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COMMUNICATIONS FROM THE OBSERVATORY AT LEIDEN

THE SECONDARY VARIATIONS OF U TrA, DERIVED FROM PHOTO-ELECTRIC OBSERVATIONS MADE BY J. WOLTERBEEK MULLER

BY A. G. JANSEN

Photo-electric observations of the classical Cepheid U TrA show that the light-curve can be explained by the combination of the fundamental period: $P_0 = 2^d.56844$ and a secondary period: $P_1 = 1^d.8249$. The resulting beat period is: $P_b = 6^d.304$. The variable resembles TU Cas very closely, the ratio P_1/P_0 being 0.7105 for U TrA and 0.7097 for TU Cas. As in the case of AI Vel, the light-curves appear to be a distorted image of the sum of two harmonics, the ratio of the amplitudes b/a being 0.39.

OOSTERHOFF (1957a) has shown that the light-curve of the classical Cepheid U TrA ($\equiv$HD 143999) is strongly variable and that the photo-electric observations made in the years 1953—1956 can be roughly represented by the combination of the fundamental period, $P_0 = 2^d.568438$, and a beat period, $P_b = 6^d.9794$. The determination of the beat period was difficult on account of the fact that the old observations were spread over a large number of nights and that at the most three observations in a single night were available. For many of these observations it was impossible to decide whether they are situated on a rising or on a descending branch. The beat period derived was therefore given with some reserve.

At OOSTERHOFF's request Mr J. WOLTERBEEK MULLER observed this variable intensively in 1959 with the Rockefeller Astrograph of the Leiden southern station. Long series of photo-electric observations were made in blue and yellow. Two F8-type stars, $c_1 \equiv$ HD 143431 and $c_2 \equiv$ HD 143289, on opposite sides of the variable were measured as comparison stars. The arrangement of the observations was usually as follows: c_1 in blue and yellow, U TrA in blue and yellow, c_2 in blue and yellow. After every observation of a star, the sky near that star was measured.

A large part of the readings of the registrations had been made by WOLTERBEEK MULLER, but I have checked and completed them. I am also responsible for the reduction and the discussion of the observations.

The blue and yellow magnitudes, relative to the mean magnitude of the comparison stars, have been derived in two decimals. The correction for differential extinction seldom exceeded .01 magnitude. Therefore this correction has not been applied. The heliocentric Julian Days have been computed in three decimals. The individual observations have been listed in Table 1. The phase φ in the second column was computed with the formula:

$$\varphi = {}^{d^{-1}}.389342 \text{ (J.D.hel.} - 2430000).$$

The meaning of phase ψ in the third column will be explained below. In the Figures 1a and 1b the yellow light-curve and the colour-curve are shown; the J.D. to which each curve refers, is indicated.

It is clear that both curves are strongly variable, especially near maximum. Figure 1a shows a remarkable analogy with Figure 1 in OOSTERHOFF's (1957b) article on TU Cas. Therefore it seems nearly certain that the complicated light-variation can be explained by the combination of a fundamental and a secondary period. To find the beat period and the corresponding phases ψ, I have constructed Figure 2, which shows ten harmonic curves, each resulting from two harmonics with the same frequency, with a ratio of the amplitudes of 2 to 1 and with phase differences increasing from .0, .1, to .9. The amplitude and the

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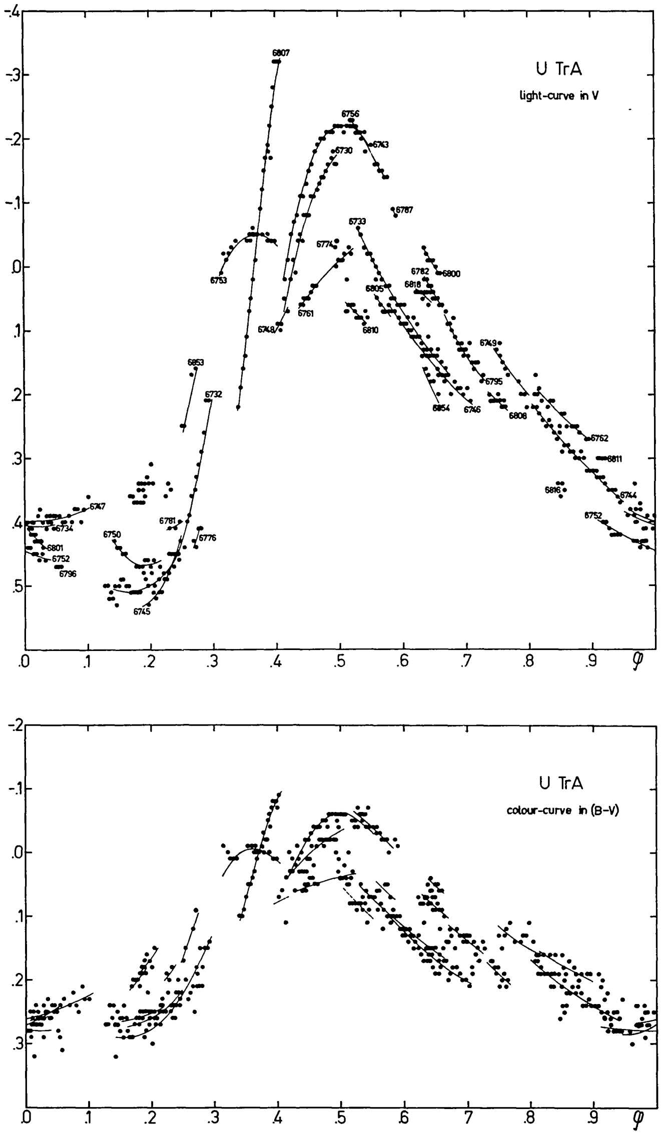
FIGURES 1a AND 1b



shift in phase of maximum or minimum can be read from the vector diagram given in Figure 3. It is clear that the phase differences indicated in Figure 2 can be identified with the phase ψ, in which the beat period is unity.

It proved to be easy to assign phases ψ to many of the line segments in Figure 1a. There is only one difficulty. In Figure 2 the range in magnitude is the same for maximum and minimum, while in the light-curve of U TrA the range of maximum is considerably larger than that of the minimum. This phenomenon has been explained by WALRAVEN (1955) for AI Vel in terms of magnitude- and phase-distortions. I could partly correct for these distortions by some tilting and shifting of diagram 2, when it was superposed on Figure 1a. I found a beat period of 6·31 days, while OOSTERHOFF had adopted a value of 6·98 days. A solution by least squares yielded the value: $P_b = 6^d.317 \pm .007$ (m.e.). The mean error of one assignment of ψ proved to be only $\pm .03$ or $\pm .04$; for ψ_0 (the phase at J.D. hel. 2436730.000) I found $.606 \pm .010$ (m.e.).

