Adv. Space Res. Vol. 6, No. 6, pp. 57–60, 1986
Printed in Great Britain.

0273-1177/86 $0.00 + .50

AN ANALYTIC MODEL FOR SOLAR FLARE DEVELOPMENT

J. S. Kaastra

Laboratory for Space Research, P.O. Box 9504, 2300 RA Leiden, The Netherlands

ABSTRACT

Recent results in solar flare analysis using integrated field properties rather than fully two- (or three) dimensional magnetic field equations are reported. The flare is described by the mutual Lorentz forces of a static background field on a rising filament current system and a current sheet far below that filament. The well-conducting solar surface with its high inertia can be represented formally as a mirror plane for the coronal or chromospheric current systems. The start of the flare is described by the well-known Van Tend-Kuperus mechanism, where a current filament meets a critical height above which static force balance is impossible. Reconnection and magnetic field dissipation occur at the induced current sheet which is situated well below the filament.

MODEL DISCUSSION

There are two ways to describe the solar flare phenomenon. The first is to consider the full two- or three dimensional field equations (Maxwell, Navier-Stokes and an energy equation) and to solve these equations using numerical techniques and the proper boundary conditions. Although progress has been made with this method, it suffers from many dificulties varying from numerical instabilities, commputer memory limitations to various physical instabilities and the complexity of the multitude of physical processes involved.

In this contribution we consider the second possibility, namely a description using integrated properties of the flare, and a reduction of the basic equations retaining only the most important terms.

The magnetodynamics of the flare can be modeled simply using the interactions resulting from the mutual Lorentz forces between a number of current systems /1/. The total magnetic field in the flaring region can be divided into a number of components, corresponding to different magnetic field sources (current systems). The first component is the background magnetic field of the active region, whitch in its most simple form may be represented by a line dipole placed below the surface of the sun. Although small local flux changes of the background magnetic field during a flare are frequently reported, these changes are relatively small compared to the global bipolar structure, which changes on a much slower time scale. As a first approximation the background field can be taken time-independent therefore.

The boundary layer of the well-conducting and highly inertious photosphere acts as a perfect mirror for fields with a coronal source varying rapidly compared to typical photospheric convection times /2/. For this reason the attraction on a preflare current filament by the background magnetic field may be balanced by the repulsive force of the photosphere on this current; this repulsive force may be described formally by placing a virtual mirror current of opposite direction and strength below the photospheric boundary layer.

The equilibrium current I of such a preflare filament is given by

$$I = 4\pi h B_d(h)/\mu_0 \equiv f(h) \qquad (1)$$

where h is the height of the filament above the surface and $B_d(h)$ the horizontal magnetic field component of the background field only perpendicular to the neutral line of the active region, at the position of the filament. It can be shown that above a certain critical height h_c force balance is impossible /3/ and any small disturbance will cause a rapid rise of the filament.

Far below the rising filament a magnetic neutral line develops /1/, where due to the large induced electric fields caused by the changing magnetic topology a current sheet is formed /4/. The current in this sheet which grows steadily is parallel to the filament current /1/ and acts effectively as a brake on the rising filament.

At the site of the current sheet magnetic reconnection occurs. Because of the presence of the photospheric boundary layer, also a formal mirror current sheet must be introduced below the surface of the photosphere.

The position of the current sheet is determined by the condition that it is centered at the neutral line of the total (time varying) magnetic field of all other current components /1/, including background field, filament and mirror filament ànd mirror current sheet. The forces upon the filament are also (nearly) in balance, resulting in a steady (or slightly accelerated) rise of the filament. The balance of the Lorentzforces upon the filament is equivalent to the condition that the filament, as in the preflare situation, is situated at a magnetic neutral line of the sum of all field components except its own field; the only difference to the preflare situation is now that apart from the background field and the mirrorfilament field also the currentsheet- mirrorcurrent sheet system must be considered in the filament force balance. For a strictly two-dimensional translational symmetric situation, which is a fair first approximation to the elongaled geometry of a large two ribbon flare, both condition of force balance upon filament and current sheet can be written down explicitly; after some simple algebra, the strength J of the filament current and I of the sheet current can be expressed in the height h of the filament and s of the sheet:

