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ARE MOLECULAR CLOUDS IN VIRIAL EQUILIBRIUM?

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ABSTRACT

A correlation between CO luminosities and virial theorem masses of Galactic molecular clouds has been claimed to prove conclusively that molecular clouds are in gravitational virial equilibrium. This correlation is an artifact of the size–line width relation exhibited by molecular clouds, which does not require clouds to be virialized. Virial theorem masses do not provide an estimate of $N(H_2)/I_{CO}$. Comparison of the virial theorem–derived values of $N(H_2)/I_{CO}$ with the γ-ray value shows that the masses of lower luminosity clouds are considerably overestimated by virial theorem analyses; these clouds are probably strongly influenced by nongravitational forces.

Subject headings: galaxies: The Galaxy — interstellar: molecules

I. INTRODUCTION

A major fraction of the mass of the interstellar medium is in the form of molecular gas. Studies of this gas through emission in the $J = 1\text{–}0$ line of the CO molecule have shown that it is distributed throughout the Galaxy in discrete clouds (e.g., Solomon and Sanders 1985, and references therein). It is well established that molecular clouds are the active star-forming components of the interstellar medium. The properties of molecular clouds are thus of great importance for an understanding of the structure of the interstellar medium and the processes involved in star formation.

If it is *assumed* that the gas motions of a cloud are confined solely by its gravitational attraction, the velocity dispersion and size of the cloud can be used as a measure of its gravitational potential. If further assumptions are made about the mass distribution, an estimate of the cloud mass can be made based upon the extent of observable CO emission and the velocity dispersion. This is the sense in which "virial theorem masses," M_{VT}, are conventionally extracted from molecular line data (see MacLaren, Richardson, and Wolfendale 1988). More generally, virial equilibrium may involve other, nongravitational forces (e.g., magnetic fields; the pressure in the intercloud medium). Throughout this *Letter* I use the term "gravitational virial equilibrium" to distinguish the case where only the kinetic and gravitational potential energy terms in the virial theorem are important from more general virial equilibrium: the term M_{VT} refers to masses calculated assuming gravitational virial equilibrium.

It has been found in recent CO surveys (Solomon *et al.* 1987; Scoville *et al.* 1987) that cloud CO line luminosities are correlated with M_{VT}; this correlation has been cited as proof that molecular clouds are in gravitational virial equilibrium. If correct, this would be an important statement about the internal dynamical state of molecular clouds with fundamental consequences for their origin and evolution. In fact, it can be shown that the correlation between M_{VT} and CO luminosity is solely an artifact of an empirical correlation between size and line width, which does *not* require molecular clouds to be in gravitational virial equilibrium.

In § II of this *Letter* I derive the relation that is expected between CO luminosities L_{CO} and virial theorem masses M_{VT} of molecular clouds, regardless of their actual masses and dynamical state, when the empirical size–line width dependence is taken into account; this relation agrees precisely with that found by Solomon *et al.* (1987). In § III the implications of this result for molecular cloud masses and dynamics are discussed.

II. CO LUMINOSITIES AND VIRIAL THEOREM MASSES

Solomon *et al.* (1987) calculate their virial theorem masses assuming that only the self-gravity of the clouds is important, from

$$M_{VT} = 3f_p \frac{S\sigma_v^2}{G} M_\odot , \tag{1}$$

where S is the cloud "size," defined as the geometric mean of the spatial dispersions observed for a cloud in Galactic latitude b and longitude l; σ_v is the line-of-sight velocity dispersion averaged over the cloud; $G = 1/232$ in units of km s^{-1}, parsecs, and solar masses; and f_p is a "projection factor" that depends on the density distribution within the cloud. Solomon *et al.* (1987) take $f_p = 2.9$.[1]

The existence of a relation between line width and size for molecular clouds was first suggested by Larson (1981) and has since been found in a number of studies of both giant molecular clouds and dark clouds (Leung, Kutner, and Mead 1982; Myers 1983; Dame *et al.* 1986). For their sample of 273 Galactic molecular clouds, Solomon *et al.* find a correlation between line-of-sight velocity dispersion and cloud size S such that

$$\sigma_v = S^{0.5}\ \text{km s}^{-1} \tag{2}$$

with S in parsecs and σ_v in km s^{-1}. The gravitational virial mass (eq. [1]) is then

$$M_{VT} = 2000\sigma_v^4\ M_\odot . \tag{3}$$

The CO luminosity of a cloud which obeys equation (2) can be estimated as follows: the luminosity in the $J = 1\text{–}0$ line of a spherical cloud of radius R is $L_{CO} = \pi R^2 (2\pi)^{1/2} \sigma_v \bar{T}_A$, where $\bar{T}_A$ is the observable antenna temperature averaged over the cloud and a Gaussian line profile has been assumed. In order to use equation (2) in this expression, it is necessary to relate the

