A new determination of the precession and the constants of galactic rotation

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A NEW DETERMINATION OF THE PRECESSION AND THE CONSTANTS OF GALACTIC ROTATION

by H. R. MORGAN and J. H. OORT

Following a suggestion made at the Paris colloquium on astronomical constants an attempt has been made to estimate the best value for the precession to be used in discussions of proper motions and the earth's structure. A correction of $+0''.75$ is proposed to NEWCOMB's general precession in longitude at 1900, derived on the assumption that no correction to his planetary precession is required. Some additional corrections to FK3 and GC motions are indicated in the last part of the article. A notable discordance remains with the observed motion of the equinox. From the various sets of proper motions considered the following values were found for the constants of galactic rotation: $A = +0.020$ km/sec.ps, $B = -0.007$ km/sec.ps.

At the international conference on astronomical constants held in Paris in the spring of 1950 we were asked to prepare a note on the constant of precession, stating the value which in our opinion could best be used for discussions of proper motions [1]). It was unanimously agreed at this conference not to revise the constant in the Ephemerides, nor in the preparation of catalogues of stellar positions.

The work on the new N30 catalogue [2]) having just been finished, the moment appears to be suitable for reaching a provisional conclusion as to the best value for the constant of precession.

We now dispose of two fundamental systems of motions that are practically independent of each other, and which may be considered as having approximately the same weight, with regard to systematic as well as accidental errors. Of the two old systems GC and FK3, which rest largely on the same observations, the FK3 system is now generally considered to be the more reliable; this is certainly so for the periodic errors in which we are interested for the present article. In order to obtain the best values for the constants of precession and galactic rotation we shall accordingly combine solutions referred to the FK3 system with those on the independent N30 system.

[1]) Cf. *B.A.* **15**, 290, 291, 1950; *Colloques Internationaux du CNRS* XXV: "Constantes Fondamentales de l'Astronomie", p. 128, 129.

[2]) This catalogue of mean positions of about 5300 stars with mean epoch around 1930 is to appear as *Astr. Pap. of Amer. Eph. and Naut. Alm.* **13**, Pt III, 1952. Besides positions it contains also proper motions derived from a comparison of the GC positions for 1900 (after application of the corrections derived in *A.J.* **54**, 3, 1948) with the N30 positions. For a fuller description of the N30 system cf. also *A.J.* **54**, 145, 1949; **56**, 97, 1951; and *B.A.* **15**, 199, 1950.

Solutions from three rather different types of stars, but all referred to the system of the FK3, have been discussed and summarized at the Paris colloquium [1]). The results have been copied in Table 1 below. Numbers in ordinary type refer to the FK3 system, those in italics refer to the N30 system.

The results in the first line were derived by EMMA T. R. WILLIAMS and A. N. VYSSOTSKY [2]) from a large material of proper motions of stars of average photovisual magnitude 11.1 collected at the McCormick Observatory, combined with Cape proper motions for southern stars in the same magnitude range. The proper motions were reduced to the FK3 system. The results marked GC $6^m.0-7^m.0$ were taken from a discussion by the same authors [3]); these had likewise been reduced to the system of the FK3. Finally, the numbers in the lines marked 'Galactic stars' refer to O- and B-type stars and supergiants of later types, i.e. to types of stars of which the distribution in space differs considerably from that of the stars making up the bulk of the material in the first two solutions [1]).

The values in italics were obtained from those on the FK3 system by first subtracting the corrections required by these quantities to reduce them from the system of the GC to that of the FK3 [4]), and then adding the corresponding corrections required to reduce

[1]) *B.A.* **15**, 222 a.f., 1950; "Constantes Fondamentales de l'Astronomie", p. 60-62. The equations used for these solutions may be found in *B.A.N.* **9**, 425, 1943, and also in *A.J.* **53**, 63, 1947.

[2]) *A.J.* **53**, 63-69, 1947; *Publ. McCormick Obs.* **10**, 17.

