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Effect of inelastic scattering on the average Coulomb-blockade peak height in quantum dots

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The average height of the Coulomb-blockade conductance peaks for chaotic elastic scattering is known to increase by a factor of 4/3 upon breaking time-reversal symmetry We calculate the temperature dependence of this factor in the regime where the inelastic scattering rate $\Gamma_{\rm in}$ is greater than the mean tunneling rate $\Gamma_{\rm el}$, which itself is less than the mean level spacing Δ Comparison to recent experimental data by Folk *et al* (Folk, Patel, Marcus, Duruoz, and Harris, cond-mat/0008052) demonstrates that $\Gamma_{\rm in}$ lies below $\Gamma_{\rm el}$ and hence also below Δ, consistent with the low-energy suppression of inelastic electron-electron scattering in quantum dots

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Inelastic electron-electron scattering in a quantum dot broadens the single-particle excitation levels by an amount $\hbar\Gamma_{\rm in}$ This broadening vanishes at low excitation energies ε and remains less than the mean level spacing Δ as long as ε is below the Thouless energy [1 2] Early Coulomb-blockade experiments by Sivan *et al* [3] agreed with this theoretical prediction, but recent experiments by Folk *et al* [4] were interpreted as inconsistent with it

Inelastic scattering can be detected by the broadening of the single-particle density of states, as was done by Sivan *et al* (Ref 3) Folk *et al* [4] instead, used the temperature dependence of the height of the Coulomb-blockade peaks in the conductance For fully elastic and chaotic scattering the average height is increased by a temperature-independent factor of 4/3 upon application of a magnetic field [5 6] Folk *et al* measured a suppression of this enhancement factor when the thermal energy kT became larger than Δ They concluded from this strong temperature dependence that the dephasing rate[7] in quantum dots is larger than $\Delta/\hbar$ at excitation energies well below the Thouless energy, in apparent contradiction with the theoretical expectation However, in the absence of a quantitative prediction for the temperature dependence of the Coulomb-blockade peak height, it is difficult to decide whether the observed temperature dependence is actually stronger than expected

What we will do here is use the semiclassical theory of the Coulomb blockade[8] to obtain the temperature dependence in the regime $\Gamma_{\rm el}\ll\Gamma_{\rm in}$, with $\Gamma_{\rm el}$ the mean (elastic) tunnel rate into the quantum dot We call this the regime of strong inelastic scattering, where "strong" means strong enough to thermalize the distribution of the electrons among the levels in the quantum dot Both $\Gamma_{\rm el}$ and $\Gamma_{\rm in}$ should be less than kT, so that we are allowed to use rate equations based on sequential tunneling The condition for the Coulomb blockade is $\Gamma_{\rm el}\ll\Delta/\hbar$ and $kT\ll e^2/C$, with C being the capacitance of the quantum dot We find that the experimental temperature dependence[4] is actually much *weaker* than predicted by the theory for strong inelastic scattering Therefore, $\Gamma_{\rm in}\lesssim\Gamma_{\rm el}\ll\Delta/\hbar$ and there is no disagreement between the experimental data of Ref 4 and the theoretical expectation of a low-energy suppression of inelastic electron-electron scattering in quantum dots [9]

The starting point of our analysis is a pair of expressions from Ref 8 for the Nth conductance peak in the two cases of purely elastic scattering ($G_{\rm el}$) and strong inelastic scattering ($G_{\rm in}$)

$$G_{\rm el}=\frac{e^2}{kT}P_{\rm eq}(N)\left\langle\frac{\Gamma^{\rm l}\Gamma^{\rm r}}{\Gamma^{\rm l}+\Gamma^{\rm r}}\right\rangle_N, \tag{1}$$

$$G_{\rm in}=\frac{e^2}{kT}P_{\rm eq}(N)\frac{\langle\Gamma^{\rm l}\rangle_N\langle\Gamma^{\rm r}\rangle_N}{\langle\Gamma^{\rm l}+\Gamma^{\rm r}\rangle_N} \tag{2}$$

