Radio spectra of non-uniform synchrotron sources with internal absorption

Bruyn, A.G. de

Astron. & Astrophys. 52, 439–447 (1976)

ASTRONOMY AND ASTROPHYSICS

Radio Spectra of Non-uniform Synchrotron Sources with Internal Absorption

A. G. de Bruyn*
Sterrewacht Leiden, The Netherlands

Received October 3, 1975, revised May 10, 1976

Summary. In this paper we investigate the radio spectra of non-uniform synchrotron sources at frequencies where the sources are subject to internal absorption. Two absorption mechanisms are considered: synchrotron self-absorption and thermal absorption by an ionized gas. Both axially (disks) and spherically symmetric sources are discussed and their spectra derived using a semi-analytical approach. If the magnetic field strength, the relativistic electron density coefficient and the thermal electron density have a power-law dependence on radius, we find that a power-law spectrum forms at frequencies below that at which the whole source becomes optically thin. The spectral index in this frequency range depends strongly on the rate with which the various quantities decrease with radius, but it is almost independent of the optically thin spectral index. The effective source diameter is shown to be a function of observing frequency. Some examples are given of numerically calculated spectra for spherically symmetric sources. We discuss several observational methods that may allow one to choose between the various absorption mechanisms that could operate in actual extragalactic sources with low frequency flattenings or turnovers. Variability in non-uniform sources is briefly discussed.

We suggest that the flat spectra in the central cores of radio galaxies are formed in non-uniform quasi-stationary sources that are maintained by continuous supply of magnetic and particle energy. The flattening of the spectra of some Seyfert galaxies and related objects at low frequencies could be due to internal free-free absorption in sources with a large radial extent.

Key words: compact radio sources — internal absorption — flat radio spectra

I. Introduction

The radio spectra of many compact extragalactic sources are observed to flatten or turn over at low frequencies. In the majority of cases this has to be attributed to internal absorption. Observations of compact radio components in quasars and radio galaxies with VLBI techniques and arguments concerning the nature of the rapid radio variability in many of them have indicated that synchrotron self-absorption has to be invoked. However, it has also become clear that the low-frequency spectra of the sources generally differ from that expected of a single uniform source and the theoretical maximum slope of +2.5 is rarely observed. Instead one often observes sources with undulating spectra (e.g., 3 C 273, BL Lac). The spectra of those sources have been satisfactorily explained as due to the superposition of the spectra of many self-absorbed components, peaking at different frequencies. Nevertheless many sources exist that, within the observational uncertainties, have straight spectra that are flat or inverted (see, for example, the extensive compilation of radio spectra by Véron et al. (1974)). Such spectra could result from the superposition of spectra of a large number of components but they can also be produced in a single but radially extended non-uniform source as we will show in this paper.

We will not restrict ourselves to the case of synchrotron self-absorption but also discuss sources where an ionized thermal gas mixed with the relativistic plasma is causing the absorption. The *mixture* is essential because ionized gas surrounding a synchrotron source causes a sharp exponential turnover at low frequencies which, except for some galactic sources (where it is due to intervening H II), is rarely observed. The sources for which we think that free-free absorption may be relevant are the disks of spiral galaxies, the central regions of Seyfert galaxies and radio galaxies whose nuclei exhibit intense optical emission lines.

In Section II of this paper we will, from simple analytical arguments, derive a relation for the spectral index of non-uniform sources at frequencies where they suffer from internal absorption. This problem was treated earlier by Braude (1967) who, however, restricted most of his analysis to a special case [expressed

* Now at Hale Observatories, 813 Santa Barbara Street, Pasadena, California 91101, USA

by his Equation (16)] which, in this author's view, severely limits the generality of the results. In Section III we will compare our theoretical results with the outcome of numerical calculations which, for one particular case, were performed earlier by Condon and Dressel (1973). Sections IV and V are devoted to a comparison of the different geometries and absorption effects and to a discussion of the presently available data in the light of the models of Section II.

II. Source Parameters and Model Calculations

In this section we will derive the relationship between the flux density and frequency of a non-uniform synchrotron source at frequencies where the source is partially opaque. We will first do this for a simple disk-like geometry and then extend the calculations to spherically symmetric sources. In each geometrical model we will consider two cases: i) the synchrotron emitting plasma is absorbing its own radiation (synchrotron self-absorption, henceforth SSA) and ii) the synchrotron emission is absorbed by an ionized thermal plasma (free-free absorption, henceforth FFA). We will first discuss the case of SSA and then, by changing the relation for the absorption coefficient, extend it to FFA.

Before continuing we give some qualitative arguments that provide physical insight in how the spectra are formed.

In both geometrical models the volume emissivity and re-absorption coefficient decrease rapidly as a function of distance from the center of the source. At a given frequency the largest contribution to the observed radio emission therefore comes from parts of the source at a given geometrical depth or radius (where the optical depth into the source reaches order unity). Since this geometrical depth or radius depends on frequency, the effective source size will be a function of frequency.

It is not possible to obtain in the general case a completely analytic solution for the shape of the integrated spectrum as has been found for the free-free emission of non-uniform but isothermal H II regions (Barlow and Wright, 1975; Olnon, 1975; Marsh, 1975). The reason for this is that in our models the ratio of emission to absorption (whether SSA or FFA) depends on position which implies that the "equivalent kinetic temperature" of the relativistic electrons contributing to the radiation at some frequency varies with both position and frequency.

