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Photographic observations of the integrated brightness of the solar crescent near totality of the total eclipse of the sun on June 19, 1936, by J. G. Ferwerda, J. Uitterdijk and A. J. Wesselink,

discussed by *A. J. Wesselink.*

1. *Introduction and summary.*

Trustworthy information as to the variation of the surface brightness over the sun's disc has been obtained for the central part (up to 95 % of the radius from the centre) by direct measurements, scintillation having but little disturbing effect[1]). This disturbing effect of the scintillation becomes important near the limb, where the results for the surface brightness from direct measures come out systematically too small. Observations of the integrated intensity of the solar crescent during the partial phase of an eclipse of the sun can be used to determine the law of darkening independent of the scintillation. In that case the integrated intensity of the solar crescent as a function of the time[2]) determines the law of darkening when the variation of the geometric elements of the crescent with time is assumed to be known[3]).

In practice only the part of the eclipse function near totality is sensitive to the law of darkening near the limb.

It follows that observations of the eclipse function *near totality* are useful for the determination of the law of darkening *near the limb*.

Direct measures on the central part of the disc and observations of the eclipse function supplement each other in the determination of the complete variation of the surface brightness from centre to limb.

The members of the expedition were J. G. Ferwerda, J. Uitterdijk and the writer. We left the Leiden Observatory on June 6th, for the north-western outskirts of the Caucasus in order to observe the total eclipse of the sun on June 19, 1936 in the way mentioned above.

Beloretchenskaia, 60 km north of Maikop and lying nearly on the central line, was chosen for observing site. We travelled over Warszawa, Kiev, Charkov, Rostov, Armavir with arrival at Beloretchenskaia on June 13. It was through the careful preparations by Mr. Uitterdijk that the many frontiers could be crossed without serious delay.

At Beloretchenskaia we had our lodgings in a building belonging to the airport. Through the courtesy of the director we could dispose of a room at the south-east, which proved to be most useful as an observing room.

Of the many expeditions sent for the observation of this eclipse we met at Beloretchenskaia a Soviet mission from the University of Charkov, under the guidance of Professor Barabashev and a French expedition from the Société astronomique de France, of which Mr. Bidault de l'Isle was the head. The collaboration with these expeditions has been most cordial and the members of our expedition keep a good memory of the days spent jointly in the eclipse camp.

The three chronometers were compared regularly with radio time signals; mainly those sent by Warszawa and Paris-Pontoise have been used.

During the eclipse the observations were carried out as planned. Unfortunately very thin clouds occasionally projected themselves on the sun and only poor results were expected.

A provisional investigation of a part of the developed material showed the disturbing effect of the clouds. However, the colour of the sun's light had

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[1]) Schwarzschild and Villiger, *Ap. J.* **23**, 284 (1906). Abbot, *Smithsonian Annals*, Vol. II, part III, 203 (1908), Vol. IV, ch. VII, 217 (1922). Moll, Burger and van der Bilt, *B.A.N.* No. 91, 83 (1926).

[2]) Shortly called eclipse function, a graph representing it eclipse curve.

[3]) Julius, *Ap. J.* **37**, 225 (1913). Moll and van der Bilt, *B.A.N.* No. 30, 171 (1922). Minnaert, *Monthly Notices*, **89**, 197 (1928).

apparently not been appreciably affected by the clouds and was found to be constant over the extreme 5½ percent of the radius of the sun's disc.

Notwithstanding the fact that the observations thus seemed to be of poor quality it was decided to measure the complete material in the Schilt photometer and to discuss the results thoroughly for the following reasons:

1°. To verify the constancy of the colour near the limb of the sun's disc.

2°. To test the new photographic method employed. The discussion has led to the conviction that the method may be used with advantage while some improvements are proposed in an appendix.

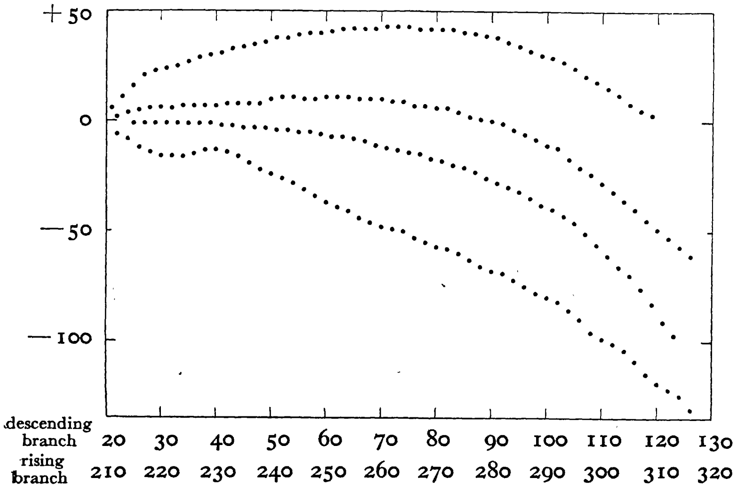
The constancy of the colour over the extreme 5½ percent of the sun's radius as mentioned above was confirmed by the measurements. It could further be shown that the disturbing clouds had influenced the sun's light in a selective way, the sun's light becoming slightly bluer when seen through the clouds.

The part of the eclipse curve from one minute before second contact to second contact has been disturbed least by the clouds. It has been analysed for the law of darkening on the assumption that the effect of the clouds could be neglected.

Though there is evidence that this view is correct the results should still be considered with some reserve. Approximative formulae for the eclipse function as a function of the geometric elements of the crescent and the law of darkening have been derived.

FIGURE 1.

Ordinates: numbers of exposures minus 15 × t (in seconds) + const.
Abscissae: t
The curves from top to bottom correspond to the films:
$\lambda_{\rm eff} = \mu{\cdot}567$, rising branch $\lambda_{\rm eff} = \mu{\cdot}454$, rising branch
,, = ·567, descending branch ,, = ·454, descending branch



Our results have been compared with an extrapolation towards the limb of a formula derived by HERTZSPRUNG.

2. *The method.*

For the photographic photometry use was made of amateur movie cameras. Six identical cameras of mark *Agfa movex* were carried on the expedition. Four of them have actually been used at the eclipse, while the remaining two were kept in reserve. The relative aperture used was 1 : 3½, the focal length was 20 mm. The focus could be adjusted so as to obtain sharp pictures of objects having distances larger than one metre. The film width was 16 mm. Single exposures were made on rectangular fields of size 7·57 × 10·42 mm. Successive fields touched by their longer sides. The apparatus could be operated only with a speed of about 15 exposures a second. The maximum capacity of the filmholder was 12 metres film. The rotating sector serving as a shutter had a free space extending over 150 degrees, so that the exposure time on a single exposure was fixed at about 1/36 of a second. The apparatus were driven by a spring motor. The complete film of 12 metres could be exposed in 2 minutes without rewinding the spring. During this interval the speed proved to be reasonably constant. In Figure 1 the rates of the four cameras during the eclipse observations are shown. Ordinates are number of exposures actually made, diminished by 15 × t (in seconds), with an arbitrary zeropoint. Each division in the abscissae corresponds to 10 seconds.

The speed at any moment is given by the tangent to the curve. A constant speed of 15 exposures a second would be represented by a straight horizontal line. Directions of tangents under ± 45° correspond to speeds of 15 ± 2½ exposures a second. The curves were obtained as a by-product, the time of each single exposure being known. Of the four cameras used two contained orthochromatic films and two ordinary photographic ones. The orthochromatic film was Gevaert safety negative film, the ordinary photographic film was Gevaert safety positive film. For both emulsions the relative spectral sensitivity as a function of wavelength has been published by the writer[1]).

In the two cases yellow filters of different spectral transmission have been used. The yellow filter used in combination with the orthochromatic film was of the kind OG 1 and consisted of two pieces having a combined thickness of 3·5 mm. The yellow filter used in combination with the positive film was 2 mm thick and of the kind GG 5. Both designations

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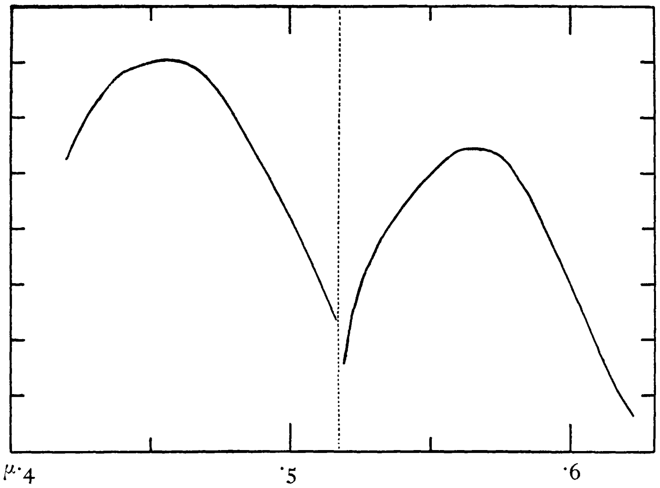
[1]) *B.A.N.* No. 294, 125 (1937).

OG 1 and GG 5 are according to Schott's list[1]). The relative sensitivity as a function of the wavelength of the combinations of photographic emulsions and filters just mentioned is shown in Figure 2 for both cases. The wavelengths of maximum sensitivity, which will be approximately equal to the effective wavelengths, are μ·454 and μ·567.

FIGURE 2.
Relative spectral sensitivity of the combinations of emulsions and filters used.
Abscissa: wavelength
Each division in the ordinates corresponds to one magnitude



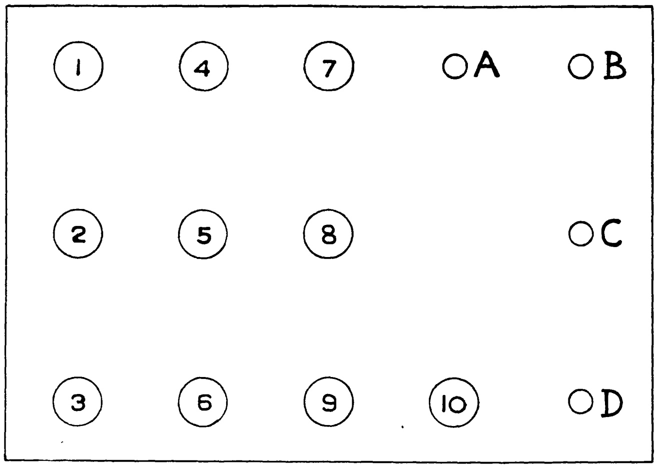
As a consequence of the narrowness of these spectral sensitivity curves the effective wavelengths are very well defined. They are practically independent of the colour of the light, which is of importance since the comparison lamps are considerably redder than the sun's crescent (about 1^m in the scale of colour-index as defined by the two wavelengths used, corresponding to $1^m{\cdot}5$ on the ordinary scale used in stellar photometry).

One pair of cameras was used before and another after totality in such a combination that simultaneous observations at the two different wavelengths were made.

The used pair was replaced by the pair by which the rising branch of the eclipse curve has been observed during the one and a half minute of total eclipse.

The problem of measuring a considerable range of intensities with a constant exposure time was solved in the following way. The movie cameras were to photograph the images of the sun reflected in a set of ten convex mirrors having different known curvatures and mounted on a board. Consequently a series of images of different but definite intensities was obtained in each exposure.

FIGURE 3.
Sketch showing arrangement of mirrors and lamps.

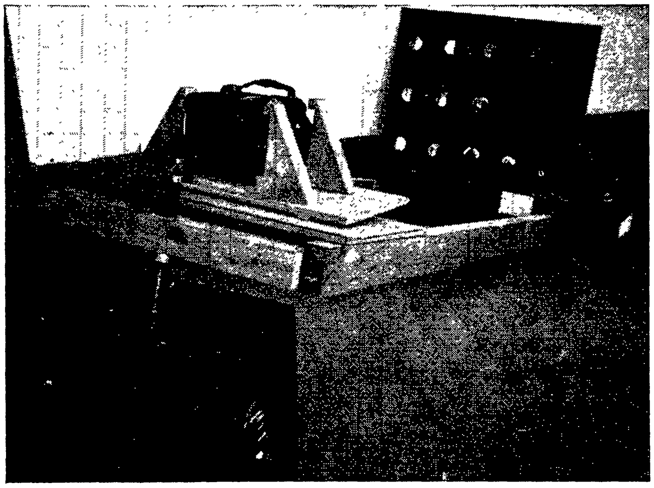


During the brightest phases nearly all reflection images were overexposed on the films. Only images reflected by the most curved mirrors then gave a measurable photographic density. At phases relatively near totality the light reflected by most of the mirrors did not make any photographic impression. Then only the flattest mirrors produced images of just measurable photographic density. At intermediate phases the images produced by some mirrors having an intermediate curvature could be measured, while the images from stronger and less curved mirrors were invisible, respectively overexposed.

The mirrors were ordinary spectacle glasses aluminized on one side and about 4 cm in diameter. Besides the ten mirrors four lamps were photographed on the films.

Figure 3 is a sketch of the arrangement of the mirrors (designated 1 to 10 in order of increasing curvature) and of the lamps (designated *A* to *D*).

FIGURE 4.
The apparatus.



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[1] *Jena colored optical filter glasses*, list 4892 E.

The lamps *B* and *C* were mounted parallel in a circuit with an accumulator (2 volt), a rheostat and an amperemeter; *C* was about $^{\rm m}{\cdot}5$ brighter than *B; B* and *C* served as comparison intensities, of which constancy was the chief claim. The time was fixed by means of the lamps *A* and *D*.

The board carrying the mirrors and the lamps was attached to a wooden frame in which the two cameras could be given a fixed position with respect to the mirrors.

Figure 4 is a photograph of the complete apparatus, showing mirrors, lamps and cameras. The whole apparatus could easily be adjusted in altitude by means of the big screw shown in the foreground of Figure 4.

For the adjustment in azimuth no special arrangement was found necessary. For the purpose of pointing the apparatus at the sun a rectangular piece of card-board was attached to the frame at the camera end; during the observations its shadow had to match a strip of white paper of the same dimensions at the bottom end of the board with the mirrors. The mirrors had been adjusted in such a way that in each of them a complete image of the crescent could be seen from the place of the camera lenses. To make this possible for both cameras simultaneously, one of them was placed upside down against the other; because of the eccentric position of the lens on the Agfa movex camera their lenses were then in contact, which made their relative parallax as small as possible. To prevent the light of the sun from being reflected in the glass-bulbs of the lamps, a screen was placed above each of them which threw its shadow on the lamp. It is clear that the presence of reflection images of the sun on the lamps *B* and *C* in particular, would be very disturbing since it would make the comparison intensities dependent on the intensity of the sun's light.