The spectral average of the elastic tunnel rate $\Gamma_p^{\rm l\,r}$ into the left or right reservoir is defined by

$$\langle\Gamma^{\rm l\,r}\rangle_N=\sum_p\Gamma_p^{\rm l\,r}[1-F_{\rm eq}(E_p|N)]f(E_p-\mu) \tag{3}$$

The equilibrium distributions $P_{\rm eq}(N)$ and $F_{\rm eq}(E_p|N)$ give, respectively, the *a priori* probability to find N electrons in the quantum dot and the conditional probability to find level p occupied by one of the N electrons (These functions are obtained from the Gibbs distribution in the canonical ensemble) The function $f(E_p-\mu)$ is the Fermi-Dirac distribution, with μ an externally tunable parameter that depends linearly on the gate voltage

If $\Gamma_{\rm in}\ll\Gamma_{\rm el}$ one may neglect inelastic scattering and use Eq (1), while if $\Gamma_{\rm el}\ll\Gamma_{\rm in}$ one should use Eq (2) The key difference between the two equations is that for $G_{\rm el}$ the fraction $\Gamma_p^{\rm l}\Gamma_p^{\rm r}/(\Gamma_p^{\rm l}+\Gamma^{\rm r})$ as a whole is averaged over the spectrum, while for $G_{\rm in}$ the numerator and denominator are averaged separately Since the spectral average extends over about kT/Δ levels, the difference between $G_{\rm el}$ and $G_{\rm in}$ vanishes if kT becomes less than Δ

In a chaotic quantum dot, the tunnel rates $\Gamma_p^{\rm l}$ and $\Gamma_q^{\rm r}$ fluctuate independently according to the Porter-Thomas distribution $P(\Gamma)\propto\Gamma^{\beta/2-1}\exp(-\beta\Gamma/2\Gamma_{\rm el})$ (We assume tunneling through two equivalent single-channel point contacts, with energy-independent mean tunnel rate $\Gamma_{\rm el}$) The index $\beta=1$ (2) in the presence (absence) of a time-reversal-symmetry breaking magnetic field The mean height $\overline{G}_{\rm el}^{\rm max}$ of the Coulomb-blockade peak for elastic scattering increases upon breaking time-reversal symmetry, by a temperature-

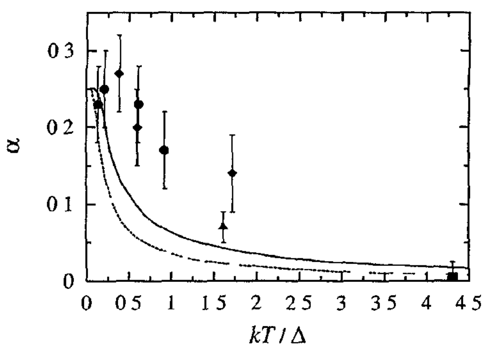

FIG 1 Temperature dependence of the parameter α defined in Eq (4) The curves are calculated from Eq (2), either for spin degenerate levels (solid) or for nondegenerate levels (dashed) The markers with error bars are experimental data for GaAs quantum dots from Folk *et al* (Ref 4) The area of the dot is 0 25 0 7, 3, and 8 μm^2 for, respectively, circle, diamond, triangle, and square markers

independent factor of 4/3 [5,6] Inelastic scattering introduces a temperature dependence, which we can study using Eq (2)

