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## Investigation of variations in the period of sixteen bright short-period eclipsing binary stars

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## INVESTIGATION OF VARIATIONS IN THE PERIOD OF SIXTEEN BRIGHT SHORT-PERIOD ECLIPSING BINARY STARS

BY K. K. KWEE

During the years 1951 to 1954, 57 epochs of minimum for 16 bright short-period eclipsing variables have been observed photoelectrically. The observations gave mean errors ranging from  $\pm 0.0008$  to  $\pm 0.0014$  for the resulting epochs. The period variations of the 16 systems are described and discussed in sections 8 and 9. Two kinds of period variations were found: small and rapid fluctuations, which are probably caused by distortions of the light-curve, and variations extending over long periods of time. The latter, which have been known for several years, can be interpreted as a sequence of more or less abrupt changes in period. There are indications that most of the systems discussed are near a state of dynamical instability. A general formula (4) is derived expressing variations of the period in variations of the total orbital momentum and of the masses of the components. The case of transfer of mass from one component to the other is considered. If this is the cause of the observed changes in the periods, an amount of mass of  $10^{-5}$  or  $10^{-6}$  times the mass of the emitting component should be involved. Finally, a number of objections are given against the transfer of mass as the only cause of the observed changes in period.

### 1. Introduction

Variations in the periods of eclipsing variables have always puzzled investigators of variable stars. Periodic variations in the period of those systems which have periods of two days or more can often be satisfactorily explained by the existence of a third component in the system. For systems with short periods, however, the problem is more intricate. Often the same mechanism as for long-period systems was adopted to explain the observed period variations. But as the observations covered more and more time and as their accuracy increased, the complexity of the problem grew also. Especially the progress in the technique of observation plays an important role in this development. With the early visual and photographic techniques the accuracy of the observed times of minimum was low and it took a long time before any change in the period of an eclipsing binary could be detected. For this reason only large variations in the period could be noticed. In order to obtain a better insight in the true character of these variations in period, accurate observations are badly needed.

At the suggestion of Prof. P. TH. OOSTERHOFF a programme was set up in the autumn of 1951 of photoelectric observations of minimum times for a

number of bright short-period eclipsing variables for several years. The observations were discontinued in the summer of 1954, mainly on account of the bad atmospheric conditions at the Leiden Observatory, where all observations were made. In these three years a total of only 57 epochs of minimum have been obtained, but the results derived therefrom indicate that a similar investigation carried out in a good climate would be worth while and might lead to important conclusions.

### 2. The systems investigated

The eclipsing variables which were included in this investigation were selected according to the following three criteria:

a) The apparent magnitude in maximum light should be brighter than  $10^m.0$ . This restriction was necessitated by the visual power of the guiding telescope of the reflector used. In those years guiding could not yet be done at the Newtonian focus of the primary mirror.

b) The period should be shorter than  $0.75$ . This limiting value was taken more or less arbitrarily.

c) The declination of the variable star should be larger than  $-15^\circ$ . For the Leiden Observatory this limiting declination corresponds to a zenith distance

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of about  $67^\circ$  in the meridian. For lower declinations the corrections for differential extinction are too complicated.

In the *General Catalogue of Variable Stars* (KUKARKIN and PARENAGO 1948) seventeen systems, classified as EB and satisfying these criteria, were found. Sixteen of these are listed in column 1 of Table 1. For the seventeenth system, W UMa, the results have already been published (KWEI 1956b), together with a discussion of a complete photoelectric light-curve.

### 3. The instruments

The telescope used was the 18" Zunderman reflector of the Leiden Observatory with the same photometer as was used by A. B. MULLER (1953) for his second series of photoelectric observations of XZ Cyg. A detailed description and discussion of the optical and electrical parts of this equipment have been given by H. VAN WOERDEN (1957) in his article about SV Cam. In addition to this it should be mentioned that on January 20, 1954, the amplifier was replaced by another one, of which the input condenser can be varied. This improvement opened the possibility to choose an adequate time constant for each amplification and to adjust it to the magnitude of the star as well as to the noise fluctuations caused by the atmospheric conditions. This new amplifier was built with the same kind of electronic units as the former one, so that the physical properties remained the same as described by H. VAN WOERDEN.

As a rule no colour filters have been applied; only when the spectral type of the comparison star differed too much from that of the variable, as was the case with 44i Boo, an orange filter, Corning 3482, was placed in front of the photo-multiplier. Neutral filters, Schott NG 3, NG 4, NG 8 and several combinations of the three, have been used when the magnitude differences were too large.

### 4. The comparison stars

The comparison stars were chosen so as to have about the same magnitude and spectral type as the variable. In Table 1 some data of the comparison stars are listed, together with those of the variable stars. Columns 2 and 3 give the Henry Draper Catalogue number and the Bonner Durchmusterung number, respectively, columns 4 and 5 the approximate coordinates (epoch of 1950), and the last two columns the spectral type and the photometric magnitude of the stars taken from the Henry Draper Catalogue; for ER Ori and U Peg the magnitude was taken from the Bonner Durchmusterung.

For 44i Boo no comparison star with the same magnitude and spectral type could be found, so three stars were successively tried, using the orange filter

TABLE I  
Eclipsing systems investigated and their comparison stars.

Star	HD No	BD No	$\alpha$ (1950)	$\delta$ (1950)	Sp	<i>m</i>
OO Aql	187183	+ 8 4224	<sup>h m</sup> 19 45.8	+ 09 13	G5	9.3
	187377	+ 8 4228	19 47.0	+ 09 05	G5	9.5
44i Boo	133640	+ 48 2259	15 02.2	+ 47 51	G0	6
	133962	+ 48 2262	15 03.8	+ 48 21	A0	5.59
	133483	+ 48 2258	15 01.3	+ 47 56	G5	8.8
	133268	+ 48 2256	15 00.2	+ 48 08	G	9.4
DO Cas	16506	+ 59 529	02 37.5	+ 60 20	A2	8.46
	17356	+ 59 548	02 45.8	+ 60 24	F0	8.58
	15785	+ 59 513	02 31.1	+ 60 19	B	8.41
VW Cep	197433	+ 75 752	20 40.5	+ 75 24	G5	7.62
	199476	+ 74 889	20 53.9	+ 74 33	G5	7.9
GO Cyg	196628	+ 34 4095	20 35.4	+ 35 16	A0	8.27
	196771	+ 35 4197	20 36.3	+ 35 28	A2	8.27
YY Eri	26609	- 10 858	04 09.8	- 10 36	G5	8.8
	26600	- 9 844	04 09.6	- 09 24	G0	8.7
AK Her	155937	+ 16 3130	17 11.7	+ 16 25	F8	8.3
	155676	+ 16 3123	17 10.2	+ 16 34	F8	8.6
SW Lac	216598	+ 37 4717	22 51.4	+ 37 39	G5	8.8
	216342	+ 37 4710	22 49.3	+ 38 25	K2	8.9
V502 Oph	150484	+ 0 3562	16 38.8	+ 00 36	G0	8.9
	150732	+ 0 3569	16 40.3	+ 00 11	G0	8.5
V566 Oph	163611	+ 5 3547	17 54.4	+ 05 00	F5	7.46
	163697	+ 4 3558	17 54.9	+ 04 53	F5	8.50
ER Ori		- 8 1050	05 08.8	- 08 37		9.0
		- 8 1051	05 08.9	- 08 41		9.0
U Peg		+ 15 4915	23 55.4	+ 15 40		9.3
		+ 14 5078	23 55.2	+ 15 11		9.3
RS Sct		- 10 4814	18 46.4	- 10 18	F5	9.8
	174085	- 10 4816	18 46.5	- 10 39	F5	9.4
AG Vir	104350	+ 13 2481	11 58.5	+ 13 17	A0	8.6
	105163	+ 13 2490	12 04.0	+ 12 42	A2	9.0
	104381	+ 13 2482	11 58.7	+ 12 39	A0	6.93
AH Vir	106400	+ 12 2437	12 11.8	+ 12 06	K0	8.7
	106187	+ 12 2434	12 10.6	+ 12 13	K0	8.9
BF Vir	120166	+ 0 3102	13 45.3	- 00 21	A0	9.68
	119933	- 0 2736	13 43.8	- 01 00	A2	9.7

or the neutral filter combinations mentioned in the preceding section. The comparison star HD 15785 for DO Cas was used by Mr C. J. VAN HOUTEN, whose observations are also mentioned in this article. Moreover, neutral filters have been used for the observations of DO Cas, VW Cep, V566 Oph, RS Sct and AG Vir.

BD + 35° 4197 was used as a comparison star for

GO Cyg. However, in his article about this variable, which reached the author after the present programme was concluded, OVENDEN (1954) suspected this comparison star of short-period variability. In a private communication OVENDEN states that the internal deviations of his measurements of this star were much larger than could be accounted for by observational errors and fluctuations of the extinction. Therefore on September 24, 1956, the suspected variability of the star was investigated by checking the intensity of BD + 35° 4197 against OVENDEN's other comparison star BD + 34° 4098. In total 70 measurements of each star were made covering a time interval of 7 hours. When the proper extinction corrections had been applied, the magnitude difference of the two stars remained constant with a mean deviation of <sup>m</sup>.01. We can also test the constancy of BD + 35° 4197 by comparing the magnitude differences ( $m_v - m_c$ ) for the same phases on the three nights: 10-X-1951, 23-X-1952, and 16-VIII 1953, on which GO Cyg was observed for the present programme. If the magnitude of BD + 35° 4197 is subject to short-period fluctuations, the differences of ( $m_v - m_c$ ) for different nights must also show these fluctuations. When these differences were plotted it appeared that no fluctuations larger than <sup>m</sup>.02 were present and that the difference remained constant with a mean deviation of <sup>m</sup>.01. We may conclude that BD + 35° 4197 was constant at the times we observed it, and that it did not affect the determination of the minimum epochs of GO Cyg significantly. It should be remarked, however, that LIAU (1935) also suspected BD + 35° 4197 of variability, while PAYNE-GAPOSCHKIN (1935) found no evidence of variability. Therefore it seems indicated that special precautions be taken if this star is used as a comparison star for GO Cyg.

### 5. The observations

The observations were made from August, 1951, to June, 1954. As the only aim of this investigation was to determine accurate times of minimum, no attention was paid to the zero points of the magnitudes derived on different nights. The observations were made by measuring alternately the intensities of the comparison star and of the variable, while the sky was measured between each pair of star measurements. The time needed for one such cycle: comparison star — sky — variable star — sky, varied from one to three and a half minutes, depending mainly on the magnitude of the star and on the use of neutral filters.

