remain silent". Tarski was clearly aware of this consequence of the Doctrine of Categories and he chose to formulate it in terms that must seem highly congenial to Hintikka:

The language of a complete system of logic should contain — actually or potentially — all possible semantical categories which occur in the logic of the deductive sciences. Just this fact *gives to the language* mentioned a certain 'universal' character, and it is one of the factors to which logic owes its fundamental importance for the whole of deductive knowledge.¹³

Let us now review the categorical status of Tarski's examples, concerning the use of quotation-mark names, with respect to the doctrine of semantical categories. The following schema is then obtained:

expressions of a semantical cate- gory	express (stand for)	entities of an onto- logical category (type)
'Mary'	stands for	Mary
''Mary''	stands for	'Mary'

which yields the uncomfortable conclusion that 'Mary' belongs to both semantical and objectual categories. In his *Nachwort* Tarski abjures his faith in the Doctrine of Semantic Categories.¹⁴ Indeed, his adherence had been confined to the linguistic level, with respect to the semantic theories, but he had not followed suit in the corresponding ontological framework; in his formalized metatheory, for example, the quantifiers are not restricted to types (ontological categories) but range over one single universe as in Frege or Hilbert. Given this abjuration, Tarski need not worry about the double categorical status of expressions; there is only one category of individuals anyway, namely the universe of objects.¹⁵

¹²See *Tractatus* 5.62 for the passage cited. The matter under discussion is most lucidly spelt out in Wolfgang Stegmüller, *Das Wahrheitsproblem und die Idee der Semantik*, Springer Verlag, Wien 1957, p. 84. See also James C. Morrison, *Meaning and Truth in Wittgenstein's Tractatus*, Mouton, The Hague, 1968, pp. 79-80, for basically the same point with respect to the *Tractatus* and the impossibility of a hierarchy of metalanguages.

¹³WB, p. 220 [80]. My italics.

¹⁴WB, p. 268 [133].

¹⁵This is perhaps a place where the influence of the reism of Tarski's teacher Kotarbiński can be seen. Only the denotata of names exist so there is only one category of objects, namely

For a constructivist the Doctrine of Semantic Categories, with the concomitant structure of types, is something to take most seriously; one should here compare the *Intuitionistic Type Theory* offered by Per Martin-Löf.¹⁶ To my taste the position in which a constructivist would find himself, when adopting the Tarskian framework, using expressions with a double categorical status as certain entities, as well as expressions, is at least as uncomfortable as that in which the medievals found themselves concerning the alleged ambiguity of the *suppositio materialis* (which latter position, if and when forced to a choice, I would prefer): an expression is not just an object and therefore not an element of an ontological category.

One way to resolve the problem might be to let "expressions" be uninterpreted physical marks on paper (as is the case in strict, possibly uncharitable, readings of Hilbert).¹⁷ This, however, is not how Tarski views the matter. First, he is explicitly aware of the need to let an "expression" be a type, rather than a token piece of matter, and he devotes a lengthy footnote to qualifying his remarks on this score.¹⁸ Secondly, Tarski explicitly, and very firmly, rejects the notion of a "formal" language where the "signs" have no material [German *inhaltlichen*] sense:

We shall always ascribe ... meanings to the signs ... in the languages we consider. The expressions we call sentences ... remain sentences after the signs which occur in them have been translated into colloquial language.¹⁹

Thus we must ask ourselves what the categorical status is of whatever it is the universe of things. The precise nature of the relationship between Tarski's views and those of his teachers is highly complex and has to be left for another occasion.

¹⁶Bibliopolis, Naples, 1984. In this connection, see also its elegant linguistic use by Aarne Ranta, 'Intuitionistic Categorical Grammar', *Linguistics and Philosophy* 14 (1991), pp. 203-239, building forth on the seminal work by K. Ajdukiewicz, 'Die syntaktische Konnexität', *Studia Philosophica* 1 (1936), pp. 1-27, and Richard Montague, *Formal Philosophy*, R. Thomason (ed.), Yale U. P., New Haven, 1974.