[3]) *A.J.* **53**, 72-75, 1947; *Publ. McCormick Obs.* **10**, 27.

[4]) These corrections were derived in *B.A.N.* **9**, 425 (Table 1), 1943. Similar corrections have been computed by EMMA T.R. WILLIAMS and A.N. VYSSOTSKY, *l.c.* Table 6. 1. The two sets of results show accurate accordance.

TABLE 1

Corrections to the constants of precession and galactic rotation (centennial motions)

Material	Fund. system	Δk	m.e.	Δn	m.e.	A	m.e.	B	m.e.
		″	″	″	″	″	″	″	″
McCormick, Cape	FK3	−·41	±·07	+·60	±·05	+·43	±·05	−·17	±·04
	N30	−·*23*		+·*58*		+·*46*		−·*14*	
GC $6^{m}\cdot 0-7^{m}\cdot 0$	FK3	−·44	±·07	+·42	±·07	+·41	±·09	−·13	±·07
	N30	−·*26*		+·*40*		+·*44*		−·*10*	
Galactic stars	FK3	−·29	±·08	+·32	±·09	+·35	±·09	−·21	±·06
	N30	−·*14*		+·*44*		+·*46*		−·*11*	
Weighted average	FK3	−·40	±·04	+·49	±·04	+·40	±·06	−·17	±·04
	N30	−·*22*		+·*50*		+·*45*		−·*12*	

TABLE 2

Changes in Δk, Δn, A and B corresponding to the transition from the GC to the N30 system (centennial motions)

Material	$\delta\Delta k$	(w)	$\delta\Delta n$	(w)	δA	(w)	δB	(w)
	″		″		″		″	
entire sky	+·08	(18)	−·02	(24)	+·11	(14)	+·15	(18)
$\delta < -30°$ half weight	+·01	(16)	−·02	(19)	+·04	(12)	+·14	(15)
galactic zone	−·04	(6)	−·07	(6)	+·20	(6)	+·26	(9)
μ_α only	+·27	(10)	+·33	(5)	−·07	(4)	−·24	(4)
μ_δ only	—		−·13	(17)	+·03	(8)	+·26	(13)

from the GC system to the N30 system. These latter corrections were determined from the systematic differences between the N30 and the GC proper motions, using equations of condition as given in *B.A.N.* **9**, 425, or in *A.J.* **53**, 63 [1]). Table 2 lists various solutions. In the first three the equations for μ_α and μ_δ have been combined, while the last two solutions are from μ_α and μ_δ separately. In the second solution the southernmost quarter of the sky has been given half weight; this solution, while giving more adequate weights, has the disadvantage that the lack of symmetry introduces systematic errors due to the dependence of mean parallax on galactic latitude, which dependence was not taken into account. The third solution refers to stars distributed evenly along the galactic circle. The relative weights with which the unknowns come out of the various solutions have been indicated in parentheses.

Actually, the stars used by the McCormick astronomers in their solution for faint stars are not evenly distributed over the sky; they were deemed, however, to approach this condition sufficiently to permit the application of the corrections derived from a uniformly distributed material.

[1]) In equations (B) on p. 64 of this latter article the last term in the equation for μ_α'' should read $+\cdot 87\,e\,Q$ instead of $-\cdot 87\,e\,Q$, while the last term in the equation for μ_δ'' should read $-\cdot 87\,a\,Q$ instead of $+\cdot 87\,a\,Q$.

In the FK3 system the solutions of Δn and B from μ_α and μ_δ separately gave very accordant results (cf. WILLIAMS & VYSSOTSKY, *A.J.* **53**, 66). A new solution for Δn on the GC system showed similar agreement: for the McCormick-Cape material Δn was found to be $+''\cdot 53$ and $+''\cdot 50$ as derived from the periodic terms in μ_α and μ_δ, respectively; for the $6^{m}\cdot 0-7^{m}\cdot 0$ GC stars the corresponding values were $+''\cdot 44$ and $+''\cdot 43$. On the N30 system, however, the agreement is poor, as may be seen by applying the corrections given in the last two lines of Table 2. The difference is rather larger than might have been expected from the mean errors, and seems to indicate that the periodic terms in μ_α may contain considerable systematic error. It is probable that periodic errors have been introduced in the older catalogues by insufficient elimination of daily periods in the clocks.