The observation of a minimum was considered successful when on one night a complete minimum including descending and ascending branches had

been observed. Series of observations during which clouds or other disturbances interfered have not been used in this article.

### 6. The reductions

The star intensities on the Brown recorder sheets, from which the sky intensities were subtracted, were read off with an accuracy of about .5 percent. The magnitude differences ( $m_v - m_c$ ) were derived in three decimals and were corrected for differential extinction. An approximate value for the extinction coefficient, which was sufficient for this purpose, was obtained from the intensity variation of the comparison star during the time of observation. In cases where this was not possible, a mean value of <sup>m</sup>.6 for the extinction coefficient was adopted. The values of the observed extinction coefficient range from <sup>m</sup>.3 to <sup>m</sup>.3; on most nights, however, this value lies between <sup>m</sup>.4 and <sup>m</sup>.8.

The times were measured in seconds of civil time and were finally computed in five decimals of a Julian day, after reduction to the sun had been applied.

In Table 5 all the individual observations have been listed: the two columns represent the heliocentric Julian days and the magnitude differences ( $m_v - m_c$ ) corrected for differential extinction. In Figure 2 four arbitrarily chosen samples of an observed minimum have been plotted.

### 7. The computation of the epochs

The method used for the derivation of the epochs of minimum has been described in a former article (KWEE and VAN WOERDEN 1956a). Magnitudes, equidistant in time, were computed by linear interpolation between the observed magnitude differences. The total number of interpolated magnitudes was made about equal to the total number of observations used. Three sums,  $s(T_1)$ ,  $s(T_1 + \frac{1}{2}\Delta t)$  and  $s(T_1 - \frac{1}{2}\Delta t)$ , were derived, from which the epoch and its mean error were computed.

For different minima of one variable star, the same phase interval has always been used for the computation of the epoch. Where the observations covered too short a phase interval to fulfil this condition, a correction has been applied to the obtained epoch. This correction, which in general appeared to be rather insignificant with respect to the mean error, was derived experimentally from the observed complete minima of the star. In the case of a real asymmetry in the minima, differential effects on the epochs caused by incompleteness of the observed minima could thus be avoided as much as possible.

The results are given in Table 2. The meaning of the different columns is:

*Column 1:* designation of the variable, arranged

TABLE 2  
Observed minima

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Designation	Date	Min. Hel. J.D.	m.e. of epoch	$M$	$N$	m.e. of one obs.	$\Delta P$	$O - C$
OO Aql	5 X 1951	2433925.37418 <sup>d</sup>	.00029 <sup>d</sup>	93	63	.026 <sup>m</sup>	.164 <sup>P</sup>	+ .00079 <sup>d</sup>
	30 VI 1952	2434194.48196	.00019	48	37	.014	.156	- .00010
	10 VIII 1953	2434600.42550	.00016	79	57	.018	.164	- .00016
44i Boo	29 IV 1952	2434132.51692	.00031	160	61	.007	.237	- .00068
	5 VII 1952	2434199.47151	.00036	127	73	.016	.233	+ .00090
	10 IV 1953	2434478.53146	.00076	56	56	.015	.233	+ .00070
	3 V 1953	2434501.56168	.00047	76	56	.007	.233	- .00091
	7 I 1954	2434750.62786	.00082	66	28	.013	.226	+ .00007
	21 I 1954	2434764.55410	.00032	67	31	.007	.233	+ .00009
	25 I 1954	2434768.57067	.00085	78	37	.013	.233	- .00053
1 II 1954	2434775.53238	.00061	58	40	.012	.233	- .00193	
DO Cas	6 X 1951	2433926.45752	.00012	186	141	.009	.183	+ .00023
	15 X 1951	2433935.35748	.00015	274	159	.011	.183	- .00044
	15 IX 1953	2434636.45601	.00016	114	110	.011	.183	+ .00013
VW Cep	8 IX 1951	2433898.44198	.00037	86	47	.015	.201	- .00037
	12 X 1951	2433932.39698	.00019	176	61	.009	.200	- .00006
	16 X 1951	2433936.29364	.00020	63	53	.009	.202	+ .00016
	13 VIII 1953	2434603.41956	.00016	79	46	.007	.201	.00000
GO Cyg	10 X 1951	2433930.40600	.00021	161	101	.010	.135	- .00068
	23 X 1952	2434309.38563	.00033	102	65	.006	(.135)	+ .00013
	16 VIII 1953	2434606.53982	.00015	102	90	.008	.136	+ .00045
YY Eri	26 IX 1953	2434647.59263	.00020	56	41	.014	.259	- .00003
AK Her	11 III 1954	2434813.59075	.00037	60	47	.015	.203	+ .00043
	26 IV 1954	2434859.53645	.00025	73	52	.011	.200	.00000
SW Lac II	3 X 1951	2433923.54460	.00022	81	61	.019	.217	- .00179
	I 8 X 1951	2433928.51748	.00015	124	58	.016	.217	- .00011
	I 11 X 1951	2433931.40381	.00008	137	66	.011	.214	- .00028
	I 12 XII 1951	2433993.30416	.00015	86	70	.018	.214	+ .00057
	I 15 IX 1952	2434271.37095	.00013	102	69	.016	.214	+ .00072
	II 10 VIII 1953	2434600.58995	.00029	68	50	.011	(.217)	- .00220
V502 Oph	17 V 1953	2434515.46166	.00084	38	32	.006	(.180)	- .22639
	7 IV 1954	2434840.54647	.00029	57	48	.010	.180	- .24300
V566 Oph	17 V 1953	2434515.56423	.00069	38	29	.016	(.180)	+ .00033
	3 VIII 1953	2434593.39559	.00021	61	47	.009	.180	- .00014
ER Ori	9 X 1951	2433929.61754	.00020	79	51	.013	.195	+ .00247
	22 I 1954	2434765.41663	.00027	64	49	.020	.197	- .00228
	2 II 1954	2434776.42586	.00043	55	47	.020	.189	- .00161
U Peg	4 X 1951	2433924.54967	.00020	66	60	.013	.222	- .00007
	17 X 1952	2434303.45450	.00009	107	85	.008	.222	- .00031
	3 XI 1953	2434685.35860	.00027	58	48	.016	.227	+ .00047
RS Sct	8 VIII 1953	2434598.46301	.00020	42	38	.016	.094	+ .01200
AG Vir	14 III 1952	2434086.41948	.00036	133	92	.018	.195	+ .00185
	17 IV 1952	2434120.47868	.00037	145	94	.015	.195	+ .00065
	19 IV 1953	2434487.42968	.00046	99	87	.021	.197	- .00097
	2 II 1954	2434776.62146	.00036	112	103	.017	.193	- .00127
AH Vir	22 III 1952	2434094.41509	.00026	65	28	.009	(.242)	+ .00070
	18 IV 1953	2434486.44716	.00021	89	72	.014	.242	+ .00001
	10 V 1953	2434508.45296	.00009	78	76	.008	.242	- .00018
	8 IV 1954	2434841.39628	.00021	63	58	.012	(.242)	+ .00055
BF Vir	16 IV 1952	2434119.49335	.00020	100	90	.021	.228	- .00068
	8 V 1954	2434871.53121	.00141	60	23	.029	(.228)	+ .00084



alphabetically according to constellation. For SW Lac I means primary and II, secondary minimum.

*Column 2:* observing date; for observations after midnight the date of the preceding day is given.

*Column 3:* computed epoch of minimum in heliocentric Julian days.

*Column 4:* mean error of the epoch.

*Column 5:* total number  $M$  of observations of the variable made during the night.

*Column 6:* number  $N$  of observations used for the computation of the epoch.

*Column 7:* mean deviation of a single observation from a theoretical symmetrical light-curve. This quantity can easily be derived from the mean error of the epoch. When no asymmetry is present in the eclipse curve, this quantity can be used as a measure for the quality of the night. Otherwise it gives an upper limit for the mean error of one observation.

*Column 8:* phase interval  $\Delta P$  from which the resulting epoch was derived. Parentheses mean that the observations did not cover the required interval, and that a correction as described above has been applied.

*Column 9:* residuals of the observed epochs computed with light-elements which will be discussed in the following section.

### 8. Description of the periods

**OO Aquilae:** Earlier observations are too scarce to give reliable information about possible variations of the period. Therefore the following new linear light-elements were derived including all known observed times of minimum (for references see WOOD 1953).

$$\text{Min.} = \text{Hel. J.D. } 2433925.37339 + .50679600 E. \\ \pm .00016 \pm .00000006 \text{ m.e.}$$

The residuals of the three new epochs from these elements are given in column 9 of Table 2. Although the first residual is rather large, there is no evidence yet of a variable period.

**44i Bootis:** Variations in the period of this eclipsing variable have been discussed extensively by BINNENDIJK (1955), by ABRAMI and CESTER (1956) and by SCHMIDT and SCHRICK (1957). The epochs published by BINNENDIJK, who had received preliminary results from the present investigation, differ slightly from those given here, owing to the fact that for the present paper the epochs were recomputed with a somewhat smaller phase interval, about equal for all minima. The differences between the two sets of epochs are quite insignificant and do not affect any of his conclusions. In column 9 of Table 2 the new residuals from BINNENDIJK's elements:

$$\text{Min.} = \text{Hel. J.D. } 2434132.5176 + .26781204 E \\ \text{are given.}$$

**DO Cassiopeiae:** Besides a few earlier visual and photographic observations, photoelectric observations have been published by SCHNELLER and DAENE (1952). From their list of individual observations the author has computed one well determined epoch of minimum: Hel. J.D.  $2433937^d.41144 \pm ^d.00045$  m.e. Furthermore, Mr C. J. VAN HOUTEN has made some photoelectric observations of this system at the Leiden Observatory. He has kindly put his measurements at the disposal of the present author, who has derived the following well observed epoch from this material: Hel. J.D.  $2434269^d.47501 \pm ^d.00047$  m.e.

The five photoelectric epochs, together with older visual and photographic observations, by AHNERT, HOFFMEISTER, ROHLFS and VAN DE VOORDE (1947), and by LORETA (1940), give the following new light-elements:

$$\text{Min.} = \text{Hel. J.D. } 2433926^d.45729 + .68466595 E. \\ \pm .00021 \pm .00000040 \text{ m.e.}$$

The residuals of the new epochs from these elements are given in column 9 of Table 2; those of the epochs by SCHNELLER and DAENE and by VAN HOUTEN are  $-^d.00050$  and  $+^d.00008$ , respectively. All the earlier observations are satisfactorily represented by these elements and consequently there is no indication for a change in the period.

