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Intrinsic Colours and Physical Properties of Cepheids

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Summary. This paper presents a discussion of extensive photometry in the Walraven *VBLUW* system for 170 Cepheids in the southern Milky Way. It is shown that there exists a narrow intrinsic Cepheid locus in the $(V-B)-(B-L)$ diagram. Using this locus, colour excesses are determined for 156 Cepheids. For a number of Cepheids no consistency can be obtained for the unreddened colours in all three two-colour diagrams. Most of these discrepancies are probably caused by the presence of companions. It is suggested that at least 25% of all Cepheids is member of a binary system. A comparison with the colour excesses of other studies reveals systematic differences: on the average, the *VBLUW* excesses are about $0^m.05$–$0^m.10$ smaller in $E(B-V)$. The difference with the colour excesses of Sandage and Tammann is even larger ($\sim 0^m.15$) and there is evidence of period-dependent effects in those excesses; use of the *VBLUW* colour excesses will therefore probably affect the Cepheid distance scale.

Using model atmosphere spectra from Kurucz, temperatures, gravities, bolometric lightcurves, radius variations, and the equilibrium values of these quantities are derived from the intrinsic colours of 98 Cepheids with $P<11^d$. The results support the adopted reddening corrections, and indicate that the application of hydrostatic-equilibrium model atmospheres to the continuum colours of Cepheids is not too unrealistic.

Key words: Cepheids — *VBLUW* photometry — colour excesses — physical properties

Introduction

In a previous paper ("Paper I"; Pel, 1976) extensive photometry in the Walraven *VBLUW* system has been presented for more than 150 Cepheids in the southern Milky Way. These observations were made in 1970 and 1971 with the five-channel *VBLUW* photometer and the 91-cm "Lightcollector" of the Leiden Southern Station at Hartebeespoortdam, South Africa. Together with the Cepheids, 240 stable supergiants were measured; the *VBLUW* data for those stars can also be found in Paper I. Including the Cepheids that had been observed by Walraven et al. (1964), we now have well-covered light and colour curves for 170 galactic Cepheids in the *VBLUW* system. With only a few exception these are all "classical" Cepheids. The necessary foundation for the interpretation of this large material was laid in Paper II (Lub and Pel, 1977), which dealt with the photometric properties and stability of the *VBLUW* system, and with calibration of the *VBLUW* colours in terms of physical parameters by means of theoretical spectra.

In the present paper, Paper III, we finally come to the principal aim of these observations: the determination of colour excesses and reddening-free colours for the Cepheids, and the derivation of physical properties of the Cepheids from their intrinsic colours. Information of this kind is essential for a discussion of the Cepheid distances and for a comparison with pulsation theory. The discussion in this paper will be limited however to the derivation of colour excesses and physical parameters. The application of these to the problems just mentioned will be the subject of future papers.

The discussion will be divided into two parts. In Part 1 the Cepheid colours are corrected for interstellar reddening and the *VBLUW* colour excesses are compared with those from other studies on Cepheids. In Part 2 physical quantities are obtained by comparing the reddening-free Cepheid colours with the theoretical colours that were computed in Paper II from the grid of model atmosphere fluxes by Kurucz (1975). In this way, the cyclic behaviour of temperature, gravity, luminosity, radius and the equilibrium values for these quantities are determined for the Cepheids with periods shorter than 11 days.

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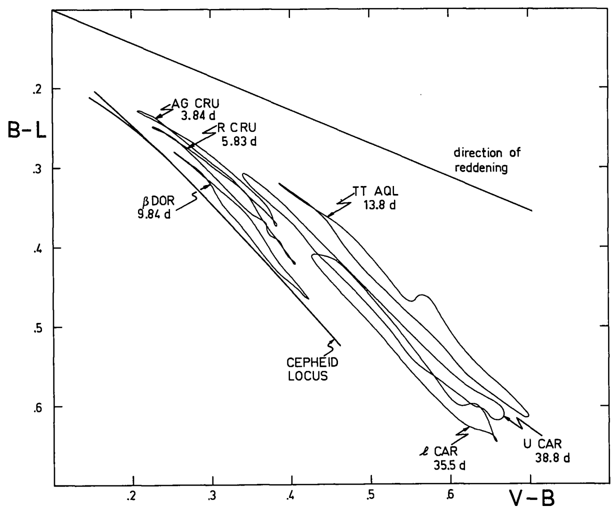


Fig. 1. The loops of six bright Cepheids of different periods in the $(V-B)-(B-L)$ diagram. They are shown in their observed position. Also indicated is the intrinsic Cepheid locus (see text). Following the *VBLUW* convention, all photometric quantities throughout this paper are on a ^{10}log (intensity) scale, unless stated otherwise

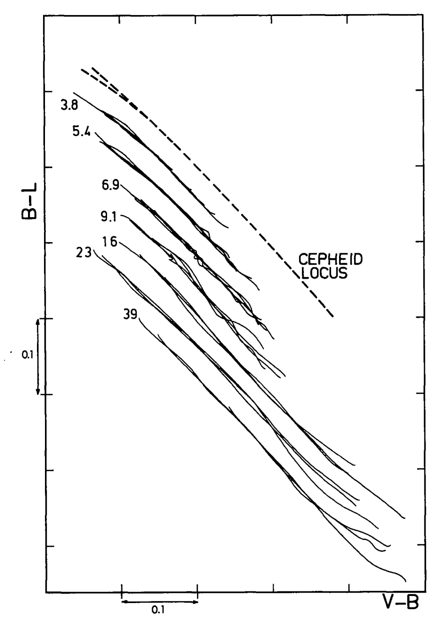


Fig. 2. The descending branches of 29 bright Cepheids, divided into seven period intervals. The average period (in days) for each of the seven groups has been indicated. In each group, the descending parts of the $(V-B)-(B-L)$ loops were shifted along the reddening line onto the position of the descending branch of βDOR. By fitting the descending branches of all groups on top of each other, the shape of the Cepheid locus as indicated in this figure is obtained. For clarity, the different groups have been shifted apart by 0.04 in the $(B-L)$ direction. The zeropoints of the $(V-B)$ and $(B-L)$ scales are arbitrary

1. Reddening Corrections

1.1. Method

The observed colours of most galactic Cepheids are affected by considerable amounts of interstellar reddening: even the brightest Cepheids often have colour excesses of several tenths of a magnitude in $(B-V)_{UBV}$. This is the main reason why there is still much uncertainty (e.g. Canavaggia et al., 1975) about the intrinsic colours of Cepheids, an uncertainty which undermines the accuracy of the Cepheid distance scale as well as our knowledge of the physical parameters of Cepheid atmospheres (an error of $0^m.1$ in E_{B-V} corresponds to about 300 K in the temperature scale). The problem is complicated by the fact that the existing Cepheid photometry is extensive (at least in the northern hemisphere) but very inhomogeneous. Moreover, most Cepheid photometry has been made in wide-band photometric systems. It is well known that such systems are not very well suited for accurate reddening determinations: stars with different spectral energy distributions, observed through the same amount of interstellar dust will show different colour excesses if the photometric passbands are very broad. In principle one can correct for such bandwidth nonlinearities, but often the correction terms are not sufficiently well defined, and in the case of Cepheids they are not only different from star to star but even from phase to phase. From this it is clear that the present Cepheid observations offer an opportunity to improve the accuracy of the intrinsic colours of Cepheids: firstly, the *VBLUW* material is large and homogeneous, and secondly, bandwidth nonlinearities in the *VBLUW* system are very small (see Paper II, Section 5).

There are two standard methods that can be used for the determination of reddening-free colours. The first is, to construct reddening-independent indices that are linear combinations of the measured colours. With our five bands, four independent colours are measured, which means that three reddening-free indices can be computed when the interstellar extinction law is known. The use of reddening-independent colour combinations has disadvantages however. While the colours were designed to separate the effects of the various physical parameters as well as possible, by combining we mix these effects again. In the second method, the observed points in the two-colour diagrams are shifted along the reddening lines (corresponding to the "known" interstellar extinction law) onto their intrinsic unreddened position. This can only be done, of course, if extra information is available which defines that intrinsic position. For the *VBLUW* photometry of classical Cepheids the latter method can be applied successfully, as was shown by Walraven (1966). I decided to follow the same procedure that Walraven used, but I made an independent determination of the required intrinsic "Cepheid locus". This I will now discuss.

As we will see in the following, the two-colour diagram that is suited most for the construction of a locus is the $(V-B)-(B-L)$ diagram. During their cycles, the Cepheids describe long, narrow loops in the $(V-B)-(B-L)$ plane. Some of these loops are shown in Figure 1 in their observed position. Apart from the irregularities that are associated with the secondary 'bumps' in the lightcurves, and the differences in amplitude for stars of different periods, the loops of most Cepheids look very similar in shape and orientation. They are always described counter-clockwise. From the observations of stars of different luminosity classes we know that for spectral types later that F0 there are practically no luminosity effects in the $(V-B)-(B-L)$ diagram (see Paper II, Fig. 9). The theoretical colours derived in Paper II (Fig. 8 of that paper) confirm this: all constant-gravity lines for $T_e<7500$ K nearly coincide. It is therefore reasonable to assume that all unreddened Cepheid loops occupy a well-defined "locus" in the $(V-B)-(B-L)$ diagram, and that the observed loops have been shifted away from this locus by different amounts of reddening. By shifting the loops back onto the locus, we can determine colour excesses. Shape and position of the Cepheid locus were determined in the following way.

a) Shape of the Locus

If the spectrum of a Cepheid can be compared at all with that of a stable supergiant, the resemblance is probably closest during the descending part of the lightcurve. I therefore decided to use only the descending branches of the $(V-B)-(B-L)$ loops. In order to see whether the shape of the locus is a function of period, 29 of the brightest Cepheids were divided into seven different period intervals, and in each interval the descending parts of the $(V-B)-(B-L)$ loops were shifted along the reddening line onto the position of the descending branch of β DOR, the brightest and least reddened of the 170 Cepheids. The result is shown in Figure 2. Within each group, the different descending branches fit very well on top of each other. Only for the interval around 9 days and at the very longest periods there are somewhat larger differences between the individual variables. For the Cepheids with $P=8$–10 days this is clearly related to the secondary bumps in the colour curves, which are very pronounced in that period range. The deviations around minimum of the long-period Cepheids are probably also due to these bumps, which at those periods start appearing at minimum (after their phase has shifted over the maximum and down the rising branch of the lightcurve between 9 and 20 days period).

The theoretical $(V-B)-(B-L)$ diagram of Paper II shows that particularly between 7000 and 6000 K the lines of constant gravity come very close together. Around $T_e=6000$ K, a temperature range through which all Cepheids pass during their cycle, the lines of $\log g=2.5$ and $\log g=1.5$ lie only a few thousandths apart. Moreover, the theoretical colours indicate that differences in microturbulent velocity have relatively little influence in the $(V-B)-(B-L)$ diagram: the microturbulence vector at 6000 K makes only a small angle with the lines of constant gravity (see Paper II, Table A3). The final shape of the Cepheid locus was therefore determined by fitting all descending branches of Figure 2 on top of each other into one narrow strip. From the theoretical colours I estimate that the width of this strip due to gravity effects is smaller than 0.005 in $^{10}\log$ (intensity), so the uncertainty caused by taking the same locus for Cepheids of different effective gravity is comparable to the photometric accuracy of the loops of the brightest Cepheids.

The locus that results from combining the different groups of Figure 2 shows a small divergence at the high-temperature end; the short-period Cepheids follow the lower branch, the maxima for the 11 to 20 days periods lie on the upper branch. The locus is forked here, because the maxima of the Cepheids in the intermediate period range of 7 to 10 days are not blue enough to reach the point of divergence, while the sudden increase in colour amplitudes at periods longer than 11 days makes the maxima of those stars very blue again. The split upper end of the Cepheid locus had already been noticed by Walraven (1966). Comparing with the theoretical $(V-B)-(B-L)$ diagram, we see that this fork shape is probably caused by gravity differences between short- and long-period Cepheids, as for $T_e>7000$ K $(B-L)$ becomes sensitive to the Stark broadening of the Balmer lines. The cool end of the locus in Figure 2 does not extend quite as far as the minimum of the Cepheids with the longest periods. As was remarked above, there are individual differences in the loops of these stars around minimum, and there are too few bright long-period Cepheids to determine accurately where the locus ends.

b) Position of the Locus

Now that the shape of the locus is known, we have to fix its exact unreddened position. For this I used the following information: i) the reddening of the brightest and least reddened Cepheid, βDOR; ii) the reddening of the least reddened cluster Cepheid, S NOR; iii) the position of the brightest Ib and Iab supergiants in the $(V-B)-(B-L)$ diagram; iv) the theoretical $(V-B)-(B-L)$ diagram.

i) The colour excess of βDOR, which has a very small reddening due to its relatively high galactic latitude ($-33°$) and small distance (~ 0.3kpc), can be estimated in the following way. As β DOR lies well outside the galactic plane, and at only a few degrees distance from the Large Magellanic Cloud, the value of the galactic reddening in the direction of the LMC is a good estimate for the colour excess of this Cepheid. The highest values of the foreground reddening towards the LMC are given by the Radcliffe astronomers: $E(B-V)_{UBV}=0\overset{m}{.}07$ (Feast et al., 1960); most other estimates give lower numbers (e.g. Dachs, 1972: $E(B-V)_{UBV}=0\overset{m}{.}03$). I adopted $E(B-V)_{UBV}=0\overset{m}{.}05$ for β DOR, which corresponds to $E(V-B)=0.024$ in the logarithmic units of the *VBLUW*-system. This value is consistent with the linear polarization of β DOR which gives a lower limit to its colour excess.