The combination of the fundamental period with this beat period gives a good representation of the observations in Table 1. Also the old observations

FIGURE 2

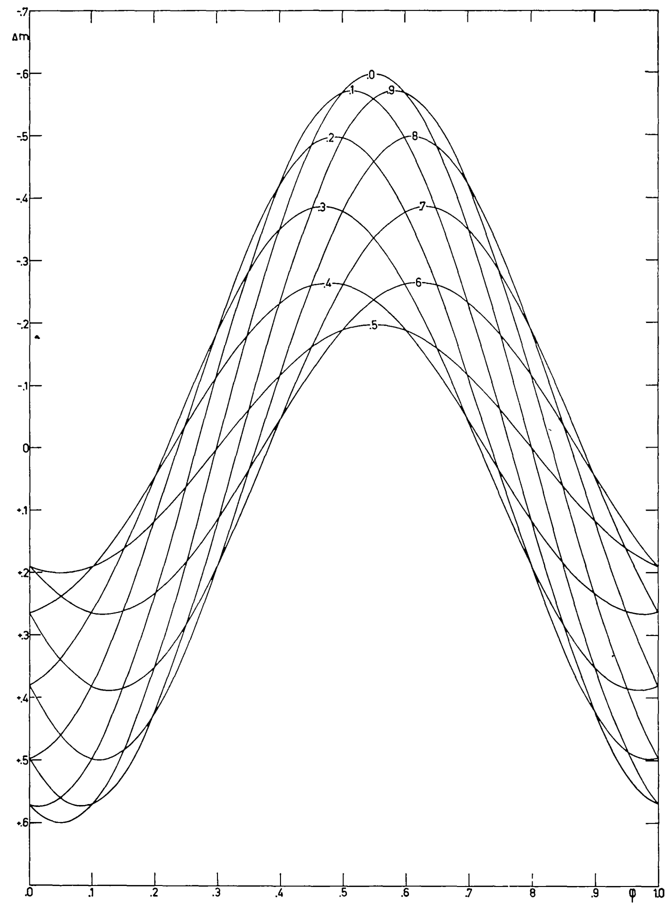


FIGURE 3

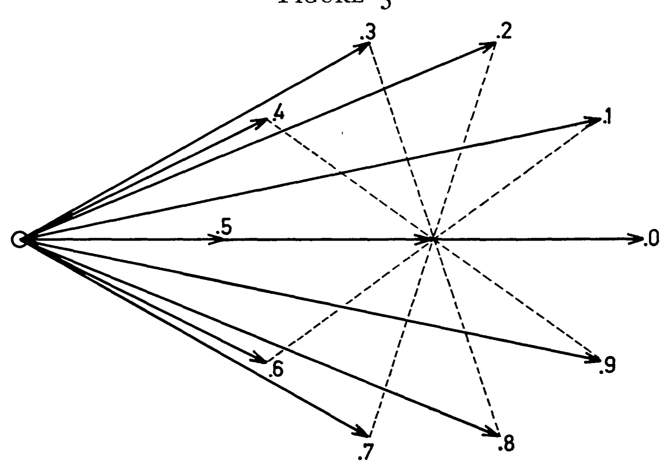
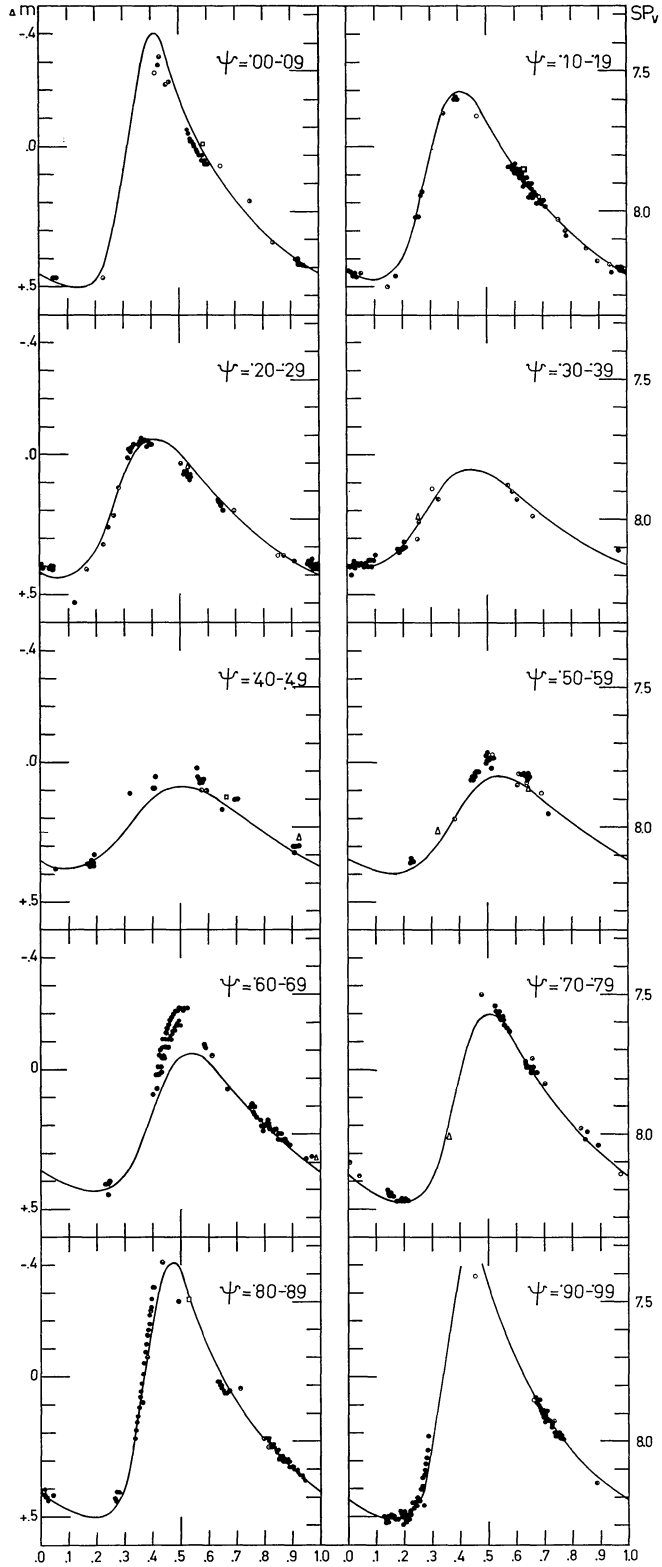
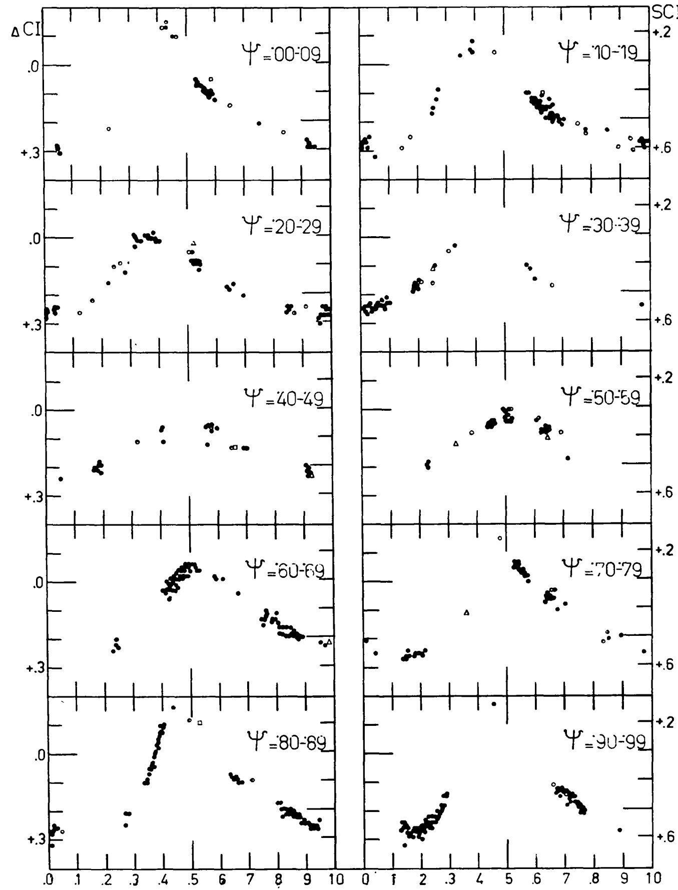