On the whole, Table 1 shows a very satisfactory agreement, among the various categories of stars as well as between the two systems N30 and FK3.

However, there is one set of data that we have not used, so far, namely the direct determinations of the equinox. In the above solutions the mean motion in right-ascension, Δk, remaining after allowance for galactic rotation and solar motion, has been considered as an independent quantity. In reality, however, Δk is related to Δn by the equation

$$\Delta k - \Delta n \operatorname{ctg} \varepsilon + \Delta\lambda = -\Delta e, \qquad (1)$$

where $\Delta n = \Delta p_1 \sin \varepsilon$ ($\sin \varepsilon = 0{\cdot}398$, $\operatorname{ctg} \varepsilon = 2{\cdot}304$), Δp_1 is the correction to NEWCOMB's general precession, $\Delta\lambda$ is the correction required by NEWCOMB's planetary precession, while Δe is the correction required by the motions in right-ascension as a consequence of the error of NEWCOMB's motion of the equinox (Δe is usually called the correction required by NEWCOMB's motion of the equinox). New computations of the planetary precession have indicated no sensible deviations from the values used by NEWCOMB. The modern determinations of the equinox have been reviewed by one of us at the Paris symposium [1]); combined with the older determinations they indicated a correction $\Delta e = +0''{\cdot}2$. Combining this with $\Delta k = -''{\cdot}31$ as found from Table 1, and putting $\Delta\lambda = 0$, equation (1) yields $\Delta n = -''{\cdot}05$, a value differing widely from that found in Table 1 from the periodic terms in μ_α and μ_δ. This discrepancy has long been known, and has formed an important difficulty in deciding about the correction required by NEWCOMB's general precession.

It is perhaps useful to state a little more clearly the relation between equation (1) and the equations of condition from which the results in Table 1 were obtained. Instead of solving for the four unknowns tabulated we might have started with a solution for the three constants Δn, A and B, making use of the periodic terms only. The results would have been identical with those in Table 1. We would then still have to satisfy one more equation of condition, in which the observed quantities are the total mean motion in right-ascension of the stars concerned ($\bar{\mu}_\alpha$), and the correction to NEWCOMB's motion of the equinox deduced from the equinox observations. This equation may be written

$$\Delta n \operatorname{ctg} \varepsilon + 0{\cdot}465\, B - \Delta\lambda = \bar{\mu}_\alpha + \Delta e. \qquad (2)$$

It is identical with (1).

From a mathematical point of view it would have been logical to have combined this equation from the start with the equations for the periodic terms in μ_α and μ_δ, and to have determined the three unknowns from this combined set (putting $\Delta\lambda = 0$). It appears preferable, however, to make the solution in two steps, as we have done, because we have no good data from which to estimate the weight of the right-hand member in the above equation as compared to the weights of the periodic terms used in the first solution; we only know that it must be much smaller than these latter weights. It seemed better, therefore, to consider the value of Δn as determined from (1) or (2) as an independent determination. It is true that, as may be seen from (2), it is slightly tied up with B, but, considering the very low weight of (2) and the relatively small coefficient of B in this equation, a combined solution could not have appreciably altered the result for B.

We may consider the same problem geometrically, by decomposing the precessional and galactic rotations each into a rotation around an axis in the plane of the equator and a rotation around an axis perpendicular to the equator. It is evident that the determination of the latter component will be greatly impaired by the difficulty of measuring the equinox, so that it seems reasonable to base the solutions primarily on the rotation components around axes in the equator. For the galactic rotation, the axis of which is inclined only 28° to the equator, practically no weight can be added by considering also the component around the polar axis; for the precession, however, the conditions are reversed, so that it is not at once evident that the rotational component around the equatorial axis should be left out of consideration.