1. *The Case of Synchrotron Self-absorption* (SSA)

a) Axially Symmetric Disk-like Sources. Consider an axially symmetric disk whose inner and outer boundary have radii of r_0 and R respectively (see Fig. 1). A point in the disk is specified by its radius r and height z. The disk-thickness h varies according to $h(r)=h_0(r/r_0)^{\varepsilon}$, where $h_0 \ll r_0$. The magnetic field strength B and the relativistic electron density coefficient K are functions of r only and vary according to $B=B_0(r/r_0)^{-m}$ and $K=K_0(r/r_0)^{-n}$ respectively. Throughout the source we assume the relativistic electron momentum distribution to be isotropic and to have an energy distribution $N(E)dE=KE^{-\gamma}dE$. For an observer located along the symmetry axis (face-on view) the observed radiation spectrum can be written as

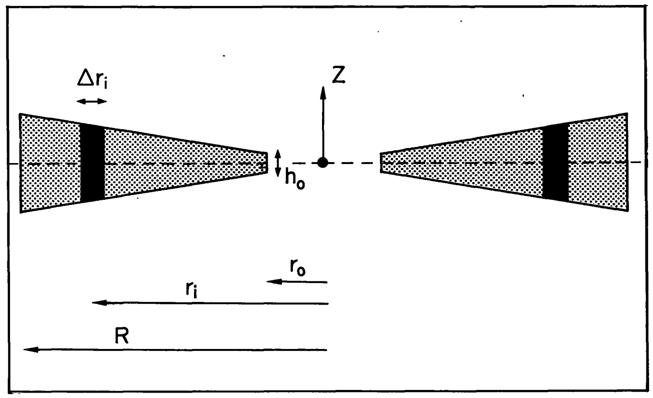

Fig. 1. Disk geometry used in Section II.1a. Rotational symmetry around the z-axis is assumed. For simplicity, $\varepsilon=1$ in this diagram

$$S_\nu = \int_{r_0}^{R} 2\pi r S_\nu(r)dr \tag{1}$$

where

$$S_\nu(r) \propto \int_{-h(r)/2}^{+h(r)/2} \varepsilon_\nu(r) e^{-\int_z^{h(r)/2} k_\nu(r)dz} dz \tag{2}$$

is the intensity at radius r. The volume emissivity $\varepsilon_\nu(r)=aKB^{(\gamma+1)/2}\nu^{(1-\gamma)/2}$ and the re-absorption coefficient $k_\nu(r)=bKB^{(\gamma+2)/2}\nu^{-(\gamma+4)/2}$ are given by Ginzburg and Syrovatskii (1965). Because we took $\varepsilon_\nu(r)$ and $k_\nu(r)$ independent of z, Equation (2) can readily be integrated to give

$$S_\nu(r) \propto \{\varepsilon_\nu(r)/k_\nu(r)\} \cdot \{1-e^{-k_\nu(r)h(r)}\}. \tag{3}$$

Putting in the relationships for $\varepsilon_\nu(r)$ and $k_\nu(r)$ and substituting (3) into (1) leads to

$$S_\nu \propto \int_{r_0}^{R} r^{1+m/2}\nu^{2.5}\{1-e^{-c r^{\varepsilon-n-m(\gamma+2)/2}\nu^{-(\gamma+4)/2}}\} dr \tag{4}$$

By making the following transformation we bring the ν-dependence out of the exponential and integral term.

$$r^{\varepsilon-n-m(\gamma+2)/2}\nu^{-(\gamma+4)/2}=t$$

or

$$r=t^{1/(\varepsilon-n-m(\gamma+2)/2)}\nu^{(\gamma+4)/(2\varepsilon-2n-m\gamma-2m)} \tag{5}$$

substitution of (5) into (4) leads to

$$S_\nu \propto \nu^{2.5+\frac{(2+m/2)(\gamma+4)}{(2\varepsilon-2n-m\gamma-2m)}} \int_{t(r=r_0)}^{t(r=R)} t^{\frac{(2+m/2)}{(\varepsilon-n-m(\gamma+2)/2)}-1} \cdot \{1-e^{-ct}\} dt. \tag{6}$$

The integral still depends on ν because the integration boundaries depend on ν according to (5). However, $t(r=r_0)\to\infty$ if $r_0\to 0$ and $t(r=R)\to 0$ if $R\to\infty$ assuming that

$$\varepsilon-n-m(\gamma+2)/2<0\,. \qquad \text{(A)}$$

If the integral converges for $t(r=r_0)\to\infty$ and $t(r=R)\to 0$ it becomes independent of ν. Convergence is obtained for

$$-1<(2+m/2)/(\varepsilon-n-m(\gamma+2)/2)<0\,. \qquad \text{(B)}$$

Since, for physical reasons, $2+m/2>0$, the right-hand side of (B) will be fulfilled because of (A). The left-hand side reads[1]

$$2+\varepsilon-n-m(\gamma+1)/2<0\,. \qquad \text{(C)}$$

If (C) holds, (A) automatically holds. Note that if $\varepsilon-n-m(\gamma+2)/2>0$, convergence is still obtained if (B) holds. However, this can only be the case if $2+m/2<0$ or $m<-4$, which is physically unreal.

If (C) holds Equation (6) simplifies to

$$S_\nu\propto\nu^{(8+5\varepsilon-5n-3m-2m\gamma+2\gamma)/(2\varepsilon-2n-m\gamma-2m)}\,. \qquad (7)$$

The disk spectrum thus adopts a power-law form whose index is less than the $+2.5$ expected for a uniform source. The condition (C) guarantees that the limiting slopes of (7) are $+2.5$ and $(1-\gamma)/2$ (as they should be, of course).