FIGURE 4a



△ 1953 ⊙ 1956
□ 1954 ● 1959
○ 1955

FIGURE 4b



from the years 1953 to 1956 given in OOSTERHOFF's paper (1957a), are represented reasonably well, but they show a phase shift of about .4 with respect to the new observations. From the combination of both sets of observations I derived:

$$P_b = 6^d.3041 \pm {}^d.0010 \text{ (estimated m.e.)}.$$

The values of ψ, computed with the formula:

$$\psi = {}^{d-1}.158626 \text{ (J.D. hel. } - 2430000)$$

for the new and the old observations are given in the third column of Tables 1 and 2 respectively. The range of the light-curve will be largest for $\psi = .947$.

In the Figures 4a and 4b we have shown the yellow magnitudes and the colours of Tables 1 and 2, arranged in ten equal intervals of the phase ψ. Before plotting the two sets of observations in the same diagram we have adjusted the zeropoints of the magnitudes and colours as well as we could, namely:

$$V = {}^m.00 \rightarrow SPv = 7^m.77 \text{ and } \Delta CI = {}^m.00 \rightarrow SCI = +{}^m.31.$$

These ψ-curves are not actual light-curves, as ψ changes by about .4 in one fundamental period. The fact that the phase φ of maximum brightness decreases slowly near maximum range and then increases rather rapidly near minimum range, proves that the secondary period is shorter than the fundamental period. A comparison of the results for U TrA and TU Cas yields:

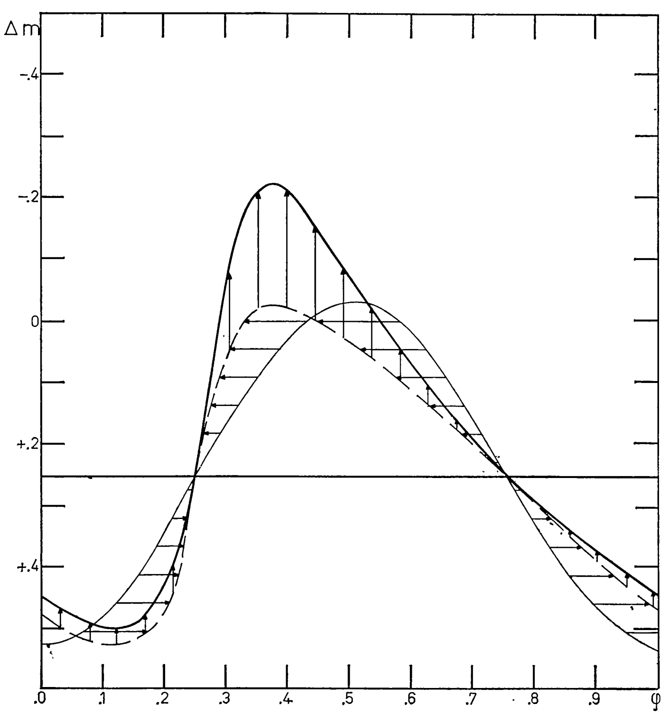
	U TrA	TU Cas
P_o	$2^d.568438$	$2^d.139295$
P_b	6.3041	5.23026
P_1	1.8249	1.5183
P_1/P_o	0·7105	0·7097

It is interesting to note how closely the ratio of fundamental period and secondary period is the same for these two Cepheids, which up to now are the only variables of this class for which a secondary period has been derived. On the other hand there is a remarkable difference with the dwarf Cepheids, where the ratio P_1/P_o, according to DETRE (1956), lies between ·77 and ·83.

Although the amount of information in Figure 4a is barely sufficient for such an investigation, I have tried to represent the light-variations of U TrA by a model of two harmonics, distorted in phase and in magnitude, as did WALRAVEN (1955) in the case of AI Vel, SX Phe and RR Lyr.

Figure 5 shows an example of these distortions; the thin line represents a harmonic curve, resulting from two harmonics with given amplitudes and a constant phase difference ($\varphi_1 - \varphi_o \equiv \psi$). The dashed line is obtained from this by shifting all points in phase φ proportionally to the instantaneous value of the harmonic. To get an observed ψ-curve, as shown by the thick line, all points of the dashed curve should be shifted upwards; this distortion in magnitude is a smooth function of the instantaneous value of the phase-distorted harmonic, as drawn in Figure 7.

FIGURE 5



From Figure 5 it will be clear that at both zero points the tangent to any observed ψ-curve coincides with that to the phase-distorted harmonic. Due to the phase-distortion, the tangent to the rising branch is much steeper than that to the falling one; the difference in slope yields the value of the phase distortion, while the amplitude of the harmonic is a simple function of the mean slope.

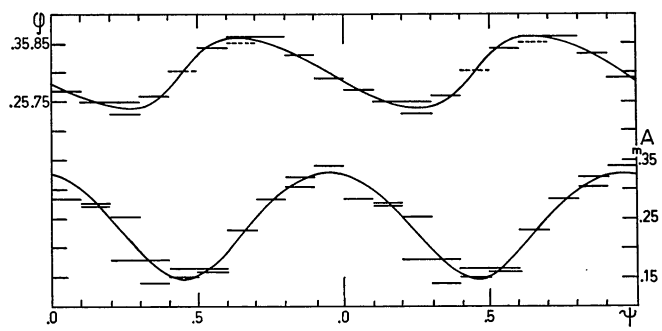
By definition the distortions vanish at the zero-points, so that the phases φ of the rising and the falling branch here differ by exactly .50. From four intervals of ψ in Figure 4a (ψ = .10 — .19, .20 — .29, .80 — .89 and .90 — .99) we derive for the zero point:

$$\Delta m = + 0^m.25\ (\pm .01) \text{ in } V.$$

Measurements of the slopes of the tangents at the zero points in as many ψ-curves as possible yield a phase distortion of 0.31 cycle/magnitude. The amplitudes A derived from the mean slopes of the tangents at the zero points are plotted as a function of ψ in the lower part of Figure 6. Interpreting this amplitude A as that of a resultant of two harmonics with amplitudes a and b and a phase difference ψ, we have

$$A^2 = a^2 + b^2 + 2ab \cos \psi.$$

FIGURE 6



From this equation we can find a and b; a least-squares solution yields:

$$a = 0^m.235 \quad \pm .005 \quad \text{in } V$$
$$b = 0^m.092 \quad \pm .007$$
$$b/a = 0.39 \quad \pm .03$$

