In order to find the distribution of optical pairs in the groups $d<5''$; $5''-10''$ and $10''-15''$ we simply have to multiply the fractions of table 7 by the numbers of optical pairs derived in tables 4 and 5.

After substracting these numbers from the observed numbers in table 6, we find the distribution of physical double stars over m and Δm, which is given in table 8.

The weight to be given to our solution of the distribution of physical double stars over the different values of Δm will be largest for those magnitudes where the percentages of the optical pairs are small relatively to the percentages of the physical ones. In other words the best results are those obtained from the brighter stars with small value of d.

For the faint double stars of the group $d=15''-30''$ the numbers of optical pairs are nearly equal to those of the physical pairs. So I have given no results for those stars as these results are entirely problematic.

In the present paper I will not give a statistical discussion of the results. The data obtained from the Vatican and Greenwich astrographic zones are now being prepared for press.

The statistical results obtained from the complete material will be fully discussed in a future paper.

The double stars of the Vatican Astrographic Catalogue, second paper, by Dr. *E. A. Kreiken.*

Introduction.

In the preceding article some formulas were derived by which we are able to determine the number of optical pairs between definite limits of magnitude m and magnitude difference Δm.

In the same article these formulae were applied to the material collected by J. SCHEINER in the *Publicationen des Astroph. Obs. zu Potsdam*, Nr 59.

The present article deals with the double star catalogue published by father J. STEIN in the *Catalogo Astrografico, sezione Vaticana*, Decl. da $+55°$ a $+65°$ Appendice III.

In this catalogue Father STEIN gives R. A., Declination, angular distance, position angle and magnitudes of all double stars with a separation $d<15''$ between the declinations $+58°55'$ and $+65°5'$, all those pairs being contained in one of the Zones of the Vatican Astrographic Catalogue.

The total number of pairs given by Father STEIN is 1265.

For the application of the method exposed in the preceding paper we also want the numbers of pairs with an angular separation from $15''$ to $30''$. The data necessary to find these numbers were placed at my disposal by Father STEIN. I have to express my sincere thanks for his extreme kindness.

The elaborate work of counting the numbers of stars between definite magnitude limits on each plate of the Vatican Astrographic Catalogue was undertaken by Dr. H. SPECKMAN. I owe him great thanks for his valuable help.

The observed distribution over m and Δm.

The distribution of the observed numbers with $d<15''$ over m and Δm is found by direct counts in Father STEIN's Catalogue. For the pairs with $d=15''-30''$ the unpublished material of Father STEIN was used. In this material the pairs on each separate plate are collected while in the Appendice III the pairs contained in a certain part of the sky are given. As the centres of the plates in the Astrographic zones are chosen in such a way, that each part of the sky is covered at least by two different plates, the total surface of all plates together is much larger than the total surface of the sky between the declinations $+58°55'$ and $+65°5'$. The correct numbers of double stars with $d=15''-30''$ may be found by multiplying the observed numbers in this group by a fraction which is equal to

$$\frac{\text{total number of square degrees of the observed part of the sky}}{\text{number of square degrees of each plate} \times \text{number of plates}}.$$

The whole material collected by Father STEIN was devided into three groups.

The first one contains the double stars with a galactic latitude $<20°$ the second with latitude between $20°$ and $40°$ while the third one contains the pairs with latitude from $40°-60°$.

TABLE 1.

Distribution of the observed numbers over m and Δm

LATITUDE $0°-20°$

$d<5''$

m Vat	0.0	0.3	0.6	0.9	1.2	1.5	1.8	2.1	2.4>	Total
7.0	0	1	0	0	0	0	0	0	0	1
8.0	0	0	1	0	0	0	0	0	1	2
9.0	1	3	1	3	1	3	2	2	2	18
10.0	6	8	5	2	1	1	7	1	11	42
11.0	15	12	6	6	4	7	2	5	9	66
12.0	24	8	12	8	3	3	2	—	—	60
13.0	13	16	7	1	2	—	—	—	—	39

TABLE 1. (*Continued.*)