It can easily be shown that for finite source boundaries the integral in (5) depends significantly on ν only for $\nu>\nu_2$ and $\nu<\nu_1$ (where $\nu_2\gg\nu_1$). In those frequency regimes the disk spectrum starts to deviate from the power-law form of (7). For $\nu\gg\nu_2$ the source is completely transparent and the spectral index is $(1-\gamma)/2$; for $\nu\ll\nu_1$ the disk is completely opaque and the index becomes $+2.5$. The extent of the intermediate frequency regime, where (7) holds, depends on the values of the parameters R/r_0, ε, γ, n and m.

In sources where B and K are not uniform in the z-direction but both have the same exponential dependence on z with a scale height $h(r)\propto r^\varepsilon$, the result that is obtained is identical to that of the uniform disk. This can be shown fairly straightforward, although with somewhat more complicated mathematics, by first substituting $y=z/r^\varepsilon$ into (2) and then making the transformation (5).

Deviations from face-on view change the axial symmetry of the calculations and in general will lead to a curved spectrum. For small deviations from face-on view and/or small ε (which determines the rate of spreading of the disk), the deviations from a power-law spectrum take place only very slowly.

[1] This condition can physically be interpreted as requiring that, at a given frequency, the emission from a cylindrical shell, with thickness proportional to radius, decreases with increasing radius (see also the spherical treatment below)

One of the key steps in deriving (7) is given in (5). The physical meaning of this transformation is that if we compensate a change in radius, r, by a change in frequency, ν, such that t is constant, the optical depth through the disk (which is proportional to t) remains the same. Equation (5) therefore also expresses the change of the effective size of the source as a function of frequency. We emphasize this point here because a similar transformation will be used in the spherical case that we discuss below.

b) Spherically Symmetric Sources. The spherical case is more difficult to analyze than the disk-type geometry because of the different optical depth encountered by different parts of the source at a given radius. The quasi-analytical procedure that is used below to derive the spectrum of a spherical source, however, gives more physical insight into how the final result is obtained. By r we now denote the radius of the spherical source and B and K depend in the same way on r as before. The inner and outer boundary of the source have radii r_0 and R, respectively.

Consider a shell at radius r_1 whose thickness Δr_1 is much less than the scale lengths typifying the changes in volume emissivity and absorption coefficient. For reasons that will become apparent later we take $\Delta r_1\propto r_1$. For an observer at infinity the flux density received from this shell is determined by three quantities: 1) the emissivity in the shell, 2) its volume, and 3) the distribution of optical depth over the shell due to attenuation by other parts of the source between the shell and the observer. It is clear that 1) and 2) only determine the absolute level of the received emission and that the detailed shape of the spectrum is solely determined by 3) (for a given γ). Because at sufficiently high frequencies the spectrum of the shell must approach the optically thin value $(1-\gamma)/2$ and at sufficiently low frequencies the emission must be completely absorbed we can characterize the spectrum by a peak $(\nu_{\max}(r_1), S_{\max}(r_1))$. Now consider a second shell with radius r_2 and thickness Δr_2 where $\Delta r_2=(r_2/r_1)\,\Delta r_1$ and peak $(\nu_{\max}(r_2), S_{\max}(r_2))$.

If we demand that the spectral envelope of the sum of the spectra of all volume elements in shell 2 be *congruent* with the spectral envelope of shell 1, we must ensure that the geometrical distribution of optical depth is the same for both shells. This can be accomplished by compensating the difference in radius by a shift in frequency such that for every volume element ΔV_1 in shell 1 at frequency ν_1 the optical depth equals that of the corresponding (scaled in dimensions) volume element ΔV_2 in shell 2 at frequency ν_2 (see Fig. 2). This optical depth can be expressed as

$$\tau_{\nu_i}(\Delta V_i)=b\int_{\text{l.o.s.}} r_i^{-n-m(\gamma+2)/2}\nu^{-(\gamma+4)/2}ds_i \qquad (8)$$

where the integral extends along the line of sight.

From similarity arguments it follows that if $r_0 \to 0$ and $R \to \infty$, both ds_i and the integration boundaries will scale with r_i. The condition for congruence thus becomes

$$r_i^{-n-m(\gamma+2)/2}\nu^{-(\gamma+4)/2} \cdot r_i = \text{constant}.$$

This tells us that the spectrum of shell 2 must be displaced in frequency from that of shell 1 by an amount

$$\nu_{max}(r_2)/\nu_{max}(r_1) = (r_2/r_1)^{(2-2n-m\gamma-2m)/(\gamma+4)}. \quad (9)$$

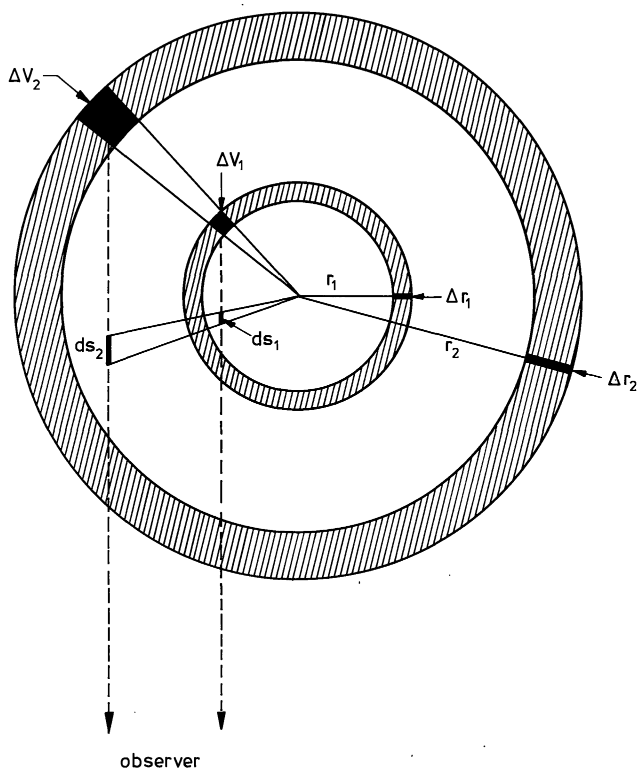

Fig. 2. Schematic two-dimensional illustration of the quantities used in the spherical source model