$$I = (f(h) + g\, f(s)) / (1 + g^2) \tag{2}$$

$$J = (f(s) - g\, f(h)) / (1 + g^2) \tag{3}$$

where f(h) is defined by (1) and

$$g = g(h,s) \equiv 4\, hs / (h^2 - s^2)\ . \tag{4}$$

The rise of the filament, although fast compared to the preflare development, is slow compared to the Alfvén velocity, and the sound velocity is also small compared to the Alfvén velocity except in the current sheet where magnetic field dissipation occurs. Both statements allow us to consider the flare development everywhere away from the locations of the currents as a continuous series of quasi-static equilibria: time enters equation (2) and (3) only implicitly by the dependence h(t) and s(t).
The electrical field E can be calculated from Maxwell's equation using the potential function of the magnetic field:

$$E = -\partial A / \partial t \tag{5}$$

where

$$B = \nabla \times A\ . \tag{6}$$

At the site of the current sheet, the potential function A is given by /1/

$$A_s = A_d\,(z) + \frac{\mu_o}{2\pi}\left\{ I(h,s) \ln\left(\frac{h+z}{h-z}\right) + J(h,s)\left[0.5 + \ln\left(\frac{2z+2s}{b}\right)\right] \right\}\Big|_{z=s} \tag{7}$$

where b is the halfwidth of the current sheet which is assumed to be much smaller than s, and

$$B_d\,(z) \equiv -\,\partial A_d / \partial z\ . \tag{8}$$

The dissipation P at the current sheet can be evaluated simply from

$$P = -\,J E_s L \tag{9}$$

where E_s follows from (5) and (7) and L is the length of the system along the neutral line.

The explicit time dependence of the flare can be obtained by calculating the resistance of the current sheet; this results in a relation between E_s and J which will not be discussed here further.

I will conclude with a specific example. For the background magnetic field a line dipole at depth d below the photosphere is taken, resulting in

$$f(h) = K\, hd/(d+h)^2 \tag{10}$$

Contour lines of constant I and J as a function of h and s are shown in figure 1 and 2. The critical height h_c for this specific choice is $h_c = d$, namely the height at which (10) has its maximum. At the start of the flare, the filament height is h_c and the current sheet height $s = 0$, with $J = 0$ (reconnection has not yet started). Let us take as an example the solution where the decrease of I is minimal; the resulting solution is shown in figure 3. It can be seen that the height of the current sheet is small compared to the filament height, in agreement with the observation that the distance between the flare ribbons is small compared to the filament height /5/. The magnetic topology in a plane perpendicular to the neutral line is shown in figure 4. The energy source is situated at the current sheet, where magnetic reconnection occurs and magnetic energy is transformed into heat and fast particles.

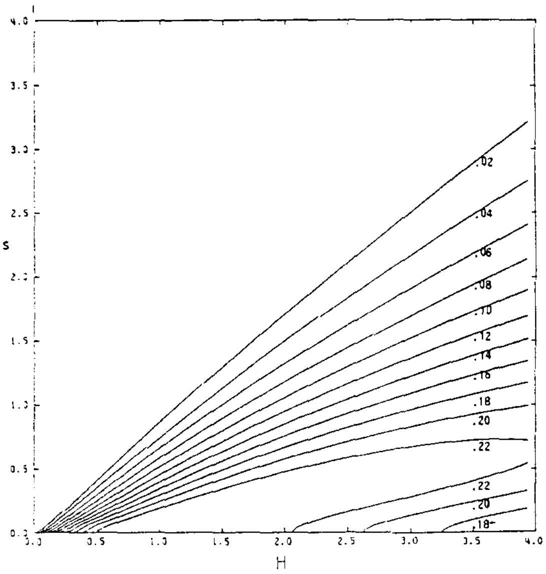

Fig. 1. Value of the equilibrium current of the filament as a function of filament height h and current sheet height s, in units of K.