The great difference between the corrections to the general precession as determined from these two components calls for a reconsideration of the various kinds of errors that may have entered into the two determinations.

Let us first consider the determinations from the terms varying with α (Table 1). Three different causes of the discrepancy may be considered:

1. The periodic terms in μ_α and μ_δ are due to systematic errors of a different kind than an error in the precession.
2. They are due to systematic motions of the stars considered.
3. They are due to an error in the theory of the precession.

1. At first sight some support to this supposition seems to be given by the large difference between the corrections Δn as found in the N30 system from μ_α and μ_δ, respectively (cf. p. 380). This difference is, however, probably due to the much greater uncertainty of the solution from μ_α. Not only is the average coefficient with which Δn enters into the normal equations only half as large as in the case of μ_δ, but, in addition, the systematic errors proportional to $\sin\alpha \sin\delta$ in μ_α are likely to be higher than the systematic errors proportional to $\cos\alpha$ in μ_δ. The weight of the solution from μ_α may therefore be taken at most 1/5 that of the solution from μ_δ; in reality the ratio is probably even smaller.

In order to judge whether the term $\Delta n \cos\alpha$ found in μ_δ could possibly be ascribed to systematic error we must make an estimate of the amount of periodic errors of this type that may be expected. This can be done by considering systematic corrections of this form that are needed by individual catalogues in order to reduce them to the fundamental system. In this way

[1]) *B.A.* **15**, 201, 1950.

it has been estimated [1]) that the mean error of Δn_δ due to *systematic* errors of the catalogues used in the construction of the FK3 system is only $\pm''{\cdot}036$. It is unlikely that it would be higher for the N30 system. This discussion comprises all systematic errors except those that vary systematically with the epoch. The possibility of the latter exists certainly for the right-ascensions, where the observing methods have undergone considerable changes; for the declinations it cannot perhaps be altogether excluded for the motions depending upon older catalogues, but it seems very unlikely that there could be any large effect in the N30 system. One error of this type in the FK3 and GC motions has been pointed out [2]): Due to the fact that no corrections for latitude variation were applied before 1890 the μ_δ of these systems must contain a periodic error of about $+0''{\cdot}14 \cos\alpha$.

If we adopt for the FK3 system $\Delta n = +0''{\cdot}49 \pm 0''{\cdot}04$ m.e. as given in Table 1, and apply a correction for the error just mentioned, we obtain $\Delta n = +0''{\cdot}35 \pm 0''{\cdot}04$ m.e. [3]).

The mean GC and FK3 *positions* must also have been influenced to some extent by the neglect of the latitude variation. Using 36 series of fundamental declinations, 1884—1917, all freed from latitude variation effects, the correction to GC declinations at 1904 is found to be $-''{\cdot}02 \cos\alpha$. In computing the N30 proper motions the GC declinations around 1900 were corrected by $-''{\cdot}025 \cos\alpha$. It is thought, therefore, that there can be very little latitude effect in the N30 motions. From the μ_δ referred to the N30 system, values of Δn $+''{\cdot}30$ and $+''{\cdot}33$ have been derived from Wilson and Raymond's solutions from GC stars, and from Williams and Vyssotsky's investigations, respectively. These results are in good agreement with the value of $+0''{\cdot}35$ found above from GC and FK3.

Another argument indicating that the systematic effects in the proper motions in δ are due to an error in the precession rather than to an unknown systematic error of different kind, is that the variation of the systematic effects with α has just the same phase as a precession correction. The truth of this statement can be seen by considering the solutions for the constant B of galactic rotation. The term in μ_δ corresponding to this rotation has the form $0{\cdot}885\, B \sin(\alpha - 10)$, and differs, therefore, almost 90° in phase from the precession term $\Delta n \cos\alpha$. If we compute the rotation term from dynamical data that are independent of proper motions, and correct the proper motions for it, and if we solve the remaining motions for terms with $\cos\alpha$ and $\sin\alpha$ respectively, we find a negligible value for the amplitude of the $\sin\alpha$ term, indicating that the systematic errors vary just as a precession error [1]).