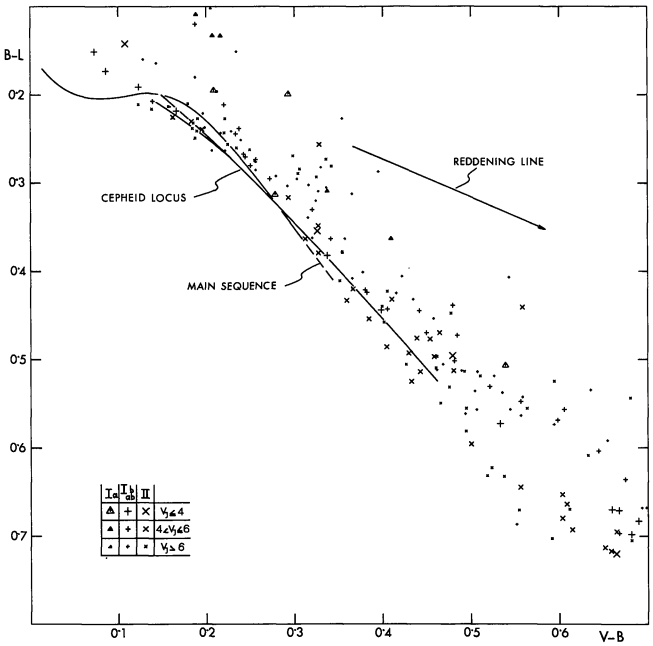


Fig. 3. The position of the Cepheid locus relative to the main-sequence and a number of non-pulsating supergiants. The supergiants are shown in their reddened position

Table 1. Normal points of the Cepheid locus

$V-B$	$B-L$	
0.460	0.524	all periods
0.370	0.421	
0.300	0.345	
0.230	0.274	
0.190	0.237	$P>10^d$
0.152	0.203	
0.200	0.249	$P\leqq 10^d$
0.170	0.227	
0.141	0.207	

Using the relation between maximum polarization percentage and colour excess from Serkowski et al. (1975), $p_{max}(\%)=9.0E(B-V)_{UBV}$, and the linear polarization of β DOR given in that same paper (0.44%), one finds $E(B-V)_{UBV}\geqq 0\overset{m}{.}05$. The adopted colour excess of β DOR is also consistent with estimates of the reddening in the direction of the galactic poles from polarization data. For stars at the south galactic pole with distances >100 pc from the plane, Appenzeller (1975) gives a minimum colour excess $E(B-V)_{UBV}\geqq 0\overset{m}{.}016$ with a most likely value of $0\overset{m}{.}03$.

ii) S NOR, in NGC 6087, is the least reddened of the known cluster Cepheids. A few authors have suggested duplicity of this variable. Breger (1970) found a zeropoint difference between his radial velocities and earlier velocity curves that could possibly be caused by binary motion. Janot-Pacheco (1976) reported indications of a faint blue companion in *UVBGR* (Lick system) data of S NOR. Although the ultraviolet colours of the *VBLUW* system are very sensitive to contaminations by blue companions, I cannot find any evidence of such a companion in the *VBLUW* colours of S NOR. Compared to other Cepheids of the same period S NOR looks completely normal, and I therefore assume that the colours of S NOR are undistorted. Breger (1970) derived $E(B-V)_{UBV}=0\overset{m}{.}18$ for S NOR. In *VBLUW* units this transforms to $E(V-B)=0.087$. This value agrees very well with the colour excess of β DOR, as the observed difference in $E(V-B)$ between these 10-days period Cepheids is 0.056. So both S NOR and β DOR lead to the same position of the locus to within a few thousandths in $(V-B)$.

iii) When the Cepheid locus is shifted to a position corresponding to $E(V-B)=0.024$ for β DOR, as shown in Figure 3, we also get good agreement with the position of the least reddened *Ib* and *Iab* supergiants in the $(V-B)-(B-L)$ diagram. The locus of Figure 3 coincides well with the blue envelope of the bright Ib and Iab stars. This confirms what Walraven (1966) found in his discussion of a more limited number of Cepheids and supergiants. Walraven's Cepheid locus agrees very closely (to within 0.01 in $V-B$ and $B-L$) with the locus in Figure 3. At the red end of the locus, a number of class II supergiants fall somewhat below the locus. This is probably due to the higher gravity of these stars as compared to the Cepheids. In fact, several of the class II stars are sometimes classified as class III giants (see references to Table 2, paper I). Also in the theoretical $(V-B)-(B-L)$ diagram the lines of higher gravity fall below the low-gravity lines for $T_e<6000$ K. From this divergence of the gravity lines at lower temperatures one might expect that the cool end of the locus is split like the upper end. This is not so, however, because the lower part of the locus is populated only by the long-period Cepheids.

iv) We can construct a "theoretical Cepheid locus" by fitting a smooth line in the theoretical $(V-B)-(B-L)$ diagram (paper II, Fig. 8) through the points ($T_e=7000$, $\log g=2.5$), ($T_e=6500$, $\log g=2$), ($T_e=6000$, $\log g=1.5$), and ($T_e=5500$, $\log g=1$). Because the $(V-B)-(B-L)$ diagram is very insensitive to gravity in this temperature range, a rough approximation for the relation between gravity and temperature of Cepheids is sufficient here. The position and shape of this theoretical locus relative to the theoretical main-sequence agree very well with the empirical locus and main-sequence of Figure 3. Around $T_e=6500$ K the theoretical locus lies about 0.01 in $(B-L)$ below the $\log g=4.5$ line, while the corresponding difference in $(B-L)$ in Figure 3 is about 0.02. In absolute position the agreement is less precise: the locus in Figure 3 is about 0.02 redder in $(B-L)$ than we would have estimated from the theoretical colours alone. The same difference between observed and theoretical $(B-L)$ colours of cool stars was already noticed in paper II (see the discussion in Section 6 of the paper). In view of the strong line-blanketing effects in the spectra of these stars, particularly in the L band, I do not find this difference

Table 2. The $E\,(V\text{–}B)$ colour excesses of the Cepheids. All numbers are in ^{10}log (intensity). Stars with known or suspected companions and stars which show peculiarities in the two-colour loops are indicated by question marks. For stars marked with (?) there is evidence for a companion, but the *VBLUW* colours look normal

T ANT .087 ?	UW CAR .194	V CEN .117	β DOR .024	SY NOR .29 ?	U SGR .182	CK SCT .37 (?)
	UX .037	TX .40:(?)		TW .56:	W .037	EV .29
U AQL .141	UY .079	UZ .095	W GEM .098(?)	GU .27 ?	X .086 ?	
SZ .255	VY .099	VW .156	DX .18:		Y .060	R TRA .06
TT .202	WW .176 ?	XX .113		Y OPH .278 ?	VY .5:	S .025
FF .058 ?	WZ .170	AZ .065	GH LUP .149 ?	BF .100(?)	WZ .18	
FM .294 ?	XX .156	BB .140 ?			YZ .117	T VEL .112
FN .219 ?	XY .184	KK .24:	T MON .06 ?	RS ORI .149	AP .058	V .117 ?
KL .057 ?	XZ .15 ?	KN .32 ?	SV .093	GQ .09:	AV .6:	RY .26 ?
336 .277	YZ .173	339 .181	TX .23 ?		AY .37	RZ .126
496 .167 ?	AQ .070	381 .083	TZ .17	X PUP .183	BB .107	ST .232
600 .37:	CC .25:	419 .065	XX .28	RS .229		SV .190
η .032	CN .17	496 .249	AC .21	VW .20	RV SCO .149(?)	SW .160
	CR .24:		CS .25 ?	VZ .227	RY .30 ?	SX .109
RW CMA .24:	DY .15	R CRU .064	CV .33 ?	WW .16 ?	482 .134	XX .23:
RY .093	ER .045	S .066	EK .24:	WX .126	500 .27	AE .29
RZ .220 ?	FI .31:	T .072		WY .102 ?	636 .078 ?	AH .0 ?
SS .240 ?	FN .25:	X .107	R MUS .044	AD .144		AP .19
TV .25	FO .22: ?	SU .489	S .11 ?	AP .084	X SCT .275(?)	AX .089
TW .160 ?	FR .134	SV .35:	RT .151 ?	AT .084	Y .35	BG .209
	GH .160	VX .36:	UU .175	BN .189	Z .195	CP .41:
U CAR .129	GX .162	VW .25 ?		CE .36:	RU .46 ?	CX .31:
V .070	GZ .17	AD .30	S NOR .080		SS .138	DR .33
Y .050	IT .083	AG .097 ?	U .473 ?	S SGE .024 ?	TY .43:	EX .39:
SX .138	ℓ .097		RS .24 ?		UZ .46:	FG .38:

very disturbing. In fact, the agreement between theoretical and observed colours is quite satisfactory.

Combining the data of i) to iv), we can put rather strict constraints on the unreddened position of the Cepheid locus. I finally adopted the locus as drawn in Figure 3 and tabulated in Table 1. This is the locus position which corresponds to $(V-B)$ colour excesses of 0.024 and 0.080 for β DOR and S NOR, respectively. The estimated systematic uncertainty in this locus is smaller than 0.01 in $(V-B)$ and $(B-L)$. Apart from S NOR there are five other cluster Cepheids for which *VBLUW* data are available (CV MON, TW NOR, RS PUP, EV SCT, U SGR). Unfortunately it is at present not possible to improve the calibration of the Cepheid locus by means of these stars. The clusters to which these Cepheids belong have not yet been observed in the *VBLUW* system. They are all much more reddened than S NOR, so the strong nonlinearity of the transformation between $E(B-V)_{UBV}$ and $E(V-B)$ makes it impossible to derive sufficiently accurate $(V-B)$ excesses from the published *UBV* colour excesses. To improve this situation, *VBLUW* observations of some of the clusters containing Cepheids are under way.

So far I have not considered the influence of abundance differences on the position of the $(V-B)-(B-L)$ loops of the Cepheids. It has been shown in paper II that differences in line-blanketing have a considerable effect on $(B-L)$: the models in the range 7000–5500 K with 1% of the solar heavy element abundance have $(B-L)$ colours that are 0.05 (at 7000 K) to 0.17 (at 5500 K) bluer than for the corresponding solar composition models. In the determination of the Cepheid locus I have assumed that all Cepheids have solar abundances, but it is obvious that this locus cannot be used for Cepheids of different composition. Fortunately, there is observational information that can help us here. Population II Cepheids can usually be distinguished from the "classical" population I Cepheids, because their lightcurves have different shapes. Moreover, the shape, slope and position of the loops of population II Cepheids in the two-colour diagrams differ considerably from the behaviour of the classical Cepheids; particularly in the $(V-B)-(B-U)$ diagram the population II loops are much more open (e.g. see Oosterhoff and Walraven, 1966, for *VBLUW* data of W VIR). Among the 170 Cepheids discussed in this paper, there are a few known or suspected population II Cepheids (e.g. UY ERI, TX DEL, κ PAV, CO PUP, DD VEL, PZ AQL, PP AQL). There stars differ clearly from the regular pattern that the other Cepheids show in the two-colour diagrams. I have therefore assumed that the criteria mentioned above are sufficient to recognize the Cepheids that are not "classical". I also assumed that the classical Cepheids are a chemically homogeneous group of stars with abundances very close to the solar values. I will come back to the problem of composition differences in the next section.