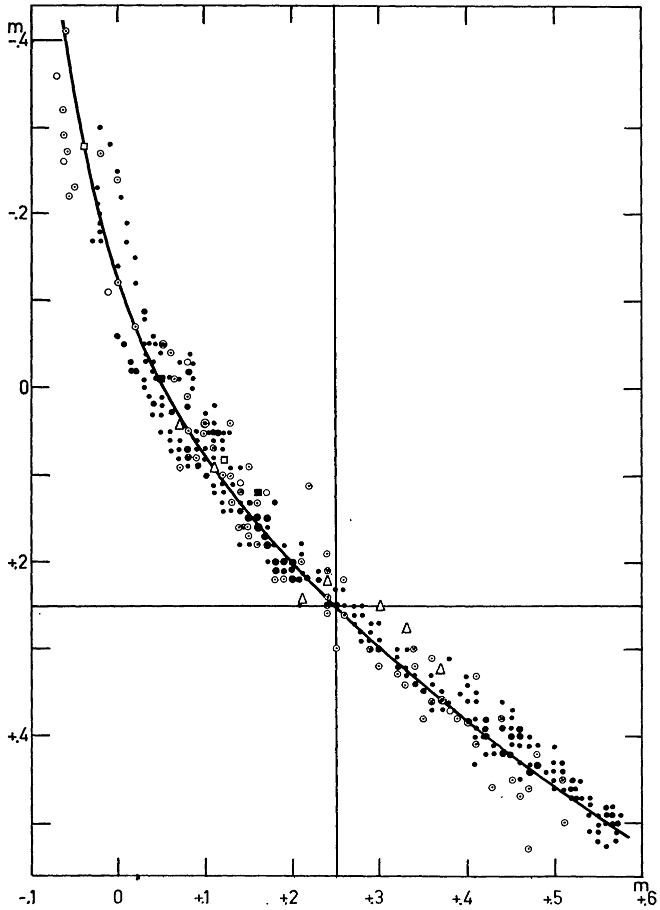
The amplitude A as a function of ψ, computed with these values for a and b, is drawn in the lower part of Figure 6. In the upper part of this figure the phases φ of the zero points are plotted against ψ. The curve in this figure is computed with the previous value of b/a, with the assumption that both for maximum and minimum range the zero points lie at φ = .30 and φ = .80.

Both the upper and the lower part of Figure 6 confirm that the range is a maximum for $\psi = .95$, as derived above.

The value of the phase distortion can also be derived from the displacement of the maxima and the minima of the observed curves, which lie vertically above the extremes of the phase-distorted curves, with regard to the points midway between the zero points. The relation between this displacement and the amplitude A yields a value of the phase distortion of 0.43 cycle/magnitude. The bad agreement of this figure with the previous one is surprising. Since the influence of this on the following calculation is small, I have adopted the mean value, 0.37 cycle/magnitude in V.

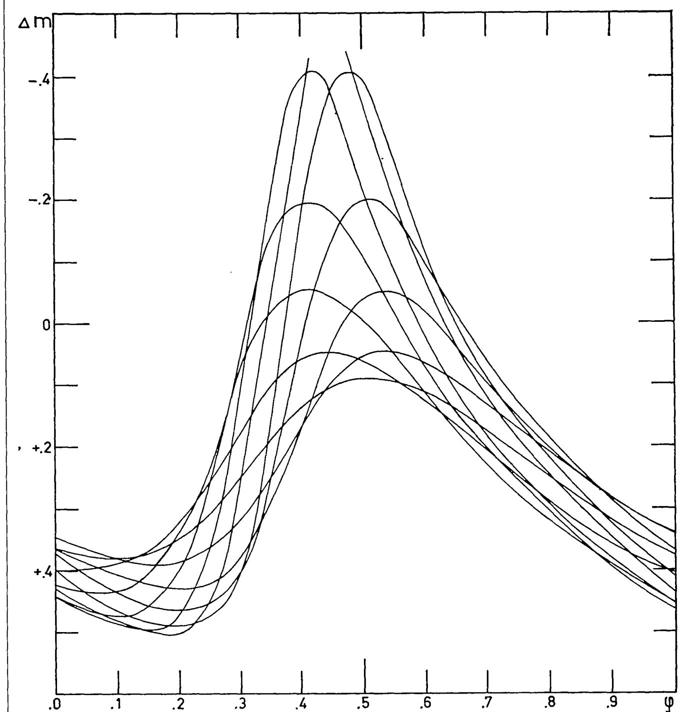
With the values obtained for the zero point, the amplitudes a and b, the phases φ of the zero point and ψ of maximum range and the adopted value for the phase distortion, I have computed phase-distorted harmonics. The observed magnitudes plotted against the values of the phase-distorted harmonics yield the magnitude-distortion curve (Figure 7).

FIGURE 7



Abscissae: Values of the phase-distorted harmonics.
Ordinates: Observed magnitudes (values of the phase- and magnitude-distorted harmonics).

FIGURE 8



The smooth free-hand curve, drawn through the points, was used to compute the 10 thin curves in Figure 4a, which are plotted again in Figure 8. If we take into account that the 10 diagrams in Figure 4a are for intervals in which ψ changes by .10, the agreement is as good as we could expect, except for $\psi = .60 - .69$, where the observations lie rather on a curve for $\psi = .75$. In all other cases the computed curves fit the observations reasonably well and the total impression of Figure 8 is almost identical to that of Figure 1a. However, the existence of a third pulsation is not out of the question.

There is no indication that the ratio b/a depends on the colour of the observations: we can represent the amplitude A as a function of ψ in blue light within the observational error simply by multiplying the magnitude scale of Figure 6 by a factor of about 1.4.

At the present time it is not possible to make a comparison with TU Cas, because the number of observations of the latter is as yet too small to derive the distortions and the b/a ratio.

It is a pleasure to thank Prof. OOSTERHOFF for his interest and advice and Mr J. TINBERGEN for reading this paper before it was printed.

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P. TH. OOSTERHOFF 1957b, *B.A.N.* **13**, 322 (No. 479).
TH. WALRAVEN 1955, *B.A.N.* **12**, 223 (No. 459).
L. DETRE 1956, *Mitt. Sternwarte Budapest*, No. 40.

