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The structure of the Ca^+ chromosphere and the observations of the K line,

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1. *The K_1 line.* The measures of the contour of the K_1 line by M. MINNAERT [1]) show a marked asymmetry, the intensity at the violet side being smaller than that at the red side, as has been pointed out by MINNAERT. This fact may be interpreted by considering the influence of the outward moving Ca^+ atoms. Consider the optical thickness due to those atoms that absorb in a frequency ν_1; these atoms have outward velocities v in the range $v_1 - \delta$, $v_1 + \delta$, if v_1 is the outward velocity that corresponds to the frequency ν_1 and δ is the mean radial velocity due to the "thermal" motion of the atoms. Hence the optical thickness in the frequency ν_1 due to the outward moving atoms is:

$$(1.1) \qquad T^r_{\nu_1} = x_{\nu_0}\, \rho\, \Delta z\,;$$

x_{ν_0} is the mass-absorption coefficient for atoms at rest with a mean "thermal" radial velocity δ; ρ is the density and Δz is an interval of length along the solar radius that corresponds to an interval in velocity v equal to 2δ. The value of Δz is equal to:

$$(1.2) \qquad \Delta z = 2\delta \frac{dz}{dv}\,;$$

the coordinate z is measured along the solar radius from a level that corresponds to the solar limb.

A theoretical value of $T^r_{\nu_1}$ may be derived if we disregard second ionisation, hence take the product ρv independent of z, and if we assume a functional relation between v and z. I take [2]) v^2 proportional to z. Then:

$$(1.3) \qquad \rho \frac{dz}{dv} \text{ is independent of } z;$$

hence $T^r_{\nu_1}$ is independent of ν_1.

An empirical value of $T^r_{\nu_1}$ may be derived from MINNAERT's numerical values. The intensities at frequencies $\nu_0 + \Delta\nu$ and $\nu_0 - \Delta\nu$ have a ratio equal to

$$(1.4) \qquad 1/1 + T^r_{\nu_0 + \Delta\nu}.$$

Using (1.4) I derive the values:

$\Delta\lambda$	v	T^r
1 A	75 km/sec	0·20
2	150	0·11
3	225	0·25
4	300	0·25
5	375	0·21
6	450	0·15
8	600	0·01
10	750	0·01

The values T^r are remarkably constant until the velocity is very large.

If the observations are accurate enough to lay much stress upon the decrease in the last two lines, we must conclude that the chromosphere is terminated by second ionisation at a height corresponding to an outward velocity of about 500 km/sec.

2. *The chromospheric K line.* The contour of the K emission line is determined by the considerations developed in a preceding note [1]). This contour depends on the optical depth measured tangential to the solar surface; I denote this quantity by T^t_ν. This quantity is equal to $x_{\nu_0}\rho$ multiplied by the length

$$(2.1) \qquad 2\,\delta r_0/v\,;$$

r_0 is the solar radius. The dependence of T^t_ν on z may be computed as in the case of T^r_ν.

[1]) *Z. f. Ph.* **45**, p. 614.
[2]) *B. A. N.* 167.

[1]) *B. A. N.* 180, p. 135.

The resulting value is:

$$(2.2)\quad T^t_\nu = 2\,x_{\nu_0}\,\rho\,\delta r_0/v = 2\,x_{\nu_0}\,\rho\,\delta\,\frac{dz}{dv}\,r_0/v\,\frac{dz}{dv} = \\ = T^r_\nu\,\frac{r_0}{v}\,\frac{dv}{dz} = \frac{1}{2}\,T^r_\nu\,\frac{r_0}{z}.$$

As T^r_ν is independent of z (if z is not too large), this formula gives the functional relation between T^t_ν and z.

The total intensity of the chromospheric line depends principally on two factors: the value of the central intensity and the width of the line.

The first factor is nearly constant so long as T^t_ν is large, say larger than 5. If z increases T^t_ν decreases and the central intensity falls off, until for small values of T^t_ν it is proportional to T^t_ν. The second factor may be determined roughly in this way: The locus of points at which T^t_ν is equal to a moderate definite number is a sphere concentric with the solar surface and with a radius $r_0 + H$. If we wish to see how the width of the line changes if we look into the chromosphere at increasing distance from the solar limb, we must fix our attention on the line segments intercepted by the sphere with radius $r_0 + H$ on the line of sight: the line width is roughly proportional to the length of the line segment. Hence it is clear that the line width changes rapidly if in going outwards from the solar limb we have traversed a distance H. These considerations make it probable that what is usually termed the height of the chromosphere is actually this height H.

Taking H equal to 14000 km and deducing from (2.2) the value of T^t_ν we get:

(2.3) at $z = H$: $T^t_\nu = 25\,T^r_\nu$, hence with $T^r_\nu = 0{\cdot}20$, $T^t_\nu = 5$

Considering the uncertainty of several elements of this deduction, the agreement between the values of the table of section 1 and the height of the chromosphere is satisfactory.