Before proceeding to the actual reddening corrections, a few remarks should be made about the reddening lfnes. It has been demonstrated in paper II that the effects of reddening on the various *VBLUW* colours are highly linear, because the *VBLUW* bands are sufficiently narrow. In particular, the slopes of the reddening lines are nearly independent of the observed energy distribution. This means that the same reddening correction can be used for all points of a Cepheid loop, and that the same reddening direction can be used for all Cepheids. As was pointed out already, this greatly simplifies the reddening corrections in our system as compared to wide-band systems, where complicated (and inaccurate) differential corrections are needed that depend on the colour differ-

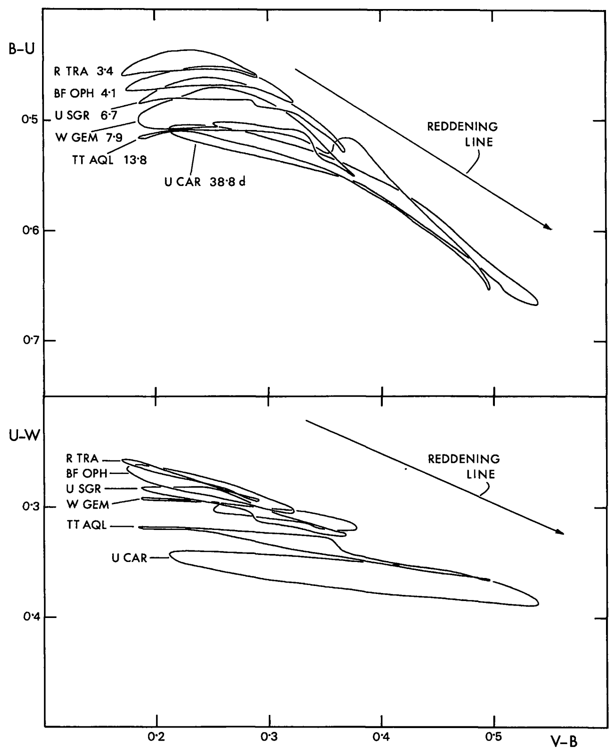


Fig. 4. The loops of seven bright Cepheids of different periods in the $(V-B)-(B-U)$ and $(V-B)-(U-W)$ diagrams, after correction for reddening. Like in the $(V-B)-(B-L)$ diagram, all loops are described counter-clockwise

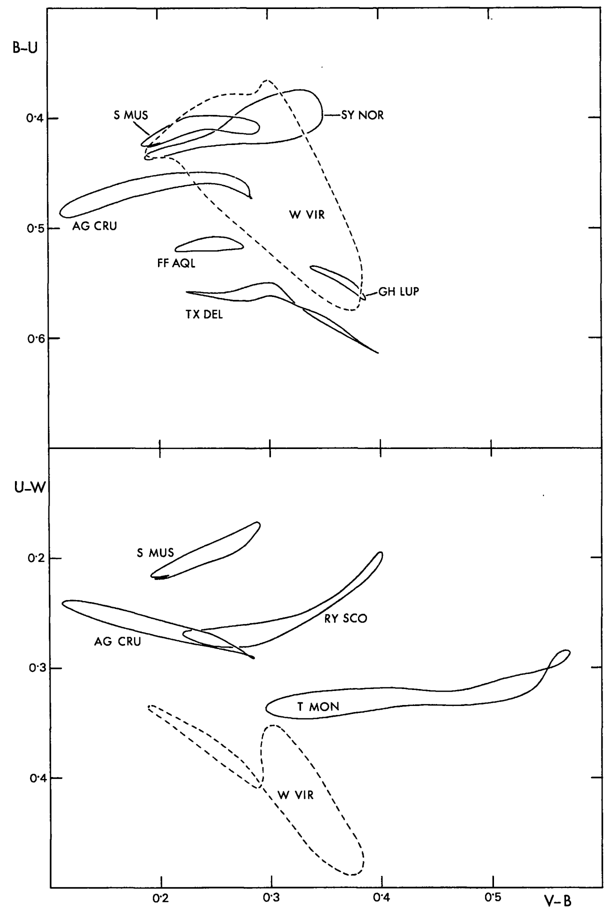


Fig. 5. The loops of some peculiar Cepheids in the $(V-B)-(B-U)$ and $(V-B)-(U-W)$ diagrams. For the behaviour of normal Cepheids see Figure 4. No reddening corrections could be made for W VIR and TX DEL, but the reddening of these stars is probably very small. The other loops are in their unreddened positions

ences between Cepheids and even on the colour variations during the cycle of each individual Cepheid. For the slopes of the reddening lines I used the colour excess ratios given in paper II (Section 5):

$$E(B-L)/E(V-B)=0.43, E(B-U)/E(V-B)=0.64, E(U-W)/E(V-B)=0.45.$$

1.2. Results

The colour excesses that were determined by shifting the descending branches of the Cepheid loops onto the locus in the $(V-B)-(B-L)$ diagram are listed in Table 2. Only the $E(V-B)$ values are given [all in $^{10}\log$(intensity)]; the reddening in the other three colours can be computed from the colour excess ratios given above. The Cepheid data of Walraven et al. (1964) were transformed to the standard *VBLUW* system of 1970–1973 by applying small corrections for V and W (see paper II, Section 3). Not all Cepheids of paper I and of Walraven et al. (1964) appear in Table 2. A number of variables were excluded because they are probably not population I Cepheids. These stars are: PP AQL, PZ AQL, TX DEL, UY ERI, κ PAV, CO PUP, DD VEL. For FN AQL, KL AQL and Y OPH a colour excess is given, but it is doubtful whether these stars are population I Cepheids: the shape of the lightcurve of FN AQL is rather unusual for classical Cepheids of this period, KL AQL has a peculiar loop in the $(V-B)-(B-U)$ diagram and is sometimes classified as population II (Petit, 1960), and Y OPH is a well-known exceptional case. Six Cepheids are too faint in L to make a reliable reddening determination possible: EV AQL, AB ARA, FQ CAR, HW PUP, V470 SCO, EZ VEL. The number of Cepheids for which more or less reliable colour excesses can be determined, is 156. The accuracy of these colour excesses depends on the brightness. For the brightest Cepheids $E(V-B)$ can be determined to within ± 0.003; the reddening of the faintest Cepheids in Table 2 is not more accurate than ± 0.03 in $(V-B)$. There uncertainties refer only to the accuracy with which the $(V-B)-(B-L)$ loops of the Cepheids can be fitted onto the intrinsic locus. They do not include the uncertainty in the locus itself, and the uncertainties due to small composition differences between Cepheids or local deviations from the average interstellar extinction law.

When the $(B-U)$ and $(U-W)$ Cepheid colours are corrected by the amounts that correspond to the $(V-B)$ excesses of Table 2, the position of the unreddened loops in all two-colour diagrams is fixed. The situation in the $(V-B)-(B-U)$ and $(V-B)-(U-W)$ diagrams is

Table 3. Schematic description of the peculiarities of the Cepheids marked with ? in Table 2. A minus sign indicates that the star is too blue for its period in the corresponding colour, a plus sign means that it is too red. When the slope of a loop is indicated as too negative, this means anti-clockwise rotation with respect to the normal loop orientation

Name	$(V-B)_o$	$(B-L)_o$	$(B-U)_o$	$(U-W)_o$	Other remarks
T Ant	-!	-!	-!		Sometimes classified as pop.II Cepheid (Petit, 1960)
FF Aql			+!	+	Spectroscopic binary (Abt, 1959). Loops very small.
FM Aql			-		
FN Aql					Peculiar lightcurve. Slope of (V-B)-(B-U) and (V-B)-(U-W) loops too positive.
KL Aql	+!	+!	+!		Abnormal shape of (V-B)-(B-U) loop (8-shape). Pop.II Cepheid? (Petit, 1960)
496 Aql	(+)	(+)	(+)	(+)	Slope of (V-B)-(B-U) loop too positive.
RZ CMa	-!	-!	-!		
SS CMa	-	-	-		Faint star within the diagram (see paper I).
TW CMa			-!		Distorted loop in (V-B)-(B-U)
WW Car	-!	-!	-!		Slope of (V-B)-(B-U) loop somewhat too negative.
XZ Car					Slope of (V-B)-(B-L) loop slightly too negative. Colours look normal otherwise.
FO Car			-!		Faint star within the diaphragm (see paper I).
TX Cen					According to Plaut (1934) there is a companion at 6"($V\sim12^m$). Not detected visually nor in the colours.
BB Cen			+!		Loops are very small.
KN Cen	-!	-!	-!	-!	Spectroscopic companion (Lloyd Evans, 1968). All loops strongly distorted; too negative slope.
VW Cru			+!		Slope of (V-B)-(B-U) loop too positive.
AG Cru	(-)	(-)	(-)	(-)	Slope of (V-B)-(B-L) and (V-B)-(B-U) loops too negative.
W Gem					Faint star within the diaphragm? (see paper I). Colours look normal.
GH Lup	+!	+!	+		Loops very small.
T Mon				-	(V-B)-(B-L) loop rather open. (V-B)-(U-W) loop has too negative slope.
TX Mon	-!	-!	-!		
CS Mon			-!		
CV Mon	(-)	(-)	-		Distorted loop in (V-B)-(B-U).
S Mus	-!	-!	-!	-!	Slope of (V-B)-(B-U) and (V-B)-(U-W) loops too negative.
RT Mus	(-)	(-)	-	-	Slope of (V-B)-(B-U) loop too negative.
U Nor	-!	-!	-!	-	Slope of all loops somewhat too negative.
RS Nor			+!		
SY Nor	-	-	-!	-!	Companion at $2''.1$, $V=12^m.8$ (see paper I). All loops strongly distorted; too negative slope.
GU Nor	+	+	+		
Y Oph	-!	-!			Peculiar Cepheid (e.g.: Abt, 1954; Mianes, 1963).
BF Oph					Mianes (1963) suspected presence of red companion. VBLUW colours look completely normal.
WW Pup	-!	-!	-!		(V-B)-(B-U) loop distorted and slope too negative.
WY Pup			-		Distorted loop in (V-B)-(B-U).
S Sge			+		Spectroscopic binary (e.g. Lloyd Evans, 1968).
X Sgr	-!	-!	-!	-	Slope of (V-B)-(B-U) loop somewhat too negative.
RV Sco					Faint companion (Plaut, 1934). Colours look completely normal.
RY Sco			+!	-!	Companion, $V=12^m$ (Plaut, 1934). Strongly distorted colour curves; all loops too negative slope.
636 Sco					Spectroscopic binary (Lloyd Evans, 1968). VBLUW colours look normal.
X Sct					Very faint star within the diaphragm (see paper I). Colours look completely normal.
RU Sct	-	-	-	-	Slope of all loops is too negative.
CK Sct					(V-B)-(B-L) loop has rather negative slope. No other peculiarities.
V Vel	(-)	(-)	-!		(U-W) looks normal.
RY Vel			-	-	All loops have too negative slope, particularly (V-B)-(U-W).
AH Vel	(-)	(-)	+	+	Spectroscopic binary? (Lloyd Evans, 1968). Slope of (V-B)-(B-U) loop too negative. Loops very small.

illustrated in Figure 4. The appearance of the Cepheid loops in these diagrams is quite different from the $(V-B)-(B-L)$ loops. While in the $(V-B)-(B-L)$ plane all loops lie in the same narrow strip, Cepheids of different periods occupy different regions in the two other diagrams. Comparing the observed Cepheids colours with the theoretical colours of paper II, we see that the difference in behaviour of the loops in the three diagrams is caused by differences in effective gravity of the Cepheids: for F and G type stars $(B-L)$ is nearly independent of $\log g$ but $(B-U)$ and $(U-W)$ are very sensitive to gravity differences.

1.3. Cepheids with Abnormal Colours

The $(V-B)-(B-U)$ and $(V-B)-(U-W)$ loops of most Cepheids follow a very regular pattern. In both diagrams, the shape and position of the loops change smoothly with increasing period. There are, however, a number of variables that do not fit in this sequence. These Cepheids have been indicated by a question mark in Table 2. They are considerably too blue or too red in one or more colours in comparison with most other Cepheids of the same period, and often the shape and orientation of the loops are abnormal. A few typical cases are shown in Figure 5; in Table 3 I have listed in a schematic way the properties of the abnormal loops. Table 3 contains also the Cepheids with known or suspected companions (whether their loops look abnormal or not). Also these stars have question marks in Table 2.

In the context of the present paper, I do not want to discuss all deviating Cepheids individually. The purpose of Table 3 is to separate out those Cepheids which show inconsistencies in one or more colours, indicating that the intrinsic colours obtained by the locus method may be incorrect for some reason, and that consequently the physical quantities derived from those colours may be wrong. Some general remarks about the discordant Cepheids should be made, however.

Several stars of Table 3 have a spectroscopic or visual companion. I suspect that the deviations of most of the others are also due to the presence of companions, but companions are not the only way to explain peculiarities in the Cepheid colours. Deviations from the assumed population I composition, or incorrect reddening values due to local variations of the interstellar extinction law, can also cause deviations from the general pattern of the Cepheid loops. However, since we have three independent two-colour diagrams, it is often possible to discriminate between these three possibilities.

From Figure 8 in paper II we see that the influence of chemical composition is quite different in the three two-colour diagrams (let us call the $(V-B)-(B-U)$, $(V-B)-(B-L)$ and $(V-B)-(U-W)$ diagrams I, II and III, respectively). Diagrams I and II are both very sensitive to composition differences, and the blanketing vectors have similar size and direction. In I it is difficult to seperate the effects of gravity and composition, but in II this is possible below $T_e=7500$ K, where the gravity dependence becomes very small. Diagram III is equally sensitive to gravity as I, but $(U-W)$ is very little dependent on composition. So for Cepheids of different composition we expect large shifts in I and II, and not much difference in III. To these intrinsic shifts of the loops we have to add the errors in the reddening corrections made by using the standard Cepheid locus for Cepheids of non-standard composition. The loop of an underabundant Cepheid, which in reality should lie above the locus in diagram II, will be shifted along the reddening line by an extra amount to bring it onto the locus. This means too large reddening corrections, and consequently all colours are made too blue. For overabundant Cepheids the errors have the opposite sign. The theoretical colours indicate that the orientation of the loops in I and II will change when the composition differences are large. Cepheids with compositions that differ very significantly from solar abundances, like the W Virginis stars, do not cause such problems, however. The atmospheric parameters, and their behaviour during the pulsation are so different here (cf. W VIR in Fig. 5) that such stars are rather easily recognized and excluded from the locus fitting procedure (see the remarks at the beginning of Section 1.2). Most difficult to detect are relatively small abundance anomalies. T ANT and V VEL are such cases, where the inconsistencies between the three two-colour diagrams probably have to be explained by a small deficiency in heavy elements.

Deviations from the standard interstellar extinction law affect the reddened position of the loops in all three diagrams in the same sense (all reddening lines become steeper or flatter), but the shape and slope of the loops remain normal. The size of the shifts is proportional to the amount of reddening. With reasonable values for the variations in the extinction law (e.g. from Nandy, 1965), this means that noticeable effects can only be expected for very strongly reddened Cepheids. There are no Cepheids in Table 3 for which local deviations of the extinction law are the most likely cause of the abnormal colours, but it is obvious that such deviations will cause some scatter in the intrinsic Cepheid colours derived in this paper.