TABLE I

J.D. hel. −243 0000	phase φ	phase ψ	Δm B	Δm V	ΔCI
			m	m	m
6730.379	0.419	0.61	+ 0.07	+ 0.07	0
.391	.424	.62	.03	+ 0.02	+ 0.01
.401	.428		+ 0.05	− 0.01	+ 0.06
.413	.432		0	+ 0.01	− 0.01
.422	.436		− 0.01	− 0.04	+ 0.03
.429	.439		.02	.05	.03
.439	.443		.01	.04	.03
.447	.446		.05	.08	.03
.454	.448	.62	.07	.08	.01
.461	.451	.63	.07	.08	+ 0.01
.468	.454		.09	.08	− 0.01
.475	.457		.09	.11	+ 0.02
.486	.461		.12	.11	− 0.01
.501	.467		.14	.12	.02
.509	.470		.15	.13	.02
.517	.473	.63	.15	.14	.01
.525	.476	.64	.16	.14	.02
.534	.480		.19	.15	.04
.543	.483		.18	.16	.02
.561	.490		.19	.17	.02
.566	.492		.20	.18	.02
.573	.495		.21	.16	.05
6730.580	0.497	0.64	− 0.20	− 0.16	− 0.04
6732.271	0.156	0.91	+ 0.78	+ 0.49	+ 0.29
.285	.161	.92	.78	.50	.28
.295	.165		.79	.50	.29
.306	.169		.80	.51	.29
.315	.173		.78	.51	.27
.323	.176		.77	.51	.26
.332	.180		.78	.51	.27
.342	.183	.92	.77	.51	.26
.355	.189	.93	.77	.48	.29
.365	.192		.77	.49	.28
.376	.196		.77	.50	.27
.388	.201		.78	.48	.30
.401	.206	.93	.77	.50	.27
.417	.213	.94	.76	.49	.27
.427	.217		.74	.51	.23
.439	.221		.73	.48	.25
.450	.226		.71	.49	.22
.462	.230		.70	.45	.25
.470	.233	.94	.71	.45	.26
.479	.237	.95	.73	.47	.26
.488	.240		.69	.45	.24
.499	.245		.70	.46	.24
.524	.254		.64	.44	.20
.533	.258	.95	.63	.40	.23
.542	.261	.96	.61	.39	.22
.552	.265		.57	.36	.21
.566	.271		.53	.35	.18
.573	.273		.52	.33	.19
.583	.277		.46	.31	.15
.592	.281	.96	.47	.29	.18
.602	.285	.97	.41	.26	.15
.614	.289		.36	.21	.15
6732.623	0.293	0.97	+ 0.35	+ 0.21	+ 0.14
6733.234	0.531	0.07	− 0.01	− 0.06	+ 0.05
.244	.535		+ 0.02	.05	.07
.258	.540		.03	.03	.06
6733.271	0.545	0.07	+ 0.05	− 0.02	+ 0.07
6733.283	0.550	0.07	+ 0.05	− 0.02	+ 0.07
.293	.554	.08	.06	− 0.01	.07
.306	.559		.09	0	.09
.317	.563		.09	+ 0.01	.08
.330	.568		.11	.02	.09
.344	.574	.08	.13	.03	.10
.357	.579	.09	.14	.03	.11
.369	.583		.14	.05	.09
.381	.588		.15	.05	.10
.393	.593		.16	.06	.10
.407	.598	.09	.18	.06	.12
.419	.603	.10	.18	.06	.12
.434	.609		.20	.08	.12
.447	.614		.23	.10	.13
.458	.618		.22	.08	.14
.474	.624	.10	.22	.09	.13
.493	.632	.11	.25	.14	.11
.511	.639		.28	.13	.15
.520	.642		.26	.11	.15
.530	.646		.28	.13	.15
.541	.650	.11	.29	.14	.15
.558	.657	.12	.25	.13	.12
.572	.662		.30	.15	.15
6733.587	0.668	0.12	+ 0.30	+ 0.16	+ 0.14
6734.329	0.957	0.24	+ 0.67	+ 0.39	+ 0.28
.340	.961		.68	.38	.30
.354	.967	.24	.66	.39	.27
.365	.971	.25	.67	.40	.27
.429	.996	.26	.66	.41	.25
.440	.000		.66	.39	.27
.449	.004		.68	.40	.28
.458	.007		.66	.41	.25
.467	.011	.26	.67	.41	.26
.528	.035	.27	.66	.41	.25
.537	.038		.66	.40	.26
.547	.042	.27	.64	.40	.24
6734.557	0.046	0.28	+ 0.65	+ 0.41	+ 0.24
6743.209	0.414	0.65	+ 0.06	+ 0.02	+ 0.04
.221	.419		+ 0.02	− 0.01	.03
.233	.424		− 0.02	.05	.03
.246	.429	.65	.05	.07	.02
.260	.434	.66	.09	.08	+ 0.01
.273	.439		.11	.11	0
.286	.444		.13	.11	− 0.02
.298	.449		.15	.13	.02
.308	.453	.66	.18	.15	.03
.321	.458	.67	.20	.16	.04
.333	.463		.22	.18	.04
.345	.467		.23	.19	.04
.358	.472		.25	.20	.05
.370	.477	.67	.25	.20	.05
.383	.482	.68	.27	.21	.06
.395	.487		.27	.21	.06
.407	.492		.27	.21	.06
.418	.496		.28	.22	.06
.430	.501	.68	.28	.22	.06
.441	.505	.69	.28	.22	.06
.453	.509		.27	.21	.06
6743.465	0.514	0.69	− 0.28	− 0.22	− 0.06

TABLE 1 (*continued*)