The closing and opening of the current through the time lamp *D* was effected by a relay in connection with a chronometer[1]), *D* flashed every sidereal second for half a second and was extinguished during the remaining half second. The beats of the chronometer that were recorded in this way on the films were moreover recorded on a chronograph roll.

According to a definite code Mr. Uitterdijk sent a short current through lamp *A*. These same signals were recorded on the chronograph roll. They were sufficiently numerous to guarantee the zeropoint of time, even if a camera had occasionally stopped. In this way for about every fifteenth exposure the time was secured. Subsequently the times of the remaining exposures were obtained by linear interpolation.

Mr. Uitterdijk started the counting of the seconds two minutes before the expected moment of second contact. This moment has been taken as zeropoint of the time *t* (in sidereal time), $t = 0$ corresponds to $3^{\rm h}56^{\rm m}57^{\rm s}{\cdot}7$ G.M.T. One hundred seconds before second contact ($t = 20$) the cameras were put into operation.

The last observations by the second set of cameras were made at $t = 314^{\rm s}$ or 98 seconds following third contact.

Between the first and last exposure 294 seconds or nearly five minutes had passed. Of this interval 90 seconds fell during totality, during which time no observations have been made.

The observations were on purpose out of focus. Since the range over which the focal distance could be changed was insufficient a negative lens of strength—3 had been placed in front of the camera lens; in this way the film was 1·2 mm inside the focus. The images were circular, having a diameter of ·34 mm at full aperture of the diaphragm. The distribution of the photographic density inside an image proved to be reasonably uniform. The window of the observing room acted as a diaphragm by which the influence of the sky light was reduced.

3. *The measures.*

The four films were developed in the laboratory of the Multifilm at Haarlem under the supervision of its director Mr. J. C. Mol. Before the development of the eclipse films, the developer had been tested by the development of parts of the films taken at the general repetition on June 18, the day before eclipse day. The developer was borax-methol-hydrochinon.

It was already mentioned in the introduction that after a crude examination of the material by means of eye estimates, it was decided to measure the material in the Schilt photometer.

Mr. Zunderman, chief instrument maker at the Leiden Observatory, constructed an apparatus by means of which a part of the film could be clamped between two glass plates; the measurements were then carried out in the same way as is usual on a stellar photograph[2]).

Fourteen exposures could be measured in a row, before a shift of the film became necessary. For control the last measured exposure of a row was measured a second time as the first one of the next row. The diaphragm used had a diameter of 6·06 mm corresponding to ·19 mm on the film. It was thus smaller than the size of an image, the latter being ·34 mm.

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[1]) Of mark Paul Ditisheim.

[2]) *B.A.N.* No. 10, 51 (1922). *Publ. Kapteyn Astr. Lab. Groningen,* No. 32 (1924).

Messrs. UITTERDIJK, FERWERDA, KOOREMAN and the writer took about equal shares in the measurements, the total number of settings being 30 000.

In order to avoid personal errors of measurement in the colour of the sun's light simultaneous observations on different films have been measured by the same person.

In order to discover such a personal error a number of exposures on the same film have been measured by different persons. However, no evidence of a personal error was found, so that in the discussion no distinction has been made with regard to the person who did the measuring (compare Table 5).

The great number of exposures allowed the measurers to be very critical. Images having the slightest defects have been rejected throughout.

4. *The reduction.*

The galvanometerreadings had now to be reduced to magnitude differences between the integrated light of the sun's crescent and the lamps.

The factor by which the integrated intensity of the sun's light is reduced at a point having a distance Δ from the surface of a mirror with radius R, is found as follows.

We neglect losses of light by reflection. Then it is known that the surface brightness of object and image are equal under all circumstances.

The total intensities of object and image are therefore proportional to their apparent areas, or to the squares of the angular dimensions.

For a point situated on the surface of a mirror the angular dimensions of object and image are equal and so are their total intensities.

The image of the sun's crescent is formed halfway between the centre and the surface of the mirror, hence:

ratio of angular dimensions of direct sun to image:

$$\frac{R/2+\Delta}{R/2} = 1 + 2\frac{\Delta}{R} \quad . \quad . \quad . \quad (1)$$

ratio of total intensities:

$$\left(1 + 2\frac{\Delta}{R}\right)^2 \quad . \quad . \quad . \quad . \quad . \quad (2)$$

or the reduction of the sun's brightness in magnitudes:

$$m' = 5 \log\left(1 + 2\frac{\Delta}{R}\right) \quad . \quad . \quad . \quad (3)$$

Numerical data in connection with the mirrors are collected in Table 1.

Mirror 10 has never been used. From the irregular behaviour of image 1 (from mirror 1) on the films taken after third contact it is concluded that by improper guiding only a part of the sun's crescent has been visible from the cameras occasionally. The results from mirror 1 have therefore been excluded altogether from the discussion.

TABLE 1.

mirror	radius R	distance from cameras Δ	from formula (3) m'	square of distance from axis δ^2	distance correction $F\delta^2$	$m'+F\delta^2$ m_c
	mm	cm	m		m	m
1	2050	106	1·54	21	·27	1·81
2	1035	104	2·39	14	·18	2·57
3	581·8	104	3·30	21	·27	3·57
4	383·6	104	4·05	10	·14	4·19
5	248·9	103	4·83	3	·04	4·87
6	150·0	103	5·83	10	·14	5·97
7	93·09	104	6·85	6	·08	6·93
8	62·94	102	7·62	0	·00	7·62
9	39·91	102	8·58	6	·08	8·66
10	27·73	102	9·35	10	·14	9·49

The reduction has been carried out separately for the four films. As a consequence of the unexpected steep gradations it was found difficult to derive the relations between galvanometerreadings and magnitudes from the eclipse material alone. Therefore the galvanometerreadings have been converted into provisional magnitudes by the aid of a normal table published in *B.A.N.* No. 318. Then differences in provisional magnitudes between images of successive brightness on the same exposure were formed.They showed a slight but distinct variation with the photographic density (e.g. the sum of the provisional magnitudes), whereas this difference would be exactly constant in the case of a linear relationship between provisional magnitudes of the table and normal magnitudes.

Table 2 shows this run in the case of mirrors 7 and 8 for the film at $\lambda = \mu{\cdot}454$ taken before second contact.

The values of the differences in magnitude in the third column have been obtained from those in the second column after division by the gradation. Nevertheless a linear relation between m and m_{pr}

TABLE 2.

Test of the table published in *B.A.N.* No. 318 for use in the reduction of the eclipse films in the case of measures of images 7 and 8 on the film of the descending branch at $\lambda_{eff} = \mu{\cdot}454$.

m_{pr} (8) + m_{pr} (7)	m_{pr} (8) — m_{pr} (7)	m (8) — m (7)
m	m	m
3·10	2·40	·77
3·71	2·37	·76
4·20	2·36	·76
4·65	2·33	·75
5·03	2·31	·74
5·41	2·31	·74
5·78	2·30	·74
6·09	2·29	·73
6·41	2·25	·72

TABLE 3.

mirror	δ^2	m'	preceding second contact				following third contact				O—C of least squares solutions for gradation G and the constant F determining the distance correction $F\delta^2$ in magnitudes			
			$\lambda = \mu{\cdot}454$		$\lambda = \mu{\cdot}567$		$\lambda = \mu{\cdot}454$		$\lambda = \mu{\cdot}567$		preceding second contact		following third contact	
			Δm_{pr}	m_{pr}	Δm_{pr}	m_{pr}	Δm_{pr}	m_{pr}	Δm_{pr}	m_{pr}	$\mu{\cdot}454$	$\mu{\cdot}567$	$\mu{\cdot}454$	$\mu{\cdot}567$
		m	m	m	m	m	m	m	m	m	m	m	m	m
2	14	2·39		·00		·00		·00		·00	— ·02	+ ·06	— ·08	+ ·01
			3·05		2·31		2·76		2·08					
3	21	3·30		3·05		2·31		2·76		2·08	·00	— ·01	+ ·02	— ·01
			1·89		1·25		1·93		1·20					
4	10	4·05		4·94		3·56		4·69		3·28	+ ·03	— ·02	— ·01	— ·01
			2·31		1·48		2·00		1·40					
5	3	4·83		7·25		5·04		6·69		4·68	·00	— ·05	·00	— ·01
			3·46		2·24		3·06		2·12					
6	10	5·83		10·71		7·28		9·75		6·80	— ·02	·00	+ ·11	+ ·03
			2·96		2·08		3·31		2·06					
7	6	6·85		13·67		9·36		13·06		8·86	+ ·03	— ·01	+ ·01	— ·01
			2·37		1·31		2·07							
8	0	7·62		16·04		10·67		15·13			— ·02	+ ·04	+ ·01	
							3·45							
9	6	8·58						18·58					— ·08	

has been assumed. The error introduced by this procedure is only small since extreme galvanometer-readings have been excluded from the discussion on purpose. The interval of provisional magnitudes used is from $^{\mathrm{m}}{\cdot}60$ to $4^{\mathrm{m}}{\cdot}00$ corresponding to the interval in galvanometerreading from 2·8 cm to 21·3 cm (fog reading 25 cm).

Table 3 gives the differences in provisional magnitudes, Δm_{pr}, between images from mirrors of successive curvature for each of the four films. For the sake of homogeneity and with regard to the slight dependence on the photographic density they refer to a definite value of the sum of the two m_{pr}[1]).

The values m_{pr} in this table were then derived from their first differences Δm_{pr}, where the m_{pr} of the image 2 has been put equal to zero.

On the assumption of a constant gradation for the different exposures on the same film the m_{pr} should be linear functions of the m' (4th column Table 1 and 3rd column Table 3) after a correction for the position of the mirror would have been applied. This distance correction is assumed to be proportional to the square of the distance δ from the optical axis. Equations of condition have been formed

$$\frac{1}{G}\, m_{\mathrm{pr}} - F\,\delta^2 + \text{const.} = m' \quad . \quad . \quad (4)$$

Here G stands for the gradation, F is the constant in the expression for the distance correction $F\delta^2$.

The results of the least squares solutions for the unknowns $1/G$ and F are given with their mean errors in Table 4. It is seen that the gradations for the films with the shorter and longer effective wavelengths are three, respectively two, times as large as that of the normal table. The gradation of the normal table is representative for an average stellar photograph.

The values of δ^2 that have been used in the computations are in the 5th column of Table 1 and in the 2nd column of Table 3.

The results for F derived from the different films are in satisfactory agreement. The mean value is

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[1]) This value has been $4^{\mathrm{m}}{\cdot}70$. The galvanometerreading corresponding to half this value is 12·5 cm, which is half the fog reading.

TABLE 4.

Results of least squares solutions for gradation G and constant F in the distance correction $F\delta^2$.

film	$1/G$	m.e.	F	m.e.
$\lambda_{\mathrm{eff}} = \mu{\cdot}454$, descending branch	·317	± ·005	+ 1·06	± ·41
„ = ·567, „ „	·472	± ·008	+ 1·52	± ·41
„ = ·454, rising „	·327	± ·004	+ 1·37	± ·37
„ = ·567, „ „	·496	± ·009	+ 1·32	± ·44

1·32 ± ·21 (m.e.) and has been adopted in the computation of the distance correction $F\delta^2$ given in column 6 of Table 1.

In column 7 of Table 1 the values m_c by which an image is fainter than the direct sun's image in the centre of the field are given, $m_c = m' + F\delta^2$.

Comparison lamp *B* proved to be rather faint on most exposures. It has therefore been excluded from the discussion and only *C* has been used throughout.

The definitive differences in magnitude *m* between the sun's crescent and lamp *C* were then calculated with the formula:

$$m = -m_c + \frac{1}{G}\left(m_{\text{pr}} - m_{\text{pr}}(C)\right) \quad . \quad . \quad (5)$$

In Table 5 the first column gives the number of the normal of ten exposures, the second column contains the eclipse time *t*, the third column gives *m*. In the fourth column the mirrors that have been used in the determination of *m* are given.

Normals with an asterisk have been derived from the same piece of film as the normals with the same number but without an asterisk, but they were measured by another person. The absence of a personal error is evident.

In Table 6 the results for every single exposure obtained during about 10 seconds near the contacts are given.

Figures 5 and 7 show the data collected in Table 5 graphically. Normals with an asterisk have not been put on the diagrams. Figures 6 and 8 show graphically the data contained in Table 6.

In all cases the upper curve corresponds to $\lambda_{\text{eff}} = \mu{\cdot}454$, whereas the lower curve represents the observations at $\lambda_{\text{eff}} = \mu{\cdot}567$.

At the bottom of Figures 5, 6, 7, and 8 the smooth curve represents the difference between the eclipse curves at the two wavelengths or the colour-index of the sun's crescent as a function of the time.

The uneclipsed sun has been found roughly 13^{m} and 12^{m} brighter than the lamp *C* respectively at $\lambda = \mu{\cdot}454$ and $\lambda = \mu{\cdot}567$. The observations have thus been started on the descending branch,

TABLE 5 *a*.

Observations at $\lambda_{\text{eff}} = \mu{\cdot}454$ preceding second contact. Each normal contains ten exposures.