¹⁷'Neubegründung der Mathematik', I, *Abh. aus dem Math. Sem. der Hamburg. Univ.* 1 (1922), p. 22 and 'Über das Unendliche', *Math. Annalen* 95 (1926), p. 171. See also the exchange between Aloys Müller and Paul Bernays on 'Zahlen als Zeichen' in *Math. Annalen* 90. Stephan Körner, *The Philosophy of Mathematics*, Hutchinson, London, 1960, Ch. V. 1, gives an exposition of the standard arguments against the "marks-on-paper" view.

¹⁸WB, p. 156 [9].

¹⁹WB, p. 167 [21]. In this, of course, he was following his other teacher Leśniewski. An important difference from Leśniewski comes out in footnote 56) on pp. 68-69 of the *German version only*. Here Tarski makes the important observation, which he clearly perceives as a criticism, that the language of Leśniewski's system 'ist nicht als etwas potentiell "Fertiges" gedacht, sondern als etwas "Wachsendes": es sind nicht im vorhinein alle Zeichen und Sprachformen vorgesehen, welche in den Sätzen des Systems erscheinen können'. (The same is the case for the Type Theory of Martin-Löf, *op. cit.*, footnote 16 above.) This circumstance, according to Tarski, makes the system 'ein überaus undankbares Objekt für methodologische und semantische Untersuchungen'. Strong words, but words that were not repeated in the corresponding footnote 2, p. 210, of the English translation. In this connection, see also Jan Woleński and Peter Simons, 'De Veritate', pp. 391-442, especially § 9.VIII, pp. 425-426, in K. Szaniawski (ed.), *The Vienna Circle and the Lvov-Warsaw School*, Kluwer, Dordrecht, 1989.

that is named by a quotation-mark name. There are various obvious alternatives here. One was expressed with particular force by Lord Peter Wimsey, the hero of the detective novels by Dorothy L. Sayers:

I always have a quotation for everything — it saves original thinking.²⁰

From this we see that what is quoted, that is, what is referred to by quotation-mark name, must express a thought, or, derivatively, in the case of a subsentential component, must have content. This was known also to the medieval theorists of material supposition. In William of Sherwood's *Introduction to Logic* in the section on the *Division of Supposition* we find:

Supposition is, on the one hand, material and, on the other hand, formal. It is called material when an expression (*dictio*) stands for either the very sound (*vox*), or for the expression itself that consists of sound and signification (meaning), for example, when we say

well consists of four letters

or

Mary is a proper name.²¹

In the first of these examples the word 'well' materially supposits for the *vox*, for the purely physical aspect of the expression in question. This we realise by considering the equally correct example:

glub consists of four letters.

Here there can be no question of the word 'glub' having anything but its purely physical aspect as material supposition. The other alternative, that involves signification, can be safely ruled out, since the word 'glub' in question has been deliberately chosen for being senseless, that is, for its lack of signification. In the second example, though, the word 'Mary' has to be a proper name, something that a member of the ontological category of material ink-stains is not; here also signification is called for.

The distinction needed was formulated with maximal clarity by Peter Abelard, as that between *vox* and *sermo*, that is, between the expression merely as matter and as a semantic element, that is, as matter with meaning.²² Strictly

²⁰ *Have His Carcase*, New English Library (paperback edition), London, 1974, p. 63.

²¹ Ch V, § 2. Quoted from the valuable selection of texts concerning material supposition offered by J. M. Bocheński in his *Formale Logik*, Karl Alber, Freiburg/München, 1956. The translation from Bocheński's German is my own and I have adapted William's examples to fit those used by Tarski.

²² I owe my awareness of these matters to the exposition offered by my former colleague L. M. de Rijk in his studies of Peter Abelard's solution of the problem of universals, now conveniently collected in his *Through Language to Reality* (ed. E. P. Bos), Variorum Reprints, Northampton, 1989, Essays II, III, IV, and VI.

speaking, in order to serve, both William's and Peter Abelard's distinctions should be made to accommodate the type/token distinction with respect to the expressions in question. Such accommodation has already been accomplished by St. Vincent Ferrer, who further subdivided the material supposition into general (*communis*) and singular (*discreta*).²³

Material supposition	singular	general
<i>vox</i>	Glub is printed in black	Glub has four letters
<i>sermo</i>	Mary is printed in black	Mary is a proper name