$d = 5'' - 10''$

	0.0	0.3	0.6	0.9	1.2	1.5	1.8	2.1	2.4 >	Total
7.0	0	0	0	0	0	0	0	0	0	0
8.0	0	0	0	1	0	0	0	2	7	10
9.0	1	0	1	1	3	3	4	3	16	32
10.0	3	3	9	6	4	6	4	5	14	54
11.0	7	7	18	6	13	5	4	2	10	72
12.0	38	21	12	9	9	9	1	3	1	103
13.0	22	21	13	6	3	4	—	—	—	69

$d = 10'' - 15''$

	0.0	0.3	0.6	0.9	1.2	1.5	1.8	2.1	2.4 >	Total
7.0	0	0	0	0	0	0	0	1	1	2
8.0	0	0	0	0	0	0	0	2	10	12
9.0	0	0	1	1	2	1	2	1	16	24
10.0	2	1	1	4	2	3	3	4	15	35
11.0	7	10	5	10	9	5	3	3	20	72
12.0	27	17	14	11	6	10	4	3	6	98
13.0	40	31	14	9	9	4	—	—	1	108

$d = 15'' - 30''$

	0.0	0.3	0.6	0.9	1.2	1.5	1.8	2.1	2.4 >	Total
7.0	1	1	1	1	0	0	1	1	20	26
8.0	0	2	1	2	2	0	0	2	44	53
9.0	0	2	0	7	2	3	3	9	82	108
10.0	7	1	11	15	18	17	16	12	115	212
11.0	43	24	27	28	39	59	29	31	121	401
12.0	92	80	84	81	69	56	24	32	20	548
13.0	167	166	88	57	54	37	2	5		576

LATITUDE $20° - 40°$

$d < 5''$

m Vat	0.0	0.3	0.6	0.9	1.2	1.5	1.8	2.1	2.4 >	Total
7.0	0	0	0	0	0	0	0	0	0	0
8.0	0	0	0	0	0	0	0	0	0	0
9.0	0	0	0	0	0	0	0	0	4	4
10.0	1	0	1	1	2	1	0	0	4	10
11.0	1	0	1	0	1	0	0	0	0	3
12.0	4	1	1	0	2	0	—	—	—	8
13.0	2	1	1	0	0	—	—	—	—	4

$d = 5'' - 10''$

	0.0	0.3	0.6	0.9	1.2	1.5	1.8	2.1	2.4 >	Total
7.0	0	0	1	0	0	0	0	0	0	1
8.0	0	0	0	0	0	0	0	0	2	2
9.0	0	1	1	0	0	2	0	0	0	4
10.0	0	1	1	2	2	1	1	1	4	13
11.0	4	0	5	0	0	2	1	0	1	13
12.0	9	4	1	1	0	0	0	0	0	15
13.0	4	2	0	0	0	—	—	—	—	6

$d = 10'' - 15''$

	0.0	0.3	0.6	0.9	1.2	1·5	1.8	2.1	2.4 >	Total
7.0	0	0	0	0	0	0	0	0	2	2
8.0	0	0	0	0	0	0	0	0	1	1
9.0	0	0	0	0	0	0	0	1	1	2
10.0	0	0	1	0	0	0	0	0	1	2
11.0	0	0	1	1	1	0	1	2	2	8
12.0	4	3	3	0	1	1	—	—	—	12
13.0	3	1	1	0	0	—	—	—	—	5

$d = 15'' - 30''$

	0.0	0.3	0.6	0.9	1.2	1.5	1.8	2.1	2.4 >	Total
7.0	0	0	0	0	0	0	0	0	0	0
8.0	0	0	0	0	1	0	0	2	4	7
9.0	0	0	1	1	0	0	1	0	4	7
10.0	2	1	0	1	0	1	2	1	13	21
11.0	5	1	4	7	0	3	1	1	8	30
12.0	11	5	13	8	6	3	0	1	—	47
13.0	17	12	2	3	4	—	—	—	—	38

LATITUDE $40° - 60°$

$d < 5''$

m Vat	0.0	0.3	0.6	0.9	1.2	1.5	1.8	2.1	2.4 >	Total
7.0	0	0	0	0	0	0	0	0	0	0
8.0	0	0	0	0	0	0	0	0	0	0
9.0	0	1	0	0	0	0	0	0	2	3
10.0	0	0	5	1	1	5	5	3	3	23
11.0	3	1	1	1	0	0	0	0	0	6
12.0	8	0	2	2	1	0	1	0	0	14
13.0	2	2	0	0	0	0	—	—	—	4