J.D. hel. −243 0000	phase φ	phase ψ	Δm B	Δm V	ΔCI
			m	m	m
6743.477	.519	0.69	− 0.27	− 0.22	− 0.05
.488	.523		.26	.22	.04
.500	.528	.69	.26	.22	.04
.513	.533	.70	.27	.21	.06
.524	.537		.26	.20	.06
.536	.542		.25	.21	.04
.547	.546		.23	.19	.04
6743.559	0.551	0.70	− 0.22	− 0.19	− 0.03
6744.221	0.808	0.81	+ 0.39	+ 0.22	+ 0.17
.232	.813		.41	.22	.19
.244	.817		.42	.23	.19
.259	.823	.81	.43	.24	.19
.272	.828	.82	.44	.25	.19
.283	.833		.45	.24	.21
.293	.837		.46	.25	.21
.305	.841		.46	.26	.20
.316	.845	.82	.46	.27	.19
.328	.850	.83	.47	.26	.21
.338	.854		.49	.29	.20
.348	.858		.50	.28	.22
.361	.863		.50	.28	.22
.372	.867		.50	.29	.21
.383	.872	.83	.51	.29	.22
.393	.875	.84	.51	.30	.21
.404	.880		.52	.30	.22
.415	.884		.53	.29	.24
.426	.888		.54	.30	.24
.437	.893		.55	.32	.23
.449	.897	.84	.56	.32	.24
.465	.903	.85	.56	.32	.24
.476	.908		.57	.33	.24
.489	.913		.58	.33	.25
.501	.918	.85	.59	.33	.26
.513	.922	.86	.59	.34	.25
.526	.927		.61	.35	.26
.539	.932		.60	.35	.25
.551	.937		.60	.35	.25
.561	.941		.62	.36	.26
6744.572	0.945	0.86	+ 0.60	+ 0.37	+ 0.23
6745.220	0.197	0.97	+ 0.79	+ 0.53	+ 0.26
.238	.204		.76	.51	.25
.248	.208		.78	.52	.26
.260	.213	.97	.76	.51	.25
.270	.217	.98	.75	.51	.24
.283	.222		.74	.49	.25
.301	.229		.72	.48	.24
.317	.235		.69	.45	.24
.327	.239	.98	.69	.45	.24
.337	.243	.99	.67	.45	.22
6745.348	0.247	0.99	+ 0.65	+ 0.43	+ 0.22
6746.201	0.579	0.12	+ 0.17	+ 0.07	+ 0.10
.211	.583	.12	.17	.07	.10
.223	.588	.13	.17	.07	.10
.240	.595		.20	.08	.12
.249	.598		.22	.09	.13
.260	.602		.22	.09	.13
.270	.606	.13	.23	.09	.14
.280	.610	.14	.23	.10	.13
.292	.615		.26	.11	.15
6746.301	0.618	0.14	+ 0.25	+ 0.11	+ 0.14
6746.312	0.623	0.14	+ 0.26	+ 0.11	+ 0.15
.322	.627		.27	.11	.16
.333	.631	.14	.29	.12	.17
.343	.635	.15	.28	.13	.15
.354	.639		.30	.14	.16
.366	.644		.30	.14	.16
.380	.649		.37	.18	.19
.394	.655	.15	.33	.14	.19
.404	.658	.16	.35	.17	.18
.415	.663		.34	.17	.17
.424	.666		.35	.18	.17
.433	.670		.37	.17	.20
.442	.673		.36	.17	.19
.452	.677	.16	.38	.20	.18
.468	.683	.17	.36	.18	.18
.479	.688		.38	.19	.19
.491	.692		.39	.19	.20
.503	.697		.40	.20	.20
.519	.703	.17	.40	.19	.21
6746.532	0.708	0.18	+ 0.40	+ 0.21	+ 0.19
6747.205	0.970	0.28	+ 0.62	+ 0.38	+ 0.24
.219	.976	.28	.64	.37	.27
.232	.981	.29	.65	.41	.24
.242	.985		.66	.41	.25
.251	.988	.29	.66	.39	.27
.311	.012	.30	.65	.40	.25
.323	.016		.67	.43	.24
.334	.021		.65	.39	.26
.343	.024	.30	.65	.38	.27
.357	.029	.31	.64	.40	.24
.372	.035		.62	.39	.23
.385	.040		.65	.39	.26
.396	.045		.64	.39	.25
.407	.049	.31	.64	.39	.25
.423	.055	.32	.63	.39	.24
.438	.061		.63	.40	.23
.449	.065		.64	.40	.24
.462	.070		.63	.39	.24
.475	.075	.32	.62	.40	.22
.486	.080	.33	.62	.38	.24
.498	.084		.63	.38	.25
.513	.090		.61	.40	.21
.523	.094	.33	.61	.38	.23
6747.542	0.101	0.34	+ 0.59	+ 0.36	+ 0.23
6748.317	0.403	0.46	+ 0.16	+ 0.09	+ 0.07
.330	.408		.15	.09	.06
6748.343	0.413	0.46	+ 0.16	+ 0.05	+ 0.11
6749.208	0.750	0.60	+ 0.26	+ 0.13	+ 0.13
.221	.755		.27	.12	.15
.232	.759	.60	.28	.15	.13
.242	.763	.61	.28	.16	.12
.253	.768		.28	.17	.11
.298	.785	.61	.32	.18	.14
.309	.789	.62	.33	.20	.13
.321	.794		.33	.20	.13
.332	.798		.33	.22	.11
.348	.805		.33	.19	.14
6749.366	0.812	0.62	+ 0.35	+ 0.17	+ 0.18

TABLE I (*continued*)

J.D. hel. −243 0000	phase φ	phase ψ	Δm B	Δm V	ΔCI
			m	m	m
6750.214	0.142	0.76	+ 0.70	+ 0.43	+ 0.27
.225	.146		.71	.44	.27
.238	.151	.76	.70	.44	.26
.250	.156	.77	.69	.45	.24
.261	.160		.71	.45	.26
.307	.178	.77	.73	.47	.26
.317	.182	.78	.72	.47	.25
.329	.187		.72	.47	.25
.341	.191		.71	.46	.25
.352	.196		.71	.46	.25
.364	.200	.78	.72	.47	.25
.377	.205	.79	.72	.46	.26
6750.402	0.215	0.79	+ 0.71	+ 0.47	+ 0.24
6752.208	0.918	0.08	+ 0.66	+ 0.40	+ 0.26
.220	.923		.68	.40	.28
.233	.928		.68	.41	.27
.244	.932		.70	.42	.28
.254	.936	.08	.70	.42	.28
.267	.941	.09	.70	.42	.28
.277	.945	.09	.70	.42	.28
.332	.966	.10	.70	.43	.27
.343	.971		.70	.43	.27
.356	.976		.70	.44	.26
.364	.979		.71	.43	.28
.371	.982		.72	.43	.29
.379	.985		.70	.43	.27
.387	.988	.10	.71	.44	.27
.430	.005	.11	.72	.44	.28
.441	.009		.73	.44	.29
.453	.014	.11	.72	.45	.27
.466	.019	.12	.71	.45	.26
.476	.023		.73	.46	.27
.489	.028		.74	.45	.29
6752.502	0.033	0.12	+ 0.71	+ 0.46	+ 0.25
6753.224	0.314	0.24	0 :	+ 0.01	− 0.01:
.235	.318		+ 0.01:	− 0.02:	+ 0.03:
.245	.322		− 0.01	.01	0
.255	.326		.01	.02	+ 0.01
.267	.330	.24	.02	.03	.01
.284	.337	.25	.03	.04	+ 0.01
.331	.355	.25	.05	.04	− 0.01
.342	.360	.26	.06	.05	− 0.01
.352	.364		.06	.06	0
.361	.367		.05	.05	0
.372	.371		.05	.05	0
.391	.379	.26	.05	.05	0
.402	.383	.27	.05	.03	− 0.02
.416	.388		.05	.05:	0 :
.431	.394		.03	.04	+ 0.01
6753.443	0.399	0.27	− 0.03	− 0.04	+ 0.01
6756.333	0.524	0.73	− 0.28	− 0.23	− 0.05
.341	.527		.28	.21	.07
.351	.531		.27	.21	.06
.361	.535	.73	.26	.21	.05
.370	.539	.74	.26	.20	.06
.380	.543		.25	.18	.07
.391	.547		.23	.19	.04
.401	.551		.22	.18	.04
.411	.555		.21	.18	.03
6756.420	0.558	0.74	− 0.20	− 0.16	− 0.04