The displacement in flux density between the two shells at corresponding positions in the spectrum (e.g. ν_{max}) can be calculated straightforwardly from the different emissivities and volumes. Hence

$$S_{max}(r_2)/S_{max}(r_1) = \{\varepsilon_{\nu_{max}(r_2)}(r_2)/\varepsilon_{\nu_{max}(r_2)}(r_1)\} \cdot (r_2/r_1)^3$$

or

$$S_{max}(r_2)/S_{max}(r_1)$$
$$= (r_2/r_1)^{3-n-m(\gamma+1)/2}\{\nu_{max}(r_2)/\nu_{max}(r_1)\}^{(1-\gamma)/2}. \quad (10)$$

Since we took $\Delta r_i \propto r_i$ the radii of successive shells will be separated by a constant factor. From (9) and (10) we then deduce that the spectral peaks of successive shells are separated by a constant factor in both frequency and flux density and can be connected by a straight line in a full logarithmic diagram. Summation of the congruent spectra of all shells in the source then produces an integrated spectrum with the same slope as this straight line, if the following condition holds[2]

$$3-n-m(\gamma+1)/2<0. \quad \text{(D)}$$

(Note that if (D) holds and $2+m/2>0$, true for physical reasons, the exponent in (9) is also less than zero.) The spectral index of the overall spectrum is now found by eliminating (r_2/r_1) from (9) and (10), yielding

$$S_\nu \propto \nu^{(13-5n-3m-2m\gamma+2\gamma)/(2-2n-2m-m\gamma)}. \quad (11)$$

Fot those values of n, m and γ for which (D) is satisfied, the limiting slopes of (11) are $+2.5$ and $(1-\gamma)/2$ (as they should be).

If the radial extent of the source is finite, as has to be the case in reality, the key-step in our approach: the spectra of individual shells are congruent, starts to break down. This is because the similarity argument used to derive (9) is not valid anymore: radiation from the shells near the inner boundary of the source will "profit" more from the central hole than shells at larger radii. A similar consideration applies to the radiation from shells near the outer boundary. However, the effects of a central hole or outer edge of the source will only manifest themselves in the integrated spectrum at very high frequencies (where the observer can see down to the inner boundary) or very low frequencies (where most of the emission comes from the outer parts of the source). There the spectrum will bend to its high- and low-frequency limits, which are $(1-\gamma)/2$ and $+2.5$ respectively. Over an intermediate frequency range the power-law form (11) holds. The magnitude of this regime depends on R/r_0, n, m and γ and can be calculated from (9) by substituting (R/r_0) for (r_2/r_1).

2. *The Case of Free-free Absorption* (FFA)

The only modification to the Equations in II.1 concerns the absorption coefficient which now equals (e.g. Osterbrock, 1974)

$$k_\nu(r) = d n_e^2(r)\nu^{-2} f(T_e, \nu).$$

In the following we will neglect the weak dependence on frequency that is included in the term $f(T_e, \nu)$. We will assume that the thermal electron density in the source varies according to $n_e = n_0(r/r_0)^{-p}$. We do not consider changes in the electron temperature or volume filling factor in the source because the physics behind such variations is less clear than those in radial changes of the gas density. If required, such changes (if of the power-law form) can be included in the parameter p. The FFA analogue of (5) (disk-geometry) is

$$r = t^{1/(\varepsilon-2p)}\nu^{2/(\varepsilon-2p)}. \quad (5^*)$$

[2] This condition is the spherical equivalent of (C); see also footnote 1

Carrying through the analysis with this substitution yields for the disk spectrum in the FFA case, if $r_0 \to 0$ and $R \to \infty$

$$S_\nu \propto \nu^{(8+5\varepsilon-4n-2m-2p-\varepsilon\gamma-2m\gamma+2p\gamma)/(2\varepsilon-4p)}. \quad (7^*)$$

The limitations on the values of ε, p, n, m and γ for the power-law form (7*) to be valid is expressed by

$$-1<(2+2p-n-m(1+\gamma)/2)/(\varepsilon-2p)<0. \quad (\mathrm{B}^*)$$

This restriction implies that only those combinations of ε, n, m, p and γ are permitted that yield a spectral index α in (7*) that lies inbetween $(1-\gamma)/2$ and $(5-\gamma)/2$. Similarly the FFA analogue of (9) (spherical geometry) becomes

$$\nu_{\max}(r_2)/\nu_{\max}(r_1)=(r_2/r_1)^{(1-2p)/2}. \quad (9^*)$$

Equation (10) is the same for FFA as for SSA, hence the spherical source spectrum in the case of FFA becomes

$$S_\nu \propto \nu^{(13-4n-2m-2p-\gamma-2m\gamma+2p\gamma)/(2-4p)}. \quad (11^*)$$

The limits on n, m, p and γ for (11*) to hold are

$$3-n-m(\gamma+1)/2 \lesseqgtr 0 \quad \text{for} \quad 1-2p \lesseqgtr 0. \quad (\mathrm{D}^*)$$

These restrictions imply that the spectral index α in (11*) has to be greater than $(1-\gamma)/2$. No limitations exist on how positive α can become, contrary to the disk model. For finite source boundaries, in both the disk and spherical geometry, the same general considerations apply as in the SSA case. For the FFA models we find that the extent of the intermediate frequency range over which (7*) and (11*) hold depends only on ε and p. One noteworthy difference between the SSA and FFA models is that realistic source models may be constructed in the FFA case which do have a power-law spectrum but also have an effective source size that is *increasing* with frequency (if $\varepsilon-2p>0$ (5*) or $1-2p>0$ (9*)).