$d = 5'' - 10''$

	0.0	0.3	0.6	0.9	1.2	1.5	1.8	2.1	2.4 <	Total
7.0	0	0	0	0	0	0	0	0	0	0
8.0	0	0	0	0	0	1	0	0	1	2
9.0	0	1	1	0	1	3	0	2	2	10
10.0	0	2	2	1	2	3	7	0	4	21
11.0	3	2	4	2	0	4	0	0	0	15
12.0	11	5	1	0	1	0	0	0	0	18
13.0	1	0	0	0	0	0	—	—	—	1

$d = 10'' - 15''$

	0.0	0.3	0.6	0.9	1.2	1.5	1.8	2.1	2.4 <	Total
7.0	0	0	0	0	0	0	0	0	0	0
8.0	0	0	0	1	0	0	0	0	3	4
9.0	1	0	0	0	0	0	0	1	1	3
10.0	0	0	0	0	0	0	0	0	0	0
11.0	1	0	0	1	1	3	1	0	0	7
12.0	4	1	1	1	0	0	0	0	0	7
13.0	2	1	0	0	—	—	—	—	—	3

$d = 15'' - 30''$

	0.0	0.3	0.6	0.9	1.2	1.5	1.8	2.1	2.4 <	Total
7.0	2	0	0	0	0	0	0	0	5	7
8.0	0	1	0	0	0	0	0	1	3	5
9.0	0	1	1	2	0	1	0	0	8	13
10.0	3	1	5	0	1	1	0	2	2	15
11.0	4	4	1	2	1	5	1	0	2	20
12.0	14	9	10	2	7	0	0	1	—	43
13.0	21	14	3	2	—	—	—	—	—	40

For each separate group the observed distribution is given in table 1. So for $d < 15''$ the values in table 1 represent the numbers of pairs in a certain area of the sky, while the values for $d > 15''$ represent the numbers of pairs contained on all plates of which the centre falls within this same area.

A. The numbers of optical pairs between definite limits of magnitude.

The magnitudes used in table 1 are on the scale used in the Vatican Catalogue. In order to find the international photographic magnitudes corresponding to the limits adopted in table 1, I have determined in the different galactic latitudes the mean numbers of stars $N(m)$ brighter than a given magnitude m. These numbers were found from the star counts of Dr. H. SPECKMAN. The results are given in table 2. By using the tables of SEARES, VAN RHIJN, JOYNER and RICHMOND [1]), we find the magnitudes on the international scale corresponding to the Vatican

[1]) *Astroph. Journal* LXII, table XVII, 1925.

magnitudes. We see, that there is a slight difference between the results in the different latitudes. In the higher latitudes the observation of the fainter stars has been more complete than in the lower ones.

In table 3 I have compared the values observed by SEARES *c. s.*, with the values derived from the formula

$$\log N(m) = a + b\ (m - 10) + c\ (m\ 10)^2 \qquad (1)$$

(The values of a, b and c in the different latitudes have been indicated in the bottom of the table). The differences between the two sets of values are always very small.

From the values in the tables 2 and 3 and from the formula (2).

$$D_0(m) = \frac{d^2}{r^2}\ A(m) \left\{ \frac{A(m) - 1}{2} + N(m_l) - N(m) \right\} \qquad (2)$$

which was derived in the preceding paper, we find the numbers of optical pairs between definite limits of magnitude.

From our table we see, that nearly all faint double stars of the group $d = 15'' - 30''$ are optical pairs. This might have been anticipated from the results of Father STEIN [1]) and Dr. GROOT [2]). When treating the double stars in the Potsdam zone we had to apply a rather important correcting factor to allow for the different densities of the separate plates and the irregularities in the distribution of the stars over the surface of each plate. There is however a large difference between the Vatican Catalogue and the Potsdam one. The number of plates from which J. SCHEINER has derived his results is much smaller than those contained in the complete programme of the Potsdam Catalogue, as only for a small number of plates the results were published in the first four volumes used by SCHEINER. Some areas in the Potsdam zone are almost completely observed while in other ones there is nearly an absolute lack of observations. Therefore the numbers of double stars in the Potsdam zone were derived in such a way, that these numbers are equal to the sum of the double stars on each separate plate. The observed surface had to be taken as the number of plates × the area of each plate expressed in square degrees.

