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Citation

Hertzsprung, E. (1923). On the relation between mass and absolute brightness of components of double stars. *Bulletin Of The Astronomical Institutes Of The Netherlands*, *2*, 15. Retrieved from https://hdl.handle.net/1887/5753

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Note: To cite this publication please use the final published version (if applicable).

BULLETIN OF THE ASTRONOMICAL INSTITUTES OF THE NETHERLANDS.

1923 September 15 | Volume II. | No. 43.

COMMUNICATIONS FROM THE OBSERVATORY AT LEIDEN.

On the relation between mass and absolute brightness of components of double stars, by *Ejnar Hertzsprung*.

The number of stars for which both mass and absolute brightness are known with some accuracy is only small and conclusions based on this scanty material are naturally uncertain. Nevertheless it is of interest to see, what we at present know about individual stars in this respect.

In Table I I have collected the double stars for which the apparent brightness of both components, the parallax and sufficient orbital elements are known to calculate the angular mass or the parallax corresponding to a given mass. In addition the spectroscopic binary and eclipsing variable β Aurigae has been included, on the supposition that this double star is a member of the Ursa major group. For the parallax of 80 Tauri that of the system of the Hyades has been adopted. The parallax of α Aurigae has been determined by combination of the radial velocities with interferometer measures. The rest of the parallaxes contained in Table I are trigonometric.

TABLE I.

	m_1	m_2	p	P	α	M_1+M_2	$\frac{M_2}{M_1}$	M_1	M_2	$\log M_1$	$\log M_2$	$m_1 + 5 \log p$	$m_2 + 5 \log p$
	m	m	″	a	″								
⊙	−27·9					1·		1		·00		− ·33	
α Aurigae	·74	1·24	·063	·28	·054	7.50	·79	4·18	3·32	·62	·52	−5·26	−4·76
β Aurigae	2·82	2·82	·025			4·72	.98	2·38	2·34	·38	·37	−5·19	−5·19
α Can. maj.	−1·58	8·44	·373	49·3	7·55	3·41	·393	2·45	·96	·39	−·02	−3·72	6·30
α Can. min.	·48	12·	·308	39·0	4·05	1·50	·33	1·13	·37	·05	−·43	−2·08	9·44
80 Tauri	5·76	8·96	·027	148·3	1·04	2·57	·39	1·85	·72	·27	−·14	−2·08	1·12
ζ Herculis	3·04	6·54	·109	34·5	1·35	1·60	·43	1·12	·48	·05	−·32	−1·77	1·73
α Centauri	·33	1·70	·748	78·8	17·65	2·11	·85	1·14	·97	·06	−·01	− ·30	1·07
η Cassiop.	3·67	7·41	·184	507·6	12·21	1·13	·27	·89	·24	−·05	−·62	− ·01	3·73
70 Ophiuchi	4·28	5·98	·189	87·9	4·56	1·82	·89	·96	·86	−·02	−·07	·66	2·36
85 Pegasi	5·86	11·06	·095	26·3	·82	·93	1.78	·33	·60	−·48	−·22	·75	5·95
ξ Bootis	4·80	6·82	·164	152·8	4·83	1·09	·87	·58	·51	−·24	−·29	·87	2·89
μ Herculis BC	10·21	10·71	·110	43·2	1·30	·88	1	·44	·44	−·36	−·36	5·42	5·92
o_2 Eridani BC	9·74	11·14	·202	180·0	4·79	·41	1	·21	·20	−·68	−·70	6·27	7·67
Krüger 60	9·43	10·93	·260	50·4	2·68	·43	·47	·30	·14	−·52	−·85	6·51	8·01

The apparent visual magnitudes (Harvard scale) of the two components are designated by m_1 and m_2 and their masses by M_1 and M_2, while p is the parallax and P and α respectively the period in years and the angular semimajoraxis of the visual orbit of the double star. The absolute magnitudes of the two components are then $m_1 + 5 \log p$ and $m_2 + 5 \log p$.