**VW Cephei:** This system has frequently been observed for nearly 30 years, during which time interval the variation in the period has changed sign twice. A description of this has been given recently by SCHMIDT and SCHRICK (1955). The present four epochs fit rather well in this description. The residuals from the elements used by SCHMIDT and SCHRICK:

$$\text{Min.} = \text{Hel. J.D. } 2424658^d.759 + ^d.27831993 E$$

are for the four minima successively:  $+^d.01794$ ,  $+^d.01791$ ,  $+^d.01809$ , and  $+^d.01114$ . The accuracy of the three later epochs is very high, owing to the exceptionally good atmospheric conditions. New light-elements were computed from the four new epochs only, to be used for predictions in the near future:

$$\text{Min.} = \text{Hel. J.D. } 2433898.44235 + .27831710 E. \\ \pm .00019 \pm .00000012 \text{ m.e.}$$

This formula leaves the residuals which are given in column 9 of Table 2.

As has already been mentioned, the minima were observed under excellent conditions. No asymmetry seems to be present during mid-eclipse, while at the brighter end the descending branch is steeper than the ascending one, owing to the unequal height of the maxima. On October 12, 1951, the maximum preceding the primary minimum was about  $^m.08$

brighter than the following maximum, which can also be seen from Figure 2.

**GO Cygni:** The present three minima have residuals of  $+^d.00179$ ,  $+^d.00277$ , and  $+^d.00324$  with the light-elements given by OVENDEN (1954):

$$\text{Min.} = \text{Hel. J.D. } 2433861^d.499 + ^d.7177626 E.$$

They show a systematic increase with time. An improvement of the elements from all available observations (for references, see OVENDEN 1954, KULIKOVSKY 1939 and POHL 1950) resulted in:

$$\text{Min.} = \text{Hel. J.D. } 2433930^d.40668 + ^d.71776291 E. \\ \pm .00024 \pm .0000020 \text{ m.e.}$$

These elements yield the residuals which are given in column 9 of Table 2. They still show a slight systematic run. In view of the existing dispute about the constancy of the comparison star BD  $+ 35^\circ 4197$  (see section 4), it would be premature to interpret these deviations as a variation of the period.

**YY Eridani:** Besides earlier photographic and visual observations by JENSCH, LAUSE and BODOKIA (for references see CILLIÉ's publication), photoelectric observations by CILLIÉ (1951) and by HURUHATA, DAMBARA and KITAMURA (1953) are available. The present photoelectric epoch gives a large residual of  $+^d.00263$  with CILLIÉ's light-elements:

$$\text{Min.} = \text{Hel. J.D. } 2427364^d.440 + ^d.32149510 E.$$

The photoelectric epochs of CILLIÉ and HURUHATA combined with the present one are well represented by the elements:

$$\text{Min.} = \text{Hel. J.D. } 2433580^d.54783 + ^d.32149588 E. \\ \pm .00008 \pm .0000007 \text{ m.e.}$$

The residual in column 9 of Table 2 was computed with this ephemeris. The new period is significantly different from the value derived by CILLIÉ. It is impossible to derive a satisfactory linear expression for the light-elements, covering the earlier observations also. We conclude therefore that the period has changed, and has increased since the time when the older observations were made.

**AK Hercules:** Earlier visual and photographic observations, which already show a slow continuous variation in the period, have been thoroughly discussed by Mrs E. J. WOODWARD (1941). She also gave the following elements:

$$\text{Min.} = \text{Hel. J.D. } 2422977^d.254 + ^d.42152207 E.$$

More recent photoelectric observations have been made by SEYFERT and MASON (1951) and by LABS and STOCK (1953). According to information received

from Dr L. BINNENDIJK of the Flower and Cook Observatories, the epoch published by SEYFERT and MASON is not heliocentric. This epoch should read: Hel. J.D.  $2433515^d.718$ , having a residual of  $-^d.0093$  from WOODWARD's elements. The data in the publication by LABS and STOCK also need a correction. The formulæ and residuals they give do not agree with each other. Dr BINNENDIJK, who received a list of the individual observations from one of the authors, kindly communicated to me the correct epoch, which he had recomputed himself. This epoch, Hel. J.D.  $2434153^d.4827$ , leaves a residual of  $-^d.0075$  from WOODWARD's elements.

The residuals of the present two epochs from WOODWARD's elements are, successively,  $-^d.00298$  and  $-^d.00318$ . The four photoelectric epochs can be represented fairly well by a linear formula:

$$\text{Min.} = \text{Hel. J.D. } 2434813^d.59032 + ^d.42152412 E. \\ \pm .00049 \pm .0000029 \text{ m.e.}$$

The residuals of the two new epochs from these elements are given in column 9 of Table 2; those of the epochs by SEYFERT and MASON, and by LABS and STOCK are  $+^d.00043$  and  $-^d.00086$ , respectively.

It is clear that after a continuous decrease of the period until 1940, the variation has changed sign and that the period has increased again considerably, which fully confirms similar conclusions by SEYFERT and MASON, and by LABS and STOCK.

**SW Lacertae:** The period changes of this variable star have also been discussed extensively by WOODWARD (1941 and 1951). The residuals of the present epochs from her light-elements:

$$\text{Min.} = \text{Hel. J.D. } 2423372^d.780 + ^d.32071464 E$$

are, successively:  $+^d.05473$ ,  $+^d.05653$ ,  $+^d.05643$ ,  $+^d.05886$ ,  $+^d.06605$ , and  $+^d.07148$ . The expectation that the period would decrease further after WOODWARD's observations, has not been realized. More recently BROWNLEE (1957) has made photoelectric observations. Normal epochs have been formed from his tabulated observed minima. Weight factors from 1 to 4 have been assigned to the individual epochs. The results are:

$$\begin{aligned} \text{Min. I:} & \text{Hel. J.D. } 2434668.74513 \pm .00022 \text{ m.e.} \\ \text{Min. II:} & \text{Hel. J.D. } 2434668.90534 \pm .00034 \text{ m.e.} \\ \text{Min. I:} & \text{Hel. J.D. } 2435037.57504 \pm .00056 \text{ m.e.} \\ \text{Min. II:} & \text{Hel. J.D. } 2435037.73428 \pm .00062 \text{ m.e.} \end{aligned}$$

Combined with the present observations the primary minima alone give the following light-elements:

$$\text{Min.} = \text{Hel. J.D. } 2434271.37023 + ^d.32072277 E. \\ \pm .00032 \pm .0000036 \text{ m.e.}$$

The residuals given in column 9 of Table 2 refer to these elements. The residuals of BROWNLEE's normal epochs are  $-^d.00061$ ,  $-^d.00076$ ,  $-^d.00189$  and  $-^d.00301$  respectively. Two conclusions can be drawn from these ten residuals: first, a quadratic term is distinctly present in the light-elements for this time interval, and secondly, the secondary minimum does not lie at  $P.5$ . The latter does not necessarily indicate orbital eccentricity, it may also occur in systems with distorted or unequal maxima. It should be noted that BROWNLEE found his minima to be asymmetric, while for the present minima no indications of asymmetry are present. The period of the system shows variations which seem to be erratic; it would therefore be worth while to keep this variable under observation in the future.

**V502 Ophiuchi:** Of this variable star only visual observations by LAUSE (1937) and photographic observations by NEKRASOVA (1943) have been published. Owing to the rather unfavourable position of the star in the sky (in declination as well as in right-ascension), we obtained only two additional epochs of minimum. The residuals given in Table 2, column 9, refer to the elements given in the *General Catalogue of Variable Stars* (KUKARKIN and PARENAGO 1948):

$$\text{Min.} = \text{Hel. J.D. } 2428684^d.493 + ^d.453419 E.$$

A discussion of the present observations will be reserved for a future publication by the author, in which an analysis of the complete light-curve of this variable, observed in the summer of 1956, will be given. The period has changed considerably; its present value is about  $P = ^d.453396$ .

**V566 Ophiuchi:** Recently FRESA (1954) published his photoelectric observations of this variable. He stated that the periods given by HOFFMEISTER and by TSESEVICH (for references see FRESA's paper) are erroneous. He derived the following elements:

$$\text{Min.} = \text{Hel. J.D. } 2430573^d.393 + ^d.4096623 E.$$

The present two epochs confirm the order of magnitude of his period. The residuals from FRESA's elements are:  $-^d.00908$  and  $-^d.01356$ , respectively.

Dr BINNENDIJK informed me that he also made photoelectric observations of this variable, and that he derived the elements:

$$\text{Min.} = \text{Hel. J.D. } 2434515^d.5639 + ^d.40964122 E.$$

The residuals in column 9 of Table 2 were computed from these elements. The few observations available are well represented by a linear ephemeris.

TABLE 3  
Primary minima of ER Orionis

Author		Min. Hel. J.D.	m.e.	$E$	$O-C_I$	$O-C_{II}$	Reference
FLORJA	vis	<sup>d</sup> 2426336.438		0	<sup>d</sup> + .0012	<sup>d</sup> + .1897	<i>Veränd. Sterne Gorki</i> 3, 86, 1931
FLORJA	"	2426387.243		120	- .0013	+ .1860	<i>Veränd. Sterne Gorki</i> 3, 86, 1931
FLORJA	"	2426409.267		172	+ .0062	+ .1969	<i>Veränd. Sterne Gorki</i> 3, 86, 1931
FLORJA	"	2426420.271		198	+ .0019	+ .1883	<i>Veränd. Sterne Gorki</i> 3, 86, 1931
FLORJA	"	2426600.2089		623	- .0033	+ .1786	<i>Veränd. Sterne Gorki</i> 4, 7, 1932
TSESEVICH	"	2427430.497	<sup>d</sup>	2584	+ .0062	+ .1671	<i>Publ. Odessa</i> 4, No. 2, 246, 1954
TECZA, SZCZYRBAK	"	2428566.460	$\pm .006$	5267	- .0009	+ .1313	<i>Acta Astr. serie c</i> , 4, 55, 1939
TECZA, SZCZYRBAK	"	2428906.445	$\pm .004$	6070	- .0025	+ .1211	<i>Acta Astr. serie c</i> , 4, 55, 1939
TECZA, SZCZYRBAK	"	2429265.481	$\pm .003$	6918	- .0059	+ .1086	<i>Acta Astr. serie c</i> , 4, 55, 1939
TSESEVICH	"	2431175.420		11429	- .0040	+ .0623	<i>Publ. Odessa</i> 4, No. 2, 246, 1954
ASHBROOK	"	2432971.895	$\pm .006$	15672	+ .0039	+ .0248	<i>A. J.</i> 58, 171, 1953
SZCZEPANOWSKA	"	2433682.354	$\pm .008$	17350	+ .0053	+ .0081	<i>Acta Astr. serie c</i> , 5, 74, 1955
KWEE	pe	2433929.61754	$\pm .00020$	17934	+ .00584	+ .00247	<i>present publication</i>
HURUHATA	"	2433960.9464		18008	+ .00344	- .00073	<i>Ann. Tokyo Astr. Obs. second series</i> , 5, 3, 1957
ASHBROOK	vis	2434366.564	$\pm .004$	18966	+ .0081	- .0063	<i>A. J.</i> 58, 171, 1953
KWEE	pe	2434765.41663	$\pm .00027$	19908	+ .02222	- .00228	<i>present publication</i>
KWEE	"	2434776.42586	$\pm .00043$	19934	+ .02316	- .00161	<i>present publication</i>
BINNENDIJK	"	2435474.62643		21583	+ .04455	+ .00213	<i>correspondence</i>