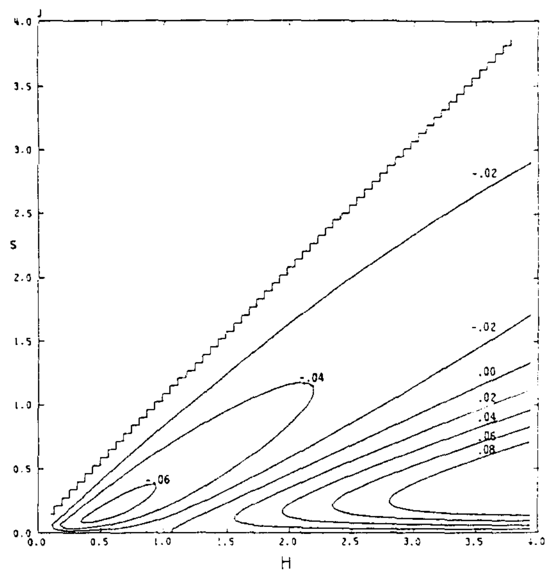

Fig. 2. Value of the equilibrium current of the current sheet as a function of h and s, in units of K.

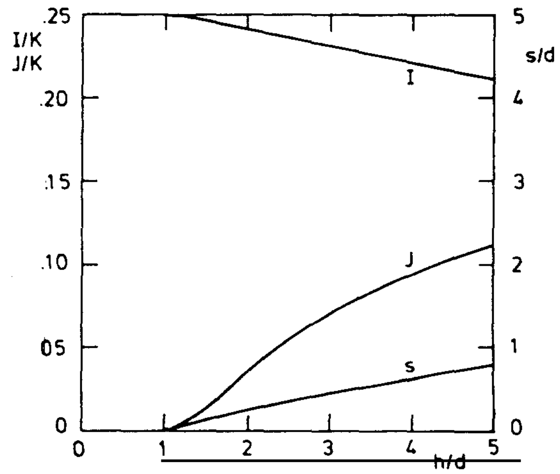

Fig. 3. Example of a flare solution for a line dipole magnetic field.

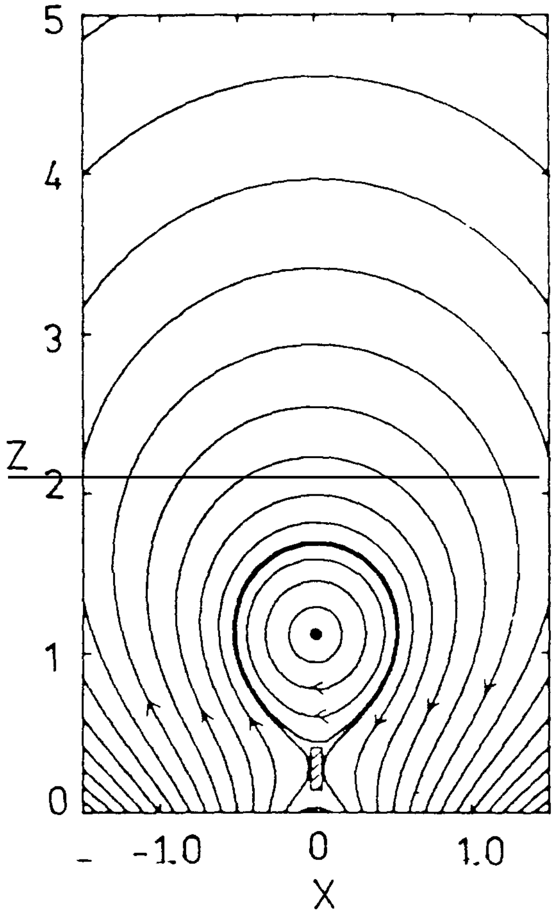

Fig. 4. Topology of the magnetic field during the flare.

REFERENCES

1. J.S. Kaastra, Solar flares. An electrodynamic Model, thesis, University of Utrecht, chapter 5 (1985).

2. M. Kuperus and M.A. Raadu, The support of prominences formed in neutral sheets, Astron. Astrophys. 31, 189 (1974).

3. W. van Tend and M. Kuperus, The development of coronal electric current systems in active regions and their relation to filaments and flares, Solar Phys. 59, 115 (1978).

4. S.I. Syrovatskii, Formation of current sheets in a plasma with a frozen- in strong meagnetic field, Zh. Eksp. Teor. Fiz. 60, 1727 (Sov. Phys. J.E.T.P. 33, 933) (1971).

5. F. Fárník, J. Kaastra, B. Kálmán, M. Karlicky, C. Slottje and B. Valníček, X-ray, H and radio observations of the two-ribbon flare of May 16, 1981, Solar Phys. 89, 355 (1983).