A graphical representation of the results is given in Figure 1. The components belonging together have been joined by straight lines. The sun is represented by an open circle. The inclination of the dotted line represents the extent, to which the absolute brightness should decrease with decreasing mass simply by the fact that a star of smaller mass has, ceteris

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paribus, a smaller surface. Thus uncertainty in the determination of the parallax will shift the dots parallel to this line.

The general decrease of absolute brightness with decreasing mass is clearly shown on the diagram. The only known case of the fainter component being the more massive is that of 85 Pegasi. Furthermore it is of interest to see, how well our sun fits in between the other stars, and that the faint companion to Sirius is extraordinary massive as compared with its small brightness.

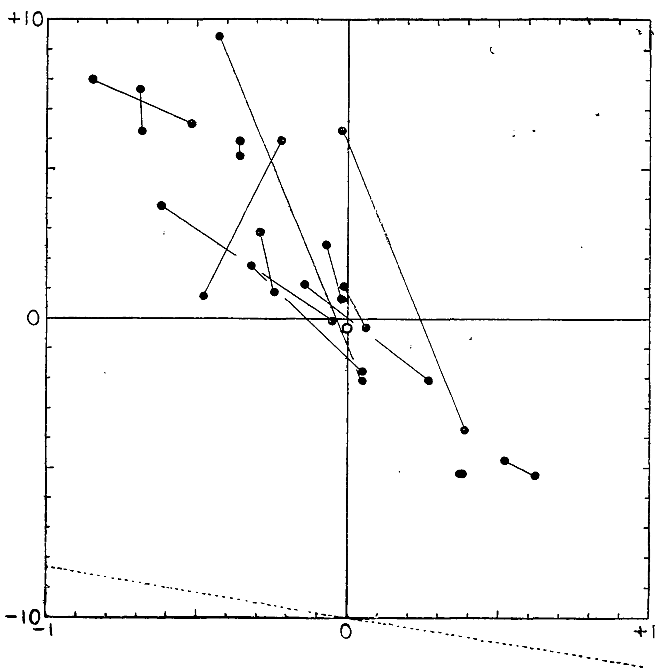

FIGURE I.

Abscissa: $\log M$, Ordinate: $m + 5 \log p$.

In the first approximation the linear relation

$$m + 5 \log p = -\frac{\log M}{\cdot 084} \qquad (1)$$

may be adopted, though the quadratic expression

$$m + 5 \log p = -11 \log M + 2 (\log M)^2 \qquad (2)$$

would represent stars of great mass rather better.

The coefficient ·084 in formula (1) is a little larger than that formerly adopted, viz. ·06 (*A. N.* 4975, **208**, 96; 1919). The absolute magnitude reduced to unit of mass, $m + 5 \log p + \frac{5}{3} \log M$, only depends on the apparent magnitude m and the angular mass M/p^3. The angular mass $(M_1 + M_2)/p^3$ of a double star is equal to α^3/P^2. If the ratio between the masses of the two components M_2/M_1 and the proportion α^3/P^2 (for a visual double star) are known, the absolute magnitude of each component reduced to the mass of the sun, $m_1 + 5 \log p + \frac{5}{3} \log M_1$ and $m_2 + 5 \log p + \frac{5}{3} \log M_2$ may therefore be calculated.

Assuming the relation between absolute magnitude $m + 5 \log p$ and mass M to be given it is evidently possible to calculate the parallax p of a double star from m_1, m_2 and $\alpha^3/P^2 = p^3{}_{(M_1 + M_2 = 1)}$. Adopting the linear formula (1) given above the parallax p is determined by

$$\cdot 86 \log p = \log p_{(M_1 + M_2 = 1)} - \tfrac{1}{3} \log (1 + 10^{-\cdot 084 (m_2 - m_1)}) + \cdot 028\, m_1 \qquad (3)$$

or (approximately)

$$\log p = 1 \cdot 163 \log p_{(M_1 + M_2 = 1)} - \cdot 3876 \log (1 + 10^{-\cdot 084 (m_2 - m_1)}) + \cdot 0326\, m_1 \qquad (4)$$