J.D. hel. −243 0000	phase φ	phase ψ	Δm B	Δm V	ΔCI
			m	m	m
6756.429	0.562	0.75	− 0.19	− 0.16	− 0.03
.438	.565		.17	.15	.02
.449	.569		.17	.15	.02
.459	.573		.16	.14	− 0.02
6756.469	0.577	0.75	− 0.14	− 0.14	0
6761.253	0.440	0.51	+ 0.12	+ 0.06	+ 0.06
.262	.443		.11	.06	.05
.270	.446		.11	.05	.06
.277	.449	.51	.10	.05	.05
.285	.452	.52	.09	.05	.04
.294	.456		.08	.04	.04
.303	.459		.08	.03	.05
.314	.464	.52	.08	+ 0.03	.05
.399	.497	.53	.03	0	.03
.408	.500	.54	.03	− 0.01	.04
.415	.503		.03	.01	.04
.423	.506		.01	.01	.02
.432	.509		.02	− 0.02	+ 0.04
.440	.513		.02	+ 0.02	0
.448	.516		.01	− 0.03	+ 0.04
6761.456	0.519	0.54	+ 0.01	− 0.02	+ 0.03
6762.205	0.810	0.66	+ 0.36	+ 0.20	+ 0.16
.215	.814		.36	.20	.16
.223	.817	.66	.37	.19	.18
.231	.821	.67	.37	.21	.16
.259	.831		.38	.22	.16
.268	.835		.40	.21	.19
.279	.839	.67	.40	.21	.19
.306	.850	.68	.41	.25	.16
.314	.853		.41	.23	.18
.324	.857		.42	.23	.19
.334	.861	.68	.42	.25	.17
.358	.870	.69	.43	.25	.18
.365	.873		.44	.25	.19
.376	.877		.45	.25	.20
.385	.881		.45	.26	.19
.414	.892	.69	.46	.27	.19
6762.423	0.895	0.70	+ 0.46	+ 0.27	+ 0.19
6774.235	0.494	0.57	− 0.03	− 0.03	0
6774.246	0.498	0.57	− 0.03	− 0.04:	+ 0.01:
6776.222	0.268	0.88	+ 0.68	+ 0.43	+ 0.25
.233	.272	.89	.65	.44	.21
.242	.276		.62	.41	.21
6776.253	0.280	0.89	+ 0.62	+ 0.41	+ 0.21
6781.262	0.230	0.68	+ 0.65	+ 0.41	+ 0.24
.284	.239	.69	.63	.41	.22
.293	.242		.65:	.45:	.20:
6781.304	0.246	0.69	+ 0.63	+ 0.40	+ 0.23
6782.301	0.635	0.85	+ 0.09	+ 0.02	+ 0.07
.311	.639		.10	.02	.08
.320	.642		.11	.03	.08
.330	.646	.85	.12	.03	.09
.339	.649	.86	.13	.04	.09
.347	.653		.13	.05	.08
.356	.656		.14	.05	.09
.365	.660		.15	.06	.09
6782.375	0.663	0.86	+ 0.16	+ 0.06	+ 0.10

TABLE I (*continued*)

J.D. hel. −243 0000	phase φ	phase ψ	Δm B	Δm V	ΔCI
			m	m	m
6784.228	0.385	0.15	− 0.22	− 0.17	− 0.05
.238	.389	.16	.22	.18	.04
6784.249	0.393	0.16	− 0.25	− 0.17	− 0.08
6786.240	0.168	0.47	+ 0.57	+ 0.36	+ 0.21
.247	.171	.48	.56	.36	.20
.256	.174		.57	.37	.20
.265	.178		.57	.37	.20
.273	.181		.56	.35	.21
.281	.184		.56	.37	.19
.290	.188		.55	.37	.18
.298	.191		.55	.36	.19
6786.306	0.194	0.48	+ 0.55	+ 0.33	+ 0.22
6787.314	0.586	0.64	− 0.07	− 0.09	+ 0.02
6787.323	0.590	0.65	− 0.07	− 0.08	+ 0.01
6795.231	0.669	0.90	+ 0.21	+ 0.07	+ 0.14
.239	.672		.21	.09	.12
.247	.675		.22	.09	.13
.255	.678	.90	.22	.10	.12
.263	.681	.91	.23	.08	.15
.271	.684		.23	.11	.12
.279	.688		.25	.12	.13
.288	.691		.26	.13	.13
.296	.694		.26	.12	.14
.304	.697		.27	.14	.13
.311	.700		.27	.13	.14
.320	.703	.91	.27	.14	.13
.328	.707	.92	.30	.12	.18
.336	.710		.29	.15	.14
.344	.713		.32	.16	.16
.352	.716		.30	.15	.15
.361	.719		.32	.15	.17
.376	.725		.32	.18	.14
6795.384	0.728	0.92	+ 0.32	+ 0.17	+ 0.15
6796.213	0.051	0.06	+ 0.75	+ 0.47	+ 0.28
.221	.054		.76	.47	.29
6796.230	0.058	0.06	+ 0.78	+ 0.47	+ 0.31
6799.226	0.224	0.53	+ 0.55	+ 0.36	+ 0.19
.235	.228	.54	.54	.34	.20
6799.245	0.232	0.54	+ 0.53	+ 0.35	+ 0.18
6800.280	0.634	0.70	+ 0.04	− 0.03	+ 0.07
.289	.638		.03:	.02	.05:
.298	.642	.70	.03	.01	.04
.305	.644	.71	.05	.01	.06
.316	.649		.04	− 0.01	.05
.326	.653		.05	0	.05
.336	.656		.07	+ 0.01	.06
6800.346	0.660	0.71	+ 0.07	+ 0.01	+ 0.06
6801.243	0.010	0.85	+ 0.69	+ 0.42	+ 0.27
.253	.013	.86	.72	.40	.32
.262	.017		.69	.42	.27
.270	.020		.68	.43	.25
.277	.023		.69	.43	.26
.285	.026		.69	.43	.26
6801.293	0.029	0.86	+ 0.70	+ 0.44	+ 0.26
6804.254	0.182	0.33	+ 0.53	+ 0.34	+ 0.19
.262	.185		.52	.34	.18
.272	.189	.33	.52	.35	.17
.281	.192	.34	.50	.34	.16
.292	.197		.51	.34	.17
.302	.201		.49	.31	.18
6804.311	0.204	0.34	+ 0.49	+ 0.34	+ 0.15
6805.220	0.558	0.48	+ 0.08	+ 0.02	+ 0.06
.229	.561	.49	.11	.05	.06
.240	.566		.18	.06	.12
.250	.570		.12	.07	.05
.261	.574		.12	.07	.05
6805.273	0.579	0.49	+ 0.13	+ 0.06	+ 0.07
6807.228	0.340	0.80	+ 0.32	+ 0.22	+ 0.10
.240	.344	.81	.29	.19	.10
.247	.347		.25	.16	.09
.256	.351		.21	.14	.07
.265	.354		.16	.11	.05
.274	.358		.12	.07	.05
.283	.361		.09	.05	.04
.291	.364		+ 0.05	+ 0.02	.03
.301	.368	.81	0	− 0.01	+ 0.01
.309	.371	.82	− 0.05	.05	0
.320	.376		.10	.09	− 0.01
.329	.379		.15	.12	.03
.337	.382		.17	.15	.02
.344	.385		.22	.17	.05
.351	.388		.25	.19	.06
.359	.391	.82	.29	.22	.07
.369	.395	.83	.33	.25	.08
.376	.397		.36	.28	.08
.383	.400		.39	.32	.07
.389	.402		.41	.32	.09
6807.395	0.405	0.83	− 0.42	− 0.32	− 0.10
6808.251	0.738	0.97	+ 0.37	+ 0.20	+ 0.17
.257	.740		.38	.21	.17
.264	.743		.39	.21	.18
.271	.746		.40	.21	.19
.278	.749		.39	.21	.18
.286	.752		.41	.20	.21
.294	.755		.41	.21	.20
.301	.758		.41	.22	.19
.309	.761	.97	.42	.21	.21
6808.317	0.764	0.98	+ 0.42	+ 0.22	+ 0.20
6810.235	0.511	0.28	+ 0.12	+ 0.07	+ 0.05
.241	.513		.12	.07	.05
.249	.516		.14	.06	.08
.257	.519	.28	.14	.06	.08
.266	.523	.29	.14	.06	.08
.273	.525		.16	.07	.09
.280	.528		.16	.08	.08
.289	.532		.16	.08	.08
.297	.535		.17	.08	.09
.307	.539		.17	.09	.08
.316	.542		.18	.07	.11
6810.324	0.545	0.29	+ 0.17	+ 0.08	+ 0.09
6811.255	0.908	0.44	+ 0.51	+ 0.32	+ 0.19

