

Substituting in these equations the numbers of Table 5:

$$X = +0''\cdot0048 \quad Y = -0''\cdot0114 \quad P \cos D = +0''\cdot0124$$

for the group  $\leq 8^m\cdot0$

$$\text{and } X = +0''\cdot0026 \quad Y = -0''\cdot0061 \quad P \cos D = +0''\cdot0066$$

for the group  $> 8^m\cdot0$ ,

the following values for  $Z$  are found:

$$Z = P \sin D = +0''\cdot0118, +0''\cdot0095, +0''\cdot0072$$

for the group  $\leq 8^m\cdot0$

$$Z = P \sin D = +0''\cdot0095, +0''\cdot0071, +0''\cdot0047$$

for the group  $> 8^m\cdot0$ ,

from which we find:

$P = 0''\cdot0171$	$0''\cdot0156$	$0''\cdot0143$	for the group	$\leq 8^m\cdot0$
$P = 0''\cdot0116$	$0''\cdot0097$	$0''\cdot0081$	" " "	$> 8^m\cdot0$
$D = 44^\circ$	$37^\circ\cdot5$	$30^\circ$	" " "	$\leq 8^m\cdot0$
$D = 55^\circ$	$47^\circ$	$35^\circ$	" " "	$> 8^m\cdot0$

Although the criterion is not very sharp, the values of  $P$  show that the ratio 1.5 is reached at a correction of nearly  $+0''\cdot009$ , in close agreement with the numbers found above.

In that case, however, the  $D$ 's do not even approximately become equal and it would seem as if other systematic errors are present, or the difference in the  $D$ 's is real. A correction of such an amount as to make the  $D$ 's of both groups equal, would largely overcompensate the ratio of the  $P$ 's.

### Further note on the causes of the discontinuous changes of the earth's rotation and their possible effect on the intensity of gravity, by *W. de Sitter*.

The fluctuations in the longitudes of the moon, the sun and planets show that discontinuous changes of the earth's moment of inertia referred to its axis of rotation to the extent of some hundred-millionths of its amount take place at irregular intervals. The moment of inertia is

$$C = \frac{2}{3} q M r_1^2 \left( 1 + \frac{2}{3} \varepsilon \right)$$

where  $M$  is the earth's mass,  $r_1$  the mean radius,  $\varepsilon$  the flattening and  $q$  is a factor of which the value is very nearly 0.50, and which depends on the inner constitution of the earth. We have (see *B. A. N.* 55, p. 99):

$$q = 1 - \frac{2}{3} \varepsilon + \frac{2}{3} J - \frac{2}{5} \left( 1 - \frac{2}{3} \varepsilon \right) \frac{\sqrt{1 + \eta_1}}{1 + \lambda_1}$$

Replacing  $J$  by  $H = \frac{C-A}{C}$ , by means of the relation

$J = qH$ , we find

$$\frac{dC}{C} = 2 \frac{dr_1}{r_1} + \frac{1}{3} dH + \frac{1}{3} d\varepsilon + d\Lambda,$$

where

$$d\Lambda = -\frac{2}{5} d \left[ \frac{\sqrt{1 + \eta_1}}{1 + \lambda_1} \right] = 0.50 d\lambda_1 - 0.16 d\eta_1$$

Similarly for the change of the acceleration of gravity  $g$  expressed in the same variables we find

$$\frac{dg}{g} = -2 \frac{dr_1}{r_1} + \left[ \frac{1}{2} dH - \frac{2}{3} d\varepsilon + 0.015 d\Lambda \right] (1 - 3 \sin^2 \varphi) - 0.0069 \left[ \frac{1}{2} \frac{dr_1}{r_1} + \frac{d\omega}{\omega} \right] \cos^2 \varphi,$$

$\varphi$  being the latitude. The second line is the change of the centrifugal force.

All these differential formulas have been derived on the assumption that the earth is approximately built up according to the theory of CLAIRAUT. By this same theory we have  $\eta_1 = 5J\varepsilon - 3$ , from which we would find

$$d\Lambda = \frac{1}{2} d\lambda_1 - 116 d\varepsilon + 119 dH.$$

But it is doubtful whether the approximation of the actual constitution of the earth to that according to hydrodynamic equilibrium is sufficiently close to warrant the application of this formula. Moreover it still contains the unknown correction  $d\lambda_1$ .

From these formulas we can draw two conclusions.