The second right hand term of equation (4) varies only slowly with the difference in magnitude between the two components, $m_2 - m_1$. The values are for

$m_2 - m_1 =$	0	1	2	3	4	5
	−·117	−·101	−·087	−·075	−·064	−·054
	6	7	8	9	10	
	−·046	−·039	−·032	−·027	−·023	

The parallaxes derived from the two assumptions represented by the formulae (1) and (2) will be designated $p_{(1)}$ and $p_{(2)}$ respectively.

If the quadratic formula (2) for the relation between $m + 5 \log p$ and M is adopted, the calculation of the corresponding parallax is a little more complicated. The figures given in Table 2 may serve as a help for this calculation. By aid of the given quantities $m_2 - m_1$ and $m_1 + 5 \log p_{(M_1 + M_2 = 1)}$ the value of $\log M_1$ is found by double interpolation. The absolute magnitude of the brighter component $m_1 + 5 \log p$ is determined by $\log M_1$ according to formula (2) and from the difference between $m_1 + 5 \log p_{(M_1 + M_2 = 1)}$ and $m_1 + 5 \log p$ the proportion $p_{(M_1 + M_2 = 1)}/p$ is found, which again determines $M_1 + M_2$.

A few examples will show the differences between the various ways of calculation of a hypothetical parallax mentioned above. Thus from the data of the 4 double stars contained in Table 3 it is seen that, as was to be foreseen, the linear formula (1) $\log M = -\cdot 084\, (m + 5 \log p)$ gives smaller parallaxes for the first two double stars of high luminosity than does the supposition $M_1 + M_2 = 2$, while the contrary is the case with the last two double stars of low luminosity.

The results obtained by adopting the quadratic expression (2) do not, as far as the double stars of low luminosity are concerned, differ materially from those calculated by making use of the linear formula (1), while for the double stars of high luminosity the

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TABLE 2.

log M_1	values of $m_1 + 5 \log p_{(M_1+M_2=1)}$ for $m_2 - m_1 =$ 0	1^m	2^m	3^m	4^m	5^m	10^m	∞	$m_1 + 5 \log p$
	m	m	m	m	m	m	m	m	m
— 1·0	11·84	11·78	11·73	11·69	11·65	11·62	11·49	11·33	13·00
— ·9	10·52	10·46	10·42	10·37	10·34	10·30	10·17	10·02	11·52
— ·8	9·25	9·19	9·14	9·10	9·06	9·02	8·89	8·75	10·08
— ·7	8·02	7·96	7·91	7·86	7·82	7·78	7·66	7·51	8·68
— ·6	6·82	6·77	6·71	6·66	6·62	6·59	6·46	6·32	7·32
— ·5	5·67	5·61	5·55	5·51	5·47	5·43	5·30	5·17	6·00
— ·4	4·56	4·49	4·44	4·39	4·35	4·31	4·18	4·05	4·72
— ·3	3·48	3·42	3·36	3·31	3·27	3·23	3·10	2·98	3·48
— ·2	2·45	2·38	2·33	2·28	2·23	2·19	2·06	1·95	2·28
— ·1	1·46	1·38	1·33	1·28	1·23	1·19	1·07	·95	1·12
·0	·50	·43	·37	·32	·27	·23	·11	·00	·00
·1	— ·41	— ·48	— ·55	— ·60	— ·65	— ·69	— ·81	— ·91	— 1·08
·2	— 1·28	— 1·36	— 1·43	— 1·48	— 1·53	— 1·57	— 1·69	— 1·79	— 2·12
·3	— 2·12	— 2·20	— 2·26	— 2·32	— 2·37	— 2·41	— 2·53	— 2·62	— 3·12
·4	— 2·91	— 2·99	— 3·06	— 3·12	— 3·17	— 3·21	— 3·33	— 3·41	— 4·08
·5	— 3·66	— 3·75	— 3·82	— 3·88	— 3·93	— 3·97	— 4·08	— 4·17	— 5·00
·6	— 4·38	— 4·46	— 4·54	— 4·60	— 4·65	— 4·69	— 4·80	— 4·88	— 5·88
·7	— 5·05	— 5·14	— 5·22	— 5·28	— 5·33	— 5·37	— 5·48	— 5·55	— 6·72
·8	— 5·68	— 5·78	— 5·86	— 5·92	— 5·97	— 6·01	— 6·12	— 6·19	— 7·52
·9	— 6·28	— 6·38	— 6·46	— 6·52	— 6·57	— 6·61	— 6·72	— 6·78	— 8·28
1·0	— 6·83	— 6·94	— 7·02	— 7·09	— 7·13	— 7·17	— 7·28	— 7·33	— 9·00
1·1	— 7·34	— 7·45	— 7·54	— 7·60	— 7·66	— 7·70	— 7·79	— 7·85	— 9·68
1·2	— 7·82	— 7·93	— 8·02	— 8·09	— 8·14	— 8·18	— 8·27	— 8·32	— 10·32
1·3	— 8·25	— 8·37	— 8·46	— 8·53	— 8·58	— 8·62	— 8·71	— 8·75	— 10·92
1·4	— 8·64	— 8·77	— 8·87	— 8·94	— 8·99	— 9·02	— 9·11	— 9·15	— 11·48
1·5	— 9·00	— 9·14	— 9·23	— 9·30	— 9·35	— 9·39	— 9·47	— 9·50	— 12·00