No. of normal	*t*	brightness	mirrors used	No. of normal	*t*	brightness	mirrors used	No. of normal	*t*	brightness	mirrors used	No. of normal	*t*	brightness	mirrors used
	s	m			s	m			s	m			s	m	
1	21·8	−7·40	7, 8	37	46·5	−7·94	8	72	75·2	−7·80	7, 8	108	105·2	−6·25	6
2	22·6	−7·45	7	38	47·3	−7·98	8	73	76·0	−7·78	7, 8	109	106·1	−6·16	6
3	23·4	−7·49	7	39	48·0	−8·00	8	74	76·8	−7·76	7, 8	110	107·1	−6·05	6
4	24·2	−7·46	7, 8	39*	48·0	−8·02	8	75	77·8	−7·72	7, 8	111	108·1	−5·94	6
5	25·0	−7·49	7, 8	40	48·9	−8·05	8	76	78·5	−7·70	7, 8	112	108·9	−5·86	6
6	25·7	−7·48	7, 8	41	49·6	−8·05	8	77	79·3	−7·67	7, 8	102*	101·4	−6·56	6
7	26·4	−7·51	7, 8	42	50·3	−8·07	8	78	80·2	−7·66	7, 8	103*	102·0	−6·49	6
8	27·2	−7·54	7	43	51·0	−8·08	8	79	80·9	−7·62	7, 8	104*	102·8	−6·44	6
9	27·8	−7·53	7	44	51·8	−8·08	8	80	81·8	−7·58	7, 8	105*	103·6	−6·39	6
10	28·5	−7·52	7	45	52·7	−8·10	8	81	82·6	−7·57	7, 8	106*	104·3	−6·31	6
11	29·3	−7·53	7	46	53·5	−8·11	8	82	83·4	−7·55	7, 8	107*	105·0	−6·27	6
12	30·1	−7·50	7, 8	47	54·5	−8·12	8	83	84·2	−7·50	7, 8	108*	105·8	−6·19	6
13	30·7	−7·53	7, 8	48	55·4	−8·14	8	84	85·1	−7·48	7, 8	109*	106·6	−6·11	6
14	31·4	−7·49	7	49	56·5	−8·16	8	85	86·0	−7·44	7, 8	110*	107·4	−6·02	6
15	32·0	−7·50	7	50	57·6	−8·14	8	86	86·9	−7·43	7	111*	108·1	−5·92	6
16	32·7	−7·50	7	51	58·4	−8·14	8	87	87·7	−7·40	7	112*	108·9	−5·86	6
17	33·3	−7·53	7	52	59·3	−8·15	8	88	88·6	−7·34	7	113	109·6	−5·79	5, 6
18	33·9	−7·56	7	53	60·0	−8·13	8	89	89·4	−7·32	7	114	110·3	−5·71	5
19	34·5	−7·58	7	54	60·9	−8·11	8	90	90·2	−7·28	7	115	111·0	−5·61	5
20	35·1	−7·58	7	55	61·9	−8·12	8	91	90·9	−7·24	7	116	111·6	−5·52	5
21	35·7	−7·62	7	56	62·8	−8·09	8	92	91·7	−7·19	7	117	112·2	−5·43	5
22	36·3	−7·60	7, 8	57	63·6	−8·07	8	93	92·4	−7·18	7	118	112·9	−5·34	5
23	37·0	−7·64	7, 8	58	64·6	−8·04	8	94	93·3	−7·14	7	119	113·6	−5·22	5
24	37·5	−7·68	7, 8	59	65·6	−8·04	8	95	94·1	−7·06	7	120	114·4	−5·13	4, 5
25	38·1	−7·68	7, 8	60	66·5	−8·02	8	96	95·2	−6·99	7	121	115·1	−5·00	4, 5
26	38·8	−7·71	7, 8	61	67·2	−7·99	8	97	96·5	−6·90	6, 7	122	115·8	−4·83	4, 5
27	39·4	−7·73	7, 8	62	67·9	−7·98	8	98	97·3	−6·86	6, 7	123	116·7	−4·64	4
28	40·0	−7·78	7, 8	63	68·6	−7·95	8	99	98·0	−6·81	6, 7	124	117·5	−4·46	3, 4
29	40·6	−7·79	7, 8	64	69·4	−7·94	8	100	98·7	−6·78	6, 7	125	118·3	−4·22	3, 4
30	41·3	−7·83	7, 8	65	70·2	−7·92	8	101	99·4	−6·72	6, 7	126	119·1	−3·96	3, 4
31	42·0	−7·79	8	66	70·8	−7·91	8	102	100·1	−6·69	6, 7	127	119·8	−3·65	2, 3
32	42·7	−7·82	8	67	71·5	−7·88	8	103	101·1	−6·57	6	128	120·6	−3·25	2, 3
33	43·4	−7·83	8	68	72·3	−7·86	8	104	102·1	−6·50	6	129	121·3	−2·72	2
34	44·2	−7·84	8	69	73·0	−7·86	8	105	103·0	−6·43	6				
35	44·9	−7·85	8	70	73·7	−7·85	7, 8	106	103·8	−6·37	6				
36	45·7	−7·91	8	71	74·4	−7·83	7, 8	107	104·4	−6·30	6				

TABLE 5 *b*.

Observations at $\lambda_{\text{eff}} = {}^{\mu}{\cdot}567$ preceding second contact. Each normal contains ten exposures.

No. of normal	t	brightness	mirrors used	No. of normal	t	brightness	mirrors used
	s	m			s	m	
1	21·3	— 6·34	5, 6	64	68·3	— 7·11	6, 7
2	21·9	— 6·42	6	65	69·3	— 7·08	6, 7
3	22·6	— 6·46	6	66	70·2	— 7·06	6, 7
4	23·3	— 6·47	6	67	71·1	— 7·04	6, 7
5	24·0	— 6·49	6	68	72·2	— 7·02	6, 7
6	24·8	— 6·49	6	69	73·1	— 6·98	6, 7
7	25·6	— 6·50	6	70	74·1	— 6·92	6, 7
8	26·4	— 6·52	6	71	75·0	— 6·93	6, 7
9	27·1	— 6·52	6	72	75·8	— 6·84	6, 7
10	27·8	— 6·53	6	73	77·1	— 6·85	6, 7
11	28·5	— 6·54	6	74	78·5	— 6·84	6, 7
12	29·2	— 6·52	6	75	79·7	— 6·77	6, 7
13	29·9	— 6·51	6	76	80·9	— 6·72	6, 7
14	30·6	— 6·51	6	77	81·7	— 6·70	6, 7
15	31·2	— 6·57	6	78	82·8	— 6·66	6, 7
16	31·9	— 6·54	6	79	84·0	— 6·64	6, 7
17	32·7	— 6·53	6, 7	80	85·5	— 6·56	6, 7
18	33·4	— 6·54	6, 7	81	86·5	— 6·55	6
19	34·0	— 6·60	6, 7	82	87·9	— 6·50	6
20	34·7	— 6·60	6, 7	83	89·2	— 6·44	6
21	35·3	— 6·62	6, 7	84	90·5	— 6·39	6
22	36·0	— 6·66	6, 7	85	91·5	— 6·36	6
23	36·6	— 6·68	6, 7	86	92·4	— 6·28	6
24	37·2	— 6·71	6, 7	87	93·6	— 6·22	6
25	37·9	— 6·75	6, 7	88	94·4	— 6·18	6
26	38·5	— 6·78	6, 7	89	95·5	— 6·10	6
27	39·2	— 6·84	6, 7	90	96·3	— 6·04	5, 6
28	39·9	— 6·87	6, 7	91	97·2	— 5·96	5, 6
29	40·5	— 6·86	6, 7	92	98·2	— 5·92	5, 6
30	41·2	— 6·88	6, 7	93	99·4	— 5·82	5, 6
31	41·8	— 6·90	6, 7	94	100·3	— 5·78	5, 6
32	42·5	— 6·92	6, 7	95	101·5	— 5·70	5, 6
33	43·1	— 6·92	6, 7	96	102·4	— 5·60	5, 6
34	43·8	— 6·96	6, 7	97	103·4	— 5·54	5, 6
35	44·4	— 6·98	6, 7	98	104·7	— 5·42	5
36	45·0	— 7·00	6, 7	99	106·1	— 5·26	4, 5
37	45·7	— 7·01	6, 7	100	107·1	— 5·14	4, 5
38	46·3	— 7·06	6, 7	101	108·3	— 5·00	4, 5
39	47·0	— 7·08	6, 7	93*	102·4	— 5·61	5, 6
40	47·7	— 7·12	6, 7	94*	103·1	— 5·55	5, 6
41	48·4	— 7·14	6, 7	95*	103·9	— 5·51	5, 6
42	49·4	— 7·19	6, 7	96*	104·6	— 5·43	5, 6
43	50·0	— 7·20	6, 7	97*	105·4	— 5·34	5
44	51·1	— 7·26	8	98*	106·0	— 5·27	4, 5
45	52·1	— 7·22	7	99*	106·8	— 5·20	4, 5
46	53·1	— 7·26	7, 8	100*	107·5	— 5·08	4, 5
47	54·1	— 7·28	7, 8	101*	108·2	— 5·03	4, 5
48	55·5	— 7·30	6, 7, 8	102	108·9	— 4·96	4, 5
49	56·2	— 7·30	6, 7, 8	103	109·7	— 4·86	4, 5
50	56·8	— 7·30	6, 7, 8	104	110·5	— 4·76	4, 5
51	57·7	— 7·31	6, 7, 8	105	111·4	— 4·61	3, 4, 5
52	58·6	— 7·31	6, 7, 8	106	112·2	— 4·50	3, 4, 5
53	59·4	— 7·29	6, 7, 8	107	112·9	— 4·40	3, 4, 5
54	60·1	— 7·28	6, 7, 8	108	113·6	— 4·27	3, 4
55	60·8	— 7·28	6, 7, 8	109	114·3	— 4·15	3, 4
56	61·7	— 7·22	6, 7	110	115·0	— 4·02	3, 4
57	62·5	— 7·22	6, 7, 8	111	115·8	— 3·90	3, 4
58	63·4	— 7·24	7, 8	112	116·5	— 3·72	3, 4
59	64·2	— 7·23	7, 8	113	117·2	— 3·52	3, 4
60	65·1	— 7·22	7, 8	114	118·0	— 3·32	2, 3
61	65·8	— 7·19	7, 8	115	118·7	— 3·08	2, 3
62	66·6	— 7·16	6, 7, 8	116	119·4	— 2·83	2
63	67·4	— 7·12	6, 7	117	120·1	— 2·51	2

TABLE 5 *c*.

Observations at $\lambda_{\text{eff}} = {}^{\mu}{\cdot}454$ following third contact. Each normal contains ten exposures.

No. of normal	t	brightness	mirrors used	No. of normal	t	brightness	mirrors used
	s	m			s	m	
1	220·4	— 2·47	2	61*	260·5	— 7·04	7
2	221·0	— 2·88	2, 3	62*	261·3	— 7·06	7
3	221·6	— 3·26	2, 3	63*	262·0	— 7·05	7
4	222·3	— 3·65	3	64*	262·7	— 7·02	7
5	223·1	— 3·92	3, 4	65*	263·5	— 7·01	7
6	223·7	— 4·12	3, 4	66*	264·3	— 6·99	7
7	224·4	— 4·24	4	67*	265·2	— 6·96	7
8	225·1	— 4·43	4, 5	68*	265·9	— 6·92	7
9	225·7	— 4·52	4, 5	69*	266·9	— 6·93	7
10	226·3	— 4·62	4, 5	70	267·8	— 6·92	7
11	227·0	— 4·68	4, 5	71	268·6	— 6·88	7
12	227·7	— 4·79	4, 5	72	269·3	— 6·87	7
13	228·3	— 4·86	5	73	270·0	— 6·90	7
14	228·9	— 4·93	5	74	270·7	— 6·93	7
15	229·6	— 5·00	5	75	271·4	— 6·94	7
16	230·2	— 5·05	5	76	272·1	— 6·93	7
17	230·9	— 5·11	5	77	272·9	— 6·91	7
18	231·5	— 5·14	5	78	273·6	— 6·90	7
19	232·1	— 5·16	5	79	274·3	— 6·88	7
20	232·8	— 5·22	5	80	275·0	— 6·85	7
21	233·5	— 5·28	5	81	275·8	— 6·85	7
22	234·3	— 5·36	5	82	276·4	— 6·85	7
23	235·0	— 5·45	5, 6	83	277·2	— 6·86	7
24	235·6	— 5·51	6	84	278·0	— 6·88	7
25	236·3	— 5·63	6	85	278·8	— 6·89	7
26	237·0	— 5·67	6	86	279·5	— 6·91	7
27	237·6	— 5·76	6	87	280·0	— 6·95	7
28	238·2	— 5·84	6	88	280·8	— 7·00	7, 8
29	238·8	— 5·90	6	89	281·5	— 7·02	7, 8
30	239·5	— 5·91	6	90	282·2	— 7·08	7, 8
31	240·2	— 6·01	6	91	283·0	— 7·12	7, 8
32	240·8	— 6·04	6	92	283·7	— 7·12	7, 8
33	241·4	— 6·04	6	93	284·5	— 7·16	7, 8
34	242·1	— 6·06	6	94	285·2	— 7·22	7, 8
35	242·7	— 6·07	6	95	285·9	— 7·26	7, 8
36	243·3	— 6·04	6	96	286·6	— 7·28	7, 8
37	243·9	— 6·02	6	97	287·4	— 7·29	7, 8
38	244·6	— 6·00	6	98	288·3	— 7·31	7, 8
39	245·3	— 5·93	6	99	289·0	— 7·33	7, 8
40	246·1	— 5·89	6	100	289·7	— 7·33	7, 8
41	246·8	— 5·86	6	101	290·4	— 7·34	7, 8
42	247·6	— 5·87	6	102	291·1	— 7·35	7, 8
43	248·3	— 5·87	6	103	291·8	— 7·40	7, 8
44	249·0	— 5·89	6	104	292·6	— 7·46	7, 8
45	249·7	— 5·95	6	105	293·4	— 7·49	7, 8
46	250·4	— 6·06	6	106	294·2	— 7·53	8
47	251·1	— 6·12	6	107	294·8	— 7·60	8
48	251·7	— 6·26	6	108	295·6	— 7·64	8
49	252·5	— 6·49	6	109	296·4	— 7·65	8
50	253·3	— 6·44	7	110	297·2	— 7·61	8
51	254·0	— 6·59	7	111	298·0	— 7·53	8
52	254·7	— 6·71	7	112	298·9	— 7·50	8
53	255·4	— 6·83	7	113	299·7	— 7·48	8
54	256·1	— 6·93	7	114	300·5	— 7·48	8
55	256·8	— 6·98	7	115	301·2	— 7·49	8
56	257·5	— 6·99	7	116	302·0	— 7·49	8
57	258·2	— 6·97	7	117	302·9	— 7·50	7, 8
58	258·9	— 6·99	7	118	303·6	— 7·54	8
59	259·7	— 6·99	7	119	304·4	— 7·54	8
60	260·3	— 7·04	7	120	305·1	— 7·56	8
61	261·0	— 7·06	7	121	306·0	— 7·66	8
62	261·7	— 7·05	7	122	306·9	— 7·80	8
63	262·3	— 7·03	7	123	307·6	— 7·94	8
64	263·0	— 7·01	7	124	308·5	— 7·93	9
65	263·8	— 7·00	7	125	309·5	— 7·99	9
66	264·5	— 6·99	7	126	310·3	— 8·08	9
67	265·2	— 6·96	7	127	311·1	— 8·12	9
68	265·8	— 6·93	7	128	312·1	— 8·18	9
69	266·5	— 6·90	7	129	313·0	— 8·27	9

Table 5 *d*.

Observations at $\lambda_{\text{eff}} = \mu{\cdot}567$ following third contact.
Each normal contains ten exposures.