III. Numerical Calculations

To check the validity of the equations derived in Section II and to determine the spectra in the transition regions we numerically integrated the radiative transfer problem for spherical source models with various values for the parameters m, n, p and γ. Because of the rotational symmetry of the source with respect to a line joining its centre and the observer, the calculations could be restricted to two dimensions: we computed the intensity at varying projected radii as a function of frequency.

Similar calculations were performed earlier by Condon and Dressel (1973) for the case of SSA only. Our models and calculations differ in some aspects of Condon and Dressel's. Firstly, we preserved spherical symmetry in the calculations and, secondly, we used a larger range in radius ($R/r_0=100$) and included no flattening of the values of B, K and n_e near the inner radius. The reason for the latter was that by letting these quantities approach an asymptotical value near the inner boundary we would create different effective values of n, m, and p at different radii thereby masking the straight power-law dependence of flux density on frequency that we wanted to test. Note that we have no singularity problem at the centre since the inner boundary of the source, r_0, has a finite value, and no radiation is emitted from regions interior to r_0.

To check that the grid size that was used to calculate the resultant spectral intensity (the minimum grid size was determined by the available computing time) had no significant influence on the final spectral index in the opaque frequency regime, we compared the numerical calculations at a frequency where the source is completely transparent, with an analytic integration of the emission from r_0 to R at the same frequency. This comparison indicated that the numerical procedure used was accurate to better than 1%, enough for our purpose.

A representative set of spectra for both SSA and FFA models is shown in Figures 3 and 4. In the four different cases of SSA models one can clearly see how the frequency range over which the power-law spectrum

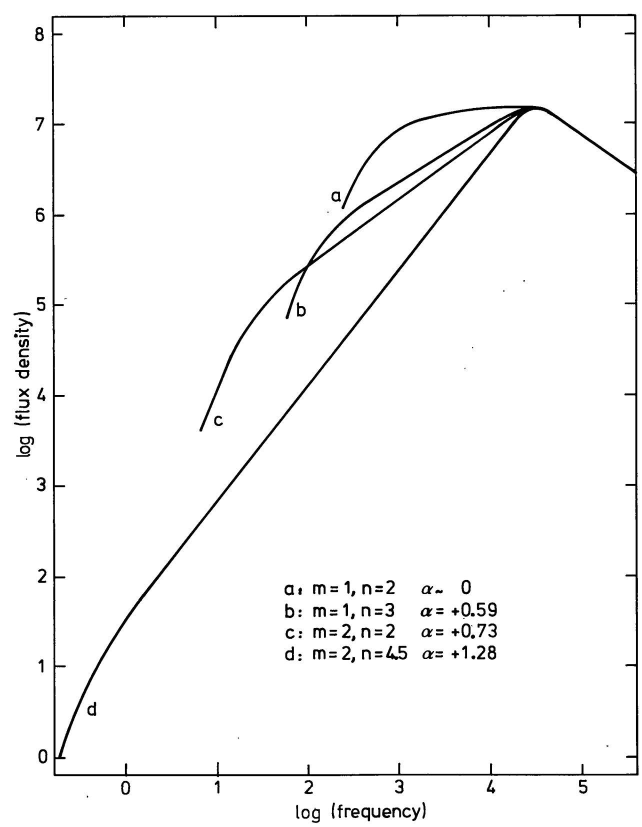

Fig. 3. Four examples of a numerically calculated spectrum of a spherically symmetric source with synchrotron self-absorption. The parameters m, n and α are defined in the text. The scales are in arbitrary units. The optically thin spectral index is -0.75 ($\gamma=2.5$)

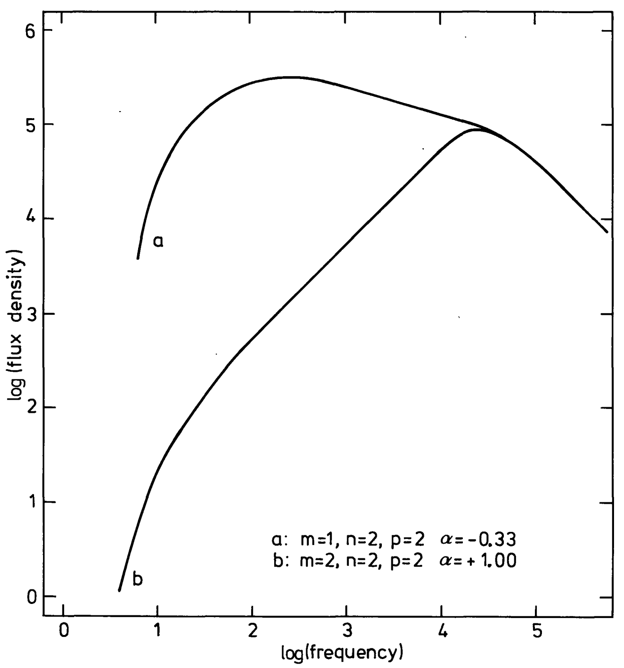

Fig. 4. As in Fig. 3 but here the absorption is due to thermal gas (FFA). The parameters m, n, p and α are defined in the text. The optically thin spectral index is -1 ($\gamma=3$)

forms is correlated with the spectral index $\alpha(n, m, \gamma)$ as is expected from (9). The agreement with theory is excellent except for the ($n=2$, $m=1$) model where there is not a completely flat spectrum regime as expected from (11). This is due to the fact that for this particular set of values the range in radius is too small to clearly separate the predicted power-law spectrum from the transition regions to the high- and low-frequency limits. From Figures 3 and 4 one can also see that these transition regions generally occupy a relatively narrow frequency range.

In real sources these transition regions will be broader because the magnetic field stregth and the particle density cannot change discontinuously at the inner or outer boundary. From the numerical calculations of the intensity as a function of projected radius we could also determine the change in source diameter as a function of frequency. We find that, *in the region where the power-law spectrum holds*, there is perfect agreement with the theoretical expectations as given by (9) and (9*). That is, the source diameter θ (defined as the "diameter" where the surface brightness drops to a constant fraction, say 1/2, 1/4 or 1/10 of the central surface brightness[3], is proportional to $\nu^{2/(1-2p)}$ (for FFA) and $\nu^{(\gamma+4)/(2-2n-2m-m\gamma)}$ (for SSA). Although we have no numerical computations for disk-type models, it seems only natural that Equations (5), (7), (5*) and (7*) also hold.

[3] The central surface brightness is in our models (where we have a hole in the centre of the source) not equal to the maximum surface brightness. However, for frequencies below the spectral peak the difference is small because emission from the immediate surroundings of the central hole then does not contribute significantly to the spectrum

IV. General Discussion

The results obtained in Section II and illustrated in Section III show that it is possible to explain the flat and/or inverted spectra of sources with internal absorption as due to their non-uniformity. An observational problem is that up till now only few extragalactic sources have had their spectrum determined with a sufficient accuracy over a wide frequency range to decide whether they are truly straight, which would be suggestive of continuous (single component) sources, or undulating, indicative of more than one component. The only real difference between such sources seems to lie, however, in the continuous or discontinuous character of their formation. This is because approximately flat spectra resulting from the superposition of spectra of many discrete concentric sources (each produced by a separate event), still require that the average magnetic field strength and relativistic electron density decrease from the inner to the outer component with a similar dependence on radius as in continuous sources, that is, much slower than $B \propto r^{-2}$ and $K \propto r^{-2-\gamma}$ as in the spherical, uniformly and adiabatically expanding, flux-conserving model.

One of the original assumptions in Section II was that the index γ is uniform throughout the source. If this assumption is violated, for example due to radiation losses or a curved injection spectrum, the results will not differ very much for the following reasons: 1) the range in energy of electrons contributing to the observed radio spectrum is small due to the fact that at lower frequencies the radiation comes from regions of the source that also have a lower magnetic field strength, and 2) the spectra in the partially opaque frequency regime are rather insensitive to the value of γ. A second assumption, isotropy of the particle momentum distribution throughout the source, may have larger consequences if it is not obeyed. This could be the case if the particles are streaming down a gradient in the magnetic field and conservation of the adiabatic invariant affects a redistribution of the electron pitch angles.

a) Other Geometries

Although the calculations presented in Sections II and III apply only to sources with spherical or axial symmetry they can easily be extended to other geometries. Direct physical reasoning shows that the dimensionality of the source (determining the rate of increase of volume with "radius") is more important for the shape of the spectrum than the detailed structure. This is shown, for example, by the fact that if we substitute $\varepsilon=1$ (disk thickness proportional to radius) into (7) or (7*), we arrive at the same result as for the spherically symmetric sources [Equations (11) and (11*)]. Thus it can also be seen that for sources shaped like a double cone with a small opening angle, the optically thick spectrum will also be given by (11) (SSA) ot (11*) (FFA) because such a source structure can be regarded as a small azimuthal section of a disk with $\varepsilon=1$.

b) Observational Distinction between SSA *and* FFA

i) If from an opaque source with known flux density the angular size can be determined by direct (VLBI observations) or indirect means (variability timescale plus a known distance), its brightness temperature can be determined. If this falls within the range 10^{10}–10^{12} K it seems likely that SSA is the dominant absorption mechanism. Values much less than 10^{10} K would, however, exclude SSA.

ii) Linear polarization of the emission from compact opaque sources excludes FFA (and the Razin-Tsytovich effect) because the large electron densities implied by these mechanisms would lead to complete Faraday depolarization (e.g. Aller, 1970). Care has to be taken, however, when applying this argument to sources with complex spectra and several spatially separated components, because it is not always clear in which component the polarized radiation originates.

iii) Measurements of the optical emission line intensities of galactic nuclei containing radio cores enable an estimate of the importance of FFA depending on the size of the radio-source/thermal-plasma mixture. Let us illustrate this with a numerical example. Take a hypothetical nucleus with an extended distribution of gas at a temperature of 10^4 K and with a radial electron density variation $n_e=n_0(r/r_0)^{-2}$ (which is not critical for this argument). Assume the H_β-luminosity is $6\,10^{41}$ erg/s and arises under case B conditions (Pengelly, 1964). This is a typical value for Seyfert galaxy nuclei (Anderson, 1970; Weedman, 1972; distances have been calculated using $H=50$ km/s/Mpc); for radio galaxies no quantitative data are available as yet. Now the effective size of the source will depend on frequency according to (9*) with $p=2$. If we then calculate the radius down to which the optical depth equals unity at a frequency of 5 GHz, we find a value of about 9 pc ($\propto I_{H_\beta}^{1/2}\nu^{-1}$). This means that at 5 GHz most of the observed radio emission from the source must come from regions outside ~ 10 pc. This inner radius is independent of possible clumping although the total gaseous mass involved is not. For a filling factor of 10^{-3} the integrated gas content from r_0 to $100\,r_0$ (10–1000 pc in this example) is of the order of $5\,10^6\,M_\odot$ and $n_0=10^5\,\mathrm{cm}^{-3}$, not unreasonable values. This example illustrates that FFA can influence the spectrum of the radio cores in galactic nuclei for linear sizes much greater than the size required for SSA to be important (typically 0.1–1 pc).