Qualitatively, the effect of inelastic scattering on the 4/3-enhancement factor can be understood as follows The spectral average $\langle \quad \rangle_N$, defined precisely in Eq (3), can be approximated by an average over kT/Δ levels around the Fermi energy in the quantum dot containing N electrons If $kT \gg \Delta$ the spectral average becomes equivalent to an ensemble average The ensemble averages of Γ_p^l and Γ_p^r are both equal to the β-independent value Γ_{el}, so the peak height (2) for strong inelastic scattering simplifies to $G_{in} \approx \frac{1}{2}\Gamma_{el}(e^2/kT)P_{eq}(N)$ — independent of whether time-reversal symmetry is broken or not This explains why the enhancement factor drops from 4/3 to 1 as kT becomes larger than Δ in the case of strong inelastic scattering

For a quantitative comparison, we have plotted in Fig 1 the temperature dependence of the parameter

$$\alpha = 1 - \bar{G}_{in}^{\max}(\beta=1)/\bar{G}_{in}^{\max}(\beta=2), \tag{4}$$

which drops from 1/4 to 0 as kT becomes larger than Δ The solid curve is for equally-spaced spin degenerate levels ($E_{2p}=E_{2p-1}=p\Delta$, $\Gamma_{2p}=\Gamma_{2p-1}$) Because the spin degeneracy might be lifted spontaneously,[10] we also show for comparison the case of equally spaced nondegenerate levels ($E_p=p\Delta/2$, all Γ_p's independent) In either case Δ is defined as the mean level spacing of a single spin degree of freedom We see that the temperature dependence is stronger for nondegenerate levels An even stronger temperature dependence (not shown) is found if, instead of equally spaced levels, we would use a Wigner-Dyson distribution The data points are the experimental results of Folk *et al*,[4] for GaAs quantum dots of four different areas The values of Δ used are those given in Ref 4, estimated from the area A and the two-dimensional density of states ($\Delta = 2\pi\hbar^2/mA$, with m the effective mass of the electrons) There is therefore no adjustable parameter in the comparison between theory and experiment

It is clear from Fig 1 that the experimental temperature dependence is much weaker than the theoretical prediction, regardless of whether we include spin degeneracy or not We have found that the theory would fit the data within the error bars if we would rescale kT/Δ by a factor of 3 (with spin degeneracy) or a factor of 5 (without spin degeneracy) Such a large factor is beyond the experimental uncertainty in level spacing or temperature We conclude that the inelastic scattering rate is well below Γ_{el} and $\Delta/\hbar$ for a range of energies within kT One possible explanation of the deviation of our theoretical curves from the experimental data would be that only the high-lying levels have equilibrated, while the low-lying levels have not Such an explanation would be consistent with the scenario put forward in Ref 2, according to which the discreteness of the spectrum prevents the low-lying levels to equilibrate on an arbitrarily long time scale

We conclude with two suggestions for future research on this topic From the theoretical side, it would be useful to generalize Ref 8 to an arbitrary ratio of Γ_{el} and Γ_{in} [going beyond the two limits of large and small Γ_{el}/Γ_{in} given in Eqs (1) and (2)] From the experimental side, it would be of interest to compare data for the temperature dependence of α for different values of Γ_{el}, that is to say, for different heights of the tunnel barriers separating the quantum dot from the electron reservoirs We would expect the data points in Fig 1 to approach the theoretical curves as the tunnel barriers are made higher and higher, giving more precise information on the rate of inelastic scattering

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[7] Folk *et al* (Ref 4) interpret their data in terms of the dephasing rate Γ_ϕ, but we would argue that their experiment is more sensitive to the inelastic scattering rate Γ_{in} than to Γ_ϕ It is inelastic scattering that destroys the $\frac{4}{3}$-enhancement of the conductance peaks — not dephasing Indeed, the theories of Refs 4–6 use a model of "sequential tunneling," in which there is no phase coherence at all between the electron entering and leaving the quantum dot It is irrelevant for the $\frac{4}{3}$ enhancement whether

such phase coherence exists or not One can conclude from the experimental data that $\Gamma_{in} \lesssim \Gamma_{el}, \Gamma_{\phi} \ll \Delta/\hbar$, but the relative magnitude of Γ_{el} and Γ_{ϕ} remains undetermined

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