No. of normal	t	bright-ness	mirrors used	No. of normal	t	bright-ness	mirrors used
	s	m			s	m	
1	222·0	— 2·49	2	59*	261·8	— 6·12	5, 6
2	222·6	— 2·75	2	60*	262·5	— 6·07	5, 6
3	223·3	— 2·97	2	61*	263·2	— 6·06	5, 6
4	223·9	— 3·17	2	62*	264·0	— 6·03	5, 6
5	224·6	— 3·37	2	63*	264·7	— 6·00	5, 6
6	225·3	— 3·44	2, 3	64*	265·4	— 5·97	5, 6
7	225·9	— 3·54	2, 3	65*	266·0	— 5·94	5, 6
8	226·5	— 3·64	2, 3	66*	266·8	— 5·94	5, 6
9	227·1	— 3·74	2, 3	67	267·4	— 5·91	5, 6
10	228·0	— 3·82	2, 3	68	268·1	— 5·86	5, 6
11	228·7	— 3·88	2, 3	69	268·8	— 5·83	5, 6
12	229·4	— 3·95	2, 3	70	269·4	— 5·84	5, 6
13	230·2	— 4·00	2, 3, 4	71	270·0	— 5·86	5, 6
14	230·8	— 4·04	2, 3, 4	72	270·6	— 5·90	5, 6
15	231·5	— 4·10	3, 4	73	271·2	— 5·92	5, 6
16	232·1	— 4·10	3, 4	74	271·8	— 5·92	5, 6
17	232·7	— 4·14	3, 4	75	272·5	— 5·90	5, 6
18	233·3	— 4·19	3, 4	76	273·2	— 5·88	5, 6
19	234·0	— 4·28	3, 4	77	273·8	— 5·85	5, 6
20	234·8	— 4·40	3, 4	78	274·5	— 5·82	5, 6
21	235·5	— 4·51	3, 4	79	275·2	— 5·79	5, 6
22	236·2	— 4·57	3, 4, 5	80	275·9	— 5·79	5, 6
23	237·0	— 4·67	3, 4, 5	81	276·5	— 5·78	5, 6
24	237·9	— 4·77	3, 4, 5	82	277·2	— 5·79	5, 6
25	238·6	— 4·81	3, 4, 5	83	277·9	— 5·84	5, 6
26	239·3	— 4·84	3, 4, 5	84	278·6	— 5·85	5, 6
27	240·0	— 4·92	3, 4, 5	85	279·2	— 5·83	5, 6
28	240·7	— 4·93	3, 4, 5	86	279·9	— 5·84	5, 6
29	241·3	— 4·93	3, 4, 5	87	280·6	— 5·88	5, 6
30	242·1	— 4·93	3, 4, 5	88	281·3	— 5·92	5, 6
31	242·9	— 4·92	3, 4, 5	89	282·0	— 5·97	5, 6
32	243·5	— 4·89	3, 4, 5	90	282·7	— 6·02	5, 6
33	244·2	— 4·86	3, 4, 5	91	283·4	— 6·04	5, 6
34	244·8	— 4·82	3, 4, 5	92	284·1	— 6·06	5, 6
35	245·4	— 4·75	3, 4, 5	93	284·8	— 6·16	6
36	246·1	— 4·71	3, 4, 5	94	285·5	— 6·20	6
37	246·7	— 4·68	3, 4, 5	95	286·2	— 6·26	6
38	247·3	— 4·68	3, 4, 5	96	286·9	— 6·28	6
39	248·1	— 4·65	3, 4, 5	97	287·6	— 6·29	6
40	248·9	— 4·65	3, 4, 5	98	288·3	— 6·31	6
41	249·4	— 4·67	3, 4, 5	99	289·0	— 6·30	6
42	250·0	— 4·73	3, 4, 5	100	289·8	— 6·31	6
43	250·7	— 4·80	3, 4, 5	101	290·4	— 6·31	6
44	251·3	— 4·87	3, 4, 5	102	291·2	— 6·32	6
45	251·8	— 5·02	3, 4, 5	103	291·8	— 6·37	6
46	252·5	— 5·24	4, 5	104	292·5	— 6·42	6
47	253·1	— 5·38	4, 5	105	293·2	— 6·47	6
48	253·7	— 5·55	4, 5	106	294·0	— 6·50	6
49	254·5	— 5·74	5, 6	107	294·8	— 6·53	6, 7
50	255·3	— 5·88	5, 6	108	295·7	— 6·60	6, 7
51	256·2	— 6·00	5, 6	109	296·5	— 6·58	6, 7
52	256·8	— 6·05	5, 6	110	297·2	— 6·56	6, 7
53	257·4	— 6·06	5, 6	111	298·0	— 6·47	6, 7
54	258·2	— 6·06	5, 6	112	298·7	— 6·42	6, 7
55	258·8	— 6·04	5, 6	113	299·6	— 6·39	6, 7
56	259·5	— 6·04	5, 6	114	300·5	— 6·39	6, 7
57	260·2	— 6·09	5, 6	115	301·2	— 6·37	6, 7
58	260·9	— 6·11	5, 6	116	302·0	— 6·37	6, 7
59	261·7	— 6·10	5, 6	117	302·7	— 6·35	6, 7
60	262·4	— 6·07	5, 6	118	303·4	— 6·38	6, 7
61	263·2	— 6·04	5, 6	119	304·3	— 6·38	6, 7
62	264·0	— 6·01	5, 6	120	305·0	— 6·40	6, 7
63	264·8	— 6·00	5, 6	121	305·7	— 6·50	6, 7
64	265·5	— 5·96	5, 6	122	306·4	— 6·62	6, 7
65	266·2	— 5·94	5, 6	123	307·2	— 6·76	6, 7
66	266·9	— 5·94	5, 6	124	307·9	— 6·84	6, 7
				125	308·5	— 6·92	6, 7
58*	261·2	— 6·12	5, 6	126	309·2	— 6·98	6, 7

Table 6 *a*.

Single observations at $\lambda_{\text{eff}} = \mu{\cdot}454$.
Some seconds before second contact.

t	bright-ness	mirrors used	t	bright-ness	mirrors used
s	m		s	m	
111·29	— 5·55	5	116·67	— 4·66	4, 5
·36	— 5·55	5	·75	— 4·64	4, 5
·43	— 5·50	5	·83	— 4·61	4, 5
·50	— 5·55	5	·92	— 4·59	4, 5
·57	— 5·55	5	117·00	— 4·57	4
·64	— 5·51	5	·08	— 4·57	4
·71	— 5·50	5	·17	— 4·53	4
·79	— 5·54	5	·25	— 4·55	4
·86	— 5·49	5	·36	— 4·47	4
·93	— 5·50	5	·42	— 4·48	3, 4
112·00	— 5·48	5	·50	— 4·48	3, 4
·07	— 5·44	5	·58	— 4·47	3, 4
·14	— 5·43	5	·67	— 4·41	3, 4
·21	— 5·44	5	·75	— 4·40	3, 4
·29	— 5·42	5	·83	— 4·38	3, 4
·36	— 5·33	5	·92	— 4·38	3, 4
·43	— 5·38	5	118·00	— 4·31	3, 4
·50	— 5·47	5	·15	— 4·24	3, 4
·57	— 5·37	5	·23	— 4·25	3, 4
·64	— 5·39	5	·31	— 4·22	3, 4
·71	— 5·37	5	·38	— 4·21	3, 4
·79	— 5·38	5	·46	— 4·14	3, 4
·86	— 5·33	5	·54	— 4·16	3, 4
·93	— 5·29	5	·62	— 4·16	3, 4
113·00	— 5·35	5	·69	— 4·12	3, 4
·08	— 5·34	5	·77	— 4·09	3, 4
·15	— 5·29	5	·85	— 4·04	3, 4
·23	— 5·28	5	·92	— 4·02	3, 4
·31	— 5·28	5	119·08	— 3·96	3
·38	— 5·25	5	·15	— 3·89	3
·46	— 5·26	5	·23	— 3·87	3
·54	— 5·25	5	·31	— 3·84	3
·62	— 5·16	5	·38	— 3·85	3
·69	— 5·20	5	·46	— 3·91	3
·77	— 5·21	5	·54	— 3·78	3
·85	— 5·16	5	·62	— 3·77	3
·92	— 5·27	5	·69	— 3·70	3
114·00	— 5·21	5	·77	— 3·63	3
·08	— 5·16	5	·85	— 3·65	2, 3
·17	— 5·16	4, 5	·92	— 3·63	2, 3
·25	— 5·19	4, 5	120·00	— 3·57	2, 3
·33	— 5·14	4, 5	·14	— 3·56	2, 3
·42	— 5·05	5	·21	— 3·45	2, 3
·50	— 5·10	4, 5	·29	— 3·37	2
·58	— 5·06	4, 5	·36	— 3·38	2
·67	— 5·08	4, 5	·43	— 3·40	2
·75	— 5·06	4, 5	·50	— 3·31	2
·83	— 5·06	4, 5	·57	— 3·26	2
·92	— 5·06	4, 5	·64	— 3·23	2
115·00	— 5·00	4, 5	·71	— 3·17	2
·08	— 4·98	4, 5	·79	— 3·09	2
·15	— 4·99	4, 5	·87	— 3·07	2
·23	— 4·98	4, 5	·93	— 3·01	2
·31	— 4·98	4, 5	121·00	— 2·96	2
·38	— 4·94	4, 5	·08	— 2·91	2
·46	— 4·90	4, 5	·15	— 2·83	2
·54	— 4·92	4, 5	·23	— 2·84	2
·62	— 4·88	4, 5	·31	— 2·76	2
·69	— 4·90	4, 5	·38	— 2·67	2
·77	— 4·84	4, 5	·46	— 2·62	2
·85	— 4·81	4, 5	·54	— 2·51	2
·92	— 4·78	4, 5	·62	— 2·44	2
116·00	— 4·82	4, 5	·69	— 2·40	2
·17	— 4·76	4, 5	·77	— 2·36	2
·25	— 4·72	4, 5	·85	— 2·31	2
·33	— 4·71	4, 5	·92	— 2·26	2
·42	— 4·73	4, 5			
·58	— 4·69	4, 5			

TABLE 6 *b*.

Single observations at $\lambda_{\rm eff} = {}^{\mu}\cdot 567$.
Some seconds before second contact.

t	bright-ness	mirrors used	*t*	bright-ness	mirrors used
s	m		s	m	
113·00	— 4·40	3, 4, 5	117·08	— 3·56	3, 4
·08	— 4·38	3, 4, 5	·17	— 3·54	3, 4
·15	— 4·34	3, 4, 5	·25	— 3·50	3, 4
·23	— 4·38	3, 4	·33	— 3·48	3, 4
·31	— 4·29	3, 4	·42	— 3·46	3
·38	— 4·35	3, 4	·50	— 3·44	2, 3
·46	— 4·30	3, 4	·58	— 3·35	3
·54	— 4·36	3, 4	·67	— 3·46	2, 3
·62	— 4·30	3, 4	·75	— 3·39	2, 3
·69	— 4·24	3, 4	·83	— 3·38	2, 3
·77	— 4·26	3, 4	·92	— 3·36	2, 3
·85	— 4·26	3, 4	118·00	— 3·29	2, 3
·92	— 4·18	3, 4	·08	— 3·29	2, 3
114·00	— 4·20	3, 4	·15	— 3·27	2, 3
·08	— 4·21	3, 4	·23	— 3·22	2, 3
·15	— 4·20	3, 4	·31	— 3·19	2, 3
·23	— 4·16	3, 4	·38	— 3·16	2, 3
·31	— 4·15	3, 4	·46	— 3·18	2, 3
·38	— 4·16	3, 4	·54	— 3·16	2, 3
·46	— 4·15	3, 4	·62	— 3·15	2, 3
·54	— 4·09	3, 4	·69	— 3·12	2, 3
·62	— 4·10	3, 4	·77	— 3·05	2, 3
·69	— 4·10	3, 4	·85	— 3·04	2, 3
·77	— 4·08	3, 4	·92	— 2·96	2, 3
·85	— 4·09	3, 4	119·00	— 2·96	2, 3
·92	— 4·03	3, 4	·08	— 2·99	2
115·00	— 4·02	3, 4	·15	— 2·95	2
·08	— 4·00	3, 4	·23	— 2·94	2
·15	— 3·98	3, 4	·31	— 2·90	2
·23	— 4·00	3, 4	·38	— 2·85	2
·31	— 3·97	3, 4	·46	— 2·85	2
·38	— 3·96	3, 4	·54	— 2·81	2
·46	— 3·94	3, 4	·62	— 2·78	2
·54	— 3·94	3, 4	·69	— 2·70	2
·62	— 4·00	3, 4	·77	— 2·62	2
·69	— 3·98	3, 4	·85	— 2·65	2
·77	— 3·88	3, 4	·92	— 2·65	2
·85	— 3·84	3, 4	120·00	— 2·56	2
·92	— 3·83	3, 4	·08	— 2·54	2
116·00	— 3·86	3, 4	·15	— 2·51	2
·08	— 3·86	3	·23	— 2·41	2
·15	— 3·84	3, 4	·31	— 2·44	2
·23	— 3·80	3, 4	·38	— 2·32	2
·31	— 3·74	3, 4	·46	— 2·31	2
·38	— 3·72	3, 4	·54	— 2·22	2
·46	— 3·72	3, 4	·62	— 2·19	2
·54	— 3·72	3, 4	·69	— 2·14	2
·62	— 3·70	3, 4	·77	— 2·18	2
·69	— 3·66	3, 4	·85	— 2·08	2
·77	— 3·63	3, 4	·92	— 2·00	2
·85	— 3·63	3, 4	121·08	— 1·86	2
·92	— 3·62	3, 4	·15	— 1·87	2
117·00	— 3·58	3, 4	·23	— 1·78	2

TABLE 6 *c*.