**ER Orionis:** In Table 3 all the observed primary minima of this variable have been compiled: columns one to four and column seven give the author, the epoch of minimum, its mean error, the number of the epoch, and the reference, respectively. The last epoch given in the table was kindly communicated to me by Dr BINNENDIJK, who also made photoelectric observations of this system. In column five the residu-

als are given of the observed epochs from the elements by TSESEVICH (1947):

$$\text{Min.} = \text{Hel. J.D. } 2426336^d.4368 + ^d.4233955 E.$$

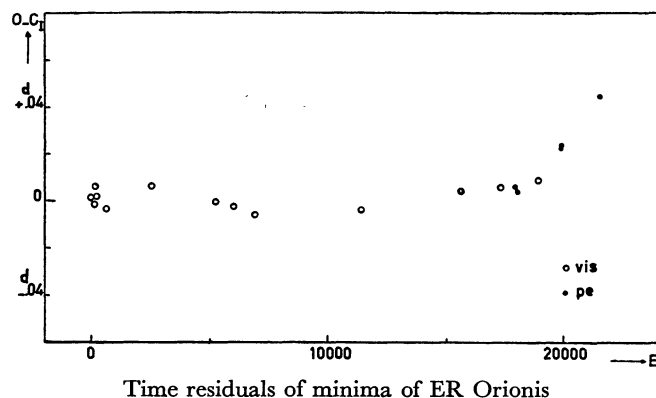
They have been plotted against  $E$  in the accompanying Figure 1. This figure shows that the observations satisfied TSESEVICH's elements fairly well up to about 1952 ( $E = 18500$ ). Only a small quadratic term may



be added to obtain a still better representation. After 1952 the period has increased very strongly. When the photoelectric minima alone are used for the computation of new light-elements, we obtain:

$$\text{Min.} = \text{Hel. J.D. } 2434765.41891 + .42340620 (E - 19908) \\ \pm .00118 \pm .0000082 \text{ m.e.}$$

FIGURE 1



The residuals of the observations from these elements are given in column 6 of Table 3 and also in column 9 of Table 2. The periods in the two formulae differ by about 1 second and the residuals ( $O - C_{II}$ ) show that a considerable quadratic term is still present in the time interval covered by the photoelectric observations. Figure 1 also shows that the change in the period around  $E = 18500$  must have taken place very rapidly.

**UPegasi:** A list of observed minima for this variable has been given by RECILLAS and WOODWARD (1945). Furthermore, observations have been made by TSESEVICH (1954), SZCZEPANOWSKA (1955), LAFARA (1952) and HURUHATA, KITAMURA, NAKAMURA and TANABE (1957b). All these observations show that the period has decreased continuously. LAFARA, who also made photoelectric observations, computed light-elements from his observations combined with the epoch by RECILLAS and WOODWARD. He found:

$$\text{Min.} = \text{Hel. J.D. } 2430260.6790 + .3747821575 E.$$

The present three photoelectric epochs, with residuals of  $+ .00030$ ,  $+ .00037$ , and  $+ .00145$ , are not represented accurately by these elements. A small systematic positive deviation is present, which indicates a small increase of the period. New elements derived from LAFARA's observations and the present ones, are:

$$\text{Min.} = \text{Hel. J.D. } 2433182.85525 + .374782246 E. \\ \pm .00030 \pm .0000016 \text{ m.e.}$$

The residuals from these elements are given in column 9 of Table 2.

**RS Scuti:** With its declination of  $-10^\circ$ , this variable lies near the limit of what can be observed at the Leiden Observatory. Its right-ascension is also very unfavourable, as the star culminates at midnight during the summer. Due to these circumstances, only one complete minimum has been observed in the present programme. Its accuracy, however, is high, due to the steep branches of the minimum.

Earlier observations have been compiled by PIOTROWSKI (1936). Their residuals computed with KORDYLEWSKI's light-elements (also used by PIOTROWSKI):

$$\text{Min.} = \text{Hel. J.D. } 2424814.2400 + .664237 E,$$

show large individual deviations, as was also noted by PIOTROWSKI. As the star is situated in one of the Franklin-Adams fields of which plates are available at the Leiden Observatory, Mr VAN HOUTEN has kindly investigated the variable on these plates at the request of the author. He found eleven primary minima, which are given in the accompanying Table 4, together with the epoch number  $E$  and the residuals from KORDYLEWSKI's elements.

TABLE 4  
Photographic minima of RS Scuti  
found by C. J. VAN HOUTEN on Franklin-Adams plates.

Min. Hel. J.D.	$E$	$O - C$
$d$		$d$
2427979.323	+ 4765	- .0063
2427985.296	+ 4774	- .0114
2427988.634	+ 4779	+ .0054
2428772.439	+ 5959	+ .0107
2428788.362	+ 5983	- .0080
2429015.529	+ 6325	- .0100
2429049.412	+ 6376	- .0031
2429077.323	+ 6418	+ .0099
2429097.249	+ 6448	+ .0088
2429407.450	+ 6915	+ .0111
2429546.273	+ 7124	+ .0086

The present photoelectric epoch gives a residual of  $+ .01200$  from the same elements. This large positive value may indicate a change in period, because no linear relation can be found which represents all observations satisfactorily. More accurate observations are required to confirm this conclusion.

**AG Virginis:** This variable has been discussed recently by WOOD (1946), who also gave references to earlier observations. WOOD's light-elements:

$$\text{Min.} = \text{Hel. J.D. } 2426418.9852 + .6426462 E$$

yield residuals of  $+^d.02247$ ,  $+^d.02142$ ,  $+^d.02144$ , and  $+^d.02243$  for the four epochs from Table 2. It is clear that these elements do not satisfy the new observations. A new formula was derived, using only WOOD's observations and the present ones. The new elements:

$$\text{Min.} = \text{Hel. J.D. } 2434086.41763 + .64264907 E \\ \pm .00057 \pm .0000010 \text{ m.e.}$$

have the residuals which are given in column 9 of Table 2. Computed with this formula the earlier observations have residuals which are slightly positive in the mean; however, the scatter of the individual observations is too large to draw any conclusions. The residuals of the four new epochs, however, clearly show a systematic run, which is larger than can be accounted for by the mean errors. The period which fits these four epochs satisfactorily is  $P = ^d.64264648 \pm ^d.00000075$  m.e. It differs significantly from the period derived above, but is about the same as WOOD's period. Therefore, a jump in phase has probably taken place during the time interval between WOOD's observations and those of this article, which, of course, means that the period must have changed at least twice during this time.

Finally it should be mentioned that the central part of the minimum is asymmetric. Near minimum the ascending branch is less steep than the descending branch. This feature, which recurs in all four minima, was also present in WOOD's photoelectric observations. It is shown in Figure 2.

**AH Virginis:** The present four epochs have residuals of  $+^d.00564$ ,  $+^d.00549$ ,  $+^d.00532$  and  $+^d.00652$  with the light-elements given by NASON and MOORE (1951):

$$\text{Min.} = \text{Hel. J.D. } 2433389^d.811 + ^d.4075179 E.$$

They are large and systematic. New light-elements, computed from the four new epochs only, are:

$$\text{Min.} = \text{Hel. J.D. } 2434094.41439 + .40751846 E. \\ \pm .00061 \pm .00000055 \text{ m.e.}$$

The residuals computed with these elements are given in column 9 of Table 2. Although the new period does not differ significantly from NASON and MOORE's period, the new light-elements yield a large residual of  $-^d.0040$  for NASON and MOORE's photoelectric epoch. It is impossible to derive a satisfactory linear representation of all five photoelectric epochs.

The present four minima are of high quality and they all show a constant light-intensity during mid-eclipse, lasting 45 minutes (Figure 2).

**BF Virginis:** A previous note on this variable has been published earlier (KWEI 1952). Owing to rather unfavourable observing conditions (low declination),

only one new epoch of minimum was obtained. In Table 2 of this paper only the two complete minima are given. They show residuals of  $+^d.00135$  and  $+^d.00357$  from the light-elements of the previous publication:

$$\text{Min.} = \text{Hel. J.D. } 2434119^d.492 + ^d.6405755 E.$$

Improved elements were derived from all available epochs, including photographic observations by WHITNEY (1955), but omitting SOLOVIEV's first epoch. This resulted in:

$$\text{Min.} = \text{Hel. J.D. } 2434119.49403 + .64057610 E. \\ \pm .00094 \pm .00000025 \text{ m.e.}$$

The residuals given in column 9 of Table 2 of the present paper were computed with these light-elements. Except for the large residual of SOLOVIEV's first epoch, which has a rather low weight, there are no indications of a variation in the period.

### 9. Discussion of the variations in period.

Before we discuss the period variations of the systems investigated, it is necessary to give an exact definition of the period of an eclipsing variable. Although the period usually is defined as the time of one revolution of the binary system, we find it practical to define the period as the time elapsed between two successive epochs of primary minimum, a quantity which can be derived directly from the observations. If no complications in the eclipsing system are present, this time will be exactly equal to the time of revolution. We will leave the effect of constant radial velocity out of consideration. In the present case, however, where we are dealing with short-period eclipsing systems, the physical conditions are such that complications may influence the determination of the epochs, and the time elapsed between two successive epochs of primary minimum will in general not be equal to the actual time of revolution. Consequently, any variation found in the „observed period” need not be identified with a real variation of the „revolution period”.

Inspecting the data described in the previous section, we find two kinds of variation in period. The first consists in small and rapid fluctuations. They lie near the limit of what can be shown by the present observations. Except for V502 Oph and RS Sct, the residuals of the epochs of minimum, given in column 9 of Table 2, were computed with the best linear elements for the time interval covered. For OO Aql, GO Cyg, ER Ori and AG Vir these residuals show a noticeable systematic run, and for 44i Boo, DO Cas, SW Lac and AH Vir they are too large by a factor 2 or 3 if compared with the computed mean errors given in column 4 of the same table.