The ultraviolet bands of the *VBLUW* photometry provide a very sensitive tool for the detection of photometric companions that are bluer than the Cepheids. Blue companions will distort position and shape of the loops particularly in diagrams I and III. The distortion is largest around minimum light. Spectacular cases are S MUS and KN CEN, which have relatively bright blue companions. The companion of S MUS is probably an intermediate B-type star, about 1.5 to 2 magnitudes fainter in V than the mean brightness of the Cepheid. The companion of KN CEN is probably an early B star, ~ 1–1.5^{m} fainter than KN CEN. The deviations of U NOR, AG CRU and RT MUS can be explained by late B-type companions that are 2–3^{m} fainter than the variables. If we assume that these companions are at the same distance as the Cepheids, they are most likely main-sequence stars. Red companions affect the loops only noticeably when they are bright, and their distortion of the loops is less striking. The loops are displaced to the red and they become shorter and more open, but these effects are smaller than the deviations of stars like S MUS and KN CEN. AH VEL and FF AQL probably have supergiant companions, with brightness and colours close to those of the Cepheids. The effect of a companion is usually least pronounced in diagram II, and for most Cepheid-companion combinations the shift in this diagram is approximately along the direction of the undisturbed loop, which does not affect the reddening determination very much. Nevertheless, it has to be stressed that the colour excesses for Cepheids with companions may be incorrect. Moreover, in diagram I the loops can be disturbed strongly by a companion, so when we come to the derivation of physical parameters for the Cepheids—and there the $(V-B)-(B-U)$ plane is essential—the stars of Table 3 will have to be treated with caution.

When the symptoms of the Cepheids in Table 3 are examined according to the criteria discussed above, it turns out that for the majority the presence of a companion is the most plausible explanation. One can try to solve the parameters of these companions from the corrections needed to make the observed abnormal loops normal again. For sufficiently bright Cepheids, and particularly in the case of blue companions, a two-dimensional classification of the companion spectrum should even be possible. Such information would be very interesting from an evolutionary point of view, and in principle a detailed study of Cepheid binaries could provide mass and luminosity estimates for the Cepheids. The binary candidates of Table 3 therefore deserve a careful discussion. At present I have not yet made detailed solutions for the stars in Table 3; this will be the subject of a future paper.

At least 25% of the Cepheids in Table 2 show evidence of a companion. In the few cases where the companion is actually visible, the distance to the Cepheid is a few arcseconds; for the unseen companions (the majority) the distances will be smaller. At typical Cepheid distances of ~ 1 kpc these separations correspond to a few thousand astronomical units, indicating that most of the Cepheid-companion pairs are probably physical binaries. This means that at least 20 to 25% of all Cepheids are members of a binary system. Since we will have missed the fainter companions (a companion is usually undetectable in the photometry when it contributes less than 1% of the average Cepheid light) this number is only a lower limit to the real frequency of binary Cepheids. In itself it is not surprising that one out of every four Cepheids is a binary member, but most earlier estimates in the literature give smaller numbers [e.g. Abt (1959): $\sim 2\%$ in binaries; Lloyd Evans (1968): $\gtrsim 15\%$ in binaries]. My estimate of $\gtrsim 25\%$ binaries among Cepheids agrees very well with recent work by Madore (1975, and private communication), who finds a binary percentage of 20–30%. The binary nature of a considerable fraction of the Cepheids will be a source of scatter in most empirical Cepheid relations. In particular, the period-colour-luminosity relation and the distances derived from it for individual Cepheids will be affected by the presence of companions. This stresses once more the importance of a detailed study of the Cepheids in Table 3: when corrections for the influence of companions can be made from the photometry, this will improve the accuracy of Cepheid distances considerably.

1.4. Comparison with Other Colour Excess Studies

Let us now compare the colour excesses of Table 2 with those from other studies. Colour excesses of Cepheids have been determined by many different authors using different methods, and it is not possible to give a comparison here with all excesses that can be found in the literature for each individual Cepheid. But before deriving physical quantities of the Cepheids from their de-reddened *VBLUW* colours, I want to have at least a general idea of how the colour excess scale of the present study compares with that of the most widely used sources of Cepheid colour excesses. This comparison is made in Figure 6. The relevant references can be found in the legend to this figure. A comparison with the extensive work on Cepheid colours by Nikolov is not given in Figure 6, because in his last paper Nikolov adopts the colour excesses derived by Tsarevskii and Yakimova as the best available at present (Nikolov and Ivanov, 1974).

As most Cepheid photometry has been made from the northern hemisphere, the overlap of the present work with other studies is often quite limited. Nevertheless it is clear that the correlations in Figure 6 show significant differences in slope and zeropoint.

The differences in slope can be explained by the different effective wavelengths of the blue and visual passbands employed by the various authors, but more problematic are the surprisingly large differences in the zeropoints. Using the relations between the *UBV*, *uvby* and *VBLUW* colour excesses computed in paper II, lines of the appropriate slope and curvature were fitted to the data in Figure 6. For the *UBV* data (Figs. 6b to 6i), the computed relation for G2 I supergiants (see paper II, Fig. 7) was taken as an average for Cepheids. The original colour excesses of Mianes and of Parsons and Bouw are in the Lick six-colour system, but these authors give also the $(B-V)_{UBV}$ excesses. These are shown in Figure 6, and the G2 I relation for the *UBV* system was also used in those two cases. The passbands employed by Walraven et al. (1958) are not known accurately enough to compute the relation with the *VBLUW* colour excesses. A simple straight line fit was therefore made in Figure 6a. Not too much weight can be given to the zeropoint in this case: the basic assumption—identical intrinsic colours at maximum light for all Cepheids—used by Walraven et al. to determine reddening values is no longer tenable. Figure 6a is mainly included because of the almost complete overlap with the *VBLUW* survey, but there are pronounced period-dependent trends in this diagram.

The resulting zeropoints are given in Table 4. The stars with and without question marks in Table 2 (open and filled symbols in Fig. 6) showed no significant differences in the correlations, so all stars were given equal weight in the zeropoint determination. As the fitting was done visually, the uncertainties in Table 4 are only estimates. One should realize, however, that at least for the correlations with wide-band photometry (the majority in Fig. 6) the zeropoints in Table 4 are first approximations only. The *UBV* excesses cannot be related to the *VBLUW* excesses via one single transformation relation, but in fact this transformation is strongly nonlinear and dependent on spectrum and reddening of the star (see Paper II, Fig. 7). This problem causes extra uncertainty in the zeropoints of Figures 6b to 6i, and makes it difficult to detect small period- or reddening-dependent systematic effects in these figures. The fact remains however, that the *VBLUW* colour excesses are systematically smaller then those of most other studies.

Before discussing Figure 6 in some more detail, I want to point out that most colour excess studies in this figure are not independent. In particular, this figure is dominated by colour excess scales that are related to the colour excesses of Kraft (1961). Fernie reduced all Cepheid reddenings available in 1967, including the data of Walraven et al., Mianes and Williams, to a corrected

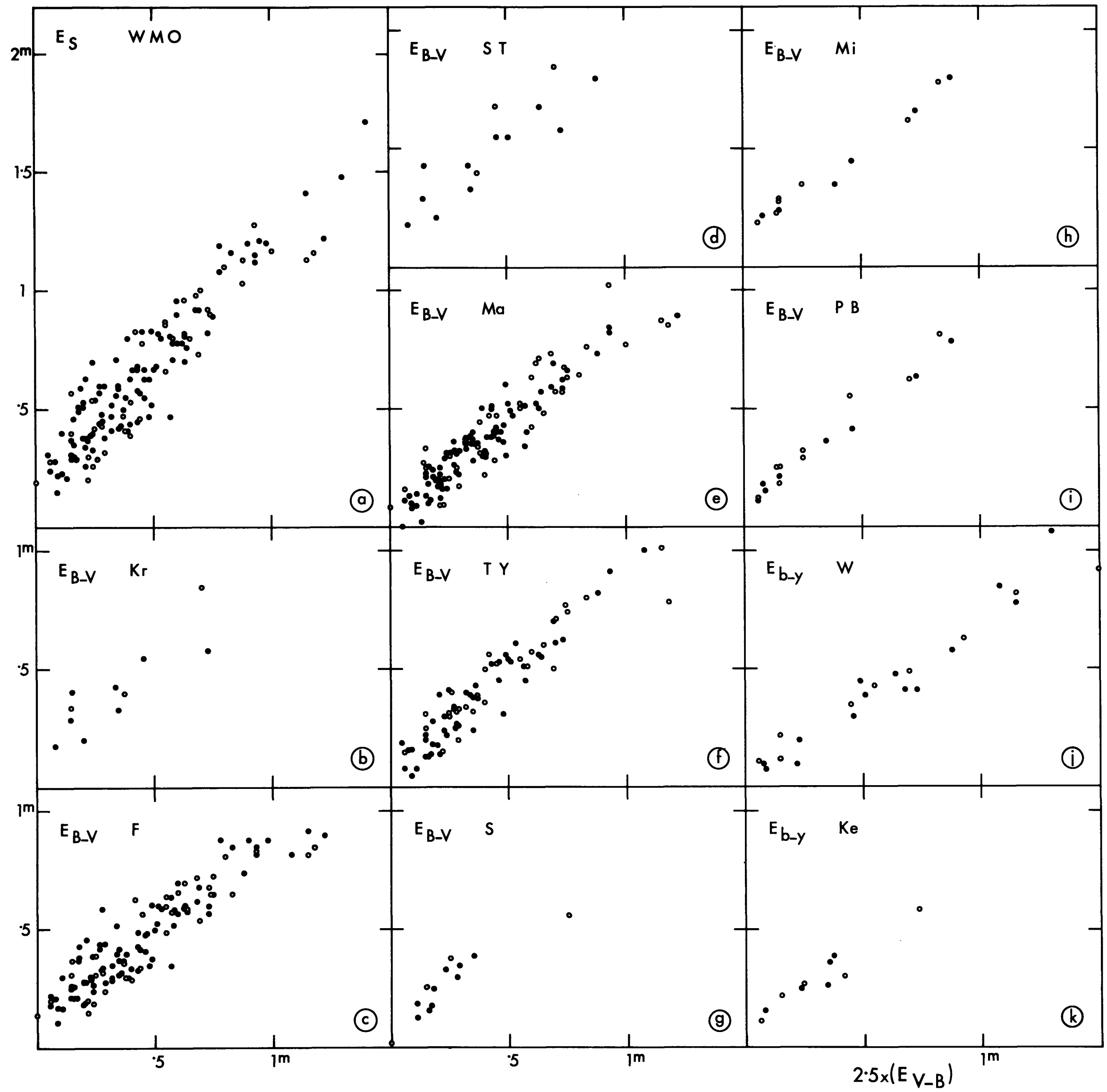


Fig. 6a—k. Comparison of the colour excesses of this paper with those from other studies. Stars with question marks in Table 2 have been indicated by open circles. The different abbreviations have the following meaning: **a** WMO: Walraven et al. (1958), **b** Kr: Kraft (1961), **c** F: Fernie (1967), **d** ST: Sandage and Tammann (1968), **e** Ma: Makarenko (1971), **f** TY: Tsarevskii and Yakimova (1970), **g** S: Schmidt (1972[a]), **h** Mi: Mianes (1963), **i** PB: Parsons and Bouw (1971), **j** W: Williams (1966), **k** Ke: Kelsall (1971)

Table 4. Zeropoints of the correlations of Figure 6. The numbers are expressed in the ordinate scales of the corresponding panels in Figure 6

Diagram	Zeropoint	Error
a WMO	$+0^m.15$	$\pm 0^m.05$:
b Kr	+0.15	±0.03
c Fe	+0.11	±0.03
d ST	+0.16	±0.03
e Ma	+0.03	±0.03
f TY	+0.07	±0.03
g S	+0.07:	±0.05:
h Mi	+0.16	±0.02
i PB	+0.09	±0.02
j W	+0.02	±0.02
k Ke	+0.09	±0.03

version of the Kraft system. Tsarevskii and Yakimova, in turn, criticized the results of Fernie (Yakimova and Tsarevskii, 1968) and derived a revised (spectral type-intrinsic colour) relation based mainly on the original *G*-band measurements of Kraft (1960), but their list of Cepheid reddenings contains also data derived from Mianes and Williams. Kelsall used Fernie's colour excesses to fix the zeropoint of his intrinsic Cepheid relation; the zeropoint of Williams' Cepheid locus was determined by the colour excesses of a number of cluster Cepheids taken from Kraft. The colour excesses of Sandage and Tammann are essentially in the original Kraft system. It should also be noted that the photometric observations used by the different authors are often identical (Fernie, Sandage and Tammann, Makarenko, Tsarevskii and Yakimova all use the data compiled in the "Tonantzintla Catalogue" of Mitchell et al., 1964), but that the observations in this catalogue have partly been transformed to the *UBV* system and therefore cannot be considered as a strictly homogeneous set of data. These relations between the various studies of Cepheids colours greatly complicate a correct evaluation of the data in Figure 6, and in fact I have not yet found a satisfactory explanation for the systematic zeropoint differences and for a number of inconsistencies in this figure.