quadratic expression (2) gives still smaller hypothetical parallaxes.

One of the most luminous double stars showing decided orbital motion is ζ Orionis. It will therefore be of special interest to see, how much the different hypothetical parallaxes differ for this object. The annual motion in position angle is + °·0801 or corrected for precession + °·0746 ± °.0091 (m. e.) Assuming the distance to be constant and equal to 2″·5 the minimum value of $p_{(M_1+M_2=1)}$ is ″·00695. Taking $p_{(M_1+M_2=1)}$ to be 1·7 time this minimum value (*A.N.* 4975, **208**, 96; 1919) we get $p_{(M_1+M_2=1)}$ = ″·0118 or $p_{(M_1+M_2=2)}$ = ·0094, while $p_{(1)}$ = ″·0055 and $p_{(2)}$ = ″·0038. The value $p_{(M_1+M_2=2)}$ = ″·016 given by JACKSON (*M. N.* **81**, 25; 1920) is based on an annual motion in position angle of °·112 not corrected for precession. The motion found by VOÛTE (*Ann. Sterrewacht Leiden*, **10**, Part 2, B [72]; 1913) is still greater, viz. °/a·102 ± °/a·0010 (m. e.) after correction for precession. *)

*) The mean error as given by VOÛTE appears to be about 10 times too small.

TABLE 3.

designation of double star		hypothetical parallaxes	$m + 5 \log p = f(M)$		app. magnitudes		abs. magnitudes formula (2)		total mass of system $M_1 + M_2$		
Bu GC		$M_1+M_2=2⊙$ (Jackson)	formula (1) linear	formula (2) quadratic	m_1	m_2	$m_1 + 5 \log p$	$m_2 + 5 \log p$	Jackson	formula (1)	formula (2)
		″	″	″	*m*	*m*	*m*	*m*			
5223	φ Ursae maj.	·0117	·0084	·0078	5·03	5·63	— 5·52	— 4·92	2	5·32	6·81
6955	ζ Bootis	·0192	·0141	·0133	4·61	4·61	— 4·77	— 4·77	2	4·92	6·00
7929	β 416	·123	·149	·149	5·99	8·49	1·86	4·36	2	1·13	1·13
7561	Σ 2026	·036	·0424	·0423	9·0	9·5	2·13	2·63	2	1·26	1·27

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