This argument fails if the mean errors have been

underestimated. However, their determination according to the method described in an earlier article (KWEE and VAN WOERDEN 1956a) seems to be quite correct, as these errors were expressed directly in the observational errors of the magnitudes. Moreover, it should be noted that VAN WOERDEN (1957) found the same kind of variations in the period of the eclipsing variable SV Cam, and SCHMIDT and SCHRICK (1955) already suspected such variations to be present in VW Cep. The epochs of 44i Boo, the most frequently observed system, show residuals up to  $^d.00200$ .

The author suggests that these variations should not be considered as fluctuations of the period of revolution, but that they are caused by distortions in the light-curves. Such distortions, giving rise to asymmetries, are quite common in the light-curves of these systems. For W UMa (KWEE 1956b) the removal of rather small  $\sin \theta$ - and  $\sin 2\theta$ -terms from the light-curve resulted already in a shift of  $^d.00030$  in the epoch of primary minimum. For systems where the slope of the branches during eclipse is less steep (for 44i Boo, for example, this slope is 6 times less steep than for W UMa), or where the sine-terms are relatively larger (VW Cep, for example, displays differences of  $^m.10$  between primary and secondary maximum), the resulting variations in the epochs may easily amount to  $^d.001$ , which is about the order of magnitude of the observed variations. Especially sine-terms of higher order in the phase angle  $\theta$  will cause considerable shifts in the epochs.

The other kind of period variations are those which extend over long intervals of time and which have been known for several years. They are clearly present in the systems 44i Boo, VW Cep, AK Her, SW Lac, U Peg and W UMa. The present observations fully confirm these variations and indicate even that there are other systems showing similar changes: YY Eri, V502 Oph, ER Ori and RS Sct. The author suspects that these variations may be found in every short-period eclipsing binary system, if only the observations are sufficiently accurate and extend far enough in time. No satisfactory explanation exists so far for this kind of period variations. They do not seem to follow any rules either: some systems, like 44i Boo, show a continuous lengthening of the period, others a shortening of the period, while several systems display variations with alternating sign (VW Cep, AK Her, SW Lac, W UMa). For most of the systems of this last category one can fit, at least approximately, a long-period variation through the observed epochs, although in no case has a whole period been covered. For SW Lac, however, the variations have a quite erratic character. Moreover, when the period variations of the other systems are

scrutinized more closely, it appears that they can also be interpreted as a sequence of more or less abrupt variations, i.e. changes in period taking place in relatively short time intervals.

Several authors, among others KUIPER (1941) and WOOD (1950), have ascribed this kind of period change to loss of mass by one or both of the components, caused by dynamical instability. The problem of dynamical instability was investigated thoroughly by KOPAL (1954), who determined maximum dimensions for the components beyond which instability will set in. At present it is very difficult, however, to determine accurately the actual dimensions from the light-curve, because the methods of solution are far too simple to deal with the complicated nature of close binaries. Recent photoelectric observations have shown that for 44i Boo (BINNENDIJK 1955) and W UMa (KWEE 1956b) both components are just inside the stability limits, while for GO Cyg (OVENDEN 1954), SW Lac (BROWNLEE 1957), ER Ori (HURUHATA, NAKAMURA and KITAMURA 1957a) and YY Eri (HURUHATA, DAMBARA and KITAMURA 1953), one of the components seems to extend beyond its stability limit. A first requirement for further investigations along these lines is a refinement in the method of solution for the dimensions of the components from the light-curve. Special attention should be paid to the rectification process for these systems. Although we can not yet conclude definitely from the photometric data that the components of such a system have surpassed their stability limits, other phenomena, such as the presence of emission lines and asymmetry in the light-curves, suggest that instability prevails. All the systems investigated in this article, except DO Cas, RS Sct and BF Vir (of these three, only DO Cas has been investigated thoroughly), show asymmetric light-curves, and for most of them variations of the light-curve from cycle to cycle have been reported.

KOPAL (1957) has shown that from the unstable secondary components of subgiant eclipsing systems material can be emitted at the inner Lagrangian point. In this case mass transfer would take place from the secondary component to the primary. If we assume that similar effects occur when the primary component is unstable, we may draw the conclusion that instability results mainly in an exchange of mass between the components. We shall now calculate the effect of such a transfer of mass on the period of the system. If  $m_1$  and  $m_2$  are the masses of the components,  $a$  the distance between their centres,  $\omega$  the orbital angular velocity and  $P$  the orbital period, the total orbital momentum is given by:

$$M = \frac{m_1 m_2}{m_1 + m_2} a^2 \omega. \quad (1)$$



According to KEPLER's third law we also have:

$$\omega^2 = G \frac{m_1 + m_2}{a^3}, \quad (2)$$

where  $G$  is the gravitational constant. Eliminating  $a$  from these two expressions and inserting  $P = 2\pi/\omega$ , we get

$$M^3 = \frac{G^2}{2\pi} \frac{m_1^3 m_2^3}{m_1 + m_2} P. \quad (3)$$

Differentiating this expression we obtain:

$$\frac{dP}{P} = 3 \frac{dM}{M} - \left( 3 - \frac{m_1}{m_1 + m_2} \right) \frac{dm_1}{m_1} - \left( 3 - \frac{m_2}{m_1 + m_2} \right) \frac{dm_2}{m_2}. \quad (4)$$

In this general formula the fractional variation of the period is expressed in the fractional variations of the total orbital momentum and of the masses of the components. On purpose we have left the term with  $dM$  in the equation. When the law of conservation of angular momentum holds for the total orbital momentum alone, we may insert  $dM = 0$ . There exist, however, also processes in which the orbital momentum of the binary system changes. One of these processes has been discussed by WOOD (1950); he considered ejection of mass, which will be lost completely for the binary system.

It is also possible that the total orbital momentum changes at the expense of rotational momentum. If, by some internal process (e.g. stellar evolution), the radius of one component changes, the axial revolution of that component will also change. As the tidal friction will tend to equalize the orbital and the rotational periods, this would lead to an exchange of momentum between the two revolutions. KOPAL (1956) has stated, however, that the effect of tidal friction is so small, that time intervals of the order of  $10^8$  or  $10^9$  years are required to make its effects felt. In the systems investigated the observed changes in the period took place in a time of 20 or 30 years. In the following we have therefore neglected the effect of an interaction between the rotational and orbital momenta, and we have simply applied the law of conservation to the orbital momentum alone.

In the case considered here, i.e. transfer of mass from component 1 to component 2, we then may put  $dM = 0$ . Furthermore we have  $dm_1 = -dm_2$ , where  $dm_1$  is a negative quantity. Equation (4) then takes the simple form:

$$\frac{dP}{P} = 3 \left( \frac{m_1}{m_2} - 1 \right) \frac{dm_1}{m_1}. \quad (5)$$

This formula proves that the sign of the period variation is determined by the ratio  $m_1/m_2$  only; it is positive if  $m_1 < m_2$  and negative if  $m_1 > m_2$ . As all

systems investigated in this paper are supposed to be contact binaries, both components may be unstable and mass transfer between the components may occur both ways. From the observations it is found that the period variations  $dP/P$  are of the order  $10^{-5}$ . For all systems in this article we may put  $\frac{1}{2} < 3(m_1/m_2 - 1) < 10$ ; this means that, if mass transfer is the only cause of the period variations, relative masses  $dm_1/m_1$  of the order of  $10^{-5}$  or  $10^{-6}$  would be involved.

We will now investigate the changes of the stability limits caused by mass transfer from one component to the other. If  $r_1$  and  $r_2$  denote the relative values of the mean radii of the critical Roche equipotentials around the components expressed in the distance between the components  $a$ , and  $\rho_1$  and  $\rho_2$  the absolute values of the same quantities, we have:  $r_1 = \rho_1/a$  and  $r_2 = \rho_2/a$ . Differentiating these expressions we finally get:

$$\left. \begin{aligned} \frac{d\rho_1}{\rho_1} &= \frac{dr_1}{r_1} + \frac{da}{a} \\ \frac{d\rho_2}{\rho_2} &= \frac{dr_2}{r_2} + \frac{da}{a} \end{aligned} \right\} \quad (6)$$

Transfer of mass will cause a variation of  $a$  as well as variations of  $r_1$  and  $r_2$ . The first can be computed by eliminating the angular velocity  $\omega$  from expressions (1) and (2). We again find after differentiating and inserting, as before,  $dM = 0$  and  $dm_2 = -dm_1$ :

$$\frac{da}{a} = 2 \left( \frac{m_1}{m_2} - 1 \right) \frac{dm_1}{m_1}. \quad (7)$$

The quantities  $r_1$  and  $r_2$  depend only on the mass ratio  $m_1/m_2$ , accordingly, they will change only if this ratio changes. The variation of  $m_1/m_2$  by mass transfer can be found by simple differentiation and substitution of  $dm_2 = -dm_1$ :

$$d \left( \frac{m_1}{m_2} \right) = \frac{m_1}{m_2} \left( \frac{m_1}{m_2} + 1 \right) \frac{dm_1}{m_1}. \quad (8)$$

The quantities  $r_1$  and  $r_2$  have been tabulated by KOPAL (1954, Table 1) for different values of the mass ratio. The variations  $dr_1/r_1$  and  $dr_2/r_2$  with  $d(m_1/m_2)$  can be evaluated numerically from this table.

Assuming that mass is lost by the lighter component, and taking for a typical contact-binary system  $m_1/m_2 = .6$ , we find from (7):  $da/a = -.8 \times dm_1/m_1$ . Equation (8) gives  $d(m_1/m_2) = +1.0 \times dm_1/m_1$ , which according to KOPAL's table corresponds to  $dr_1/r_1 = +.2 \times dm_1/m_1$  and  $dr_2/r_2 = -.2 \times dm_1/m_1$ . From equations (6), we finally find for the variations of the absolute dimensions of the critical equipotentials:  $d\rho_1/\rho_1 = -.6 \times dm_1/m_1$  and  $d\rho_2/\rho_2 = -1.0 \times dm_1/m_1$ . We see from these figures that the relative changes in the absolute dimensions of the critical surfaces remain of the same order as the relative amount of



mass transferred. This conclusion holds in general unless the mass is very unequally distributed between the two components.