Let us consider the proper name 'Mary'. There is, indeed, a difference in categorical status between, on the one hand, what results when we choose to disregard the type- and meaning-aspects of the name, namely the material aspects of the name, and a piece of matter. The piece of matter is just that: a piece of matter, whereas the material residue obtained from a given expression is not just a piece of matter. It is here presupposed that it is obtained from something which can be used to express something.²⁴ It is precisely this that marks the differences between semantical and ontological categories. Tarski's expressions are firmly entrenched in ontological categories and thus fall into the group: expression (*vox*) in general material supposition. The objectual status of Tarski's expressions comes out even more clearly when one considers their analogues within his formal treatment of the metatheory for the formal object-language. (Recall that the latter is a "real" language, with meaning!) In this treatment he defines an ontological category of metamathematical "EXPRESSIONS" by means of what is, in essence, the following induction:²⁵

- (0) *ng*, *sum*, *un*, and *in* (these are names of object-language expressions) are EXPRESSIONS.
- (1) for every natural number k , v_k (these are names of object-language variables) is an EXPRESSION.
- (2) whenever x and y are EXPRESSIONS their "concatenation" $x * y$ is an EXPRESSION.
- (3) there are no other EXPRESSIONS than those obtained from, possibly iterated, applications of (0) - (2).

The final, "extremal" clause could, of course, equally well be taken care of using a principle of Proof by EXPRESSION Induction. Thus defined the

²³See the passage from Ferrer, Bocheński, *Formale Logik*, op. cit, f. n. 15, § 27.11-12.

²⁴Usually, something *else*.

²⁵WB, Axioms 1-5, pp. 173-174 [29]. I prefer to capitalize the word here to indicate its specific, technical, usage.

EXPRESSIONs clearly constitute an ontological, rather than a semantical, category. Indeed, from a constructivist point of view, the EXPRESSIONs even form a set in the sense of Martin-Löf's Type Theory. One should note here that, by triple application of (0) and double application of (2), $in * in * ng$ is an EXPRESSION, that is, an element of the ontological category EXPRESSION, but it is not an *expression*; it does not express anything. The expression ' $in * in * ng$ ', on the other hand, does express something, namely the object $in * in * ng$ of the category EXPRESSION. In fact, for Tarski,

$$in * in * ng = '\supset\supset\neg',$$

but ' $\supset\supset\neg$ ' is not meaningful, does not signify, so the EXPRESSIONs do not form a semantical category.

It is clear from the text of Tarski that the above reading is not unduly harsh; on the contrary, in *Der Wahrheitsbegriff* he even explicitly acknowledges that there are "meaningless" expressions:

meaningful expressions which contain meaningless expressions as syntactic parts (every quotation-name of a meaningless expression will serve as an example).²⁶

The objectual status of Tarski's EXPRESSIONs is brought out very clearly by considering the fact that they belong to the universe of quantification.²⁷ These objects do not express anything, as was already stressed above, but get expressed using real expressions; they are things one talks about, but they are not the things that one uses to express oneself with. Thus, for instance, Tarski's famous truth-predicate has the universe of things for its range of significance and may rightly be applied only to a subclass of the EXPRESSIONs. Normally, one would say that it is the proposition that is the primary bearer of truth. Then, given the notion of propositional truth, one can, derivatively, introduce a notion of truth for the sentences with which propositions are expressed. If we want to be sufficiently awkward, yet a further, derived, notion of truth can be obtained, that applies to the (types of the) configurations of matter that are used to produce the sentences that express the propositions. The EXPRESSIONs are the formalized counterparts to Tarski's "expressions" and thus truth for expressions corresponds to the truth-predicate for EXPRESSIONs. Accordingly the notion of truth that Tarski defines corresponds to the the third, awkward, one.

The objectual status of the metamathematical EXPRESSIONs has been beautifully expressed by Kleene:

[The] problem of designation, which is troublesome to treat explicitly, is extraneous to the metamathematics as mathematics. The issue can be

²⁶WB, p. 162 [16] 'Meaningless expression' seems a contradiction in terms. What, precisely, could be expressed by a meaningless expression? 'Why, nothing', seems the only adequate answer for purely terminological reasons.