Because of the great importance for the Cepheid distance scale of the colour excesses of Sandage and Tammann, it is particularly disturbing that the zeropoint difference in Figure 6d is so large. In order to check for possible period-dependent effects, the data in Figure 6d were divided into three period groups ($P<7^d$, $7 \leqq P < 12^d$ and $P \geqq 12^d$) and zeropoints were determined for each period interval separately. This time individual transformation relations were used for the three period ranges: the F8 I relation for $P<7^d$, the G2 I relation for the intermediate periods, and the G6 I curve for $P \geqq 12^d$ (see Fig. 7, Paper II). Although the overlap of the present work with the data of Sandage and Tammann is limited, and the number of stars per period interval in Figure 6d small, there is evidence of a systematic trend with period: the zeropoints in the short-, intermediate-, and long-period intervals are $+0\overset{m}{.}09$, $+0\overset{m}{.}09$ and $+0\overset{m}{.}24$, respectively. The uncertainty in these zeropoints is about $\pm 0\overset{m}{.}05$. As the Sandage and Tammann colour excesses are almost identical to those of Kraft, the same fitting procedure was applied to Figure 6b. Although the number of long-period Cepheids (2) in Figure 6b is too small for a significant zeropoint determination, the data in Figure 6b are completely consistent with the zeropoints found in Figure 6d. In the relation with the colour excess scale of Tsarevskii and Yakimova, which is the most recent of a number of revisions of the original Kraft system, there is at most a very small trend with period. In Figure 6f we find zeropoints $+0\overset{m}{.}06$, $+0\overset{m}{.}09$, $+0\overset{m}{.}09$ (with uncertainties of about $\pm 0\overset{m}{.}03$) for the short-, intermediate-, and long-period Cepheids, respectively.

These results confirm the recent discussion by Canavaggia et al. (1975) of the Cepheid colour excesses obtained by Schmidt (1972b, 1973) (N.B.: these are not the colour excesses used in Figure 6g, which Schmidt derived from Hα temperatures). Canavaggia et al. point out that Schmidt's colour excesses are essentially based on the Kraft system. From a discussion of Kraft's work, revised versions of Kraft's (spectral type-intrinsic colour) relation (particularly the work of Tsarevskii and Yakimova), and reddening determination in the Lick six-colour system, they conclude that in the original Kraft system the colour excesses of long-period Cepheids are overestimated by about $0\overset{m}{.}15$.

It is not immediately clear how the Sandage and Tammann period-luminosity-colour relation (Sandage and Tammann, 1968, 1969, 1971) changes when the *VBLUW* reddening scale is used. Qualitatively, one expects a decrease in the Cepheid luminosities when the reddening corrections are made smaller and the intrinsic colours redder, but the exact numbers can only be given after a careful discussion of all factors involved in the calibration of the $P-L-C$ relation. I want to stress, however, that any revision in the luminosities and colours of Cepheids automatically affects the extragalactic distance scale and the value of the Hubble constant. I will come back to this subject in a future paper.

Of all studies represented in Figure 6, the reddening values of Makarenko and of Williams, who both use a locus method, agree most closely with those of the present paper. I want to point out, however, that there are inconsistencies in those two cases, and that the good agreement may be fortuitous.

The position of Makarenko's locus is fixed by β DOR, the same Cepheid that I used for the *VBLUW* locus, but Makarenko adopts zero reddening for this variable while I took $E(B-V)_{UBV} = 0\overset{m}{.}05$. One expects therefore a zeropoint difference between the Makarenko and *VBLUW* excesses of $-0\overset{m}{.}05$ instead of $+0\overset{m}{.}03$ as observed in Figure 6e. Closer examination of the data in this figure reveals systematic effects, however. When the data of Makarenko are divided into period intervals in the same way as described above, we find zeropoints $+0\overset{m}{.}01$, $+0\overset{m}{.}03$, $+0\overset{m}{.}06$ for the short-, intermediate-, and long-period Cepheids in Figure 6e. More serious than this trend in the zeropoint is the fact that the short-period Cepheids in Figure 6e define a relation that is significantly steeper than the computed curve for F8 I stars, while the long-period Cepheids give a relation that is much flatter than the computed G6 I relation. The slope for the intermediate-period stars is about correct. These systematic effects are probably caused by a combination of two prolems in Makarenko's $(B-V)-(U-B)$ diagram. Firstly, I expect that the use of one *UBV* locus for Cepheids of different periods is not justified: the gravity effects in Balmer jump and Balmer continuum of F-G supergiants are very large [cf. the theoretical $(B-U)$ and $(U-W)$ colours of Paper II], and the *UBV U*-band contains the information of L, U and W of the *VBLUW* system taken together. Secondly, Makarenko has to make complicated corrections for the nonlinearity of the reddening lines in the *UBV* two-colour diagram. As the angle between his Cepheid locus and the reddening direction is small, very small errors in the adopted slope and curvature of the reddening vectors can produce large effects in the colour excesses.

The shape of Williams' Cepheid locus is determined from X CYG, its position by the colour excesses of a number of cluster Cepheids taken from Kraft (1961). It is therefore surprising that the relations between my colour

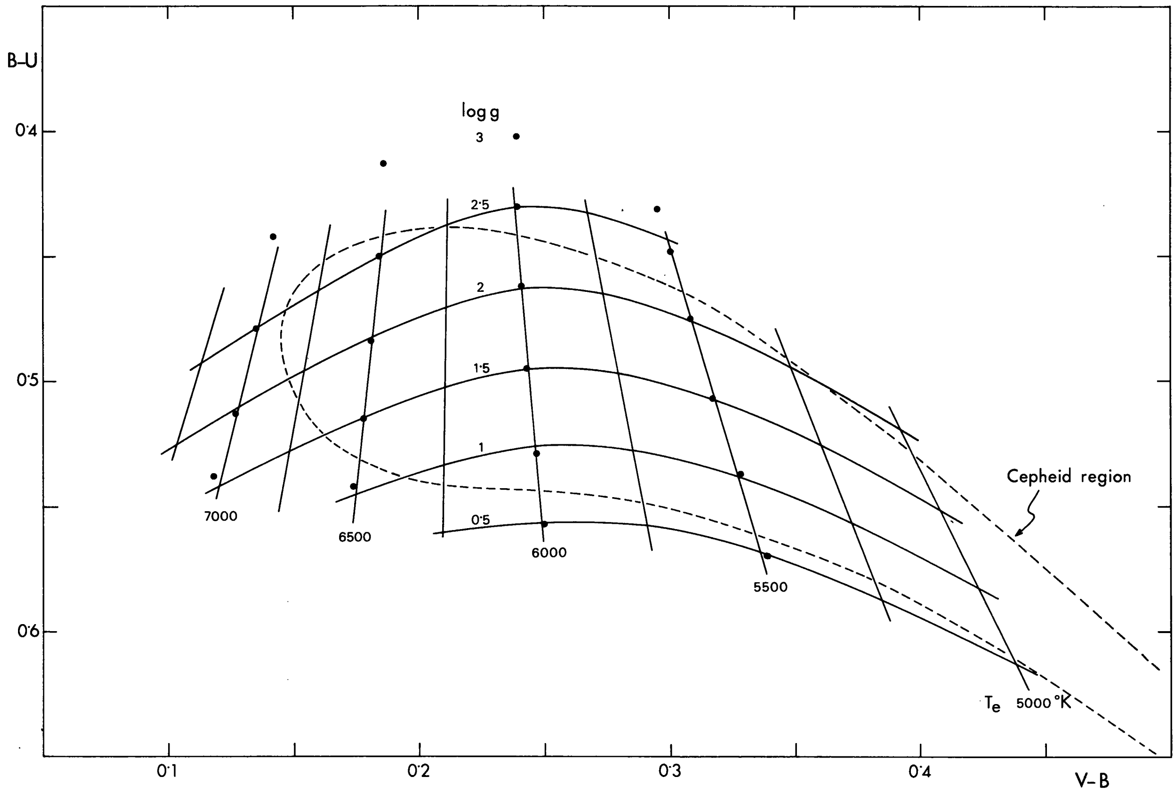


Fig. 7. The $(V-B)-(B-U)$ grid of theoretical colours, adapted for a microturbulent velocity $v_{mi}=4\ \mathrm{km\ s^{-1}}$ (see text, Section 2.1). The region covered by the Cepheid loops is indicated by a dashed line

excesses and those of Kraft and Williams differ by $0^m.12$ [on $(B-V)_{UBV}$ scale]. No explanation for this difference could be found. The excesses of Williams were reanalysed by Kelsall (1971), but this revision does not change Figure 6j noticeably. The cluster Cepheid excesses of Kraft had to be transformed by Williams from $(B-V)_{UBV}$ to $(b-y)_{\mathrm{Williams}}$. Although there may be some uncertainty in this transformation for supergiants, it is unlikely that this uncertainty can cause errors as large as $0^m.12$. Makarenko (1971) has criticized the use of X CYG for Williams' Cepheid locus, because this variable is considerably reddened and particularly because it has a wide open loop in the *UBV* two-colour diagram, but also this criticism does not offer a satisfactory explanation for the discrepancy in the zeropoints.

When the data in Figure 6 are evaluated, taking into account the remarks made above, this still leads to the suggestion that the *VBLUW* colour excesses are about $0^m.05$–$0^m.10$ too small, and that the Cepheid locus in the $(V-B)-(B-L)$ diagram should be shifted blueward by the corresponding amount. However, in view of the considerations given in Section 1.1, such a shift cannot be accepted. The physical quantities to be derived in the next section will provide an independent check on the correctness of the *VBLUW* colour excesses: any substantial error in the reddening corrections will lead to distortions of the derived radius curves, because of the sensitivity of these curves to errors in the temperature scale.

2. Physical Properties of the Cepheids with $P<11$ Days

2.1. *Derivation of the Physical Quantities*

I will now discuss the physical parameters that can be derived from the reddening-free Cepheid colours obtained in Section 1. This discussion will be based almost completely on the theoretical colours that were computed in Paper II from the model fluxes by Kurucz (1975). This immediately raises the question, whether it is possible to use models of atmospheres in hydrostatic equilibrium for the interpretation of stars which are obviously not in equilibrium. There are many phenomena in pulsating stars (differential motions, shocks, the interaction of pulsation and convection) which could make the application of these models to Cepheids unrealistic. Although I am fully aware of these problems, I have taken the point of view that it is worthwhile to see what the results of such a simple-minted interpretation are.

While the reddening determinations are made in the $(V-B)-(B-L)$ diagram because of its insensitivity to gravity differences, I will now use the $(V-B)-(B-U)$ diagram for the derivation of physical properties of the Cepheids because this diagram is very sensitive to both temperature and gravity, and because the two parameters can be well separated. The models indicate that the effects of temperature and gravity are even more orthogonal in the $(V-B)-(U-W)$ diagram. This diagram is not used, however, as the W fluxes of the models are probably not very reliable (expecially for the cooler stars; see Paper II, Section 6) and because W can only be measured with sufficient accuracy for the brightest Cepheids. The $(V-B)-(B-U)$ diagram is also very sensitive to chemical composition, but here I assume again that the "classical" Cepheids have solar composition, and that stars with different abundances can be recognized by peculiarities in their lightcurves and two-colour loops.

Most Kurucz models were computed for a microturbulent velocity $v_{mi}=2$ km s^{-1}, but for nine points in the solar abundance ($A=1$) grid also the fluxes for $v_{mi}=4$ km s^{-1} are available. As the latter value is more appropriate to Cepheids, I constructed the theoretical $(V-B)-(B-U)$ diagram corresponding to the higher microturbulence from the complete $A=1$, $v_{mi}=2$ grid and the few $v_{mi}=4$ models. The available models indicate that the effects of higher microturbulence increase approximately linearly with increasing θ_e ($=5040/T_e$) and also linearly with decreasing effective gravity $\log g_e$. I therefore adapted the colours of the $v_{mi}=2$ grid to $v_{mi}=4$ km s^{-1} by adding corrections that were interpolated and extrapolated linearly with θ_e and $\log g_e$ from the known microturbulent vectors. The resulting $(V-B)-(B-U)$ diagram is shown in Figure 7.

This way of adjusting the theoretical colours to the higher microturbulence of Cepheids is admittedly rather crude, but the small number of models with higher v_{mi} does not allow more detailed corrections. An additional and more serious uncertainty arises from the fact that there are differences in v_{mi} between different Cepheids, and moreover, that the microturbulence of Cepheids varies with phase (e.g. van Paradijs, 1972). Karp (1973) has suggested that these variations in microturbulence can be explained by variations of the velocity gradient in the Cepheid atmosphere during pulsation, but subsequent more detailed calculations by Karp indicated that velocity gradients can explain only part of the microturbulence, and that there is still no satisfactory explanation for the observed microturbulence of Cepheids (Karp, 1975). There are therefore objections to the use of the same microturbulence for all Cepheids and for all phases, but a more sophisticated procedure is not possible at present. This uncertainty due to the microturbulence affects mainly the gravities derived from the $(V-B)-(B-U)$ diagram. Fortunately, the effective temperatures are much less dependent on the adopted v_{mi}. The bolometric corrections computed from the models were also corrected for $v_{mi}=4$ km s^{-1}, but they appeared to be very unsensitive to microturbulence, as can be seen in Table A3 of Paper II. This means that the radius variations that can be derived from the lightcurves, bolometric corrections and effective temperatures, are not very sensitive to microturbulence variations.