On the other hand we have to consider the volume taken up by the amount of mass transferred from one component to the other. It is reasonable to assume that, if mass is transferred, it will come from the outer layers of the ejecting star. As the density is very low in these regions, the volume taken up by a certain amount of mass is large. For SCHWARZSCHILD's star model (HYNEK 1951) a rough computation shows that the thickness of an outer shell containing  $10^{-6}$  of the stellar mass equals about one per cent of the stellar radius. This means that by a mass transfer of  $dm_1/m_1 = -10^{-6}$  from the first component to the second, the latter will receive a quantity of diluted mass, which, if distributed equally over its surface, would cause an increase of its radius of about one per cent. It remains an open question if this mass will form a local bulge on top of the atmosphere of the second component, or if it will penetrate into deeper regions. Much depends on the velocity of the ejected mass. If this is small, the latter will be the case and the ejected mass will then adapt itself to the conditions prevailing in the inner regions of the star. This means that it will be compressed so much that it adds only a small fraction to the total volume of that component. For larger velocities of ejection, however, the matter will remain in the outer parts of the atmosphere of the second component. In this case it will extend locally beyond the critical dimensions, which, as was shown above, will hardly be influenced at all by the process of mass transfer. As a result of the initial velocity of ejection this bulge will revolve slowly around the second component. After one revolution, when it reaches the inner Lagrangian instability point again, the reverse of the process takes place and the matter will be returned to the first component. This gives rise to a change in the period of the binary system with opposite sign. In this case it is plausible that period changes with alternating sign succeed each other.

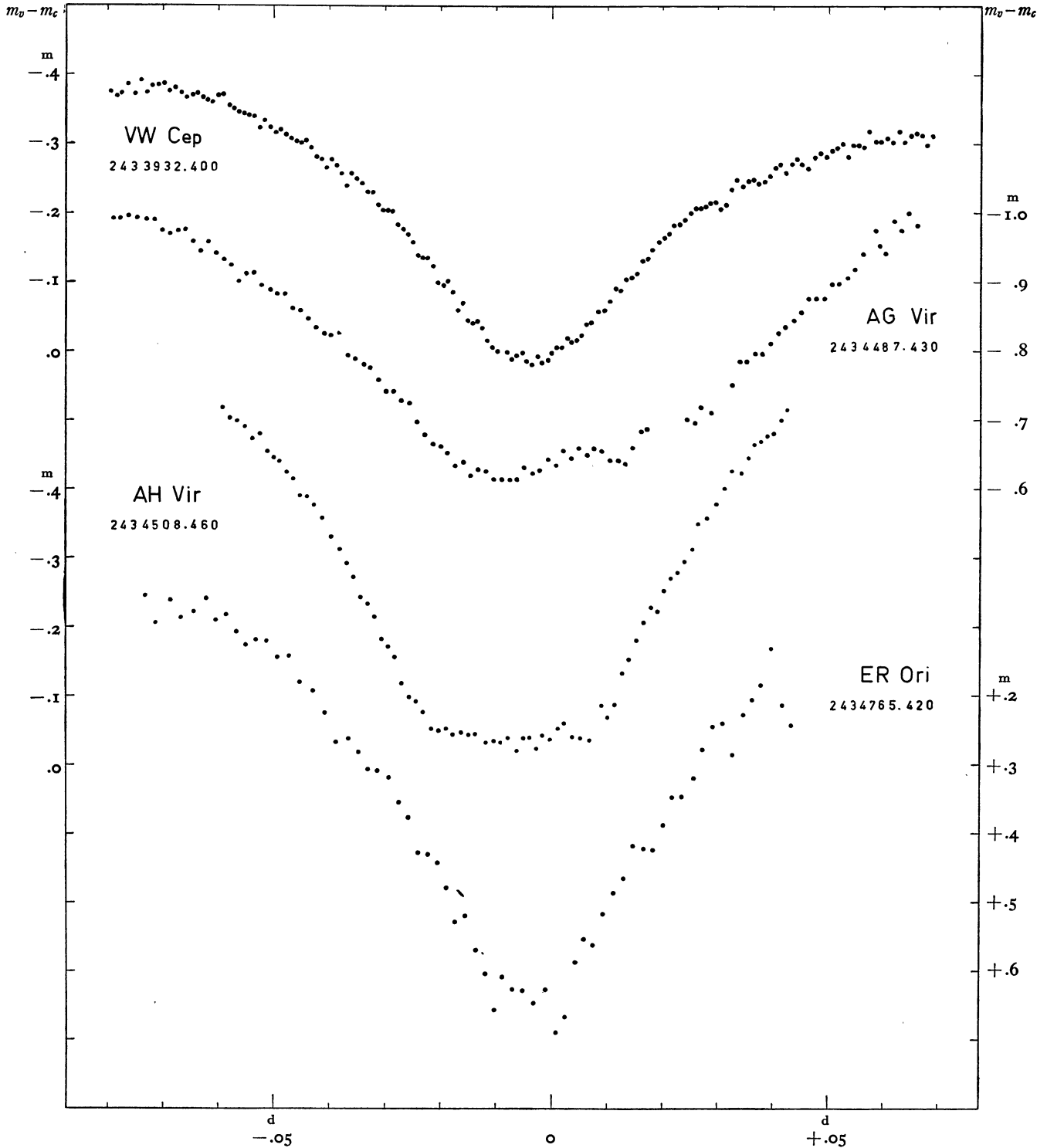
Although the process described qualitatively above may explain some of the problems related with

contact-binary systems, it is open to some criticisms. First of all, the amount of mass transferred from one component to the other, required to explain the observed period variations, is of the order of  $10^{-5}$  of the mass of the emitting component. This fraction is rather large, although emission of this amount of mass seems also to occur in shell-type Be stars. Secondly, for all systems investigated in this paper and showing period variations with alternating sign, the time between two opposite variations is of the order of 20000 revolutions, while, according to the orbits computed by KOPAL (1956 and 1957), one would expect a more rapid exchange of mass. The third objection, which is perhaps the most serious one, is that this mechanism can not explain the period variations of the semi-detached systems, in which only one component has reached the limit of instability. As both groups of systems show period variations of similar character, one would expect that the underlying cause is the same for both.

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FIGURE 2



Individual observations in four nights of different quality. The computed mean error of a single observation is : for VW Cephei  $\pm m.009$ , for AG Virginis  $\pm m.021$ , for AH Virginis  $\pm m.008$  and for ER Orionis  $\pm m.020$ . For each light-curve the time given in the figure in Heliocentric Julian days corresponds with the zero point of the abscissa.

TABLE 5

Individual observations

Table with 14 columns: Hel. J.D., m\_p - m\_c, Hel. J.D., m\_p - m\_c, Hel. J.D., m\_p - m\_c, Hel. J.D., m\_p - m\_c, Hel. J.D., m\_p - m\_c, Hel. J.D., m\_p - m\_c, Hel. J.D., m\_p - m\_c. Rows are grouped by star names: 00 Aquilae, 44i Bootis, and J.D. 2434194, 2434199, 2434501, 2434478.

TABLE 5 (continued)

Table with 14 columns: Hel. J.D., m\_p - m\_c, Hel. J.D., m\_p - m\_c, Hel. J.D., m\_p - m\_c, Hel. J.D., m\_p - m\_c, Hel. J.D., m\_p - m\_c, Hel. J.D., m\_p - m\_c, Hel. J.D., m\_p - m\_c. Rows include 44i Bootis, DO Cassiopeiae, and various J.D. entries.



TABLE 5 (continued)