The totals derived in this way will be largely influenced by the different densities of the separate plates and the irregularities in the distribution of the stars on the surface of each plate.

TABLE 2. Mean values of log N_m and the magnitudes on the international photographic scale statistically corresponding to the magnitudes on the Vatican scale.

m Vat.	Gal. latitude 0—20		Gal. latitude 20—40		Gal. latitude 40—60	
	log N_m	m Int.	log N_m	m Int.	log N_m	m Int.
7.0	9.964	7.87	9.604	7.70	9.506	7.70
8.0	0.312	8.90	0.068	8.77	0.117	9.00
9.0	0.748	9.92	0.587	9.95	0.572	10.14
10.0	1.142	10.85	0.971	10.92	0.967	11.05
11.0	1.421	11.50	1.278	11.67	1.316	11.80
12.0	1.793	12.30	1.598	12.45	1.573	12.63
13.0	1.983	12.97	1.817	13.05	1.765	13.16
		m_l = 13.27		m_l = 13.37		m_l = 13.30

TABLE 3. Comparison of the numbers observed by SEARES *c. s.* with the numbers computed with the formula $\log N_m = a + b\,(m - 10) + c\,(m - 10)^2$.

	0—20°			20—40°			40—60°		
m Int.	log N_m Obs.	log N_m comp.	$O-C$	log N_m Obs.	log N_m comp.	$O-C$	log N_m Obs.	log N_m comp.	$O-C$
7.0	9.478	9.476	+ ·002	9.250	9.249	+ ·001	9.103	9.103	·000
8.0	9.920	9.921	— ·001	9.694	9.695	— ·001	9.545	9.550	— ·005
9.0	0.358	0.360	— ·002	0.128	0.129	— ·001	9.978	9.981	— ·003
10.0	0.791	0.791	·000	0.551	0.552	— ·001	0.395	0.395	·000
11.0	1.217	1.216	+ ·001	0.964	0.963	+ ·001	0.795	0.790	+ ·005
12.0	1.634	1.633	+ ·001	1.364	1.363	+ ·001	1.178	1.174	+ ·004
13.0	2.042	2.044	— ·002	1.748	1.751	— ·003	1.538	1.539	— ·001
	$a = +·791$; $b = +·428$ $c = -·00344$			$a = +·552$; $b = +·417$ $c = -·00583$			$a = +·395$; $b = +·406$ $c = -·00826$		

[1]) Dr. J. STEIN (*l. c.*). [2]) *M. N. R. A. S.* Nov. 1927, page 50-55.

TABLE 4. Numbers and percentages of physical and optical doubles.

$\beta = 0° - 20°$ surface 544 □ degrees.

m Vat.	m Int.	$d < 5''$				$d = 5'' - 10''$				$d = 10'' - 15''$				$d = 15'' - 30''$			
		$D(m)$ Obs.	Opt. numb.	Phys. numb.	°/₀ Phys.	$D(m)$ Obs.	Opt. numb.	Phys. numb.	°/₀ Phys.	$D(m)$ Obs.	Opt. numb.	Phys. numb.	°/₀ Phys.	$D(m)$ Obs.	Opt. numb.	Phys. numb.	°/₀ Phys.
7.0	7.97	1	1	0	—	0	2	(— 2)	—	2	4	(— 2)	—	12	8	4	33
8.0	8.90	2	1	1	50	10	4	6	60	12	6	6	50	25	15	10	40
9.0	9.92	18	3	15	83	32	9	23	72	24	16	8	33	51	40	11	22
10.0	10.85	42	6	36	86	54	21	33	61	35	34	1	3	100	86	14	14
11.0	11.50	66	10	56	85	72	32	40	56	72	54	18	25	188	136	52	28
12.0	12.30	60	17	43	72	103	52	51	54	98	86	12	12	211	219	— 8	—
13.0	12.87	39	17	22	57	69	51	18	26	108	70	38	35	224	215	+ 9	4
$\beta = 20° - 40°$ surface 181 □ degrees.																	
7.0	7.70	0	0	0	—	1	0	1	100	2	0	2	100	0	0	0	—
8.0	8.77	0	0	0	—	2	0	2	100	1	0	1	100	3	1	2	67
9.0	9.95	4	0	4	100	4	0	4	100	2	1	1	50	3	4	(— 1)	0
10.0	10.92	10	0	10	100	13	1	12	92	2	2	0	0	10	8	2	20
11.0	11.67	3	1	2	67	13	2	11	85	8	3	5	63	14	14	0	0
12.0	12.45	8	1	7	78	15	3	12	80	12	4	8	67	22	21	1	5
13.0	13.05	4	1	3	75	6	2	4	67	5	3	2	40	18	18	0	0
$\beta = 40° - 60°$ surface 363 □ degrees.																	
7.0	7.70	0	0	0	—	0	0	0	—	0	0	0	—	3	1	2	67
8.0	9.00	0	0	0	—	2	0	2	100	4	0	4	100	2	2	0	0
9.0	10.14	3	0	3	100	10	1	9	90	3	1	2	67	6	5	1	17
10.0	11.05	23	0	23	100	21	1	20	95	0	2	(— 2)	0	7	8	(— 1)	0
11.0	11.80	6	0	6	100	15	1	14	93	7	2	5	71	10	12	(— 2)	0
12.0	12.63	14	0	14	100	18	2	16	89	7	3	4	57	21	19	2	10
13.0	13.16	4	0	4	100	1	1	0	—	3	2	1	33	20	10	10	50