To the grid of theoretical colours and bolometric corrections obtained in this way, the following analytic expressions were fitted:

$$\theta_e=0.812+1.008(V-B)^*+8.5\{(V-B)^*-0.012\}^3$$
$$\text{with}\quad (V-B)^*=0.337\frac{(V-B)-0.216}{(B-U)-0.157}, \tag{1a}$$

$$\log g_e=1.340-16.1(B-U)^*-7.0\{(B-U)^*\}^2$$
$$\text{with}\quad (B-U)^*=(B-U)-0.504-1.2|\theta_e-0.852|^{1.61}$$
$$+0.015(\theta_e-0.852), \tag{1b}$$

$$\text{B.C.}=0.834|\theta_e-0.695|^{3/2}+0.035(\log g_e)$$
$$-\theta_e(0.027\log g_e-0.053)-0.156. \tag{1c}$$

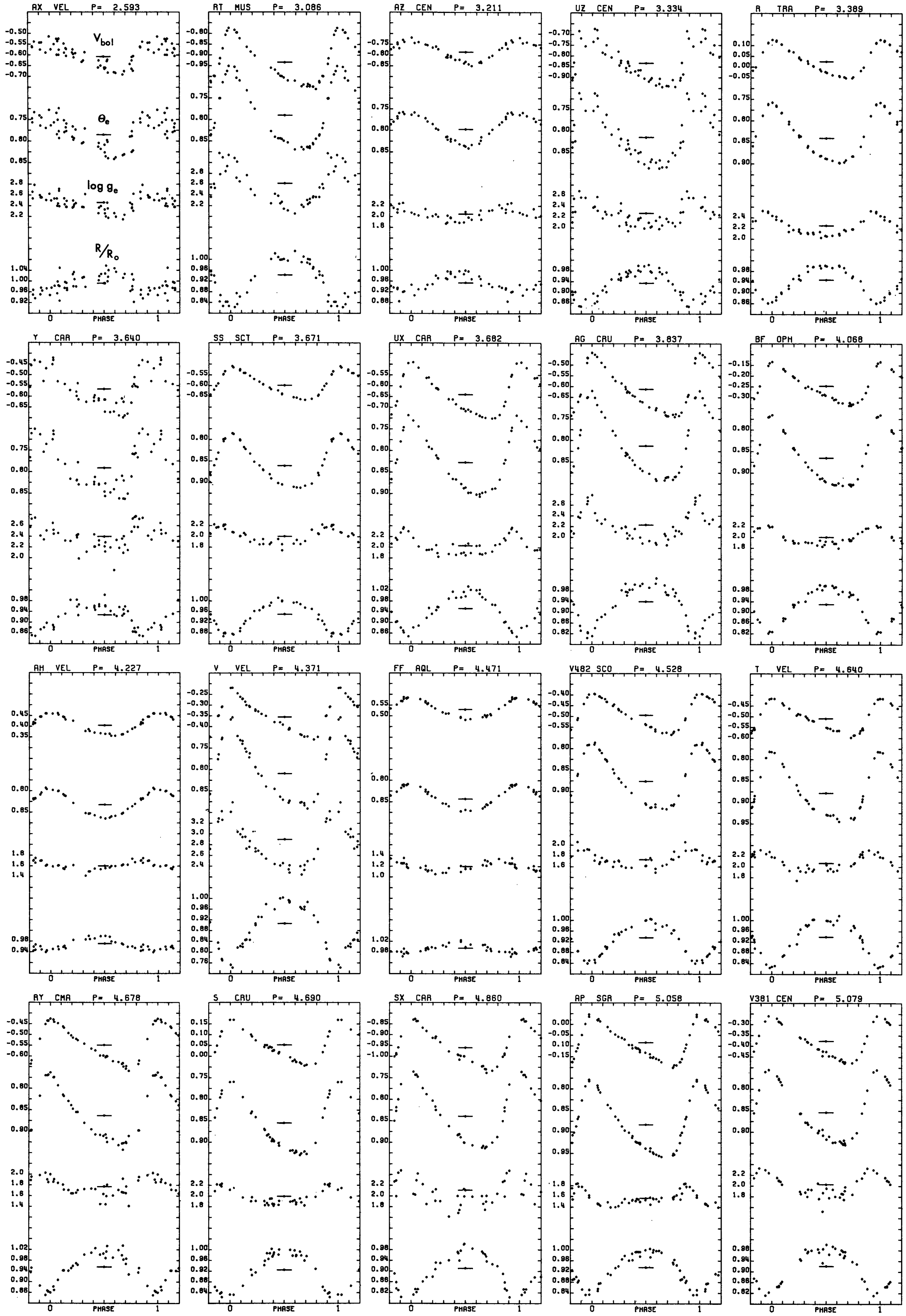
In the region covered by the Cepheid loops, these relations fit the exact values at the discrete grid points to within 0.0015 in θ_e, 0.05 in $\log g_e$, and 0.001 in B.C. (like V, the bolometric corrections are given on a ^{10}log scale; see Paper II, Appendix). This is sufficiently accurate for our purpose, and higher precision would not be meaningful considering the uncertainties mentioned before.

The analytic fit to the models given above is also shown in Figure 7. As can be seen in this figure, the grid is continued down to $T_e=5000$ K in order to cover more of the Cepheids. I realize that extrapolations of this kind are dangerous, but the *VBLUW* colours kindly computed by Dr. Bell from the model atmospheres for cool giants by Gustafsson, Bell and co-workers (Gustafsson et al., 1975; Bell et al., 1976) gave me confidence that this extrapolation is justified (unfortunately only a qualitative use of these colours was possible, as no data were available for cool, solar-abundance supergiants with sufficiently high microturbulence).

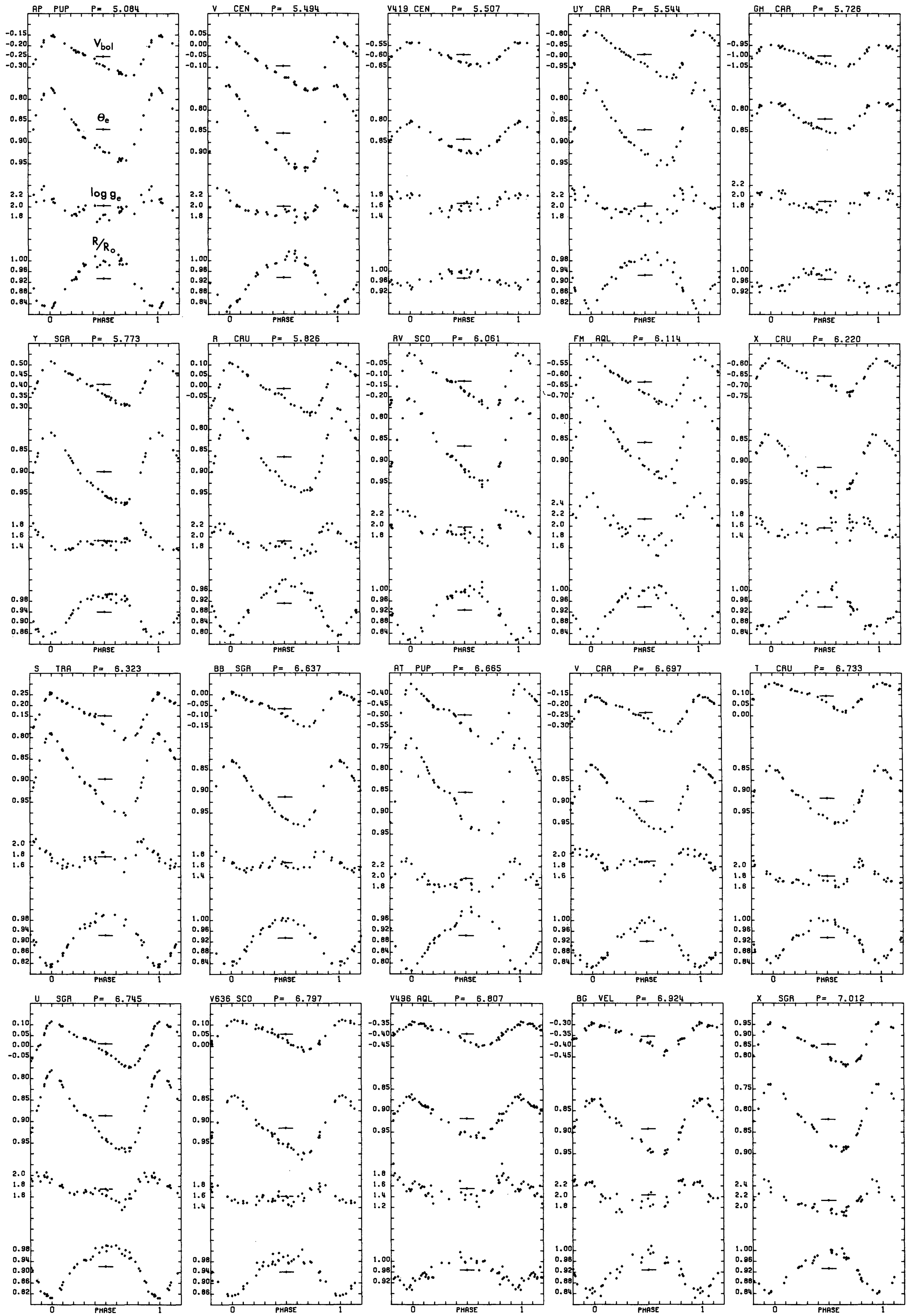
We now have all necessary ingredients for a straightforward derivation of temperature, gravity and radius variations for the observed Cepheids. From the reddening-free $(V-B)$ and $(B-U)$ colours of a Cepheid, θ_e, $\log g_e$ and bolometric corrections are computed by means of relations 1a, b, c. The bolometric corrections added to the V lightcurve give the bolometric lightcurve, which—apart from a zeropoint difference due to the unknown distance—is equal to the luminosity variation $L(t)$. At any moment we have $L\propto R^2\cdot\theta_e^{-4}$, so the relative radius variations can be computed from:

$$\frac{R}{R_0}=\{\theta_e/(\theta_e)_0\}^2\cdot\{L/L_0\}^{1/2}, \tag{2}$$

where the reference values $(\theta_e)_0$ and L_0 are arbitrarily taken at phase 0.5. Figures 8.1 to 8.3 show the results. Here the bolometric lightcurves and the behavior of temperature, gravity and radius are shown for the 60 Cepheids that are brighter than $V=-1.0$ ($V_{UBV}\lesssim 9^{\rm m}3$) at minimum and that have periods shorter than 11 days. The loops of Cepheids with longer periods fall outside the range of the model colours at minimum. All quantities are derived per individual measurement, without any kind of smoothing.



Figs. 8.1—8.3. Cyclic behaviour of the physical parameters of 60 bright Cepheids with periods shorter than 11 days. The dashes at phase 0.5 indicate equilibrium values (see text). $V_{bol} \equiv V + \text{B.C.}$ is on a ^{10}log scale. The period in days is indicated at the top of each panel


AP PUP P= 5.084
V CEN P= 5.494
V419 CEN P= 5.507
UY CAR P= 5.544
GH CAR P= 5.726
Y SGR P= 5.773
R CRU P= 5.826
RV SCO P= 6.061
FM AQL P= 6.114
X CRU P= 6.220
S TRA P= 6.323
BB SGR P= 6.637
AT PUP P= 6.665
V CAR P= 6.697
T CRU P= 6.733
U SGR P= 6.745
V636 SCO P= 6.797
V496 AQL P= 6.807
BG VEL P= 6.924
X SGR P= 7.012
V_{bol}
Θ_e
$\log g_e$
$R/R_\odot$
PHASE