Hel. J.D.	$m_p - m_c$	Hel. J.D.	$m_p - m_c$	Hel. J.D.	$m_p - m_c$	Hel. J.D.	$m_p - m_c$	Hel. J.D.	$m_p - m_c$	Hel. J.D.	$m_p - m_c$	Hel. J.D.	$m_p - m_c$
<b>DO Cassiopeiae</b>													
(continued)													
J.D. 2433935.		d	m	d	m	d	m	d	m	d	m	d	m
d	m												
40772	+ .171	47029	- .112	46778	+ .452	41787	- .199	34000	- .370	42792	- .209	32123	- .213
40849	+ .151	47103	- .116	46961	+ .459	41911	- .178	34096	- .372	42877	- .214	32219	- .207
40925	+ .159	47170	- .132	47118	+ .446	42041	- .177	34192	- .356	42976	- .216	32323	- .210
41007	+ .143	47240	- .126	47262	+ .446	42164	- .166	34283	- .351	43067	- .206	32428	- .220
41081	+ .157	47310	- .122	47374	+ .446	42288	- .135	34374	- .346	43166	- .212	32535	- .238
41158	+ .151	47377	- .137	47485	+ .437	42480	- .105	34469	- .344	43263	- .234	32635	- .246
41240	+ .147	47447	- .128	47608	+ .431	42618	- .084	34558	- .342	43357	- .248	32741	- .239
41308	+ .128	47520	- .123	47714	+ .452	42751	- .083	34655	- .340	43460	- .238	32833	- .247
41378	+ .131	47587	- .129	47816	+ .428	42864	- .077	34745	- .324	43557	- .246	32930	- .250
41444	+ .125	47656	- .132	47933	+ .426	42969	- .059	34842	- .334	43649	- .248	33033	- .247
41516	+ .125	47726	- .123	48048	+ .411	43100	- .044	34940	- .324	43751	- .242	33143	- .243
41588	+ .114	47866	- .141	48158	+ .412	43214	- .035	35035	- .316	43845	- .245	33247	- .259
41657	+ .098	47971	- .131	48268	+ .396	43329	- .016	35125	- .321	43940	- .253	33345	- .254
41731	+ .113	48049	- .139	48375	+ .392	43454	- .007	35220	- .314	44041	- .265		
41801	+ .100	48119	- .148	48478	+ .376	43574	- .002	35311	- .308	44132	- .270	J.D. 2434603.	
41867	+ .096	48194	- .157	48574	+ .363	43691	- .002	35412	- .304	44230	- .258	37567	- .344
41939	+ .088	48273	- .148	48666	+ .349	43815	+ .008	35499	- .301	44333	- .270	37789	- .338
42010	+ .104	48347	- .154	48763	+ .357	43941	+ .004	35588	- .305	44429	- .278	37914	- .314
42080	+ .079	48430	- .149	48848	+ .341	44063	+ .026	35676	- .294	44526	- .270	38035	- .314
42138	+ .088	48511	- .146	48940	+ .325	44195	+ .002	35774	- .281	44624	- .264	38182	- .304
42206	+ .083	48591	- .161	49033	+ .326	44324	+ .020	35864	- .278	44745	- .280	38326	- .298
42269	+ .086	48670	- .165	49138	+ .303	44441	+ .011	35954	- .266	44858	- .286	38466	- .288
42335	+ .064	48778	- .165	49241	+ .309	44576	- .002	36045	- .278	44964	- .281	38603	- .287
42397	+ .060	48844	- .163	49338	+ .286	44703	- .015	36136	- .269	45071	- .289	38743	- .275
42467	+ .050	48919	- .165	49438	+ .271	44820	- .022	36227	- .257	45167	- .294	38884	- .258
42532	+ .057	48987	- .164	49535	+ .255	44946	- .020	36310	- .240	45266	- .290	39023	- .256
42595	+ .046	49064	- .167	49644	+ .264	45085	- .043	36397	- .258	45361	- .286	39159	- .244
42659	+ .050	49137	- .171	49757	+ .243	45206	- .054	36490	- .250	45456	- .298	39288	- .242
42728	+ .031	49207	- .153	49857	+ .260	45322	- .054	36586	- .243	45554	- .298	39402	- .222
42791	+ .031	J.D. 2434636.		49953	+ .242	45447	- .066	36681	- .230	45657	- .295	39525	- .217
42859	+ .035	39192	+ .060	50050	+ .219	45573	- .061	36781	- .230	45757	- .298	39692	- .207
42930	+ .023	39290	+ .074	50159	+ .216	45692	- .089	36874	- .212	45862	- .303	39836	- .179
43001	+ .038	39379	+ .088	50263	+ .217	45831	- .095	36963	- .204	45963	- .303	39904	- .173
43070	+ .023	39463	+ .100	50371	+ .203	45970	- .183	37048	- .204	46071	- .308	40104	- .161
43138	+ .012	39522	+ .082	50462	+ .199	46094	- .143	37137	- .202	46178	- .302	40256	- .149
43207	+ .002	39622	+ .112	50560	+ .188	46213	- .137	37229	- .183	46285	- .318	40398	- .134
43278	+ .010	39791	+ .179	50679	+ .176	46337	- .140	37321	- .177	46382	- .302	40537	- .116
43347	+ .010	40680	+ .192	50784	+ .166	46453	- .141	37414	- .169	46491	- .312	40653	- .101
43417	+ .008	40846	+ .192	50887	+ .161	46575	- .167	37505	- .157	46590	- .316	40764	- .103
43488	- .012	40988	+ .216	50981	+ .132	46701	- .186	37594	- .139	46701	- .312	40884	- .087
43556	- .009	41115	+ .218	51079	+ .128	46833	- .186	37676	- .135	46801	- .298	41007	- .069
43626	- .023	41232	+ .232	51180	+ .124	46972	- .221	37762	- .135	46890	- .312	41133	- .048
43697	- .008	41341	+ .224	51283	+ .120	47109	- .218	37864	- .123	J.D. 2433936.		41263	- .054
43765	- .026	41438	+ .253	51420	+ .115	47251	- .201	37955	- .099	41376	- .308	41376	- .038
43837	- .035	41550	+ .257	51480	+ .104	47377	- .197	38046	- .095	41508	- .224	41508	- .027
43903	- .024	41683	+ .270	51588	+ .093	47509	- .233	38132	- .101	41635	- .225	41635	- .030
43975	- .031	41770	+ .275	51695	+ .088	47638	- .181	38220	- .085	41759	- .207	41759	- .012
44043	- .027	41867	+ .298	51805	+ .082	47774	- .237	38307	- .059	41878	- .203	41878	- .022
44117	- .031	42069	+ .322	51929	+ .087	47866	- .267	38394	- .069	41990	- .200	41990	- .021
44183	- .027	42231	+ .330	52022	+ .086	48036	- .246	38486	- .044	42134	- .176	42134	- .033
44250	- .045	42331	+ .341	52116	+ .066	48217	- .261	38573	- .040	42260	- .166	42260	- .036
44321	- .037	42430	+ .341	52216	+ .050	J.D. 2433932.		38660	- .044	42392	- .166	42392	- .045
44390	- .041	42542	+ .355			38754	- .033	38754	- .033	42516	- .136	42516	- .034
44460	- .051	42637	+ .376			38843	- .015	38843	- .015	42653	- .128	42653	- .054
44528	- .051	42739	+ .391			38931	- .005	38931	- .005	42799	- .123	42799	- .070
44602	- .038	42836	+ .382			39029	- .000	39029	- .000	42874	- .118	42874	- .078
44670	- .058	42937	+ .372	<b>VW Cephei</b>				39197	+ .002	42930	- .102	43027	- .079
44740	- .058	42994	+ .327	J.D. 2433898.		39289	- .313	39289	+ .012	43155	- .097	43155	- .066
44809	- .079	43034	+ .427	d	m	39377	- .314	39377	+ .007	43294	- .068	43294	- .104
44881	- .052	43138	+ .439	36464	- .361	39472	- .325	39472	+ .003	43424	- .062	43424	- .120
44946	- .058	43232	+ .436	36612	- .371	39577	- .330	39577	+ .015	43582	- .056	43582	- .140
45014	- .075	43334	+ .428	36762	- .382	39686	- .339	39686	+ .020	43706	- .044	43706	- .150
45084	- .085	43429	+ .462	36921	- .384	39792	- .346	39792	+ .008	43833	- .042	43833	- .156
45157	- .075	43511	+ .461	37057	- .366	39904	- .350	39904	+ .017	43964	- .022	43964	- .168
45224	- .061	43596	+ .446	37195	- .384	40016	- .357	40016	+ .013	44093	- .016	44093	- .177
45308	- .067	43694	+ .459	37386	- .382	40131	- .367	40131	+ .003	44211	- .014	44211	- .198
45383	- .081	43816	+ .456	37550	- .362	40249	- .357	40249	+ .005	44338	- .008	44338	- .211
45466	- .086	43915	+ .450	37683	- .362	40371	- .363	40371	+ .005	44475	- .014	44475	- .235
45545	- .086	44013	+ .465	37839	- .382	40484	- .378	40484	+ .019	44613	- .008	44613	- .229
45623	- .064	44115	+ .456	38093	- .355	40627	- .375	40627	+ .013	44744	- .011	44744	- .238
45707	- .091	44203	+ .470	38355	- .351	40751	- .365	40751	+ .016	44878	- .005	44878	- .246
45795	- .087	44294	+ .461	38609	- .360	40883	- .378	40883	+ .023	44998	- .014	44998	- .247
45865	- .086	44398	+ .442	38853	- .347	41018	- .378	41018	+ .029	45115	- .014	45115	- .252
45939	- .083	44492	+ .455	39093	- .347	41152	- .363	41152	+ .041	45248	- .024	45248	- .254
46004	- .083	44586	+ .454	39277	- .324	41282	- .378	41282	+ .057	45377	- .027	45377	- .270
46065	- .106	44681	+ .489	39417	- .326	41417	- .375	41417	+ .059	45510	- .047	45510	- .277
46126	- .090	44771	+ .483	39538	- .300	41551	- .369	41551	+ .072	45650	- .044	45650	- .277







TABLE 5 (continued)