Now we may take one step more: from UNSÖLD's [1]) measured contour we may derive the value of v at the height H. For the sudden diminution of intensity at $\frac{1}{3}$ Å from the central wavelength allows us to put

$$(2.4)\qquad v_H \sin\varphi = 25 \text{ km/sec},$$

φ being the angle introduced in my preceding note on this subject [2]). As the height in the chromosphere to which UNSÖLD's measures refer is 2500 km we have

$$(2.5)\quad r_0\,(\sec\varphi - 1) = 14000 - 2500 = 11500 \text{ km}.$$

Hence:

$$(2.6)\quad \tfrac{1}{2}\,\varphi^2 = 0{\cdot}016\,,\quad \varphi = 0{\cdot}18 \quad v_H = 140 \text{ km/sec}.$$

and thus:

$$(2.7)\qquad v_{km}^2 = \frac{140^2}{14000}\,z_{km} = 1{\cdot}4\,z_{km};$$

the suffix km denotes the kilometre as unit of length.

The value deduced for T^r_ν, viz. 0·20, allows a determination of the density ρ according to (1.1). As

$$(2.8)\quad \frac{dz}{dv} = 1{\cdot}4\times10^{-5}\,v,\quad x_{\nu_0} = 6\times10^{10}\quad 2\delta = 3\times10^{5}$$

we get:

$$(2.9)\qquad \rho v = 0{\cdot}8\times10^{-12}.$$

In a preceding investigation [1]) on the physical state of the chromospheric Ca^+ a critical point had been introduced, called S, at which the resistance offered by the solar gases equalled the outward acceleration and the velocity had a certain well defined value, numerically equal to 10^5 c. g. s. The value of the density at this point was found to be about 10^{-15}. I now find from empirical data by taking $v = 10^5$ in (2.9) $\rho = 10^{-17}$.

This difference must be accounted for. Two possibilities seem open:

1° the theoretical value of ρ requires revision on account of uncertainties of numerical data involved;

2° second ionisation has changed the idealised conditions assumed in the theoretical deduction.

The second possibility is contrary to the evidence deduced from the observed T^r values in section 1.

As regards the first alternative it must be remarked that the theoretical determination involves two uncertain factors. The value of ρ was such that the coefficient of resistance force offered by the solar gases to the outstreaming Ca^+ had a definite value. Hence the theoretical value of ρ deduced depends on two factors, viz. the mean molecular weight m of the solar gases and their effective atomic diameters as regards collisions with the Ca^+ atoms; this dependency is proportional to

$$\frac{m}{d^2}.$$

I had taken $m = 20$ $d = 10^{-8}$, but these values are open to revision.

[1]) *Astroph. J.* 69³, p. 216, fig. 1.
[2]) *B. A. N.* 180, p. 135, diagram.

[1]) *B. A. N.* 167.

3. *The fine structure of the K line.* The considerations on the optical thickness corresponding to a frequency ν_1, given in section **1**, are only valid if ν_1 is not too close to the frequency ν_0, for the proportionality of v^2 with z applies only for not too small values of v. If ν_1 is close to ν_0 we must use a more precise value of $\frac{dz}{dv}$ in (1.2). The differential coefficient may be taken from the theoretical treatment of *B. A. N.* 167. Hence may be inferred that the ratio

$$(3.1) \qquad \left(\frac{1}{v}\frac{dz}{dv}\right)_{v\,=\,1\text{ km/sec}} : \left(\frac{1}{v}\frac{dz}{dv}\right)_{v\,=\,\text{large}}$$

equals

$$(3.2) \qquad \frac{1}{2}\left\{1+\sqrt{1+\frac{8}{\beta}}\right\} : \frac{1}{\beta} = \frac{1}{2}\left\{\beta+\sqrt{\beta^2+8\beta}\right\}.$$

The number β measures the intensity of the outward acceleration of the atoms and has been taken equal to 1. Then the ratio (3.1) is 2, hence the ratio of the optical depths for $v = 1$ km/sec and for large velocities is 2 for $\beta = 1$ and still larger if β is larger.

The consequence of this behaviour of $T^r_{\nu_0+\Delta\nu}$ as a function of $\Delta\nu$ is that the K_3 line will have its violet wing widened without a shift (as far as this cause is concerned) of the wavelength of lowest intensity. The widening of the violet wing must shift the K_2 line to the violet relative to the K_3 line as is actually observed [1]). Also it is to be expected that the red K_2 component may be stronger than the violet component of K_2, as has in some stars actually been observed by ADAMS and JOY [2]).

The red shift of the K_3 line observed by CHARLES E. ST. JOHN [3]) does not result from the preceding considerations. Following some remarks of MILNE about this point [4]) I presume it to be caused by an asymmetrical distribution of the "thermal" radial velocities in each element of volume, which possibly may be connected with the unequality of free path for atoms moving inward and outward. However this point requires further investigation.

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1) CHARLES E. ST. JOHN, *Mt. Wilson Contr.* Nr. 48.
2) *Publ. of the Astr. Soc. of the Pac.* Oct. 1929, p. 311.
3) *l. c.*
4) *M. N.* **86**, p. 597.