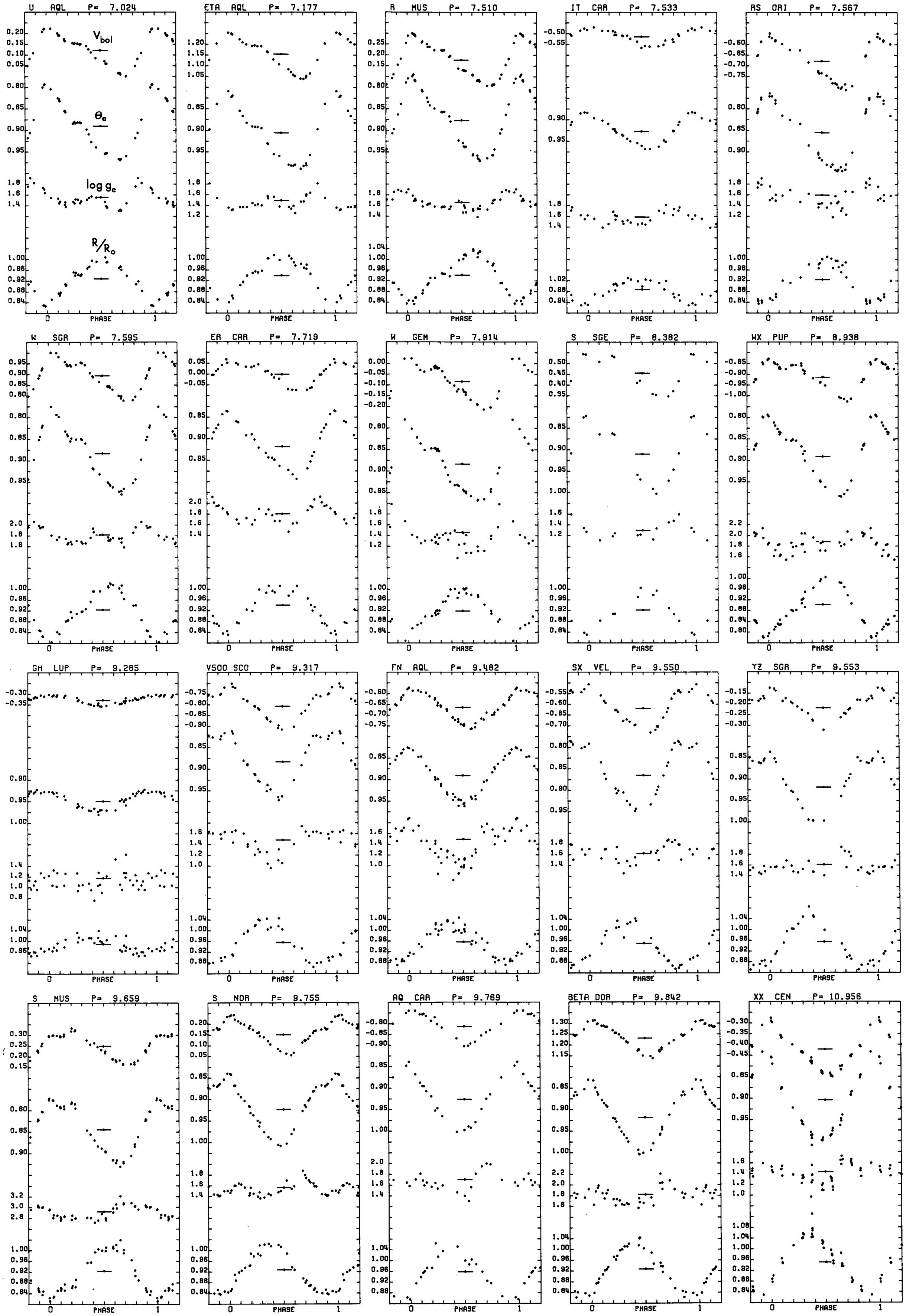

U AQL P= 7.024
ETA AQL P= 7.177
R MUS P= 7.510
IT CAR P= 7.533
RS ORI P= 7.567
W SGR P= 7.595
ER CAR P= 7.719
W GEM P= 7.914
S SGE P= 8.382
WX PUP P= 8.938
GH LUP P= 9.285
V500 SCO P= 9.317
FN AQL P= 9.482
SX VEL P= 9.550
YZ SGR P= 9.553
S MUS P= 9.659
S NOR P= 9.755
AQ CAR P= 9.769
BETA DOR P= 9.842
XX CEN P= 10.956
V_{bol}
Θ_e
$\log g_e$
$R/R_\odot$
PHASE

Table 5a. Physical properties for the brighter Cepheids with $P<11^d$, shown in Figures 8.1–8.3. For definitions of the various quantities see the text. ΔV_{bol}, $\Delta\theta$, $\Delta \log g$ and $\Delta R/\langle R\rangle$ are the amplitudes of the luminosity, temperature, gravity and radius curves, respectively. $\Delta R/\langle R\rangle$ is given in %. Cepheids with beat phenomena are indicated by B in the last column. Cepheids with question marks in Table 2 are marked in the same way here

NAME	log P	$\langle V_{bol}\rangle$	ΔV_{bol}	θ_{max}	θ_{min}	$\Delta\theta$	θ_{eq}	log g_{eq}	Δlog g	$\langle R/R_o\rangle$	$\Delta R/\langle R\rangle$	REM.
AX VEL	.414	−0.607	.119	.750	.825	.075	.785	2.47	.37	.992	7.1	B
RT MUS	.489	−0.934	.277	.676	.866	.190	.790	2.62	.85	.942	19.5	?
AZ CEN	.507	−0.785	.112	.759	.837	.078	.797	2.05	.32	.954	8.0	
UZ CEN	.523	−0.833	.225	.733	.875	.142	.814	2.28	.52	.935	14.1	B
R TRA	.530	0.026	.177	.758	.898	.140	.840	2.25	.46	.949	14.3	
Y CAR	.561	−0.566	.183	.725	.847	.122	.791	2.40	.45	.928	11.5	B
SS SCT	.565	−0.598	.155	.786	.913	.127	.861	2.01	.33	.951	13.5	
UX CAR	.566	−0.639	.265	.718	.903	.185	.827	2.04	.43	.953	16.5	
AG CRU	.584	−0.612	.293	.690	.890	.200	.812	2.24	.80	.941	20.2	?
BF OPH	.609	−0.246	.198	.766	.929	.163	.865	2.01	.33	.930	18.4	(?)
AH VEL	.626	0.405	.104	.796	.865	.069	.832	1.60	.21	.971	4.8	?
V VEL	.641	−0.355	.237	.681	.883	.202	.809	2.90	1.15	.906	26.9	?
FF AQL	.650	0.530	.102	.808	.870	.062	.843	1.20	.23	.996	4.2	?
V482 SCO	.656	−0.494	.188	.783	.940	.157	.876	1.73	.40	.936	16.0	
T VEL	.666	−0.509	.174	.782	.940	.158	.878	2.07	.36	.939	18.7	
RY CMA	.670	−0.548	.230	.767	.933	.166	.863	1.78	.35	.956	16.4	
S CRU	.671	0.053	.215	.760	.930	.170	.855	1.99	.33	.925	18.6	
SX CAR	.687	−0.959	.245	.732	.911	.179	.839	2.11	.45	.911	19.8	
AP SGR	.704	−0.084	.241	.780	.959	.179	.883	1.56	.38	.935	16.9	
V381 CEN	.706	−0.376	.220	.756	.925	.169	.854	2.03	.48	.921	18.6	
AP PUP	.706	−0.250	.190	.771	.942	.171	.869	2.03	.40	.934	19.5	
V CEN	.740	−0.045	.243	.743	.938	.195	.854	2.01	.49	.937	20.3	
V419 CEN	.741	−0.591	.107	.803	.875	.072	.842	1.67	.29	.974	5.6	
UY CAR	.744	−0.890	.222	.765	.950	.185	.870	2.02	.50	.926	19.0	
GH CAR	.758	−0.998	.100	.782	.852	.070	.820	1.91	.25	.972	6.3	
Y SGR	.761	0.410	.205	.804	.973	.169	.898	1.53	.43	.941	16.6	
R CRU	.765	−0.011	.236	.753	.944	.191	.864	1.92	.49	.912	21.3	
RV SCO	.783	−0.126	.256	.755	.949	.194	.864	1.97	.49	.928	19.8	(?)
FM AQL	.786	−0.629	.231	.750	.938	.188	.855	2.13	.93	.939	19.0	?
X CRU	.794	−0.650	.160	.836	.973	.137	.912	1.57	.32	.939	16.8	
S TRA	.801	0.153	.214	.790	.977	.187	.896	1.79	.45	.925	20.5	
BB SGR	.822	−0.066	.157	.829	.979	.150	.913	1.67	.34	.934	18.4	
AT PUP	.824	−0.495	.278	.728	.949	.221	.853	1.98	.52	.903	24.0	
V CAR	.826	−0.233	.171	.813	.965	.152	.898	1.91	.40	.923	18.6	
T CRU	.828	0.093	.134	.843	.973	.130	.916	1.83	.43	.936	16.9	
U SGR	.829	0.013	.212	.780	.966	.186	.886	1.75	.51	.922	20.7	
V636 SCO	.832	0.056	.140	.840	.979	.139	.915	1.61	.31	.941	15.0	?
V496 AQL	.833	−0.392	.106	.865	.960	.095	.918	1.56	.40	.968	10.3	?
BG VEL	.841	−0.354	.129	.825	.948	.123	.892	2.03	.49	.929	15.5	
X SGR	.846	0.859	.198	.739	.889	.150	.820	2.14	.57	.934	16.5	?
U AQL	.847	0.122	.228	.792	.969	.177	.890	1.56	.52	.929	18.3	
ETA AQL	.856	1.154	.217	.803	.985	.182	.906	1.49	.48	.940	18.3	
R MUS	.876	0.178	.242	.773	.968	.195	.876	1.66	.38	.942	20.1	
IT CAR	.877	−0.514	.087	.883	.968	.085	.927	1.59	.32	.988	9.1	
RS ORI	.879	−0.677	.246	.767	.941	.174	.855	1.60	.46	.925	17.7	
W SGR	.880	0.845	.234	.775	.974	.199	.883	1.82	.39	.925	23.0	
ER CAR	.888	−0.002	.145	.835	.990	.155	.918	1.80	.49	.941	17.9	
W GEM	.899	−0.083	.247	.780	.968	.188	.883	1.46	.60	.920	20.4	(?)
S SGE	.923	0.456	.206	.821	1.000	.179	.910	1.30	.40	.922	17.4	?
WX PUP	.951	−0.913	.192	.795	.983	.188	.891	1.88	.35	.903	24.1	
GH LUP	.968	−0.328	.047	.927	.974	.047	.949	1.18	.22	.990	5.9	?
V500 SCO	.969	−0.808	.220	.807	.969	.162	.883	1.48	.50	.956	16.6	
FN AQL	.977	−0.663	.187	.825	.956	.131	.890	1.50	.80	.957	14.1	?
SX VEL	.980	−0.618	.226	.784	.948	.164	.865	1.63	.47	.951	17.8	
YZ SGR	.980	−0.218	.199	.837	.998	.161	.919	1.61	.48	.955	19.8	
S MUS	.985	0.249	.165	.775	.925	.150	.844	2.92	.37	.925	20.5	?
S NOR	.989	0.152	.185	.840	1.008	.168	.923	1.56	.47	.928	19.6	
AQ CAR	.990	−0.813	.169	.838	1.000	.162	.926	1.70	.52	.959	19.5	
BETA DOR	.993	1.232	.179	.828	1.003	.175	.918	1.83	.46	.928	21.0	
XX CEN	1.040	−0.422	.279	.780	.997	.217	.904	1.43	.54	.950	24.1	

Table 5b. The same as Table 5a, but for Cepheids with V_{min} fainter than -1.0. Because of too much scatter, the amplitudes of the gravity curves are not given here

NAME	log P	$\langle V_{bol}\rangle$	ΔV_{bol}	θ_{max}	θ_{min}	$\Delta\theta$	θ_{eq}	log g_{eq}	$\langle R/R_o\rangle$	$\Delta R/\langle R\rangle$	REM.
EV SCT	.490	−1.408	.088	.751	.818	.067	.783	2.19	.976	6.5	
AP VEL	.495	−1.340	.177	.770	.876	.106	.826	1.81	.970	10.7	B
DX GEM	.496	−1.627	.113	.760	.837	.077	.799	1.99	.990	6.5	
GU NOR	.538	−1.483	.165	.790	.924	.134	.870	1.62	.966	12.0	?
EK MON	.597	−1.735	.141	.772	.933	.161	.863	2.38	.905	16.8	
BB CEN	.602	−1.349	.129	.800	.894	.094	.851	1.42	.949	7.2	?
GZ CAR	.619	−1.427	.110	.795	.870	.095	.835	2.06	.965	8.5	B
X SCT	.623	−1.338	.288	.693	.884	.191	.808	2.18	.924	16.2	(?)
RZ CMA	.629	−1.205	.206	.693	.848	.155	.788	3.04	.947	19.3	?
V496 CEN	.646	−1.307	.177	.752	.907	.155	.848	2.14	.934	16.3	
TV CMA	.669	−1.553	.223	.744	.951	.207	.855	2.00	.991	17.6	
DY CAR	.670	−1.846	.215	.766	.953	.187	.868	2.00	.914	20.1	
WW CAR	.670	−1.226	.260	.675	.867	.192	.792	2.74	.930	21.1	?
CN CAR	.693	−1.584	.193	.792	.954	.162	.889	1.77	.970	19.0	
WY PUP	.720	−1.544	.261	.704	.905	.201	.829	2.33	.925	23.4	?
VW CRU	.721	−1.160	.178	.820	.959	.139	.899	0.99	.950	11.8	?
UW CAR	.728	−1.100	.272	.705	.895	.190	.814	2.23	.945	18.0	
CV MON	.732	−1.437	.215	.717	.905	.188	.827	2.30	.926	21.4	?
WW PUP	.742	−1.546	.331	.686	.908	.222	.810	2.39	.938	20.8	?
RW CMA	.758	−1.750	.174	.795	.970	.175	.886	2.13	.996	19.1	
ST VEL	.768	−1.191	.194	.779	.954	.175	.878	1.83	.924	18.9	
T ANT	.771	−1.044	.312	.659	.900	.241	.802	2.79	.888	27.6	?
KL AQL	.786	−1.362	.200	.840	.999	.159	.929	0.92	.938	12.8	?
RS NOR	.792	−1.315	.233	.810	.990	.180	.906	1.18	.948	17.7	?
CS MON	.828	−1.708	.165	.762	.905	.143	.845	2.67	.912	22.0	?
XX VEL	.844	−1.588	.242	.760	.955	.195	.868	1.89	.939	20.9	
TW CMA	.845	−1.136	.202	.755	.948	.193	.851	2.26	.925	19.9	?
AE VEL	.853	−1.432	.278	.735	.943	.208	.844	1.94	.936	20.7	
GX CAR	.857	−1.051	.248	.776	.974	.198	.880	1.76	.895	21.1	
V600 AQL	.860	−1.348	.210	.780	.940	.160	.864	1.80	.920	19.1	
V336 AQL	.864	−1.251	.220	.783	.970	.187	.886	1.61	.934	21.0	
TZ MON	.871	−1.615	.213	.812	.980	.168	.906	1.54	.943	20.4	
AC MON	.904	−1.334	.193	.793	.985	.192	.883	1.65	.891	23.5	
TX MON	.940	−1.694	.208	.764	.930	.166	.843	2.58	.985	21.4	?
V339 CEN	.976	−0.785	.153	.841	1.022	.181	.925	1.60	.880	20.5	
Y SCT	1.015	−1.159	.240	.801	1.002	.201	.915	1.27	.916	25.3	
FO CAR	1.015	−1.587	.168	.820	.985	.165	.905	2.17	.932	20.5	?
FR CAR	1.030	−1.145	.206	.835	1.010	.175	.937	1.48	.947	21.4	