Single observations at $\lambda_{\rm eff} = {}^{\mu}\cdot 454$.
Some seconds following third contact.

t	bright-ness	mirrors used	*t*	bright-ness	mirrors used
s	m		s	m	
219·80	— 1·95	2	220·20	— 2·33	2
·87	— 2·06	2	·27	— 2·39	2
·93	— 2·18	2	·33	— 2·47	2
220·00	— 2·22	2	·40	— 2·47	2
·07	— 2·24	2	·47	— 2·58	2
·13	— 2·31	2	·53	— 2·60	2

TABLE 6 *c* (*continued*).

t	bright-ness	mirrors used	*t*	bright-ness	mirrors used
s	m		s	m	
220·60	— 2·66	2	225·40	— 4·42	4, 5
·67	— 2·69	2	·47	— 4·48	4, 5
·73	— 2·77	2	·53	— 4·46	4, 5
·80	— 2·78	2	·60	— 4·52	4, 5
·87	— 2·82	2	·67	— 4·52	4, 5
·93	— 2·86	2	·73	— 4·54	4, 5
221·00	— 2·87	2	·80	— 4·55	4, 5
·07	— 3·00	2, 3	·87	— 4·56	4, 5
·13	— 3·02	2, 3	·93	— 4·54	4, 5
·20	— 3·02	2, 3	226·00	— 4·54	4, 5
·27	— 3·10	2, 3	·07	— 4·57	4, 5
·33	— 3·19	2, 3	·13	— 4·60	4, 5
·40	— 3·18	2, 3	·20	— 4·62	4, 5
·47	— 3·22	2, 3	·27	— 4·59	4, 5
·53	— 3·23	2, 3	·33	— 4·62	4, 5
·60	— 3·29	3	·40	— 4·62	4, 5
·67	— 3·34	3	·47	— 4·60	4, 5
·73	— 3·38	3	·53	— 4·64	4, 5
·80	— 3·46	3	·60	— 4·67	4, 5
·93	— 3·48	3	·67	— 4·63	4, 5
222·07	— 3·55	3	·73	— 4·66	4, 5
·13	— 3·57	3	·80	— 4·66	4, 5
·20	— 3·64	3	·87	— 4·68	4, 5
·27	— 3·62	3	·93	— 4·68	4, 5
·33	— 3·63	3	227·00	— 4·66	4, 5
·40	— 3·66	3	·07	— 4·68	4, 5
·47	— 3·74	3	·13	— 4·68	4, 5
·53	— 3·73	3	·20	— 4·71	4, 5
·60	— 3·80	3, 4	·27	— 4·72	4, 5
·67	— 3·82	3, 4	·40	— 4·76	4, 5
·73	— 3·81	3, 4	·47	— 4·78	4, 5
·80	— 3·81	3, 4	·53	— 4·75	4, 5
·87	— 3·88	3, 4	·60	— 4·77	4, 5
·93	— 3·88	3, 4	·67	— 4·75	4, 5
223·07	— 3·96	3, 4	·73	— 4·78	4, 5
·13	— 3·92	3, 4	·80	— 4·82	4, 5
·20	— 3·96	3, 4	·87	— 4·82	4, 5
·27	— 4·00	3, 4	·93	— 4·83	5
·33	— 4·04	3, 4	228·00	— 4·82	5
·40	— 4·02	3, 4	·07	— 4·81	5
·47	— 4·05	3, 4	·13	— 4·82	5
·53	— 4·09	3, 4	·20	— 4·84	5
·60	— 4·10	3, 4	·27	— 4·85	5
·67	— 4·10	3, 4	·33	— 4·86	5
·73	— 4·12	3, 4	·40	— 4·84	5
·80	— 4·12	3, 4	·47	— 4·86	5
·87	— 4·13	3, 4	·53	— 4·86	5
·93	— 4·16	3, 4	·60	— 4·81	5
224·00	— 4·16	3, 4	·67	— 4·84	5
·07	— 4·21	4	·73	— 4·83	5
·13	— 4·21	4	·80	— 4·92	5
·20	— 4·22	4	·87	— 4·95	5
·27	— 4·21	4	·93	— 4·94	5
·33	— 4·18	4	229·00	— 4·95	5
·40	— 4·23	4	·07	— 4·93	5
·47	— 4·27	4	·14	— 4·91	5
·53	— 4·28	4	·21	— 4·93	5
·60	— 4·32	4	·29	— 4·97	5
·67	— 4·32	4	·36	— 4·99	5
·80	— 4·38	4	·43	— 4·98	5
·87	— 4·38	4, 5	·50	— 5·02	5
·93	— 4·39	4, 5	·57	— 5·00	5
225·07	— 4·44	4, 5	·64	— 4·97	5
·13	— 4·44	4, 5	·71	— 5·01	5
·20	— 4·48	4, 5	·79	— 5·01	5
·27	— 4·48	4, 5	·86	— 5·00	5
·33	— 4·50	4, 5	·93	— 5·02	5

TABLE 6 *d*.

Single observations at $\lambda_{\text{eff}} = \mu \cdot 567$.

Some seconds following third contact.

t	bright-ness	mirrors used	t	bright-ness	mirrors used
s	m		s	m	
221·25	— 2·11	2	225·44	— 3·55	2, 3
·31	— 2·23	2	·50	— 3·45	2, 3
·38	— 2·18	2	·56	— 3·49	2, 3
·44	— 2·20	2	·62	— 3·52	2, 3
·50	— 2·23	2	·75	— 3·46	2, 3
·56	— 2·23	2	·81	— 3·56	2, 3
·62	— 2·27	2	·88	— 3·56	2, 3
·69	— 2·33	2	·94	— 3·58	2, 3
·75	— 2·37	2	226·00	— 3·58	2, 3
·81	— 2·45	2	·06	— 3·56	2, 3
·88	— 2·40	2	·12	— 3·54	2, 3
·94	— 2·51	2	·19	— 3·54	2, 3
222·00	— 2·52	2	·25	— 3·60	2, 3
·07	— 2·57	2	·31	— 3·62	2, 3
·13	— 2·55	2	·38	— 3·65	2, 3
·20	— 2·62	2	·44	— 3·68	2, 3
·27	— 2·61	2	·50	— 3·66	2, 3
·33	— 2·64	2	·56	— 3·64	2, 3
·40	— 2·65	2	·62	— 3·63	2, 3
·47	— 2·70	2	·69	— 3·63	2, 3
·53	— 2·72	2	·75	— 3·68	2, 3
·60	— 2·72	2	·81	— 3·70	2, 3
·67	— 2·73	2	·88	— 3·68	2, 3
·73	— 2·80	2	·94	— 3·71	2, 3
·80	— 2·83	2	227·00	— 3·72	2, 3
·87	— 2·90	2	·12	— 3·71	2, 3
·93	— 2·86	2	·19	— 3·76	2, 3
223·00	— 2·89	2	·25	— 3·74	2, 3
·06	— 2·92	2	·38	— 3·77	2, 3
·12	— 2·90	2	·44	— 3·83	2, 3
·19	— 2·93	2	·62	— 3·80	2, 3
·25	— 2·90	2	·69	— 3·80	2, 3
·31	— 2·98	2	·75	— 3·77	2, 3
·38	— 3·07	2	·81	— 3·80	2, 3
·44	— 3·10	2	·94	— 3·84	2, 3
·50	— 3·02	2	228·00	— 3·86	2, 3
·56	— 3·13	2	·07	— 3·80	2, 3
·62	— 3·09	2	·20	— 3·86	2, 3
·69	— 3·12	2	·27	— 3·80	2, 3
·75	— 3·12	2	·33	— 3·84	2, 3
·88	— 3·17	2	·40	— 3·82	2, 3
·94	— 3·19	2	·53	— 3·90	2, 3
224·00	— 3·20	2	·67	— 3·82	2, 3
·06	— 3·27	2	·73	— 3·85	2, 3
·12	— 3·18	2	·80	— 3·88	2, 3
·19	— 3·23	2	·87	— 3·94	2, 3
·25	— 3·26	2	·93	— 3·91	2, 3
·44	— 3·38	2	229·00	— 3·91	2, 3
·50	— 3·34	2	·06	— 3·96	2, 3
·56	— 3·38	2	·12	— 3·92	2, 3
·69	— 3·36	2	·19	— 3·92	2, 3
·81	— 3·46	2	·25	— 3·94	2, 3
·88	— 3·44	2	·31	— 3·96	2, 3
·94	— 3·40	2	·38	— 3·96	2, 3
225·00	— 3·43	2	·44	— 3·98	2, 3
·06	— 3·38	2, 3	·50	— 3·96	2, 3
·12	— 3·40	2, 3	·56	— 3·96	2, 3
·19	— 3·42	2, 3	·62	— 3·95	2, 3, 4
·25	— 3·44	2, 3	·69	— 3·99	2, 3
·31	— 3·48	2, 3	·75	— 4·00	2, 3
·38	— 3·48	2, 3	·81	— 3·99	2, 3, 4

respectively finished on the rising branch, when the brightness of the crescent was only one percent of the brightness of the uneclipsed sun.

On the assumption that the brightness and the colour of the corona and the full moon are equal we find the total light of the corona ($14\frac{1}{2}^{m}$ fainter than the sun) $1\frac{1}{2}^{m}$ and $2\frac{1}{2}^{m}$ fainter than lamp *C* at the short, respectively long wavelength. This is about 4^{m} fainter than the faintest observation made at the eclipse[1]).

The diagrams in Figures 5, 6, 7, and 8 representing the colour-index as a function of the time show the absence of any systematic change in colour along the extreme $5\frac{1}{2}$ percent of the radius at the limb in spite of the presence of clouds.

5. *Formulae.*

The relation between the law of darkening and the eclipse function has been considered by several authors[2]).

We shall investigate this relation with a law of darkening of the form

$$\sigma = \text{const.} \times z^{\alpha} \quad . \quad . \quad . \quad . \qquad (6)$$

where σ is the surface brightness, z is the distance on the disc from the limb, α is a constant defining the law of darkening. There is a family of theoretical eclipse functions having α as a parameter. The determination of α follows from the comparison of these theoretical eclipse functions with the observed one.

Formula (6) includes as a special case $\alpha = 0$ (constant surface brightness) and $\alpha = 1$ (σ proportional to z). Both cases have been considered by MINNAERT.

In Figure 9 the centres of sun and moon are called S and M, the radii r and $r + \rho$, the distance between the centres $c =$ SM. ρ is small in comparison with r and so is c near totality. The angle PSO is φ, the extremities of the crescent correspond with an angle $\varphi_{*} = \text{arc cos } \rho/c$.

When P moves along the limb of the sun's disc the area of the infinitesimal rectangle with sides PB and $r\,d\varphi$ is proportional to PB. The mean surface brightness varies as PB^{α} and the intensity is proportional to $\text{PB}^{1+\alpha}$.

---

[1]) The comparison of the brightness of the lamp *C* with that of the uneclipsed sun was made after the return at Leiden. For this purpose the sun's light was reduced in intensity by two successive reflections on two convex mirrors. The reduction obtained in this way was about 12 magnitudes. The determination must be considered rather rough but gives an idea as to the brightness of the uneclipsed sun and of the corona as compared with the intensities that have been measured at the eclipse.

[2]) BEMPORAD, *Mem. Spettr. Ital.* **35**, 89 (1906). MINNAERT, *Monthly Notices* **89**, 197 (1928). HECKMANN und SIEDENTOPF, *Veröff. Sternw. Göttingen*, Heft 8 (1929). SCHOENBERG, *Hdb. der Astrophysik* Bd. II/1, 28.