²⁷E. g., WB, Theorem 1, p. 197 [56]. See above, footnote 15.

avoided by using only names of the formal objects, and not claiming to exhibit the objects themselves. We find it convenient to do this in the generalized arithmetic, considering now " \supset ", " \forall ", " $\forall x A(x)$ " as names of certain objects (the names are the expressions inside the quotation marks), rather than as the objects themselves. We refrain from specifying what the objects are, other than that they belong to one domain of abstract objects arranged in a certain way in relation to one another, which we are calling entities ... The objects named could be formal symbols, formal expressions, etc., ..., though we now leave this open as irrelevant for metamathematics. (While we can thus avoid the problem of designation in our metamathematics, it would have to be faced in discussing the application of the metamathematics to a particular linguistic system.)

By going over from the conception of the formal system in terms of formal symbols, treated as if they were marks on paper, to an abstract system of objects, our metamathematics (i. e. the study of the formal system) becomes a branch of pure number theory entirely on a par conceptually with the arithmetic of the natural numbers and similar mathematical disciplines.²⁸

Turning now to the author of the *Tractatus*, it is very clear that he is perfectly aware of the distinctions that have been dealt with so far, on the basis of Tarski's work and that of the medievals, though his awareness certainly did not originate in these sources. They are introduced relatively early on in the work:

3.3 Nur der Satz hat Sinn; ...

3.31 Jeden Teil des Satzes, der seinen Sinn charakterisiert, nenne ich einen Ausdruck (ein Symbol).

(Der Satz selbst ist ein Ausdruck.)

Ausdruck ist alles, für den Sinn des Satzes Wesentliche, was Sätze miteinander gemein haben können.

...

We thus know that an expression is a part of a Sentence that serves to characterize its sense.²⁹ What, then, is a Sentence? The answer is provided by thesis

3.1 Im Satz drückt sich der Gedanke sinnlich wahrnehmbar aus.

How can thought be expressed as something perceptible?

3.11 Wir benützen das sinnlich wahrnehmbare Zeichen (Laut- oder Schriftzeichen, etc.) des Satzes als Projektion der möglichen Sachlage.

...

²⁸ *Introduction to Metamathematics*, North-Holland, Amsterdam, 1952, p. 251.

²⁹ I prefer to use the technical term *Sentence* as translation of the Tractarian 'Satz'.

3.12 Das Zeichen, durch welches wir den Gedanken ausdrücken, nenne ich das Satzzeichen. Und der Satz ist das Satzzeichen in seiner projektiven Beziehung zur Welt.

How is the Sentence-sign constituted to do this?

3.14 Das Satzzeichen besteht darin, daß sich seine Elemente, die Wörter, in ihm auf bestimmte Art und Weise zueinander verhalten.

Das Satzzeichen ist eine Tatsache.

We thus know that the Sentence-signs are certain facts that are able to express Sentences in virtue of standing in a projective relation to the world. Expressions (symbols) were the meaning-characterizing constituents of Sentences. Can also they be given as perceptible?

3.32 Das Zeichen ist das sinnlich Wahrnehmbare am Symbol.

3.321 Zwei verschiedene Symbole können also das Zeichen (Schriftzeichen oder Lautzeichen etc.) miteinander gemein haben — sie bezeichnen dann auf verschiedene Art und Weise.

How does one, when presented with a sign, recognize what symbol we are dealing with ?

3.326 Um das Symbol am Zeichen zu erkennen, muß man auf den sinnvollen Gebrauch achten.

3.341 ...

... [D]as Wesentliche am Symbol [ist] das, was alle Symbole, die denselben Zweck erfüllen können, gemeinsam haben.

3.3411 Man könnte also sagen: Der eigentliche Name ist das, was alle Symbole, die den Gegenstand bezeichnen, gemeinsam haben. ...

Note here that the theses 3.321 and 3.341 state a version of the type/token distinction with respect to expressions (symbols), or perhaps better, they constitute a refinement and more exact version of this distinction. With respect to signs, the type-token distinction is clearly indicated by thesis

3.203 ...