TABLE 1 (*continued*)

J.D. hel. −243 0000	phase φ	phase ψ	Δm B	Δm V	ΔCI
			m	m	m
6811.264	0.911	0.44	+ 0.51	+ 0.30	+ 0.21
.271	.914	0.44	.50	.30	.20
.280	.917	.45	.53	.30	.23
6811.292	0.922	0.45	+ 0.52	+ 0.30	+ 0.22
6816.236	0.847	0.23	+ 0.60	+ 0.34	+ 0.26
.246	.851	.23	.60	.36	.24
.255	.854	.24	.59	.34	.25
6816.264	0.858	0.24	+ 0.59	+ 0.35	+ 0.24
6818.227	0.622	0.55	+ 0.11	+ 0.04	+ 0.07
.237	.626		.12	.04	.08
.246	.630		.12	.04	.08
.254	.633		.12	.05	.07
.262	.636	.55	.11	.04	.07
.271	.639	.56	.12	.04	.08
.279	.642		.12	.06	.06
.287	.645		.11	.04	.07
6818.296	0.649	0.56	+ 0.12	+ 0.05	+ 0.07
6827.228	0.127	0.98	+ 0.77	+ 0.50	+ 0.27
6827.239	0.131	0.98	+ 0.77	+ 0.50	+ 0.27
6827.247	0.134	0.98	+ 0.76	+ 0.52	+ 0.24
.255	.137		.77	.51	.26
.263	.140		.76	.52	.24
.271	.143		.82	.50	.32
.279	.146	0.98	.78	.53	.25
.287	.150	.99	.78	.50	.28
6827.296	0.153	0.99	+ 0.75	+ 0.49	+ 0.26
6842.234	0.969	0.36	+ 0.58	+ 0.34	+ 0.24
6849.234	0.694	0.47	+ 0.26	+ 0.13	+ 0.13
6849.245	0.699	0.47	+ 0.26	+ 0.13	+ 0.13
6853.230	0.250	0.10	+ 0.42	+ 0.25	+ 0.17
.239	.254	.10	.40	.25	.15
.270	.266	.11	.29	.17	.12
6853.284	0.271	0.11	+ 0.25	+ 0.16	+ 0.09
6854.224	0.637	0.26	+ 0.33	+ 0.16	+ 0.17
.234	.641		.34	.17	.17
.244	.645	.26	.36	.18	.18
6854.275	0.657	0.27	+ 0.36	+ 0.20	+ 0.16

TABLE 2

J.D. hel. −243 0000	phase φ	phase ψ	*SPg*	*SPv*	*SCI*
			m	m	m
4562.459	0.361	0.72	8.43	8.01	0.42
65.448	.525	.20	8.14	7.81	.33
74.340	.987	.61	8.61	8.09	.52
79.319	.926	.40	8.58	8.04	.54
80.343	.324	.56	8.44	8.01	.43
85.293	.252	.35	8.41	7.99	.42
4586.307	0.647	0.51	8.27	7.86	0.41
4907.398	0.661	0.44	8.33	7.89	0.44
17.479	.586	.04	8.12	7.76	.36
22.472	.530	.83	7.69	7.49	.20
4930.444	0.634	0.10	8.26	7.85	0.41
5251.540	0.650	0.03	8.29	7.84	0.45
5282.390	.662	.92	8.27	7.85	.42
5302.309	.417	.08	7.69	7.51	.18
15.284	.469	.14	7.93	7.66	.27
48.256	.306	.37	8.25	7.89	.36
5349.244	0.691	0.53	8.27	7.88	0.39
5567.552	0.687	0.16	8.44	7.95	0.49
73.622	.051	.12	8.85	8.22	.63
80.413	.695	.20	8.48	7.97	.51
5595.370	.518	.57	8.05	7.74	.31
5608.452	.612	.65	8.02	7.72	.30
09.377	.972	.79	8.70	8.14	.56
.464	.006	.81	8.76	8.18	.58
11.384	.753	.11	8.55	8.03	.52
.454	.780	.12	8.61	8.07	.54
12.336	.124	.26	8.87	8.30	.57
.450	.168	.28	8.71	8.18	.53
5612.556	0.209	0.30	8.57	8.10	0.47
16.322	.676	.89	8.23	7.82	.41
.395	.704	.91	8.38	7.93	.45
21.426	.663	.70	8.10	7.76	.34
.457	.675	.71	8.19	7.78	.41
22.322	.012	.85	8.77	8.18	.59
.416	.048	.86	8.77	8.19	.58
23.397	.430	.02	7.61	7.45	.16
.485	.465	.03	7.75	7.54	.21
24.302	.783	.16	8.64	8.09	.55
.487	.855	.19	8.67	8.13	.54
25.520	.257	.35	8.42	8.01	.41
26.346	.578	.48	8.24	7.87	.37
.395	.598	.49	8.24	7.87	.37
.502	.639	.51	8.21	7.84	.37
27.305	.952	.64	8.61	8.09	.52
.353	.970	.64	8.61	8.08	.53
28.368	.366	.81	8.21	7.86	.35
.408	.381	.81	7.97	7.70	.27
29.309	.732	.95	8.39	7.93	.46
30.377	.148	.12	8.87	8.27	.60
31.291	.504	.27	8.16	7.80	.36
.472	.574	.30	8.29	7.88	.41
34.257	.659	.74	8.07	7.73	.34
5636.236	.429	.05	7.66	7.48	.18
.306	0.456	0.06	7.76	7.55	0.21
5637.326	0.853	0.23	8.69:	8.13:	0.56:
.371	.871	.23	8.70	8.13	.57
.473	.911	.25	8.70	8.15	.55
38.347	.251	.39	8.54:	8.07:	.47: