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COMMUNICATIONS FROM THE OBSERVATORY AT LEIDEN.

On the orbital motion to be expected in double stars of given magnitude, separation and proper motion, by *Ejnar Hertzsprung*.

The vast number of faint double stars of moderate and large separation makes it necessary to confine accurate photographic measures to selected pairs.

One point of view is to give preference to pairs, where the first traces of orbital motion may be expected to be measurable within a reasonable time. In this respect proper motion is a much better guide than brightness.

Before many years the blinkmicroscope may have revealed practically all stars brighter than say 12^m and having a proper motion of more than $''\cdot 1$ a year. Among these objects the double stars not too close for photographic observation will have been recognized as such.

The question therefore arises, which amount of apparent orbital motion may be expected in a pair, of which the magnitude, separation and proper motion is known.

To answer this question we need the relation between $m_{\mu=1} = m + 5 \log \mu$, where μ is the annual proper motion and $m_{sv^2=1} = m + \frac{5}{3} \log s + \frac{10}{3} \log v$, where s is the apparent separation and v the apparent velocity of the orbital motion in seconds of arc a year.

In order to find this relation the two quantities $m_{\mu=1}$ and $m_{sv^2=1}$ were tabulated for a number of well known double stars and divided in groups of 10 objects each. The results thus obtained are given in Table 1, both for visual and photographic magnitudes, the latter being derived from the former by applying corrections corresponding to the spectra (A: 0^m, K: $+1^m$).

The relation between $m_{\mu=1}$ and $m_{sv^2=1}$ using photographic magnitudes is shown graphically in Figure 1.

For the present purpose we have mostly to deal with absolutely faint stars. For these it is seen that the relation between $m_{\mu=1} = m + 5 \log \mu$ and $m_{sv^2=1} = m + \frac{5}{3} \log s + \frac{10}{3} \log v$ is practically linear, with a proportion between $\Delta m_{sv^2=1}$ and $\Delta m_{\mu=1}$ of $2/3$, in which case the relation between v, s, μ and

TABLE 1.

$m_{vis,\,\mu=1}$	$m_{vis,\,sv^2=1}$	adopted mean colour-index	$m_{pg,\,\mu=1}$	$m_{pg,\,sv^2=1}$
m	m	m	m	m
9·07	5·67	1·13	10·20	6·80
7·01	3·83	1·00	8·01	4·83
5·81	3·23	·99	6·80	4·22
5·35	2·52	·93	6·28	3·45
4·90	2·66	·87	5·77	3·53
4·44	2·21	·80	5·24	3·01
4·10	2·29	·81	4·91	3·10
3·69	1·65	·71	4·40	2·36
3·36	1·41	·68	4·04	2·09
3·12	1·08	·67	3·79	1·75
2·73	·98	·64	3·37	1·62
2·33	1·27	·68	3·01	1·95
2·03	1·61	·61	2·64	2·22
1·82	1·53	·59	2·41	2·12
1·42	·99	·56	1·98	1·55
1·00	·59	·46	1·46	1·05
·73	·60	·43	1·16	1·03
·44	·13	·45	·89	·58
·11	− ·63	·47	·58	− ·16
− ·36	− 1·13	·33	− ·03	− ·80
− ·84	− ·63	·27	− ·57	− ·36
− 1·20	− 1·21	·35	− ·85	− ·86
− 1·57	− ·32	·26	− 1·31	− ·06
− 2·02	− 1·55	·33	− 1·69	− 1·22
− 2·57	− 1·91	·24	− 2·33	− 1·67
− 3·23	− 1·19	·41	− 2·82	− 1·78
− 4·11	− 2·01	·23	− 3·88	− 1·78

m turns out to be particularly simple, the expected orbital motion being directly proportional to the proper motion, when s and m are given. In fact we find:

$$\log v = \log \mu - \tfrac{1}{2} \log s - \cdot 1\,(m_{A+B} + c) \pm \cdot 3 \text{ (m. e.)} \qquad (1)$$

The constant c of this formula [1]) is about 2 when visual, and about 1 when photographic magnitudes are used. The mean error of $\log v$ thus computed is about $\pm\cdot 3$. That is to say, that mean deviations are

[1]) It is very simple to make a sliderule, on which the expected v may be read off, when μ, s and m are given.

represented by double or half the velocity of the apparent orbital motion given by the formula. In view of this uncertainty and the small influence of the magnitude on the resulting v, the use of the combined magnitude of the two components without regard to their difference in magnitude is amply sufficient.