c) Restriction of the Number of Free Parameters

If it has been possible to decide on the relevant absorption mechanism in a given source (for example, by considerations of the type given above), there are still too many free parameters to determine its physical structure from the value of its spectral index. However, if one assumes equipartition or a constant ratio between magnetic and relativistic electron energy at every point in the source, the values of n and m are related ($n=2m$). Alternatively, in sources where the magnetic field energy is dominant and the field topology is fixed in time, the adiabatic losses of outstreaming particles are zero and the parameter n is determined by the injection rate and spatial dilution only. In FFA models the magnetic coupling between thermal and relativistic plasmas relates the values of n, p and γ, depending on the propagation mode of the particles and the adiabatic losses. If future VLBI-observations are capable of measuring the source size (or some equivalent such as the baseline length (in wavelengths) at which the fringe visibility falls to a certain value) as a function of frequency we have another tool in determining the values of the parameters n, m, p and γ, because the size-dependence on frequency varies differently with these parameters than does the spectral index.

d) Radio Variability and Non-uniform Sources

It can easily be shown that (11) also describes the spectral peak evolution of an uniformly expanding cloud of relativistic electrons and magnetic fields subject to SSA. For example, when substituting $m=2$ and $n=2+\gamma$, values appropriate to the "standard" adiabatically expanding source, the original result of van der Laan (1966) is recovered. The fact that the spectral index in (11) is so much stronger dependent on n and m than on γ, therefore implies that it is a fairly crude method to determine γ by fitting the spectral peak evolution of discrete radio outbursts to the "standard" model (Kellermann and Pauliny-Toth, 1968); small changes in n and m could lead to large variations in γ determined by this method. In this connection it is worth noting that Stannard et al. (1975) suggest that the rapid radio variable BL Lac has a non-varying flat spectrum component ($\alpha\sim+0.2$) over a large frequency range. If this spectrum is interpreted as produced in an internally obsorbed radially extended source the character of the radio variability must change considerably; the decay in the ($S_{\max}$, $\nu_{\max}$)-plane of a burst of fresh electrons injected in such a source structure, for example, must be much flatter. This is actually observed at low frequencies in BL Lac (Stannard et al., 1975; Ekers et al., 1975). Similar considerations may apply to the variable radio nucleus of M 81 (de Bruyn et al., 1976).

V. Comparison with Observations

In this section we give a mostly qualitative analysis of the presently available data in the light of the models of Section II.

a) The Radio Cores of Elliptical Galaxies

Recent high-resolution aperture synthesis observations of extragalactic radio sources have established that a large percentage of the "classical" double radio sources also contains a weak third component, generally unresolved, coincident with the optical nucleus of the parent galaxy. Similar indications were found earlier by Heeschen (1970) for a small sample of nearby elliptical galaxies. The spectra of these radio cores often are remarkably flat ($\alpha \sim 0$), in contrast to the spectra of the extended radio components surrounding them (Ekers and Ekers, 1973; Colla et al., 1975). A good example is the compact radio source in NGC 4278 (Wills, 1968). If these flat spectra are not reflecting a much flatter electron energy spectrum, they have to be explained by internal absorption in non-uniform sources. Both the SSA and FFA interpretation are still valid alternatives at present. The fact that the presence of a radio core is correlated with emission lines (Disney and Cromwell, 1971; Colla et al., 1975) calls for further investigations (see alse Section IV.b.iii). Both the SSA and FFA interpretation of these flat spectra require that the magnetic field strength and the relativistic and thermal electron density decrease when one goes outwards, but much more slowly than in an adiabatically expanding source. For example, in the SSA model an $\alpha=0$ spectrum can be explained by $m=1$, $n=2$ independent of γ (this applies both for a spherical geometry and (a section of) a disk with $\varepsilon=1$). It is interesting that the latter set of values corresponds to a source where magnetic energy and relativistic electrons are continuously produced and stream away (or flow in) at uniform velocity in a loss-loss way [that is no radiation and adiabatic losses (or gains)].

The SSA interpretation of the flat spectrum cores of elliptical galaxies implies a very small linear size, about one or two orders of magnitude less than those in the much more luminous cores of quasars[4]. Because of this small linear size, future observations of variability as a function of frequency may give important clues about the structure, maintenance and dynamical evolution of these sources (see also Section IV.d).

b) Radio Sources in Seyfert Galaxies and Related Objects

The large amounts of ionized gas observed in the central regions of Seyfert galaxies suggests that the radio sources discovered in these galaxies (Wade, 1968; van der Kruit, 1971; de Bruyn and Wilson, 1976) should be the first to check for effects of FFA on the spectrum. The spectral turnovers in the cm-wavelength range of the bright Seyfert galaxies 3 C 120 and NGC 1275 have been satisfactorily explained as due to SSA. However, in the radio spectrum of NGC 1068 (Kellermann and Pauliny-Toth, 1971) there may be an indication of thermal absorption. The extended radio component in this galaxy has a spectral index $\alpha=-0.3$ at frequencies below 1 GHz, while the high-frequency spectral index is "normal" and about similar to that of other Seyfert galaxies (de Bruyn and Willis, 1974; Kojoian et al., 1976). Within the framework of the spherical FFA models of Section II.2 the change in spectral index could be explained by $p=2$, $m=1$ and $n=2$ approximately. This particular choice of m and n is also consistent with the fact that at the "transparent" frequency of 5 GHz the source emissivity decreases only slowly as a function of distance from the centre, as shown by the uniform decrease of the visibility amplitude as a function of interferometer baseline (de Bruyn and Willis, 1974).