Hel. J.D.	$m_v - m_c$	Hel. J.D.	$m_v - m_c$	Hel. J.D.	$m_v - m_c$	Hel. J.D.	$m_v - m_c$	Hel. J.D.	$m_v - m_c$	Hel. J.D.	$m_v - m_c$	Hel. J.D.	$m_v - m_c$
<b>V566 Ophiuchi</b>													
J.D. 2434515.		d	m	d	m	d	m	d	m	d	m	J.D. 2434598.	
d	m											d	m
42774	-1.037	.66131	+1.141	.39779	+3.320	.57395	+3.396	.47438	+4.444				
49783	-1.020	.66283	+1.126	.40061	+3.377	.57523	+3.377	.47532	+4.442				
49944	-1.086	.66448	+1.134	.40289	+3.354	.57973	+3.317	.47661	+4.421			.41995	-4.468
50074	-1.051	.66610	+1.104	.40490	+3.376	.58204	+3.306	.47751	+4.420			.42376	-4.431
50243	-1.047	.66792	+1.095	.40645	+3.442	.58321	+3.287	.47826	+4.394			.42649	-4.385
54841	-1.068	.66948	+1.090	.40904	+3.466	.58446	+3.298	.47907	+4.389			.42860	-4.346
54822	-1.080	.67098	+1.092	.41113	+3.480	.58567	+3.280	.47991	+4.372			.43073	-4.316
54960	-1.112	.67261	+1.070	.41309	+3.506	.58681	+3.278	.48069	+4.360			.43322	-4.249
55106	-1.149	.67444	+1.064	.41467	+3.563	.58793	+3.251	.48156	+4.352			.43484	-4.227
55247	-1.112			.41654	+3.545	.58921	+3.219	.48249	+4.329			.43651	-4.211
55358	-1.114			.41826	+3.586	.59043	+3.214	.48340	+4.330			.43826	-4.180
55462	-1.136			.41997	+3.606	.59170	+3.206	.48434	+4.318			.43971	-4.151
55585	-1.055			.42162	+3.624	.59301	+3.186	.48526	+4.314			.44155	-4.091
55724	-1.095			.42343	+3.632	.59583	+3.196	.48619	+4.288			.44340	-4.055
55840	-1.061			.42541	+3.654	.59680	+3.189	.48710	+4.291			.44567	-4.032
55953	-1.087			.42731	+3.637	.59770	+3.196	.48804	+4.286			.44774	-4.065
56054	-1.078			.42927	+3.634	.59866	+3.202	.48901	+4.272			.44947	-4.105
56168	-1.059			.43088	+3.629	.59984	+3.179	.48992	+4.264			.45111	-4.129
56281	-1.091			.43250	+3.593			.49096	+4.264			.45262	-4.202
56414	-1.083			.43396	+3.596			.49179	+4.244			.45420	-4.218
56525	-1.107			.43565	+3.582			.49265	+4.227			.45559	-4.252
56659	-1.127			.43699	+3.533			.49361	+4.212			.45701	-4.249
56830	-1.119			.43846	+3.530			.49452	+4.224			.45854	-4.267
56959	-1.121			.44003	+3.477			.49550	+4.224			.46021	-4.270
57068	-1.144			.44150	+3.505			.49640	+4.199			.46186	-4.309
57180	-1.143			.44293	+3.459			.49730	+4.195			.46339	-4.292
57285	-1.181			.44461	+3.431			.49825	+4.190			.46510	-4.270
57384	-1.192			.44613	+3.425			.49916	+4.170			.46669	-4.274
57521	-1.225			.44768	+3.399			.50012	+4.197			.46821	-4.243
57630	-1.267			.44974	+3.341			.50106	+4.175			.46985	-4.215
57756	-1.262			.45180	+3.309			.50203	+4.165			.47138	-4.186
57881	-1.281			.45329	+3.313			.50288	+4.153			.47302	-4.181
57989	-1.307			.45496	+3.309			.50379	+4.166			.47462	-4.132
58102	-1.309			.45640	+3.305			.50472	+4.171			.47631	-4.109
58215	-1.285			.45816	+3.285			.50579	+4.166			.47816	-4.039
58333	-1.257			.45984	+3.257			.50684	+4.197			.47980	-4.019
58472	-1.197			.46123	+3.255			.50792	+4.207			.48142	-4.025
58591	-1.236			.46276	+3.237			.50900	+4.236			.48311	-4.062
58702	-1.179			.46431	+3.179			.51017	+4.255			.48462	-4.087
	-1.174			.46566	+3.174			.51139	+4.258			.48666	-4.130
	-1.275			.46709	+3.275			.51261	+4.275			.48843	-4.191
	-1.279			.46846	+3.279			.51384	+4.279			.49016	-4.233
	-1.282			.46984	+3.282			.51507	+4.282			.49192	-4.285
	-1.294			.47123	+3.294			.51630	+4.294			.49369	-4.288
	-1.310			.47262	+3.310			.51753	+4.310			.49548	-4.288
	-1.331			.47401	+3.331			.51876	+4.331				
	-1.332			.47540	+3.332			.52000	+4.332				
	-1.360			.47679	+3.360			.52124	+4.360				
	-1.378			.47818	+3.378			.52248	+4.378				
	-1.369			.47957	+3.369			.52372	+4.369				
	-1.386			.48096	+3.386			.52496	+4.386				
	-1.405			.48235	+3.405			.52620	+4.405				
	-1.424			.48374	+3.424			.52744	+4.424				
	-1.443			.48513	+3.443			.52868	+4.443				
	-1.462			.48652	+3.462			.52992	+4.462				
	-1.481			.48791	+3.481			.53116	+4.481				
	-1.500			.48930	+3.500			.53240	+4.500				
	-1.519			.49069	+3.519			.53364	+4.519				
	-1.538			.49208	+3.538			.53488	+4.538				
	-1.557			.49347	+3.557			.53612	+4.557				
	-1.576			.49486	+3.576			.53736	+4.576				
	-1.595			.49625	+3.595			.53860	+4.595				
	-1.614			.49764	+3.614			.53984	+4.614				
	-1.633			.49903	+3.633			.54108	+4.633				
	-1.652			.50042	+3.652			.54232	+4.652				
	-1.671			.50181	+3.671			.54356	+4.671				
	-1.690			.50320	+3.690			.54480	+4.690				
	-1.709			.50459	+3.709			.54604	+4.709				
	-1.728			.50598	+3.728			.54728	+4.728				
	-1.747			.50737	+3.747			.54852	+4.747				
	-1.766			.50876	+3.766			.54976	+4.766				
	-1.785			.51015	+3.785			.55100	+4.785				
	-1.804			.51154	+3.804			.55224	+4.804				
	-1.823			.51293	+3.823			.55348	+4.823				
	-1.842			.51432	+3.842			.55472	+4.842				
	-1.861			.51571	+3.861			.55596	+4.861				
	-1.880			.51710	+3.880			.55720	+4.880				
	-1.899			.51849	+3.899			.55844	+4.899				
	-1.918			.51988	+3.918			.55968	+4.918				
	-1.937			.52127	+3.937			.56092	+4.937				
	-1.956			.52266	+3.956			.56216	+4.956				
	-1.975			.52405	+3.975			.56340	+4.975				
	-1.994			.52544	+3.994			.56464	+4.994				
	-2.013			.52683	+4.013			.56588	+5.013				
	-2.032			.52822	+4.032			.56712	+5.032				
	-2.051			.52961	+4.051			.56836	+5.051				
	-2.070			.53100	+4.070			.56960	+5.070				
	-2.089			.53239	+4.089			.57084	+5.089				
	-2.108			.53378	+4.108			.57208	+5.108				
	-2.127			.53517	+4.127			.57332	+5.127				
	-2.146			.53656	+4.146			.57456	+5.146				
	-2.165			.53795	+4.165			.57580	+5.165				
	-2.184			.53934	+4.184			.57704	+5.184				
	-2.203			.54073	+4.203			.57828	+5.203				
	-2.222			.54212	+4.222			.57952	+5.222				
	-2.241			.54351	+4.241			.58076	+5.241				
	-2.260			.54490	+4.260			.58200	+5.260				
	-2.279			.54629	+4.279			.58324	+5.279				
	-2.298			.54768	+4.298			.58448	+5.298				
	-2.317			.54907	+4.317								





TABLE 5 (continued)

Hel. J.D.	$m_v - m_c$	Hel. J.D.	$m_v - m_c$	Hel. J.D.	$m_v - m_c$	Hel. J.D.	$m_v - m_c$	Hel. J.D.	$m_v - m_c$	Hel. J.D.	$m_v - m_c$	Hel. J.D.	$m_v - m_c$
<b>AH Virginis</b>													
<i>(continued)</i>													
		d	m	d	m	d	m	d	m	d	m	d	m
		.49441	-.423	.39849	-.041	.40839	+ .714	.47419	+1.335	.54392	+ .931	.47278	+ .822
		.49559	-.445	.40001	-.025	.41013	+ .778	.47564	+1.312	.54536	+ .901	.47472	+ .844
J.D.2434508.		.49670	-.464	.40210	-.031	.41191	+ .761	.47709	+1.324	.54689	+ .879	.47680	+ .863
		.49790	-.469	.40349	-.035	.41370	+ .751	.47868	+1.336	.54845	+ .879	.47889	+ .887
d	m	.49905	-.477	.40487	-.031	.41531	+ .766	.47981	+1.364	.55062	+ .859	.48069	+ .912
.44505	-.043	.50023	-.481	.40626	-.037	.41685	+ .776	.48203	+1.341	.55197	+ .851	.48257	+ .943
.44636	-.044	.50150	-.500	.40793	-.015	.41850	+ .761	.48374	+1.391	.55341	+ .843	.48451	+ .936
.44821	-.032	.50261	-.515	.40932	-.028	.42007	+ .757	.48569	+1.416	.55489	+ .807	.48632	+ .990
.44962	-.034			.41050	-.057	.42161	+ .815	.48757	+1.352	.55632	+ .830	.48819	+ .942
.45091	-.032	J.D.2434841.		.41203	-.061	.42330	+ .781	.48941	+1.363	.55803	+ .849	.49055	+ .967
.45221	-.038			.41356	-.090	.42489	+ .810	.49161	+1.379	.55965	+ .826	.49285	+1.013
.45372	-.019	.35362	-.451	.41515	-.121	.42646	+ .859	.49311	+1.379	.56123	+ .819	.49541	+1.075
.45492	-.038	.35501	-.450	.41668	-.135	.42816	+ .839	.49530	+1.367	.56305	+ .848	.49764	+1.062
.45608	-.039	.35654	-.450	.41779	-.173	.42978	+ .829	.49763	+1.347	.56462	+ .784	.49951	+1.098
.45723	-.023	.35807	-.404	.41911	-.184	.43170	+ .837	.49995	+1.360	.56605	+ .772	.50194	+1.139
.45842	-.042	.35953	-.402	.42057	-.201	.43332	+ .874	.50203	+1.347	.56732	+ .790	.50500	+1.183
.45975	-.036	.36231	-.354	.42210	-.215	.43486	+ .874	.50356	+1.405			.50771	+1.230
.46102	-.052	.36432	-.303	.42362	-.244	.43637	+ .860	.50532	+1.393	J.D.2434871.		.51034	+1.292
.46227	-.060	.36550	-.280	.42522	-.275	.43823	+ .880	.50777	+1.369			.51243	+1.298
.46370	-.040	.36654	-.304	.42647	-.284	.43980	+ .903	.50946	+1.381	.42014	+ .668	.51514	+1.331
.46515	-.038	.36828	-.270	.42779	-.310	.44149	+ .881	.51126	+1.334	.42222	+ .679	.51680	+1.361
.46686	-.035	.36981	-.220	.42925	-.332	.44299	+ .933	.51299	+1.305	.42465	+ .688	.51896	+1.322
.46894	-.085	.37099	-.204	.43078	-.345	.44437	+ .941	.51469	+1.283	.42646	+ .697	.52180	+1.345
.47008	-.068	.37244	-.198	.43244	-.355	.44607	+ .950	.51596	+1.236	.42935	+ .688	.52479	+1.343
.47139	-.087	.37390	-.160	.43397	-.404	.44760	+ .950	.51787	+1.219	.43354	+ .700	.52791	+1.349
.47280	-.132	.37515	-.164	.43550	-.426	.44948	+ .982	.51943	+1.243	.43611	+ .710	.53000	+1.414
.47396	-.152	.37675	-.124	.43696	-.416	.45086	+1.004	.52102	+1.196	.43958	+ .709	.53243	+1.360
.47536	-.180	.37856	-.074	.43849	-.424	.45239	+1.004	.52234	+1.147	.44187	+ .719	.53416	+1.356
.47668	-.206	.37987	-.077	.44015	-.446	.45396	+1.043	.52377	+1.126	.44597	+ .714	.53604	+1.398
.47805	-.228	.38147	-.051	.44161	-.480	.45548	+1.090	.52538	+1.134	.44798	+ .722	.53798	+1.383
.47927	-.222	.38279	-.019	.44314	-.473	.45684	+1.094	.52679	+1.087	.45014	+ .721	.54041	+1.332
.48037	-.252	.38418	-.031	.44322	-.504	.45840	+1.120	.52822	+1.075	.45278	+ .733	.54291	+1.355
.48163	-.270	.38557	-.021	.44554	-.512	.46006	+1.098	.52981	+1.054	.45451	+ .739	.54576	+1.413
.48274	-.278	.38724	-.031			.46167	+1.116	.53129	+1.051	.45625	+ .742	.54958	+1.334
.48400	-.294	.38883	-.025	<b>BF Virginis</b>				.53279	+1.031	.45833	+ .776	.55194	+1.465
.48551	-.312	.39015	-.041					.53437	+ .992	.46021	+ .765	.55382	+1.244
.48666	-.349	.39147	-.021	J.D.2434119.		.46616	+1.158	.53593	+1.028	.46201	+ .762	.55618	+1.212
.48815	-.357	.39300	-.034	d	m	.46789	+1.210	.53757	+ .981	.46410	+ .770	.55854	+1.243
.48988	-.377	.39425	-.057			.46969	+1.245	.53916	+ .985	.46632	+ .796		
.49127	-.400	.39550	-.048	.40371	+ .727	.47139	+1.261	.54089	+ .952	.46840	+ .804		
.49266	-.425	.39703	-.025	.40650	+ .719	.47282	+1.273	.54220	+ .940	.47041	+ .831		

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