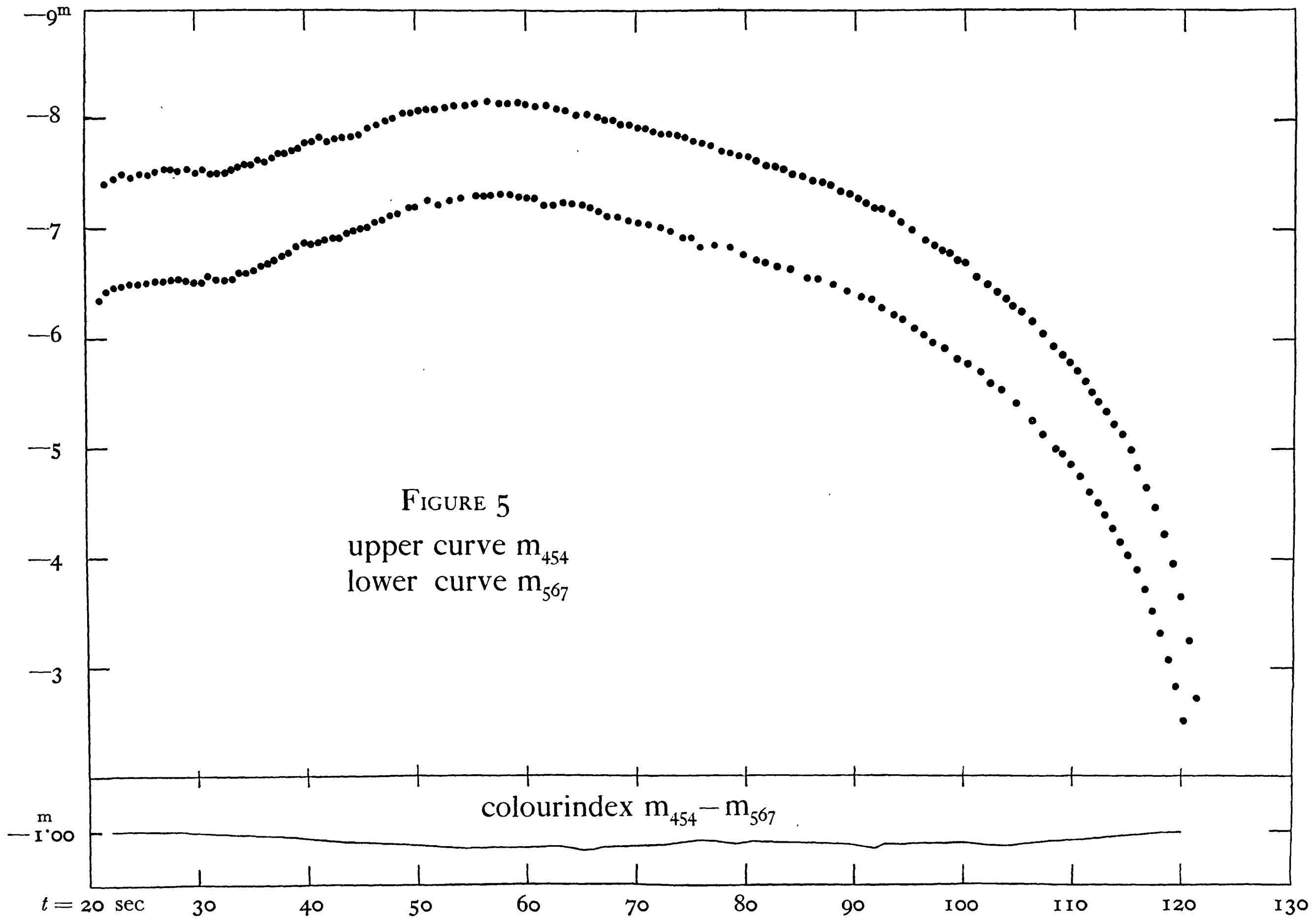


FIGURE 5
upper curve m_{454}
lower curve m_{567}

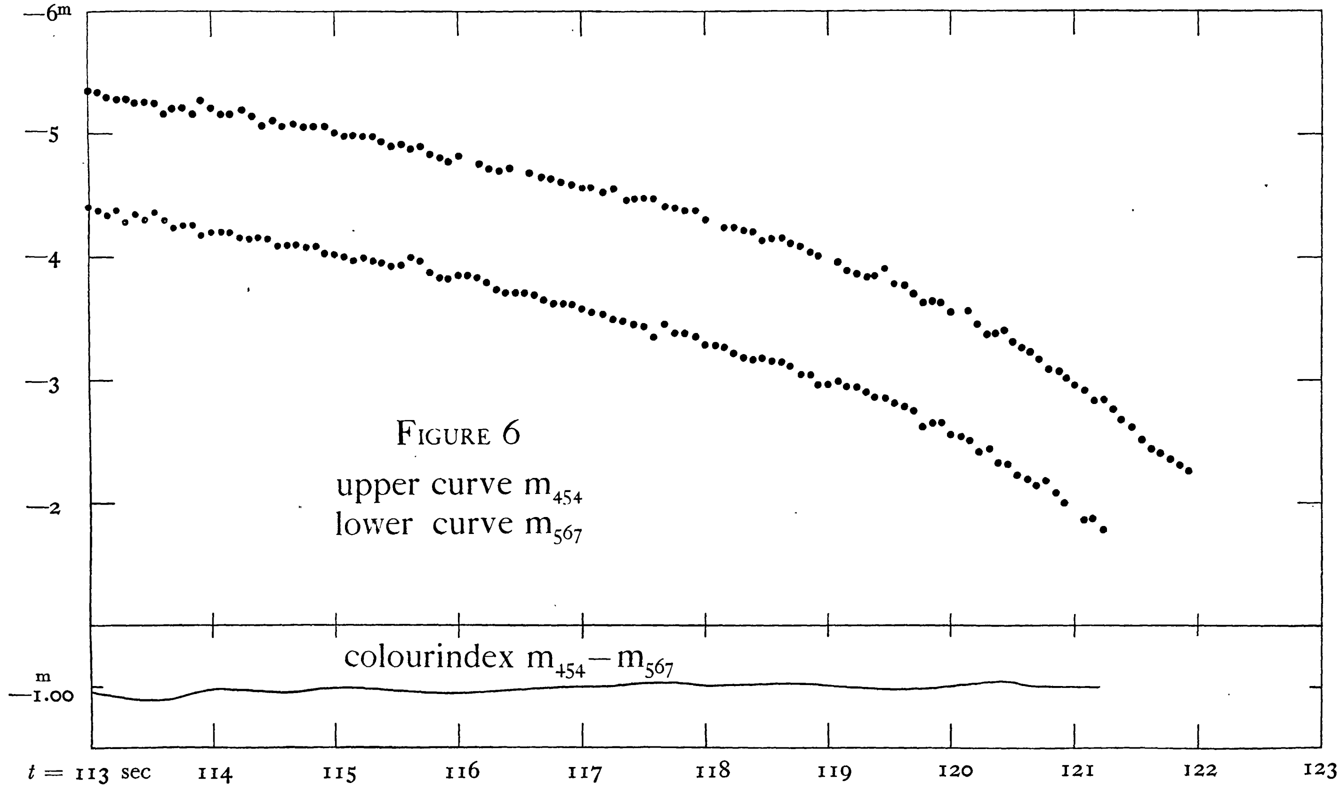


FIGURE 6
upper curve m_{454}
lower curve m_{567}

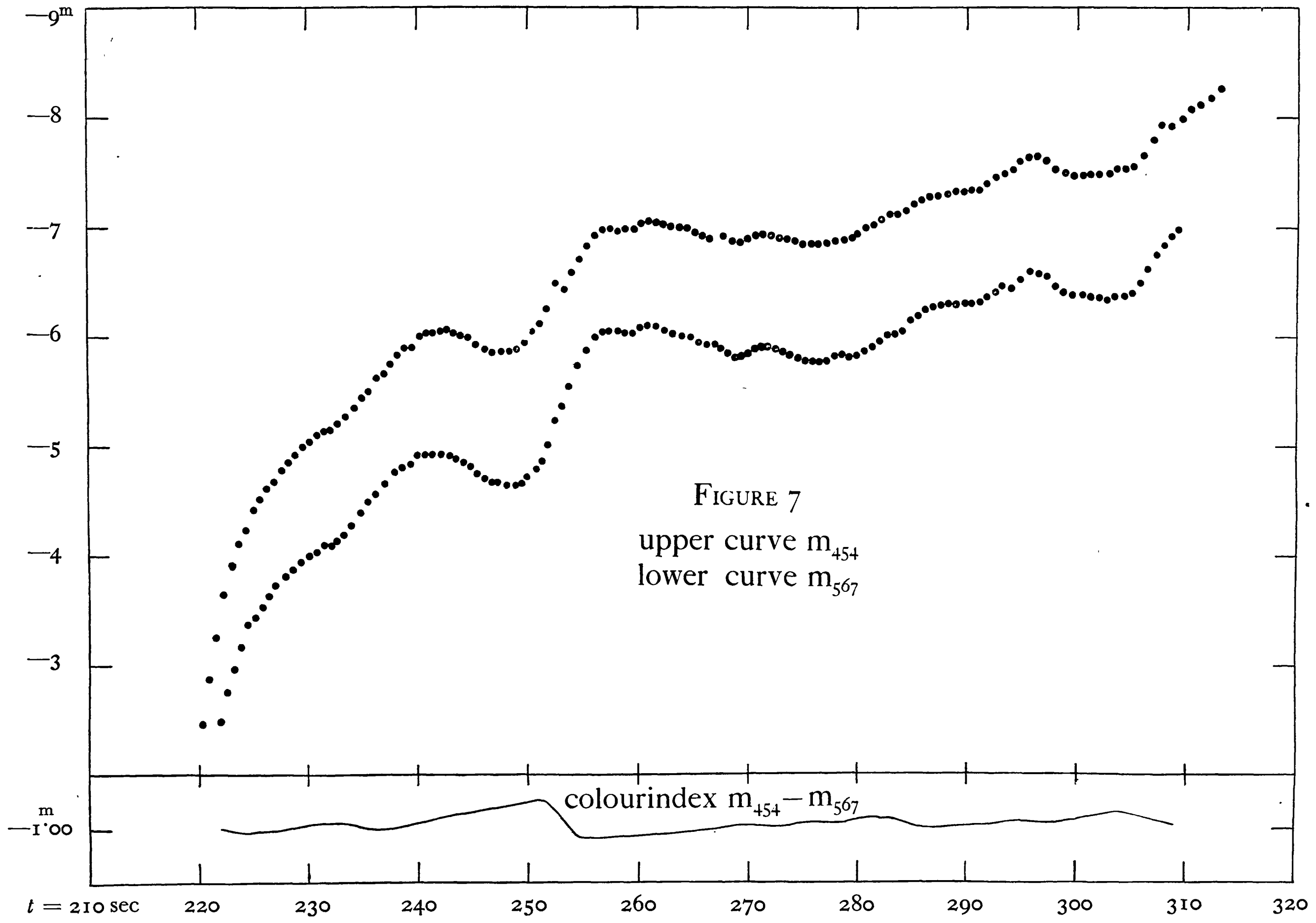


FIGURE 7
upper curve m_{454}
lower curve m_{567}

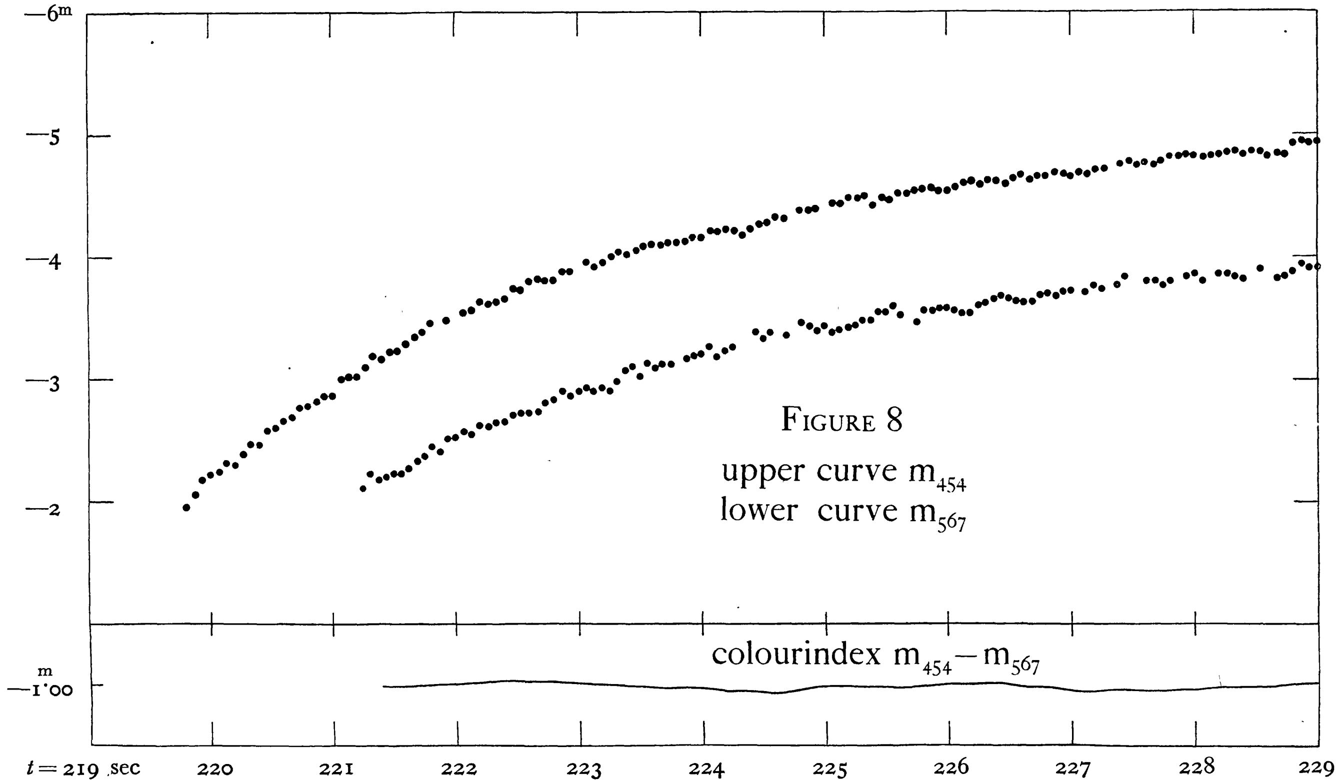
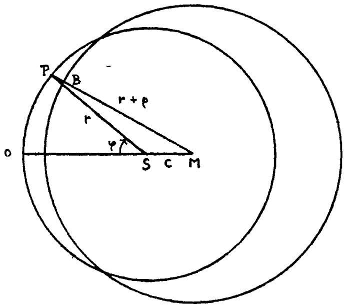


FIGURE 8
upper curve m_{454}
lower curve m_{567}

FIGURE 9.



The integrated intensity of the sun's crescent is proportional to

$$\int_0^{\varphi_*} \mathrm{PB}^{1+\alpha}\, d\varphi = \int_0^{\varphi_*} (c\cos\varphi - \rho)^{1+\alpha}\, d\varphi =$$

$$= \rho^{1+\alpha} \int_0^{\varphi_*} \left(\frac{\cos\varphi}{\cos\varphi_*} - 1\right)^{1+\alpha} d\varphi \quad . \; . \; . \quad (7)$$

The integrated intensities of solar crescents with equal φ_* but at different eclipses (different ρ) are proportional to $\rho^{1+\alpha}$.

The integrated intensities i of solar crescents at different phases of the same eclipse (ρ is constant), are proportional to

$$I = \int_0^{\varphi_*} \left(\frac{\cos\varphi}{\cos\varphi_*} - 1\right)^{1+\alpha} d\varphi \quad . \; . \; . \quad (8)$$

Some approximations to formula (8) have been made.

1°. $\varphi_* << 1$, α arbitrary.

We develop $\cos\varphi$ and $\cos\varphi_*$ into powerseries in φ, respectively φ_*, and neglect terms of order higher than the third.

We then have approximately

$$i = \text{const.} \int_0^{\varphi_*} (\varphi_*^2 - \varphi^2)^{1+\alpha}\, d\varphi = \text{const.}\, \varphi_*^{3+2\alpha} \qquad (9)$$

or expressed in magnitudes

$$m = -2\tfrac{1}{2}\,(3 + 2\alpha) \log \varphi_* + \text{const.} \quad . \; . \quad (10)$$

In the case $\alpha = 0$ (constant surface brightness) the integral (8) is

$$I_{\alpha=0} = \operatorname{tg}\varphi_* - \varphi_* = \frac{\varphi_*^3}{3} + \frac{2}{15}\varphi_*^5 + \ldots =$$

$$= \frac{\varphi_*^3}{3}\left(1 + \frac{2}{5}\varphi_*^2 + \ldots\right) \quad . \; . \; . \quad (11)$$

the term $\varphi_*^3/3$ corresponds to (9) for $\alpha = 0$.

Formula (11) expressed in magnitudes is

$$m + \frac{2}{5}\varphi_*^2 = -2\tfrac{1}{2} \times 3 \log \varphi_* + \text{const.} \qquad (12)$$

When α is very small the following formula will be more accurate than (10):

$$m + \frac{2}{5}\varphi_*^2 = -2\tfrac{1}{2}\,(3 + 2\alpha) \log \varphi_* + \text{const.} \qquad (13)$$

We remark that φ_*^2 is roughly proportional to $t - t_{\text{contact}}$. Therefore m is roughly a linear function of $\log \mid t - t_{\text{contact}} \mid$.

2°. φ_* arbitrary, $\alpha << 1$.

After the substitution $\eta = \dfrac{\cos\varphi}{\cos\varphi_*} - 1$, integral (8) is:

$$I = \int_0^{\varphi_*} \eta^{1+\alpha}\, d\varphi.$$

$$\int_0^{\varphi_*} \eta^{1+\alpha}\, d\varphi = \int_0^{\varphi_*} \eta \,.\, \eta^{\alpha} .\; d\varphi = \int_0^{\varphi_*} \eta \,.\, e^{\alpha \log \text{nat}\, \eta}\, d\varphi$$

$$= \int_0^{\varphi_*} \eta(1 + \alpha \log \text{nat}\, \eta + \ldots)\, d\varphi.$$

We introduce $$p\,(\varphi_*) = \int_0^{\varphi_*} \eta\, d\varphi = \operatorname{tg}\varphi_* - \varphi_* \qquad (14)$$

and $$q\,(\varphi_*) = \int_0^{\varphi_*} \eta \log \text{nat}\, \eta\, d\varphi \quad . \; . \; . \quad (15)$$

Neglecting powers of α higher than the first we have

$$I = p\,(\varphi_*) + \alpha\, q\,(\varphi_*) \quad . \; . \; . \; . \quad (16)$$

and $$\text{const.}\; i = p + \alpha q = p\left(1 + \alpha\frac{q}{p}\right) \quad . \; . \; . \; . \quad (17)$$

which is equivalent to

$$m + \text{const.} = -\frac{5}{2}\log p - \frac{5}{2}\log\left(1 + \alpha\frac{q}{p}\right). \quad . \quad (18)$$

Writing $m_{\alpha=0}$ for $-2\tfrac{1}{2}\log p$ we have

$$m - m_{\alpha=0} = -2\tfrac{1}{2}\log\left(1 + \alpha\frac{q}{p}\right) + \text{const.} \quad . \quad (19)$$

For $\alpha \dfrac{q}{p} << 1$ (19) reduces to the simple formula:

$$m - m_{\alpha=0} = -1{\cdot}08\, \alpha\frac{q}{p} + \text{const.} \quad . \; . \quad (20)$$

6. *Analysis of the observations.*

The observations made in the interval from one minute before second contact to second contact seem to be almost free from the disturbing effect of the clouds. The observations made in this interval of time with $\lambda_{\text{eff}} = \mu{\cdot}454$ have been analysed for the law of darkening by means of the formulae of the foregoing section.