(“A” ist dasselbe Zeichen wie “A”).

and especially in its ancestor in the *Notes on Logic*:

It is to be remembered that names are not things but classes: “A” is the same letter as “A”.³⁰

³⁰Ludwig Wittgenstein, *Notebooks, 1914-1916*, Blackwell, Oxford, 1961, p. 104.

The circumstance, that the "real" symbol is that what is common to all symbols that are able to serve the same purpose, that are able to characterize the sense of Sentences in the same way, directly leads to a central point in the Tractarian philosophy of logic.

3.344 Das, was am Symbol bezeichnet, ist das Gemeinsame aller jene Symbole, durch die das erste den Regeln der logischen Syntax zufolge ersetzt werden kann.

Such substitution has to be carried out mechanically, through calculation *am Symbol allein*. This is clear from the latter passages concerning the *Sätze der Logik* in the 6.1's and the equations of mathematics in the 6.2's, where the propositions of logic and mathematics are claimed to be recognizable by means of syntactic calculation alone. Particularly clear in this connection are the theses

6.126 Ob ein Satz der Logik angehört, kann man berechnen, indem man die logischen Eigenschaften des Symbols berechnet.

Und dies tun wir, wenn wir einen logischen Satz "beweisen". Denn, ohne uns um einen Sinn und eine Bedeutung zu kümmern, bilden wir den logischen Satz aus anderen nach bloßen Zeichenregeln.

...

and

6.1262 Der Beweis in der Logik ist nur ein mechanisches Hilfsmittel zum leichteren Erkennen der Tautologie, wo sie kompliziert ist.

It cannot be stressed enough that the mechanical calculation, that provides the proofs in logic and in mathematics, is carried out *am Symbol allein*. The internal, syntactic, properties needed to carry out such a calculation are secured, precisely because the relevant entity is not just (part of) something factual (a *Satzzeichen*), but of something factual *in a projective relation to the world*, namely the *Satz*.

It then seems that the *Tractarian* sign corresponds to the *vox* and the symbol to the *sermo*. If we compare Wittgenstein's position with that of the medievals, we see that he has *Satzzeichen*, *Satz* and *sinnvoller Satz*, and thus the third notion could perhaps be taken as the symbol in *suppositio formalis*. Matters are not as straightforward as this, though. Whether there is a difference or not between *Satz* and *sinnvoller Satz* is a difficult problem in Tractarian exegesis.³¹ For instance, in the crucial thesis 3.31, quoted above, it appears that a *Satz*

³¹ James Griffin, *Wittgenstein's Logical Atomism*, 2nd edition, O. U. P., 1964, Ch. X, gives a good attempt at a resolution of the issues. Miss Ishiguro's 'Representation: An Investigation Based on a passage in the Tractatus', in B. Freed et al. (eds.), *Forms of Representation*, North-Holland, Amsterdam, 1975, pp. 189-202, on the 'is' of representation, is also relevant here.

must have sense. But, given this, could there then be a difference between a *Satz* (and thus *with Sinn*) and a *sinnvoller Satz*? A positive answer would seem very strange, but, on the other hand, in the philosophy of logic strange things do get said; recall, for instance, Frege's having to say that the concept horse is not a concept. Whatever the answer may be, on the level of expressions (symbols) other than Sentences, it seems that we do not have all three notions, since it is not at all clear what the difference between *Satz* and *sinnvoller Satz* would amount to for subsentential parts.

Finally then, what is the position of constructive mathematics with respect to expressions? Is the relevant notion the correlate of *vox* or of *sermo*? My use of the term 'constructive mathematics' was not just due to stylistic preference (though I do prefer 'constructivism' to 'intuitionism'). Brouwer's thesis concerning the languagelessness of mathematical activity is well-known. Also, specific *intuitionistic* conceptions such as choice sequences, especially lawless sequences, resist linguistic treatment: it cannot be demanded that every lawless sequence be nameable by a linguistic expression.³²