Equilibrium values of the physical parameters are indicated in Figure 8 by dashes at phase 0.5. Following van Albada and de Boer (1975), the equilibrium luminosity L_{eq}, radius R_{eq} and effective gravity g_{eq} are defined as the time averages over one cycle $\langle L\rangle$, $\langle R\rangle$ and $\langle g_e\rangle$, respectively. For the luminosity this choice is obvious, but the true equilibrium radius may be somewhat different from $\langle R\rangle$. I expect, however, that this difference is small, as the radius variations of the Cepheids—at least for the shorter periods—are small ($\lesssim 20\%$). The effective gravity can be written as

$$g_e = \frac{GM}{R^2} + \frac{d^2R}{dt^2} \tag{3}$$

where M is the stellar mass. For a purely periodic pulsation, the acceleration term does not contribute to the time average $\langle g_e\rangle$, and we get

$$g_{eq} = \langle g_e\rangle = \left\langle \frac{GM}{R^2} \right\rangle \cong \frac{GM}{\langle R\rangle^2} \tag{4}$$

which shows that the definitions of R_{eq} and g_{eq} are consistent. With the given definitions of L_{eq} and R_{eq} the equilibrium temperature follows from $\theta_{eq}^4 \propto R_{eq}^2/L_{eq}$, or:

$$\theta_{eq} = \left(\frac{\langle L^{1/2}\cdot\theta_e^2\rangle^2}{\langle L\rangle}\right)^{1/4} = \left\langle \left(\frac{L}{\langle L\rangle}\right)^{1/2}\cdot\theta_e^2 \right\rangle^{1/2}. \tag{5}$$

Numerical data for the curves in Figures 8.1 to 8.3 are listed in Table 5a. Table 5b gives the physical parameters for a number of Cepheids that are fainter than the brightness limit $V_{min} = -1.0$. The curves of these latter stars are not included in Figure 8 because of too much scatter, and the accuracy of the numbers in Table 5b is less than for the brighter Cepheids. No physical quantities were computed for Cepheids with U values fainter than -3.0 during more than half of the cycle. The total number of Cepheids with $P<11^d$ for which physical quantities could be derived is 98. The phases of the curves in Figure 8 are mostly identical to those of Paper I and of Walraven et al. (1964), but for a number of variables from the latter paper (AQ CAR, ER CAR, IT CAR, XX CEN, S MUS, S NOR, S SGE, SX VEL) improved light-elements from Kukarkin et al. (1969, 1970) were used.

2.2. General Remarks about the Results

A detailed discussion of the physical parameters derived in this paper, the relations between these quantities, and a comparison with pulsation theory will be given in a forthcoming paper, Paper IV. I will therefore restrict myself here to a few general remarks.

Most striking in Figures 8.1 to 8.3 is the regularity in the shape of the various curves. For the temperature and gravity curves we could expect a regular behaviour from the regularity in the pattern of two-colour loops, but it was not so evident that the measured brightness and colour variations would result in radius curves that look altogether realistic and have very similar shapes and

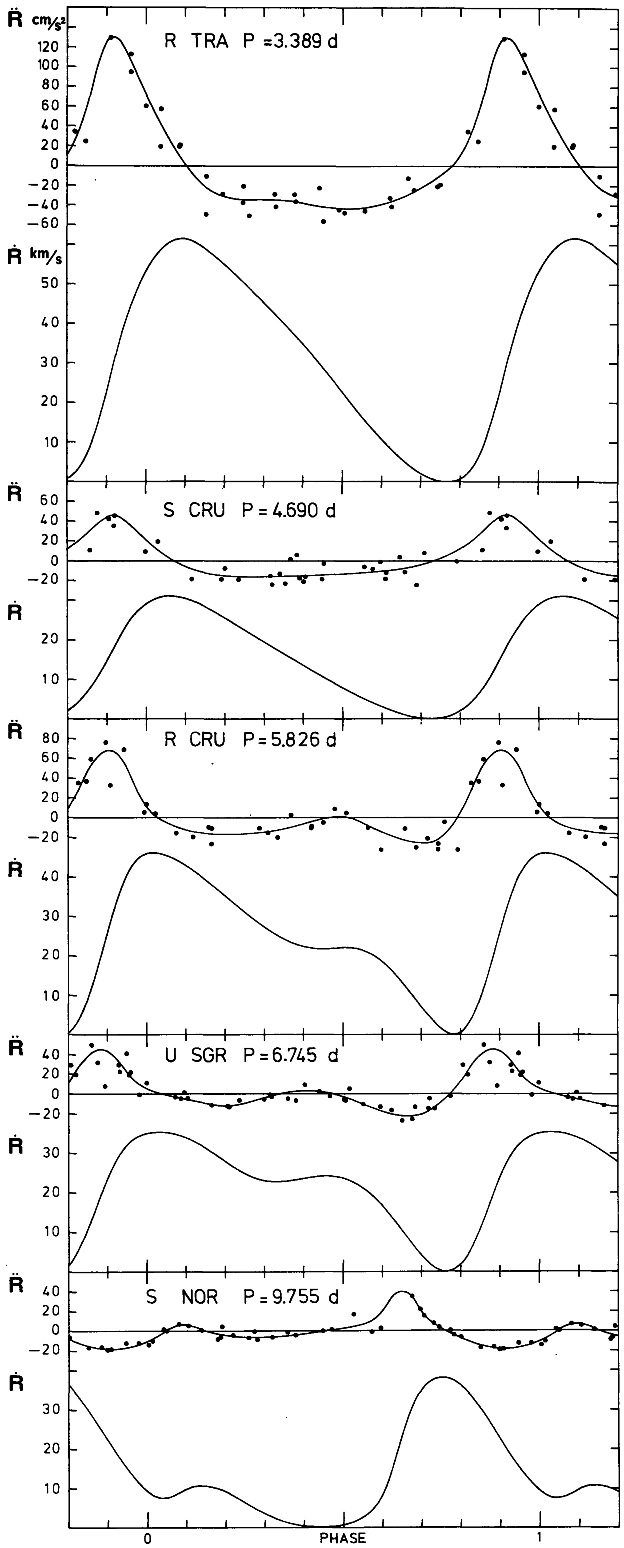


Fig. 9. The acceleration $\ddot{R}$ as a function of phase, derived from the gravity curves of five bright Cepheids. Integration of $\ddot{R}$ gives the velocities $\dot{R}$. The zeropoints of the velocity scales are arbitrary

phases for most of the variables. The shape of the radius curves is never far from sinusoidal, the maximum usually being somewhat broader and the minimum somewhat more cusped than a sine curve. Also the relative phase of luminosity and radius variations agrees well with what is known already from radial velocity measurements and from theoretical Cepheid models: on the average, maximum radius occurs about 0.5 period after, minimum radius shortly before maximum light. This gives us confidence that the temperatures derived from the theoretical colours and the reddening corrections applied to the observed colours cannot be grossly in error. As the radius curves depend strongly on the temperature ($L \propto R^2 T^4$), significant errors in the temperature scale immediately show up by distortions of the radius curves.

This is illustrated in Figure 8 by some stars that deviate from the general pattern with respect to shape and relative phase of the different curves (e.g. RT MUS, V VEL, FF AQL). Without exception these are stars which had been listed already as "peculiar" in Table 3. As could be expected, the abnormal colours of these stars have resulted in incorrect temperatures and gravities, and consequently the radius curves are also distorted. These deviating Cepheids will be discussed in more detail in Paper IV.

Of the different quantities displayed in Figure 8, the gravities are probably the least reliable. Not only is the scatter in the gravity curves rather large due to the weakness of the U intensities, but there is also the uncertainty about the effect of microturbulence variations discussed in the previous section. Furthermore, one can expect that the application of hydrostatic equilibrium models to pulsating stars causes errors particularly in the gravities. It is therefore interesting to compare the gravity curves with observed radial velocity curves. This can be done by writing Equation (3) in the following way:

$$\frac{d^2R}{dt^2} = g_e - \frac{g_{eq}}{\langle R_0^2/R^2 \rangle}\left(\frac{R_0}{R}\right)^2 . \qquad (6)$$

Both terms on the right-hand side are known at every phase, so by integrating this expression we obtain directly the behaviour of dR/dt as a function of phase. As an example, five of the best determined gravity curves were integrated in this way; this time it was necessary to fit smooth curves to the individual data points. The results are shown in Figure 9 and Table 6.

Table 6. Comparison of the velocity amplitudes derived from effective gravity variations $\left(\Delta \frac{dR}{dt}\right)$ with the amplitudes of the observed radial velocities (Δv_{rad}). A standard geometrical correction factor of 24/17 has been applied to the latter

Star	log P	$\Delta \frac{dR}{dt}$ (km s^{-1})	$\frac{24}{17} \Delta v_{rad}$ (km s^{-1})	Ref.
R TRA	0.530	61.6	50.9	Lloyd Evans (1968)
S CRU	0.671	31.1	50.8	Stibbs (1955)
R CRU	0.765	46.0	48.0	Stibbs (1955)
U SGR	0.829	35.2	56.5	Breger (1967)
S NOR	0.989	38.0	45.1	Breger (1970)

Considering the many uncertainties in the photometric gravities, it is quite satisfactory that the computed velocity curves show reasonable resemblence to the observed radial velocity curves. The observed velocity curves are not shown, but they can adequately be described as the mirror images of the lightcurves; references are given in Table 6. Shape, amplitude and phase of the velocity curves in Figure 9 are approximately correct; the stars with periods longer than five days even show bumps. There is, however, no detailed agreement with the observed radial velocities. Firstly, Table 6 indicates that the computed velocity amplitudes of the short-period Cepheids are too large, and of the longer periods too small. The gravity curves of the other Cepheids in Figure 8 confirm this. Secondly, the maxima of the computed velocity curves are up to 0.1 period later than the observed maxima (minima in the usual sign convention for v_{rad}). For S NOR the situation is not completely clear. The computed phases would be about correct if we identify the highest computed maximum with the shoulder in the observed curve (see Breger, 1970), and the second computed maximum with the main observed maximum. Finally, the secondary humps in the computed velocity curves are too pronounced compared to those in the observed curves.

It is not surprising that the accelerations derived from the gravity curves do not agree in detail with the observed radial velocities. Similar discrepancies were found by Parsons (1971[a], 1971[b]), who interpreted the phase behaviour of the observed gravities in terms of a running wave model. In fact we can conclude from the relatively small discrepancies that the use of hydrostatic equilibrium model atmospheres is not too unrealistic for the continuum colours of Cepheids. The much larger discrepancies in the gravities of RR Lyrae stars found by van Albada and de Boer (1975) indicate that the situation may be different for other types of pulsating stars. Although it is probably not justified to draw conclusions from the detailed cyclic behaviour of the photometric gravities, I have some confidence in the average gravities found for the Cepheids.

Concluding Remarks

In the first part of this paper, a homogeneous set of intrinsic colours and colour excesses has been obtained for a considerable number of Cepheids. Two problems were encountered here which need further attention. Firstly, there are the Cepheids with peculiarities in some of their colours. It is highly desirable that these stars are studied in more detail in order to find out which Cepheids have companions (and what kind of stars these companions are), and which are anomalous for other reasons. For some peculiar Cepheids it may be possible to correct the colour excesses and unreddened colours. Secondly, it is important to investigate the consequences of the present colour excesses for the Cepheid distance scale: a revision of the Cepheid distances will inevitably change the extragalactic distance scale. I will come back to both of these subjects in future papers.

In the second half of this paper, physical properties of the Cepheids were derived from the intrinsic colours. It was shown that the theoretical colours, although they refer to stable atmospheres, produce physical data which look realistic in many aspects. The obvious next step to be made, is a discussion of the results of this study from the point of view of pulsation theory. This will be the subject of Paper IV. Unfortunately, it was not possible to derive the physical characteristics of the long-period Cepheids. This I hope to do when more line-blanketed model atmospheres for cool supergiants become available.

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Note added in proof. Dr. P. S. The has recently analysed *VBLUW* measurements of the open cluster *LY*-6(work in press, Astronomy and Astrophysics 1977). He concludes that this cluster is real, that the Cepheid *TW NOR* is probably a member, and that the interstellar reddening towards the cluster is $E(V-B)=0.53\pm0.01$. This value agrees very well with the reddening derived in this paper for TW NOR [$E(V-B)=0.56\pm0.03$], and it provides an independent confirmation of the correctness of the *VBLUW* scale of Cepheid reddenings.