Recapitulating, the following arguments speak for the interpretation of the periodic μ_δ terms as precession errors rather than as other systematic errors:

a) apart from a variation-of-latitude effect that can be eliminated, systematic errors in μ_δ varying with α are generally small, the 'mean systematic error' to be expected being only about 1/10th of the effect found.

b) the independent μ_δ systems of FK3 (corrected for latitude variation) and N30 show approximately the same systematic term with $\cos\alpha$.

c) the systematic residuals in μ_δ have exactly the character that would correspond to a precession error.

2. This point has been extensively discussed in a note presented by one of us at the Paris symposium on astronomical constants. The outcome is quite definite, we believe, in denying the possibility of systematic effects simulating precession corrections of a size which would be at all comparable to the effects observed.

3. This appears improbable, but we do not feel competent to give a satisfactory judgment.

In the second place we must consider the amount of error that may have entered into the solution of Δn, or Δp_1, from equation (1). A priori it would seem more probable that the error is in the zero-point of the right-ascension motions than in the periodic terms. It is evident that the various changes in the methods of determining transits must have introduced systematic errors in the equinox. In the discussion of the motion of the equinox it has been attempted to take these systematic changes into account, although this could be done only in a very rough manner. But there are other systematic errors which were not, and could not be, taken into account. The most serious effect to be feared is that of a daily period in the clocks, which may well have considerably affected the tie between equinox and stars in all older catalogues. Such an error would be likely to have the same character in most of the old catalogues, and would only have disappeared with the introduction of constant-temperature rooms. That very considerable systematic errors in the old right-ascensions exist is indicated by the recent comparison between the N30 right-ascensions

[1]) *B.A.N.* **9**, 426, 1943.

[2]) *B.A.* **15**, 224, 1950 (also in "Constantes Fondamentales de l'Astronomie", Paris, CNRS, p. 62).

[3]) Strictly, this value depends also upon the periodic terms in μ_α. However, these latter have entered with a relatively small weight, and moreover, on the FK3 system there was very little difference between the solutions from μ_α and μ_δ separately.

[1]) In reality the solutions were made for terms with $\cos\alpha$ and $\sin(\alpha - 10)$, which however, is practically the same; the coefficient of $\sin(\alpha - 10)$ was found to be $-0''{\cdot}02 \pm 0''{\cdot}06$ m.e.

and those of the GC extrapolated to 1930. In the equatorial zone these differences vary with a semi-amplitude of as much as 0·2 seconds of arc, presumably due to errors in the older catalogues used in the GC. In view of this, the existence of a still larger systematic difference between day and night observations in these old observations is rather to be expected. A systematic difference of about half a second of arc, as would suffice to explain the discrepancy between the observed motion of the equinox and the motion that would follow from the determination of Δn, would appear to be within the limits of possibility.

As others have already pointed out, the neglect of the latitude variation has also caused systematic errors in the old determinations of the equinox. It so happens that the annual term of this variation enters with full strength into practically all of these, causing a systematic error of 0·23 seconds of arc in the equinoxes before 1890. Correction for this error is in the direction of diminishing the discrepancy considered. The correction has already partly been applied in the recent determination of Δe.

Finally, there is the possibility that the magnitude error may not have been completely eliminated in the old catalogues, so that there might be a systematic difference between catalogue stars and clock stars.

Conclusion.

In our opinion it would be reasonable for the time being to rely principally on the correction to the precessional constant as derived from the periodic terms in μ_δ. For these there are no a priori reasons to fear observational errors in the older series of observations that were systematically different from those in recent catalogues.

We have seen on p. 382 that these μ_δ results lead to an average value $\Delta n = +0''\cdot 33$.

From μ_α final values Δn $+''\cdot 28$ and $+''\cdot 31$ have been derived in the N30 system, for the GC stars and McCormick-Cape stars respectively. These are a combination with weights 1 and 2, respectively, of Δn computed from the periodic terms, and from the constant term based on a comprehensive determination of the motion of the equinox. These results from μ_α are in fair agreement with those from μ_δ, but have much less weight. It must be emphasized that the relative weights 1 and 2 used are rather arbitrary, and that the result from μ_α depends in a large measure on the choice of these weights.

Considering the above results we may provisionally adopt $+0''\cdot 30$ as the most probable value for Δn. This gives $+0''\cdot 75$ for the correction Δp_1 to NEWCOMB's general precession in longitude at 1900, derived on the assumption that no correction to his planetary precession is required.

Besides, a correction of $-0''\cdot 14 \cos\alpha$ should be applied to the FK3 and GC secular motions in declination on account of the fact that in the older catalogues no allowance has been made for latitude variation. In addition to the precession corrections mentioned the FK3 centennial motions in right-ascension require a correction of $-\Delta k + \Delta p_1 \cos \varepsilon = +1''\cdot 09$ or $+0^s\cdot 073$, the corresponding corrections to be applied to N30 being $+0''\cdot 91$ or $+0^s\cdot 061$.

For *positions* the N30 system is evidently much to be preferred. For *motions* the two systems FK3 (or GC) and N30 might be about equally reliable as far as systematic errors are concerned. The great significance of the N30 system of motions lies in the fact that it is to an important extent *independent* of the former systems of motions, and therefore gives almost independent determinations of the constants of precession and galactic rotation, as well as of the absolute magnitudes of δ Cephei variables.

Assuming that at present the best proper-motion system is the average of FK3 and N30, corrected for precession and Δk as indicated, the procedure to reduce GC motions to this average system may be summarized as follows. From *A.N.* **269**, 160, find the differences FK3 minus GC, and similarly from *A.J.* **56**, 97, the differences N30 minus GC. Add the average of these two differences to the GC motions. To the centennial motions in α so obtained add

$+0^s\cdot 021 - 0^s\cdot 020 \sin\alpha \operatorname{tg}\delta$ (seconds of time)

or $+0''\cdot 31 - 0''\cdot 30 \sin\alpha \operatorname{tg}\delta$ (seconds of arc).

To the centennial motions in declination add

$-0''\cdot 37 \cos\alpha$ [1]).

Combining the average results from the two systems as given in Table 1, it would appear that the best values for A and B now obtainable from proper motions are

$$A = +0''\cdot 43 \text{ per century} = +0\cdot 020 \text{ km/sec.ps} \pm 0\cdot 0020 \text{ (m.e.)}$$
$$B = -0''\cdot 15 \text{ per century} = -0\cdot 007 \text{ km/sec.ps} \pm 0\cdot 0015 \text{ (m.e.)}$$

As has been pointed out in an earlier article [1]), values of A and B derived from 'ordinary' stars such as make up the bulk of the GC and McCormick material, may deviate sensibly from the true values. Not too much weight should therefore be attached to the determinations from these groups. Fortunately, Table 1 shows a satisfactory agreement between the results from the 'ordinary' stars on one hand, and the 'galactic' stars on the other, so that the difficulty of deciding about rela-

[1]) *B.A.N.* **9**, 334, 1942.

[1]) Viz. the precession correction increased by half the latitude-variation correction required by the GC and FK3.

tive weights hardly arises. Of *A* various other determinations for stars with strong galactic concentration have been made from radial velocities in combination with secular parallaxes. As an average of these determinations, which are entirely independent of the above results, we may adopt +0·019 km/sec.ps. ±0·002 (m.e.), which is in excellent agreement with the above value of +0·020.

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