On the other hand the influence of these irregularities should be much smaller in the Vatican zone. In this zone the results of all plates have been published. The catalogue of Father STEIN was obtained from this complete material. Each area of the sky appears on at least two or three different plates. When it is on the edge of the one plate it will be very near the centre of a second one. So when we do not consider the number of double stars on each separate plate, but those contained in a certain area of the sky, these latter numbers will be scarcely influenced by the irregularities mentioned before. Therefore I have given in table 4 the numbers of double stars contained in certain areas of the sky. As it was anticipated the correcting factor is now found to be almost equal to unity.

A further inspection of table 4 shows, that the percentages of the total numbers which are physical pairs decrease with magnitude and with increasing value of d. This agrees with the results obtained from the Potsdam zone and it is very evident, that this also should have been expected.

Comparing the percentages of physical pairs in the different latitudes we find that in general the percentages of physical pairs are larger in the higher latitudes than in the lower ones. The number of stars per square degree in the Milky Way is much larger than in the higher latitudes. Now the numbers of optical pairs are nearly proportional to the square of the star density. So the increase of the numbers of optical double stars with increasing star density will be much larger than the increase of the physical double stars. So the relative frequency of physical pairs will decrease in the regions near the Milky Way.

Next we compute the fractions by which the distribution of the optical numbers over the different values of magnitude difference (Δm) between the two companions of the pair is determined.

We proceeded in the same way as in the preceding paper.

For the brighter magnitudes we used the formula

$$D_o\left(m, \frac{\Delta m_1}{\Delta m_2}\right) = D_o(m) \times \frac{\int_{x_1=\left(\frac{\beta}{2\gamma}+\bar{m}_g\right)\sqrt{\gamma}}^{x_2=\left(\Delta m+\frac{\beta-2\gamma\bar{m}_g}{2\gamma}\right)\sqrt{\gamma}} e^{-x^2}\,dx}{\int_{x_1=\left(\frac{\beta}{2\gamma}+\bar{m}_g\right)\sqrt{\gamma}}^{x_3=\left(\Delta m_1+\frac{\beta-2\gamma\bar{m}_g}{2\gamma}\right)\sqrt{\gamma}} e^{-x^2}\,dx} \qquad (3)$$

in which $D_o\left(m, \frac{\Delta m_1}{\Delta m_2}\right)$ represents the number of optical pairs with Δm between the limits Δm_1 and Δm_2.

For the fainter stars these fractions were derived from the distribution of the observed numbers in the group $d = 15'' - 30''$. Finally for the intermediate magnitude $m_{Vat} = 10.0 - 11.0$ we used the mean of the two sets of values obtained from formula (3) and from the observed numbers.

The results obtained in the different zones have been entered in table 5. Owing to the slight differences between the magnitude corrections in the different latitudes it was necessary to derive the fractions in each zone separately. The values of the probability integral (3) were taken from the paper of Dr. JAMES BURGESS (*Transactions Royal Society of Edinburgh* Vol. XXXIX).