*a.* It is very well possible that the changes of the moment of inertia are brought about by relatively very small readjustments of the inner layers of the earth, without producing any change in the quantities  $r_1$ ,  $H$  and  $\varepsilon$ , or any effect which would be noticeable on the surface, and without excessive work being done against, or by, gravitation, as would be the case if the change of  $C$  were attributed to  $dr_1$  only.

*b.* The acceleration of gravity will be subjected to changes at the same time as the moment of inertia. There is, however, no relation between the changes of the two quantities, they need not even be of the same order of magnitude. If the change of  $C$  were brought about wholly by  $d\Lambda$ , that of  $g$  would be negligible.

If  $g$  were not changed, a perfectly undisturbed pendulum would keep 'Newtonian' or 'mathematical' time, and would thus change its rate relatively to 'astronomical' time, whenever the earth's rotation changes. The apparent change of daily rate in 1897, and again in 1918, would have been  $0^{\circ}.0034$ . If  $g$  also changed, the apparent change of rate would be different.

These small quantities are acquiring a practical interest in view of the wonderful performance of the free pendulum clocks at Greenwich and elsewhere.

After *B. A. N.* 124 was in print, I found that the explanation of the large value of  $Q_s$  by a positive secular variation of the moon's excentricity is not new, but has already been suggested by COWELL in 1906 (*M. N.* lxvi, p. 352).

It may be remarked that the ratio of the secular increase of the excentricity to that of the mean distance, defined by the factor  $f \left( ede = \frac{1}{2} f \frac{da}{a} \right)$ , depends on the forces acting between the moon and the small secondary tidal waves in the shallow seas. It may therefore be variable, and if so  $Q_s$ , and consequently also the resulting apparent  $Q$ , will change discontinuously at the same epochs as the coefficient of tidal friction (1745 and 1870, as adopted in *B. A. N.* 124 and 127). There is perhaps some indication of this in the transits of Mercury.

I am indebted to Dr. FOTHERINGHAM for pointing out to me that BROWN's ' $T\frac{1}{2}$ ' in his paper in the *Yale Transactions* is:

$$\begin{aligned} \text{BROWN's tables} & - 2''\cdot71 + 2''\cdot02 (T + 1) \\ & - 10''\cdot71 \sin (140^{\circ}\cdot0 T + 240^{\circ}\cdot7) \end{aligned}$$

In my paper *B. A. N.* 124 p. 24 I had omitted the constant and linear terms. Adopting the corrected value as given here, the final correction to BROWN's tables given in *B. A. N.* 127, p. 50 becomes

$$\begin{aligned} \Delta L = & + 6''\cdot00 (T + 1) - 10''\cdot71 \sin (140^{\circ}\cdot0 T + 240^{\circ}\cdot7) \\ & + 5\cdot03 S + 0\cdot549 [(A) + 0\cdot23 (B)']. \end{aligned}$$

Since my values of  $B'$  in *B. A. N.* 124, p. 25, and the residuals in *B. A. N.* 127, were all computed by means of the correction  $+ 0''\cdot66 + 0''\cdot79 T$  to BROWN's adopted values, they are not affected, with the exception of the last four years. The last three values of  $B'$  in Table 1 of *B. A. N.* 124, p. 25 must be altered to  $- 11\cdot82$ ,  $(- 12\cdot21)$ ,  $(- 12\cdot20)$ .

The residuals for the meridian observations of the moon, as given in the Table *B. A. N.* 127, p. 50, for the years 1924\cdot0, and '26\cdot0 now become  $- 0''\cdot12$  and  $+ 0''\cdot13$  respectively.

In the equations of condition for the secular accelerations we must now use  $\Delta n = + 6\cdot00$  instead of  $+ 4\cdot00$ . The residuals with  $\kappa = + 0\cdot65$ ,  $\kappa' = - 0\cdot15$  then become  $+ \cdot06$ ,  $+ \cdot07$ ,  $+ \cdot03$ ,  $- \cdot34$  and  $\cdot00$ , or practically the same as before. No new solution is necessary.

## ERRATA.

*B. A. N.* 124, p. 34, first column, line 12 from bottom,  $\Delta\tau$  : for  $+ \cdot00075$  read  $+ \cdot00089$   
 127, p. 50, ,, ,, ,,  $\Delta t$  by cause (B), 1745—1870: ,,  $- 23\cdot3 S$  ,,  $+ 23\cdot3 S$ .