The eclipse curve at $\lambda_{\text{eff}} = \mu{\cdot}567$ has been found to differ by a constant (in m) only from the curve at $\lambda_{\text{eff}} = {}^{\mu}{\cdot}454$. An analysis of the curve at $\lambda_{\text{eff}} = {}^{\mu}{\cdot}567$ would therefore yield the same result as that obtained from the curve at $\lambda_{\text{eff}} = {}^{\mu}{\cdot}454$ and has therefore not been made.

From the discussion in the foregoing section it appeared that we have to know for every observed brightness the accompanying value of φ_*.

We now proceed to determine φ_* as a function of t.

Extensive use has been made of the tables and charts published by A. A. MICHAILOV concerning the elements of the 1936 eclipse[1]. By the aid of this work we find:
the geographical coordinates of our observing station

$\lambda = 39^\circ\ 54'{\cdot}7$ east of Greenwich
$\beta = +44^\circ\ 44'{\cdot}5$
G.M.T. of central eclipse $3^{\rm h}59^{\rm m}42^{\rm s}{\cdot}6$

The duration τ of the total phase at a point on the central line nearest to our observing site is $\tau = 92{\cdot}6$ solar seconds.

The position angles P_2 and P_3 of second and third contact are for our observing station (Table 4, *c* MICHAILOV l.c.):

$$P_2 = 99^\circ{\cdot}4$$
$$P_3 = 254^\circ{\cdot}4$$

Further if $2\,\theta = P_3 - P_2 - 180^\circ = 2 \times -12^\circ{\cdot}5$ the duration of the total phase at our observing station is $\tau \cos\theta = 90{\cdot}4$ solar seconds.

It is not advantageous to obtain the time of central eclipse from the G.M.T. of central eclipse as predicted by the almanacs. Such a prediction may be off by several seconds (see below).

The best procedure seems to determine the time of central eclipse, t_{central}, as the mean value of times t corresponding to the same brightness on the descending and rising branches of the eclipse curve[2].

Though the parts of the eclipse curve nearest to contact are the steepest it is better to exclude these parts from the determination as the irregularities in the profile of the moon's limb might easily introduce systematic errors.

Because of the difference between the absorptions by the clouds before second contact and following third contact the above proposed procedure has been carried out in a slightly modified form.

One of the diagrams shown in Figures 6 and 8 was reflected with respect to the time and the parts farthest from totality were matched on each other as closely as possible by shifting both curves with respect to each other in both horizontal and vertical directions.

The result was $t_{\text{central}} = 170^{\rm s}{\cdot}4 \pm {}^{\rm s}{\cdot}3$[3].

We assume the angular speeds of the sun and of the moon to be constant during the interval of time covered by the observations. Let us take as a unit of distance the angle described by a body moving with the relative angular speed (vectorial) of sun and moon in one sidereal second.

In this unit $2\,\rho' = \tau \times 1{\cdot}0027 = 92{\cdot}8$ sid. seconds.

The minimum angular distance between sun and moon is

$c_{\min} = \rho' \sin\theta = 92{\cdot}8 \times {\cdot}2166 = 10{\cdot}0$ sid. seconds.
(ρ' refers to the excess of the moon's radius over that of the sun as given by MICHAILOV).

The square of the angular distance at any moment is then:

$$c^2 = (t_{\text{central}} - t)^2 + c^2_{\min} =$$
$$= (170{\cdot}4 - t)^2 + (10{\cdot}0)^2 \quad . \quad . \quad (21)$$

The radius of the moon used in MICHAILOV's publication refers to the valleys in the profile of the moon's limb and is three quarters of a second of arc smaller than the radius as determined from occultations, which latter value is needed in our computations. The distinction between these radii is significant and corresponds to 1·5 seconds of time[4].

We thus have
$\rho = \rho' + 1{\cdot}5 = 46{\cdot}4 + 1{\cdot}5 = 47{\cdot}9$ sidereal seconds.

Then φ_* has been computed from the equation $\cos\varphi_* = \rho/c = \dfrac{47{\cdot}9}{c}$, in which c was taken from equation (21) for the normals 53 to 129 of the eclipse curve at $\lambda_{\text{eff}} = {}^{\mu}{\cdot}454$ of the descending branch.

The successive columns of Table 7 contain for the normals 53 to 129:
the number, t, $t_{\text{central}} - t$, c, $\cos\varphi_*$, φ_*, m, i, p, $-q$ and the residuals O—C between observed and computed eclipse function.

$p\ (\varphi_*)$ was found from formula (14).

$q\ (\varphi_*)$ (see formula 15) has been obtained by numerical integration for a number of values of φ_*. For other values of φ_*, q has been read from a graph drawn through the points obtained by direct computation, $q\ (0) = q\ (64^\circ{\cdot}7) = 0$; in the

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[1] A. A. MICHAILOV, "Total eclipse of the sun June 19, 1936 in U.S.S.R.", The Sternberg State Astronomical Institute.

[2] For other methods of obtaining the epoch of a symmetrical minimum, compare the methods used in the case of eclipsing variable stars, *B.A.N.* No. 166 (1929).

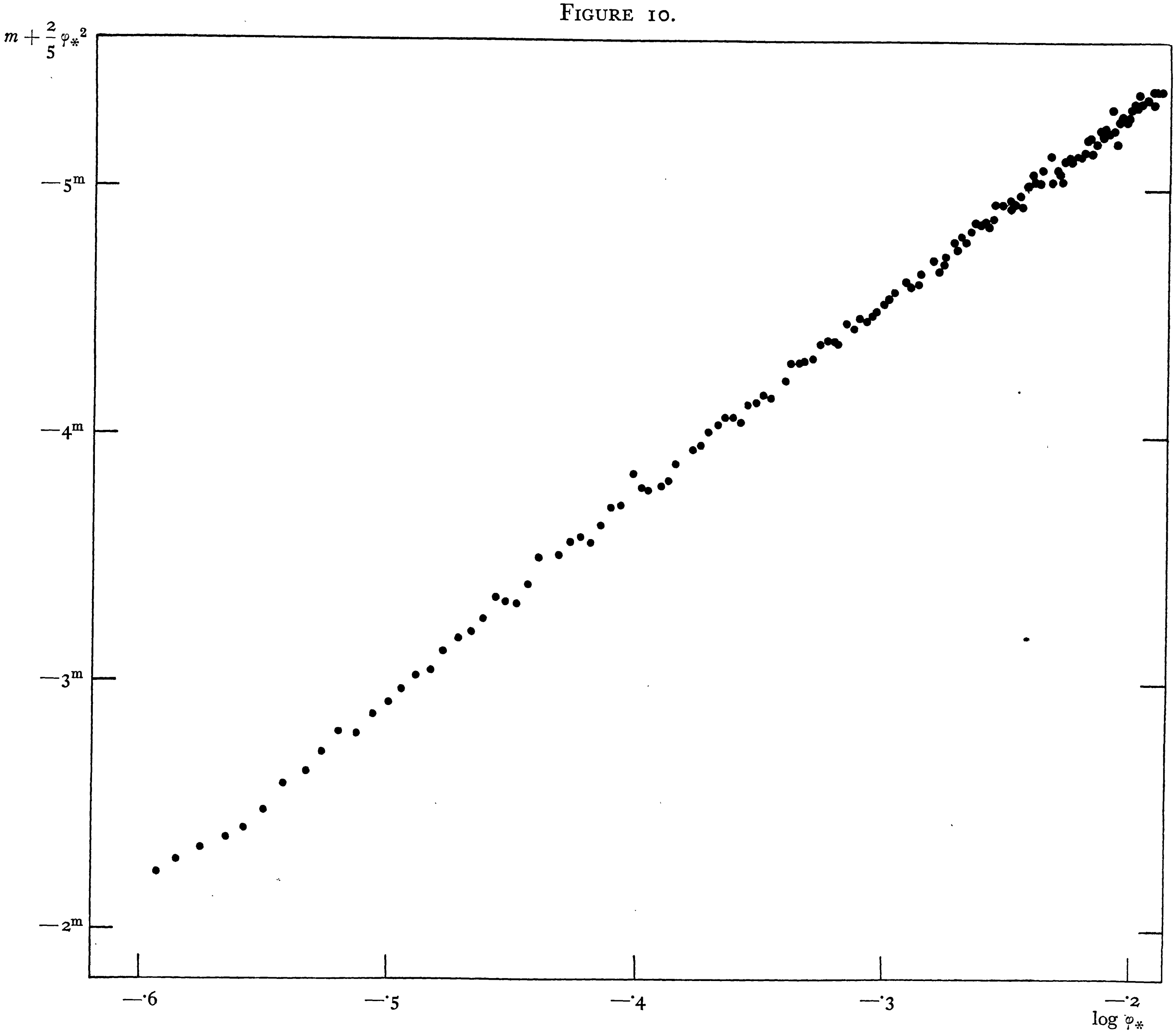
[3] Estimated uncertainty.

[4] I owe this remark to Professor BROUWER. The angular radius of the moon used by MICHAILOV reduced to a mean parallax of $3422''{\cdot}70$ is $931''{\cdot}87$. The angular radius of the moon used in occultation work for the same parallax is $''{\cdot}76$ larger viz. $932''{\cdot}63$ (compare L. J. COMRIE, *A. J.* No. 1062).

TABLE 7.

No.	t in sec	$t_{\text{central}} - t$	c in sec	ρ/c	φ°_*	m (O)	i	p	$-q$	m(O)–m(C)
53	60·0	110·4	110·9	·4320	64·40	— 8·13	1786	·9635	·022	+ ·05
54	60·9	109·5	110·0	·4355	64·18	— 8·11	1754	·9468	·033	+ ·05
55	61·9	108·5	109·0	·4394	63·93	— 8·12	1770	·9284	·049	+ ·02
56	62·8	107·6	108·1	·4431	63·70	— 8·09	1722	·9115	·060	+ ·03
57	63·6	106·8	107·3	·4464	63·49	— 8·07	1691	·8965	·070	+ ·03
58	64·6	105·8	106·3	·4507	63·21	— 8·04	1644	·8774	·082	+ ·04
59	65·6	104·8	105·3	·4550	62·94	— 8·04	1644	·8587	·096	+ ·01
60	66·5	103·9	104·4	·4587	62·70	— 8·02	1614	·8429	·106	+ ·01
61	67·2	103·2	103·7	·4619	62·49	— 7·99	1570	·8295	·117	+ ·02
62	67·9	102·5	103·0	·4651	62·28	— 7·98	1556	·8163	·126	+ ·01
63	68·6	101·8	102·3	·4682	62·08	— 7·95	1514	·8037	·135	+ ·02
64	69·4	101·0	101·5	·4719	61·84	— 7·94	1500	·7890	·143	+ ·01
65	70·2	100·2	100·7	·4757	61·59	— 7·92	1472	·7741	·152	+ ·01
66	70·8	99·6	100·1	·4785	61·41	— 7·91	1459	·7632	·158	·00
67	71·5	98·9	99·4	·4819	61·19	— 7·88	1419	·7502	·164	+ ·01
68	72·3	98·1	98·6	·4859	60·93	— 7·86	1392	·7353	·172	+ ·01
69	73·0	97·4	97·9	·4892	60·71	— 7·86	1392	·7233	·179	— ·01
70	73·7	96·7	97·2	·4929	60·47	— 7·85	1380	·7099	·187	— ·02
71	74·4	96·0	96·5	·4963	60·24	— 7·83	1355	·6977	·194	— ·02
72	75·2	95·2	95·7	·5005	59·97	— 7·80	1318	·6831	·201	— ·01
73	76·0	94·4	94·9	·5048	59·68	— 7·78	1294	·6685	·207	— ·02
74	76·8	93·6	94·1	·5089	59·41	— 7·76	1271	·6546	·214	— ·02
75	77·8	92·6	93·1	·5144	59·04	— 7·72	1225	·6365	·221	— ·02
76	78·5	91·9	92·4	·5184	58·78	— 7·70	1202	·6238	·226	— ·02
77	79·3	91·1	91·7	·5225	58·50	— 7·67	1170	·6109	·231	— ·02
78	80·2	90·2	90·8	·5274	58·17	— 7·66	1159	·5957	·237	— ·03
79	80·9	89·5	90·1	·5316	57·89	— 7·62	1117	·5830	·242	— ·02
80	81·8	88·6	89·2	·5371	57·51	— 7·58	1076	·5667	·248	— ·01
81	82·6	87·8	88·4	·5417	57·20	— 7·57	1067	·5534	·252	— ·03
82	83·4	87·0	87·6	·5467	56·86	— 7·55	1047	·5392	·257	— ·04
83	84·2	86·2	86·8	·5519	56·50	— 7·50	1000	·5249	·262	— ·01
84	85·1	85·3	85·9	·5577	56·10	— 7·48	981·8	·5091	·266	— ·04
85	86·0	84·4	85·0	·5634	55·71	— 7·44	946·2	·4941	·269	— ·03
86	86·9	83·5	84·1	·5695	55·28	— 7·43	937·6	·4785	·272	— ·05
87	87·7	82·7	83·3	·5750	54·90	— 7·40	912·1	·4647	·274	— ·06
88	88·6	81·8	82·4	·5814	54·45	— 7·34	863·0	·4491	·277	— ·05
89	89·4	81·0	81·6	·5869	54·06	— 7·32	847·2	·4360	·279	— ·05
90	90·2	80·2	80·8	·5928	53·64	— 7·28	816·6	·4223	·280	— ·05
91	90·9	79·5	80·1	·5981	53·27	— 7·24	787·0	·4102	·281	— ·05
92	91·7	78·7	79·3	·6039	52·85	— 7·19	751·6	·3974	·282	— ·03
93	92·4	78·0	78·6	·6094	52·45	— 7·18	744·7	·3856	·283	— ·06
94	93·3	77·1	77·7	·6165	51·94	— 7·14	717·8	·3706	·283	— ·06
95	94·1	76·3	77·0	·6219	51·54	— 7·06	666·8	·3596	·282	— ·02
96	95·2	75·2	75·9	·6309	50·88	— 6·99	625·2	·3417	·281	— ·01
97	96·5	73·9	74·6	·6423	50·04	— 6·90	575·5	·3200	·279	·00
98	97·3	73·1	73·8	·6489	49·54	— 6·86	554·6	·3078	·277	·00
99	98·0	72·4	73·1	·6553	49·06	— 6·81	529·7	·2965	·275	·00
100	98·7	71·7	72·4	·6618	48·56	— 6·78	515·2	·2852	·273	— ·02
101	99·4	71·0	71·7	·6680	48·09	— 6·72	487·5	·2747	·271	·00
102	100·1	70·3	71·0	·6748	47·56	— 6·69	474·2	·2635	·268	— ·02
103	101·1	69·3	70·0	·6845	46·80	— 6·57	424·6	·2481	·262	+ ·03
104	102·1	68·3	69·0	·6940	46·05	— 6·50	398·1	·2336	·258	+ ·03
105	103·0	67·4	68·1	·7032	45·32	— 6·43	373·3	·2202	·252	+ ·03
106	103·8	66·6	67·3	·7117	44·63	— 6·37	353·2	·2082	·249	+ ·02
107	104·4	66·0	66·8	·7168	44·21	— 6·30	331·1	·2012	·244	+ ·06
108	105·2	65·2	66·0	·7257	43·47	— 6·25	316·2	·1888	·238	+ ·03
109	106·1	64·3	65·1	·7358	42·62	— 6·16	291·1	·1765	·231	+ ·04
110	107·1	63·3	64·1	·7474	41·63	— 6·05	263·0	·1622	·222	+ ·05
111	108·1	62·3	63·1	·7593	40·60	— 5·94	237·7	·1484	·212	+ ·06
112	108·9	61·5	62·3	·7686	39·77	— 5·86	220·5	·1382	·204	+ ·05
113	109·6	60·8	61·6	·7776	38·96	— 5·79	207·0	·1287	·197	+ ·04
114	110·3	60·1	60·9	·7868	38·11	— 5·71	192·3	·1193	·194	+ ·03
115	111·0	59·4	60·2	·7955	37·30	— 5·61	175·4	·1107	·181	+ ·04
116	111·6	58·8	59·6	·8039	36·50	— 5·52	161·4	·1028	·174	+ ·05
117	112·2	58·2	59·1	·8104	35·86	— 5·43	148·6	·09699	·169	+ ·07
118	112·9	57·5	58·4	·8203	34·88	— 5·34	136·8	·08836	·159	+ ·05
119	113·6	56·8	57·7	·8299	33·91	— 5·22	122·5	·08039	·150	+ ·06
120	114·4	56·0	56·9	·8418	32·67	— 5·13	112·7	·07104	·139	·00
121	115·1	55·3	56·2	·8525	31·51	— 5·00	100·0	·06309	·128	·00
122	115·8	54·6	55·5	·8628	30·37	— 4·83	85·50	·05592	·120	+ ·02
123	116·7	53·7	54·6	·8772	28·69	— 4·64	71·78	·04655	·105	·00
124	117·5	52·9	53·8	·8905	27·06	— 4·46	60·81	·03857	·0928	— ·04
125	118·3	52·1	53·1	·9017	25·62	— 4·22	48·75	·03239	·0817	— ·01
126	119·1	51·3	52·3	·9158	23·68	— 3·96	38·37	·02526	·0684	— ·05
127	119·8	50·6	51·6	·9285	21·80	— 3·65	28·84	·01948	·0564	— ·04
128	120·6	49·8	50·8	·9429	19·46	— 3·25	19·96	·01368	·0433	— ·06
129	121·3	49·1	50·1	·9560	17·06	— 2·72	12·25	·00912	·0311	— ·01