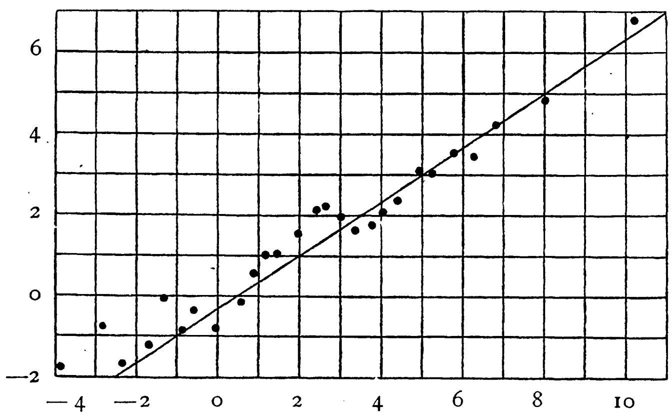

FIGURE 1.

Abscissa: $m_{pg,\,\mu=1} = m_{pg} + 5 \log \mu$

Ordinate: $m_{pg,\,sv^2=1} = m_{pg} + \frac{5}{3} \log s + \frac{10}{3} \log v$

Consider two double stars at the same distance and of the same combined magnitude m_{A+B}, the components A and B being in the one case equal and in the other case 3 magnitudes different. Taking the combined magnitude of one pair as zeropoint, the magnitude of the individual components become

$$m_{A,1} = \overset{m}{\cdot 75},\quad m_{B,1} = \overset{m}{\cdot 75},\quad m_{A,2} = \overset{m}{\cdot 07},\quad m_{B,2} = \overset{m}{3 \cdot 07}$$

For the absolutely faint stars here considered the relation between mass M and absolute magnitude $m + 5 \log p$, where p is the parallax, is from the best known stars found to be

$$\log M = - \cdot 09\,(m + 5 \log p) - \cdot 03 \qquad (2)$$

The thus computed relative masses of the individual components then are

$$M_{A,1} = \cdot 856,\quad M_{B,1} = \cdot 856,\quad M_{A,2} = \cdot 986,\quad M_{B,2} = \cdot 529$$

Hence the proportion $(M_{A,1} + M_{B,1})/(M_{A,2} + M_{B,2})$ is equal to $1 \cdot 712/1 \cdot 515 = 1 \cdot 130 = 1 \cdot 063^2$. The corresponding proportion between the two orbital motions, ceteris paribus, is 1·063, of which the logarithm is ·03 or about one tenth only of the mean error of $\log v$ as computed according to (1) and therefore practically negligible.

As examples of the application of formula (1) I have chosen the double stars listed in Table 2 and taken from the Greenwich Astrographic Catalogue Vol. 5. It is to be expected that accurate photographic measures will reveal the first trace of orbital motion in most of these pairs within a century, if such measures are made in our days. The area considered covers $\frac{1}{2}(\sin 72° - \sin 64°) = \cdot 026$ of the whole sky.

TABLE 2.

Grw. Astrogr. Cat. prec. *	$m_{pg,\,A+B}$	separation s	proper motion μ	expected projected orbital motion v
°	m	"	"/a	"/a
65 73	9·6	2·4	·141	·0079
350	7·9	1275	·133	5
888	9·0	7·0	·122	47
2540	7·2	14·7	·075	30
3487	7·8	11·0	·098	39
66 493	9·5	9·0	·121	36
1923	10·7	9·5	·131	29
2227	10·5	2·8	·076	32
2370	7·8	8·6	·146	65
4728	11·6	7·7	·162	32
4925	10·4	5·7	·122	37
5331	9·8	5·3	·090	32
7460	8·8	4·5	·122	59
67 4234	6·8	179	·105	13
5225	5·4	63·5	·500	58
8910	6·7	69	·118	24
68 314	9·4	6·7	·098	35
1481	9·8	17·0	·276	55
3900	11·3	3·3	·134	44
4762	8·5	4·3	·170	91
4823	10·5	6·0	·111	32
4932	11·7	4·7	·117	29
5947	10·0	4·8	·085	31
8895	12·1	3·0	·102	29
69 2308	8·2	5·4	·095	49
6718	10·0	27·5	·114	17
7093	9·7	2·4	·064	35
9913	6·0	9·3	·062	41
70 4336	11·2	88·5	·735	48
5223	10·3	22·4	·104	16
7061	9·8	9·8	·188	49
9608	10·5	8·2	·122	30
9699	8·9	3·5	·063	35
71 4729	9·0	4·6	1·396	646
4749	10·7	4·0	·186	63

The constants of formula (2) deviate somewhat from those of the original formula connecting mass and absolute magnitude as given in *Astron. Nachr.* 4975, Vol. 208, 96; 1919. Accordingly the formula (23) given there for the connection between trigonometric parallax p, dynamical parallax $p_h = p\,(M_A + M_B)^{\frac{1}{3}}$ and magnitudes of the components m_A and m_B is modified to

$$\cdot 85 \log p = \log p_h + \cdot 03\, m_A - \tfrac{1}{3} \log\left(1 + 10^{-\cdot 09 (m_B - m_A)}\right) + \cdot 01 \qquad (3)$$