Other galaxies having an extended radio source in their nucleus that may fall in this category are M 82, whose radio spectrum is very similar to that of NGC 1068 (Kellermann and Pauliny-Toth, 1971), and NGC 253, which has a spectral index of -0.3 between 1.4 and 5 GHz (Becklin et al., 1973; Ekers, private communication), a value that is anomalously low for an extended nonthermal source. In both galaxies there is evidence for strong optical emission lines from the central region, the intensity of which is difficult to estimate because of heavy optical obscuration.

c) Relation between Frequency and Size of Internally Absorbed Sources

An important prediction of the models in Section II, as also noted by Condon and Dressel (1973), is that the size of a source with flat or inverted spectrum should increase with decreasing frequency [Equations (5), (5*), (9) and (9*)] depending on the geometrical model and the absorption effect. Little information bearing on this aspect is available as yet. Low-frequency scintillation studies (Harris, 1974) and meter-wavelength VLBI (Erickson et al., 1972; Clark and Erickson, 1973) indicate that compact nuclei of quasars and radio galaxies still have significant flux densities left at frequencies around 100 to 200 MHz. A plausible interpretation, also suggested by these authors, is that the source sizes are much larger around 100 MHz than those observed in high-frequency VLBI measurements. Further VLBI measurements in the 21 cm to 2 cm range are now capable of testing the frequency-dependent size predictions.

[4] This is because the linear size of a SSA source at a given frequency is about proportional to the one-half power of the absolute luminosity, as can be seen by rewriting the well-known relation between ν_{max}, S_{max}, $\hat{B}$ and angular size (Kellermann and Pauliny-Toth, 1969) in terms of intrinsic parameters

Acknowledgements. I am grateful to Prof. H. van der Laan and Dr. W. W. Shane for many helpful discussions on Section II of this paper and comments on the manuscript. I thank an anonymus referee for valuable criticism which led to an improvement of the manuscript.

References

Anderson, K.S.: 1970, *Astrophys. J.* **162**, 743

Aller, H.D.: 1970, *Astrophys. J.* **161**, 19

Becklin, E.E., Fomalont, E.B., Neugebauer, G.: 1973, *Astrophys. J.* **181**, L 27

Braude, S.Ya.: 1967, *Soviet Astron. AJ.* **11**, 244

Bruyn, A.G. de, Willis, A.G.: 1974, *Astron. & Astrophys.* **33**, 351

Bruyn, A.G. de, Crane, P.C., Price, R.M., Carlson, J.B.: 1976, *Astron. & Astrophys.* **46**, 243

Bruyn, A.G. de, Wilson, A.S.: 1976, submitted to *Astron. & Astrophys.*

Clark, T.A., Erickson, W.C.: 1973, *Proc. Inst. Elec. Electron. Engrs.* **61**, 1230

Colla, G., Fanti, C., Fanti, R., Gioia, I., Lari, C., Lequeux, J., Lucas, R., Ulrich, M.-H.: 1975, *Astron. & Astrophys.* **38**, 209

Condon, J.J., Dressel, L.L.: 1973, *Astrophys. Letters* **15**, 203

Disney, M.J., Cromwell, R.H.: 1971, *Astrophys. J.* **164**, L 35

Ekers, R.D., Ekers, J.A.: 1973, *Astron. & Astrophys.* **24**, 247

Erickson, W.C., Kuyper, T.B.H., Clark, T.A., Knowles, S.H., Broderick, J.J.: 1972, *Astrophys. J.* **177**, 101

Ginzburg, V.L., Syrovatskii, S.I.: 1965, *Ann. Rev. Astron. & Astrophys.* **3**, 297

Harris, D.E.: 1974, *Astron. J.* **79**, 1211

Hirasawa, T., Tabara, H.: 1970, *Astrophys. Letters* **7**, 121

Kellermann, K.I., Pauliny-Toth, I.I.K.: 1968, *Astrophys. J.* **152**, 169

Kellermann, K.I., Pauliny-Toth, I.I.K.: 1969, *Astrophys. J.* **155**, L 71

Kellermann, K.I., Pauliny-Toth, I.I.K.: 1971, *Astrophys. Letters* **8**, 153

Kojoian, G., Sramek, R.A., Dickinson, D.F., Tovmassian, H., Purton, C.R.: 1976, *Astrophys. J.* **203**, 323

Laan, H. van der: 1966, *Nature* **211**, 1131

Marsh, K.A.: 1975, *Astrophys. J.* **201**, 190

Olnon, F.M.: 1975, *Astron. & Astrophys.* **39**, 217

Osterbrock, D.E.: 1971, Proceedings Vatican Conference on the Nuclei of Galaxies (Amsterdam, North-Holland), p. 151

Osterbrock, D.E.: 1974, Astrophysics of Gaseous Nebulae, p. 79, Freeman and Co.

Pengelly, R.M.: 1964, *Monthly Notices Roy. Astron. Soc.* **127**, 145

Rieke, G.H., Low, F.J.: 1972, *Astrophys. J.* **176**, L 95

Stannard, D., Treverton, A.M., Porcas, R.W., Davis, R.J.: 1975, *Nature* **255**, 384

Véron, M.P., Véron, P., Witzel, A.: 1974, *Astron. & Astrophys. Suppl.* **13**, 1

Weedman, D.W.: 1972, *Astrophys. J.* **171**, 5

Wills, D.: 1968, *Astrophys. Letters* **2**, 187

Wright, A.E., Barlow, M.J.: 1975, *Monthly Notices Roy. Astron. Soc.* **170**, 41