FIGURE 10.



interval $0 < \varphi_* < 64^\circ\!\cdot\!7$, $q(\varphi_*)$ is < 0 and reaches a minimum at $\varphi_* = 52^\circ\!\cdot\!2$ with the minimum value $q(52^\circ\!\cdot\!2) = -\cdot283$; for values of $\varphi_* > 64^\circ\!\cdot\!7$ we have $q(\varphi_*) > 0$.

For each normal an equation of condition was formed of the form (see formula 17)

$$px + qy = i, \text{ weight } i^{-2} \quad . \quad . \quad . \quad (22)$$

From the resulting x and y we find for the exponent in the law of darkening:

$$\frac{y}{x} = \alpha = +\cdot095 \pm \cdot005 \text{ (m.e.)}.$$

The mean error is a direct result of the computation and should be increased on account of the uncertainty in the time of central eclipse and in the adopted diameter of the moon, apart from the uncertainty of the effect of the clouds.

The residuals between observation and computation (last column of Table 7) show moreover a systematic run which suggests that a formula of the form $\sigma = \text{const.} \times z^{\alpha}$ with a value of α independent of z is a too simple representation of the actual law of darkening.

The individual observations at $\lambda_{\text{eff}} = \mu\cdot454$, given in Table 6 *a*, and represented graphically in Figure 6, have then been analysed in a different manner.

In Figure 10 the ordinates are $m + \frac{2}{5}\varphi_*^2$ and the abscissae $\log \varphi_*$.

According to formula (13) we obtain $2\frac{1}{2}\,(3 + 2\alpha)$ so that α follows at once.

A least squares solution gave as a result

$$\alpha = + \cdot 095 \quad \pm \cdot 010 \text{ (m.e.).}$$

To the perfect agreement with the result obtained above from the larger material we should not attach undue importance as the assumptions concerning the adopted values of the time of central eclipse and the radius of the moon have been the same.

The linearity of the relation between $m + \frac{2}{5}\,\varphi_*^2$ and log φ_* strengthens, however, our confidence in the correctness of these values and seems to justify the neglect of the disturbing effect of the clouds.

It is remarkable that the observed relation shown in Figure 10 does not reveal the effect of the irregularities in the moon's limb more clearly. This may be due to the happy circumstance that the moon's limb is rather smooth in the region where the second contact occurred.

7. In *B.A.N.* No. 296 E. HERTZSPRUNG has given an interpolation formula for the law of darkening towards the sun's limb as a function of the wavelength which represents the direct measurements by MOLL, BURGER and VAN DER BILT (loc. cit.) in a satisfactory manner.

Though an agreement between this formula and our results need not be anticipated it is of interest to make the comparison.

HERTZSPRUNG's formula is:

$$\sigma = \text{const.} \times \cos^{\cdot 27/\lambda}\gamma\,. \quad . \quad . \quad (23)$$

Here γ is the angle between the line of sight and the normal on the sun's surface, σ is the surface brightness as before and λ is the wavelength.

For small values of the distance on the disc from the limb z, we have

$$z = \text{const.} \times \cos^2\gamma \quad . \quad . \quad . \quad (24)$$

Consequently formula (24) may be written

$$\sigma = \text{const.} \times z^{\cdot 135/\lambda} \quad . \quad . \quad . \quad (25)$$

which is of the same form as formula (6) adopted by us in the analysis of the eclipse curve.

The values of the exponent α according to HERTZSPRUNG's formula (formula 25) for our two effective wavelengths are:

λ_{eff}	$\alpha = \cdot 135/\lambda$
$\mu\cdot 454$	$\cdot 30$
$\cdot 567$	$\cdot 24$

while our result is $\alpha = \cdot 1$.

The variation of log σ with log cos γ therefore seems less rapid at the limb than it is in the middle part of the sun's disc.

The variation in the colour, $m_{454} - m_{567}$, to be expected from an extrapolation of HERTZSPRUNG's formula is found from (compare formula 20):

$$m_{454} - m_{567} = -\,(\alpha_{454} - \alpha_{567}) \times \frac{q}{p} + \text{const.}$$

$$= -\cdot 06 \times \frac{q}{p} + \text{const.}$$

The variation of this quantity over the interval of time covered by the observations either before second or following third contact is $^{\rm m}\cdot 2$.

The constancy of the colour shown by our observations is however within a tenth of a magnitude.

8. Though the provisional investigation by eye estimates on a part of the films did not reveal any change in colour at all the results of the measurements of the complete material show the absorption of the clouds to have been selective.

At $t = 250$ (compare Figure 6) there has obviously been a strong absorption. At the same time the colour curve shows a marked variation, indicating that the light of longer wavelength has been absorbed more strongly than that of shorter wavelength. This feature of the absorption could be shown to hold generally as follows:

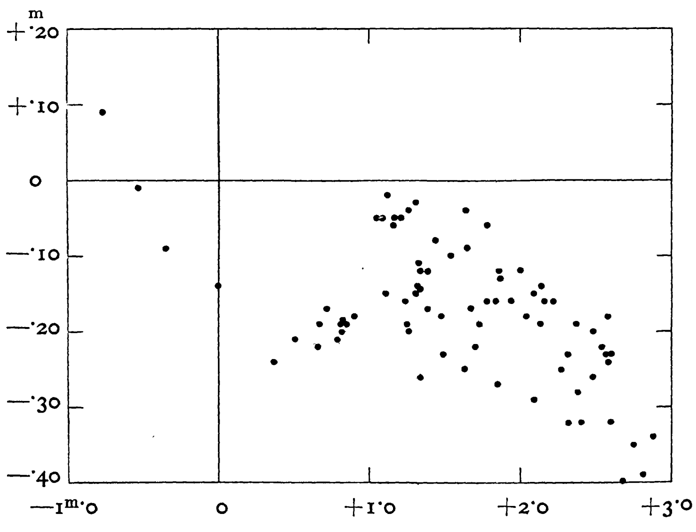
A diagram (Figure 11) was made having as abscissa the difference between two values of $m_{454} + m_{567}$ taken at times symmetrical with respect to the time of central eclipse:

$$(m_{454} + m_{567})_{\text{rising}} - (m_{454} + m_{567})_{\text{desc.}}$$

As ordinate the difference between the colour-indices at these same instants was taken:

$$(m_{454} - m_{567})_{\text{rising}} - (m_{454} - m_{567})_{\text{desc.}}$$

FIGURE 11.



On the assumption that the undisturbed eclipse curve is symmetrical with respect to central eclipse the abscissae are evidently the difference in the absorptions at $\lambda_{\text{eff}} = \mu{\cdot}454$ plus the difference in the absorptions at $\lambda_{\text{eff}} = \mu{\cdot}567$ at the two instants symmetrical with respect to the time of central eclipse. Observations nearer to the contacts than 15 seconds have been excluded, as an error in the adopted time of central eclipse would affect the diagram too seriously.

The large scatter is probably due to a change in the nature of the clouds.

In case the nature of the clouds had not changed a closer relation represented by a line passing through the origin might have been expected.

The large number of dots on the right hand side of the diagram is simply due to the fact that the absorption following third contact has been stronger than that before second contact.

In the case of a selective absorption the dots will lie either in the lower left and upper right quadrant or in the upper left and the lower right quadrant, depending on whether the light by passing the clouds has become redder or bluer.

The diagram therefore confirms our former conclusion that the absorption at $\lambda_{\text{eff}} = \mu{\cdot}567$ has been stronger than that at $\lambda_{\text{eff}} = \mu{\cdot}454$. The ordinates are in the mean $\cdot 1 \times$ the abscissae. This means that the absorption coefficients at $\lambda = \mu{\cdot}454$ and $\lambda = \mu{\cdot}567$ have been in the ratio 9 : 11.

A selective absorption by clouds in the sense as observed here has been noticed before. The conditions necessary for its occurrence seem to be rather special1).

9. In section 6 we obtained from MICHAILOV's monograph for the time of central eclipse $3^h\ 59^m\ 42^s{\cdot}6$ G.M.T.

The positions of the sun used by MICHAILOV are those from NEWCOMB's tables without any corrections. The position of the moon has been taken from BROWN's tables and corrections have been applied to both longitude and latitude:

$$\Delta\lambda = +\ 5''{\cdot}0, \qquad \Delta\beta = -\ 0''{\cdot}5.$$

Professor BROUWER kindly communicated to me what he regards as the best corrections to the positions of sun and moon adopted by MICHAILOV. He finds

$$\Delta\alpha_{☾} - \Delta\alpha_{\odot} = -\ {}^{s}{\cdot}235$$
$$\Delta\delta_{☾} - \Delta\delta_{\odot} = -\ {}''{\cdot}60$$

This corresponds to a correction of $+\ 5^s{\cdot}9$ to the time found from MICHAILOV's work, making it $3^h 59^m 48^s{\cdot}5$ G.M.T.

From the observed value $t = 170^s{\cdot}4$ we obtain the observed value in G.M.T.:

$$3^h\ 56^m\ 57^s{\cdot}7 + 170^s{\cdot}4 \times {\cdot}9973 = 3^h\ 59^m\ 47^s{\cdot}7 \text{ G.M.T.}$$

The difference between observation and prediction of the time of central eclipse is now only $-\ {\cdot}8$ seconds.

10. *Acknowledgements.*

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Mr. KOOREMAN measured part of the eclipse films.

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1) *Nature* **37**, 440 (1888). R. MECKE, *Ann. d. Physik* **61**, 471 (1920). H. BLUMER, *Zs. f. Physik* **32**, 119 (1925), **38**, 304 (1926).

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Mr. BEYERINCK generously lent an Agfa movex camera to the expedition.

11. *Appendix.*

There is a possibility of increasing the accuracy in the determination of the law of darkening by choosing (with the same equipment) the place of observation instead of on the central line, near the border of the zone of totality. It is clear that in that case the interesting phases remain longer under observation than when the station is situated on the central line. It can be shown that the corresponding increase in weight is at the most four times.

On the other hand the results will depend strongly on the relative position of the observer and the central line adopted in the reduction. In the above consideration it was assumed that this relative position is known rather more accurately than it seems possible to predict it at present. Unless one has means of determining the track of the eclipse on the earth's surface with high accuracy the suggestion of observing a total eclipse for the determination of the law of darkening at the border of the zone of totality should not be followed up.

The photographic method may be improved by using more mirrors, differing less in magnitude reducing power than those used by us at the 1936 eclipse. Even in the case of a very steep gradation (which is favourable for the accuracy) one will then be able to derive relations between galvanometer-readings and magnitudes in an independent way, that is from the material alone, thus without recourse to a mean relation as used in the present paper.

A better knowledge of the distance correction than could be obtained in the present paper may be secured by placing mirrors having equal curvature at different distances from the optical axis.

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