TABLE 5. Values of 100 × the fractions determining the distribution of the optical pairs over different values of m and Δm on the scale of the Vatican Astrograpic Catalogue.

$\beta = 0° - 20°$

mVat.	Δm 0.0	0.3	0.6	0.9	1.2	1.5	1.8	2.1	2.4	>
7.0	0.2	0.4	0.5	0.5	1.3	1.5	1.1	2.4		92.1
8.0	0.2	0.5	0.6	0.6	1.5	1.6	1.3	2.8		90.9
9.0	0.7	1.5	1.4	0.8	2.4	2.3	3.9	2.5		84.5
10.0	2.1	2.3	3.1	3.0	6.9	5.6	6.8	7.0		63.2
11.0	7.9	5.6	6.0	6.4	10.9	11.7	10.6	13.0		27.9
12.0	16.8	14.6	15.3	14.8	12.6	10.2	4.4	5.8		3.6
13.0	29.0	28.8	15.3	9.9	9.4	6.4	0.4	0.9		—

TABLE 5. (*Continued.*)

$\beta = 20° - 40°$

m	Δm 0.0	0.3	0.6	0.9	1.2	1.5	1.8	2.1	2.4	>
7.0	0.0	0.4	0.4	0.8	0.8	1.2	0.8	1.6		94.0
8.0	0.4	0.6	0.8	1.1	1.5	2.2	2.3	7.4		83.7
9.0	1.1	0.9	1.3	1.9	2.7	2.1	3.3	4.6		82.1
10.0	2·1	3·8	2.8	4.8	4.1	6.9	4.3	7.3		63.9
11.0	11.1	4.5	8.8	18.8	4.9	10.5	8.9	5.2		27.3
12.0	23.5	10.7	27.7	17.1	12.8	0.0	0.0	2.2		—
13.0	44.8	31.6	5.3	7.9	10.6					

$\beta = 40° - 60°$

m	Δm 0.0	0.3	0.6	0.9	1.2	1.5	1.8	2.1	2.4	>
7.0	0.1	0.2	0.4	0.4	0.6	0.6	1.1	1.3		95.3
8.0	0.3	1.0	0.9	1.5	1.2	2.2	2.7	1.6		88.6
9.0	1.3	1.8	2.2	3.0	2.0	4.3	3.7	6.9		74.8
10.0	3.3	2.2	2.9	6.1	7.2	6.9	7.9	14.2		49.3
11.0	13.2	13.3	5.8	12.6	8.0	19.6	10.5	8.9		8.1
12.0	32.6	21.0	23.3	4.7	16.3	0.0	0.0	2.4		—
13.0	52.5	35.0	7.5	5.0						

TABLE 6. Distribution of the physical double stars over m and Δm, m and Δm on the scale of the Vatican Astrographic Catalogue.

$\beta = 0 - 20°$ Surface 544 □ degrees.

$d < 5''$

m	Δm 0.0	0.3	0.6	0.9	1.2	1.5	1.8	2.1	2.4	>
7.0	0	1	0	0	0	0	0	0		(—1)
8.0	0	0	1	0	0	0	0	0		0
9.0	1	3	1	3	1	3	2	1		0
10.0	6	8	5	2	1	1	6	0		7
11.0	14	11	5	5	3	6	1	4		7
12.0	21	5	9	5	1	1	1	—		—
13.0	7	11	4	0	0	—	—	—		—

$d = 5'' - 10''$

m	Δm 0.0	0.3	0.6	0.9	1.2	1.5	1.8	2.1	2.4	>
7.0	0	0	0	0	0	0	0	0		(—2)
8.0	0	0	0	1	0	0	0	2		3
9.0	1	0	1	1	3	3	4	3		7
10.0	3	3	8	5	2	5	2	3		2
11.0	4	5	16	4	10	1	1	(—2)		1
12.0	29	14	4	1	2	3	(—1)	(—1)		—
13.0	7	5	5	1	(—2)	1	—	—		—

TABLE 6. (*Continued.*)