In constructive mathematics, the notion of identity is "*intensional*", and accordingly one wants this definitional, criterial, identity of objects with respect to a certain category to be decidable. That is, when α is a category and when $a : \alpha$ and $b : \alpha$, we want to be able to decide whether $a = b : \alpha$.³³ Such a decision would have to be carried out by means of syntactic calculation on the symbols 'a' and 'b'. It thus would demand, first, the careful formalization of the language used so that the structural features of the expressions, their parts and composition, become clear, and, secondly, one would have to do this in such a way that it becomes possible, in Tractarian terms, simply through mechanical substitution of signs expressing the same symbol, to bring the equation to such a form that it is directly perceivable *am Symbol allein* whether or not the two entities expressed are equal entities of the category α in question, whether or not the two signs do express the same symbol. The same must hold, *mutatis mutandis*, with respect to category membership, that is, it must be possible, given an expression 'a' to decide mechanically through calculation *am Symbol allein*, whether the object a expressed by the symbol 'a' does belong to the category α or not.³⁴

³²See A. S. Troelstra, *Choice Sequences*, O. U. P., 1977, Ch. 2. The matter of languagelessness is dealt with in my papers 'Constructions, Proofs and the Meaning of Logical Constants', *Journal of Philosophical Logic* 12 (1983), pp. 151-172, especially pp. 163-164, and 'Brouwer's Anticipation of the Principle of Charity', *Proceedings of the Aristotelian Society*, N. S., LXXXV (1984/1985), pp. i-xiv, especially § III, pp. xii-xiv.

³³I use the ':' to indicate membership of the category.

³⁴Cf. footnote 9 above for the notion of category involved and the role of the criterial, definitional identity in question. I have, at some length, gone into these issues of decidability *am Symbol allein* and their relations to the Type Theory of Martin-Löf and to the anti-realism of Michael Dummett in my paper 'Vestiges of Realism', forthcoming in the *Proceedings of the International Conference on the Philosophy of Michael Dummett*, Mussomeli, Sicily, Kluwer, Dordrecht, 1993.

Just as in the case of the *Tractatus* it is clear also here that it is the expression with its meaning, the *sermo*, that lends itself to the desired calculability. In these cases the metamathematical conception will not do. Why this is so, is easy to see: metamathematical EXPRESSIONS are made, not to be used to talk with, but to be talked about. The ideal language of the *Tractatus*, on the other hand, is one in which what can be said is said, and so its expressions are certainly there to be used. Similarly, when doing constructive mathematics, in order to speak about the mathematical objects under consideration, which, for all that matters, may well be metamathematical EXPRESSIONS, one obviously needs to use expressions with meaning, and so the EXPRESSIONS of metamathematics cannot perform as required.

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GRISS' CONTRIBUTION TO INTUITIONISM

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Abstract Griss' contribution to intuitionism presents two aspects: a new *Weltanschauung* as a basis for intuitionism, and the indication of two weak points in Brouwer's theory.

Firstly, Griss expounds his *Weltanschauung*, which starts with the original content of consciousness: the subject distinguishes himself from the object, but both cannot exist without the other. Therefore, no entity is admitted in mathematics without a mathematician who constructs it.

Then, Griss criticises the intuitionistic claim that exactness lies in the intellect: since nothing can be sharply distinguished in consciousness, only language can fix concepts.

Finally, he criticises Brouwer's definition of negation as not suitable for intuitionism: a "reasoning ending in a contradiction" does not fulfill the condition of evidence required for each step of mathematical reasoning. He suggests a definition of negation in terms of comparing two existing entities and discovering that one has more properties than the other.

G. F. C. Griss is well known for a series of papers, appeared in the 1940s, in which he criticised the intuitionistic definition of negation proposed by Brouwer. Yet, this was not his only contribution to intuitionism, but it was a part of a wider contribution which presented two aspects: a new *Weltanschauung* as a basis for intuitionism, and the indication of two weak points in Brouwer's theory. It is therefore interesting to consider all these aspects.

1. Griss' *Weltanschauung*

Griss outlined his *Weltanschauung* in the book "Idealistische filosofie", published in 1946. It was his friend, the philosopher H. J. Pos, who, in September 1940 during an exchange of ideas on negation, suggested that he should work out his *Weltanschauung* in more detail. After Griss' death, Pos complained that the clarity of the book had given the impression that it was not very profound, and that therefore it had not given rise to "the impression it merited".

Griss' *Weltanschauung* is based on the original datum that consciousness grasps by attaining its own fullness: the subject distinguishes himself from the