$d = 10'' - 15°$

m	Δm 0.0	0.3	0.6	0.9	1.2	1.5	1.8	2.1	2.4	>
7.0	0	0	0	0	0	0	0	1	(—3)	
8.0	0	0	0	0	0	0	0	2	4	
9.0	0	0	1	1	2	1	1	1	1	
10.0	1	1	0	3	0	0	0	2	—8	
11.0	3	7	2	7	3	(—1)	(—3)	4	4	
12.0	12	4	1	(—2)	(—5)	1	0	(—2)	3	
13.0	20	11	3	2	2	0	0	0	—	

$\beta = 20° - 40°$ 181 □ degrees.

m	Δm 0.0	0.3	0.6	0.9	1.2	1.5	1.8	2.1	2.4	>	0.0	0.3	0.6	0.9	1.2	1.5	1.8	2.1	2.4	>
7.0	0	0	0	0	0	0	0	0	0		0	0	1	0	0	0	0	0	0	
8.0	0	0	0	0	0	0	0	0	0		0	0	0	0	0	0	0	0	2	
9.0	0	0	0	0	0	0	0	0	4		0	1	1	0	0	2	0	0	0	
10.0	1	0	1	1	2	1	0	0	4		0	1	1	2	2	1	1	1	3	
11.0	1	0	1	0	1	0	0	0	(—1)		4	0	5	0	0	1	1	0	0	
12.0	3	1	1	0	2	0	—	—	—		8	4	0	0	0	0	0	0	—	
13.0	1	1	1	1	0	0	—	—	—		3	1	0	0	0	—	—	—	—	

$d = 10'' - 15''$

m	Δm 0.0	0.3	0.6	0.9	1.2	1.5	1.8	2.1	2.4	>
7.0	0	0	0	0	0	0	0	0	2	
8.0	0	0	0	0	0	0	0	0	1	
9.0	0	0	0	0	0	0	0	1	0	
10.0	0	0	1	0	0	0	0	0	(—1)	
11.0	0	0	1	0	1	0	0	1	2	
12.0	3	3	2	0	0	0	0	—	—	
13.0	1	1	0	0	—	—	—	—	—	

$\beta = 40° - 60°$ Surface 363 □ degrees

$d < 5''$

m	Δm 0.0	0.3	0.6	0.9	1.2	1.5	1.8	2.1	2.4	>
7.0	0	0	0	0	0	0	0	0	0	
8.0	0	0	0	0	0	0	0	0	0	
9.0	0	1	0	0	0	0	0	0	2	
10.0	0	0	5	1	1	5	5	3	3	
11.0	3	1	1	1	0	0	0	0	0	
12.0	8	0	2	2	1	0	1	0	0	
13.0	2	2	0	0	0	0	—	—	—	

$d = 5'' - 10''$

m	1.0	0.3	0.6	0.9	1.2	1.5	1.8	2.1	2.4	>
7.0	0	0	0	0	0	0	0	0	0	
8.0	0	0	0	0	0	1	0	0	1	
9.0	0	1	1	0	1	3	0	2	1	
10.0	0	2	2	1	2	3	7	0	3	
11.0	3	2	4	2	0	3	0	0	0	
12.0	10	5	0	0	1	0	0	0	0	
13.0	0	0	0	0	0	0	—	—	—	

$d = 10'' - 15''$

m	Δm 0.0	0.3	0.6	0.9	1.2	1.5	1.8	2.1	2.4	>
7.0	0	0	0	0	0	0	0	0	0	
8.0	0	0	0	1	0	0	0	0	3	
9.0	1	0	0	0	0	0	0	1	0	
10.0	0	0	0	0	0	0	0	(1—)	(— 1)	
11.0	0	0	0	1	1	2	1	0	0	
12.0	3	0	0	1	0	0	0	0	0	
13.0	1	0	0	0	0	0	—	—	—	

Generally the fractions of the optical pairs increase with increasing value of Δm. In the groups with different values of d we find the distribution of the optical pairs by multiplying the optical numbers derived in table 4 with the fractions of table 5.

By substracting these optical numbers from the observed ones, we find the distribution of the physical double stars over m and Δm. Table 6 contains the results derived in this way.

The weight of the results will diminish when the percentages of the optical pairs are increasing. So small weight should be attached to the values derived for the faint magnitudes of the group $d = 10'' - 15''$.