[1] The f_p value of Solomon *et al.* is equal to the product of the correction coefficient 1.5 for an r^{-1} density distribution (k_1 in MacLaren, Richardson, and Wolfendale 1988) and a factor $3.4/\pi^{1/2}$ to correct from the size S to an effective radius R_e; see discussion following eq. (3).

empirical cloud size S to the radius R. By defining an effective cloud radius R_e via $\pi R_e^2 = \Delta l \Delta b$, where Δl and Δb are the extent of a cloud projected in l and b, Solomon *et al.* find that on average $R_e = (3.4/\pi^{1/2})S$. Then the cloud area $A = 11.6S^2$, and so the CO luminosity is

$$L_{\rm CO} = 11.6\sqrt{2\pi}\, S^2 \bar{T}_A \sigma_v = 29\sigma_v^5 \bar{T}_A \,. \tag{4}$$

Equation (4) can be used to express σ_v in terms of $L_{\rm CO}$ and $\bar{T}_A$; the resulting expression in equation (3) gives the result

$$M_{\rm VT} = 135\, L_{\rm CO}^{0.8} \bar{T}_A^{-0.8} \,. \tag{5}$$

For a cloud-averaged temperature $\bar{T}_A = 4$ K, which is the typical value found by Solomon *et al.*, the predicted relation between calculated virial mass and CO luminosity is

$$M_{\rm VT} = 44\, L_{\rm CO}^{0.8} \,. \tag{6}$$

Nowhere in this derivation have I assumed that molecular clouds actually are in gravitational virial equilibrium; equation (6) follows solely from the observed size–line width relation (2), which does not require that clouds are virialized (see § III). Equation (6) should be compared with the "empirical" relation

$$M_{\rm VT} = 39 L_{\rm CO}^{0.81 \pm 0.03} \tag{7}$$

found by Solomon *et al.* (1987). The essentially exact agreement between equations (6) and (7), especially in view of the crude approximation to the CO luminosity used in deriving (6), shows that the relation between $M_{\rm VT}$ and $L_{\rm CO}$ found by Solomon *et al.* (1987) does not prove that molecular clouds are in gravitational virial equilibrium but is simply the expected relation for an ensemble of clouds which obey equation (2) and have similar cloud-averaged temperatures, *regardless* of their actual masses or dynamical states. Although Solomon *et al.* comment on the extremely tight correlation between $M_{\rm VT}$ and $L_{\rm CO}$ as shown in their Figure 2, this plot is in fact merely their Figure 1, the size–line width relation for the clouds in their sample, plotted in a different way: the relation merely looks tighter than the size–line width relation because the dynamic range of the plot has been increased by four orders of magnitude. The actual dispersion (in σ_v at a given cloud size, or $M_{\rm VT}$ at a fixed cloud luminosity) is identical for both relations.

Solomon *et al.* (1987) derive an expression analogous to equation (7) in the same manner as I have done here; however, they incorrectly conclude that the agreement between their derived expression and the empirical correlation between $L_{\rm CO}$ and $M_{\rm VT}$, in particular the derived exponent of 0.8, is a direct consequence of observing clouds in gravitational virial equilibrium which also obey the size–line width relation (2).

Scoville *et al.* (1987) find a somewhat different form for the relation between virial theorem mass and CO luminosity for their cloud sample: $M_{\rm VT} = (7.9 \pm 1.5)L_{\rm CO}$. Since they find a scaling of line width with size that is virtually identical to that found by Solomon *et al.* (1987), namely $\sigma_v = 0.31D^{0.55}$, where D is the geometric mean of the full extents of a cloud in l and b, they should obtain a nearly identical relation between $M_{\rm VT}$ and $L_{\rm CO}$. A derivation like the above using the equations for virial theorem mass and CO luminosity given by Scoville *et al.*[2] shows that the expected relation is $M_{\rm VT} = 161\bar{T}_A^{-0.82}L_{\rm CO}^{0.82}$; for $\bar{T}_A = 5$ K, the coefficient is 43. The reason for the discrepancy appears to be the narrow range of masses used in their fit ($M_{\rm VT} = 10^5$–$2 \times 10^6\ M_\odot$). Figure 1 shows the derived masses and CO luminosities for all clouds in the sample of Scoville *et al.* which have small distance ambiguities (either near the tangent point or associated with radio H II regions). Also shown in Figure 1 are the expected $M_{\rm VT}/L_{\rm CO}$ relation and the Scoville *et al.* relation. The expected relation fits the data as well as or better than $M_{\rm VT} = 7.9L_{\rm CO}$, especially in the lower luminosity range where most of the clouds are. Thus there is not in fact any meaningful discrepancy between the analyses of Solomon *et al.* (1987) and Scoville *et al.* (1987): both show the scaling expected as a result of the size-line width relation.

III. IMPLICATIONS FOR CLOUD MASSES AND DYNAMICS

a) Mass Determinations

The arguments of § II show that *correlations between CO luminosities and virial theorem masses do not prove that molecu-*

[2] For most of their sample, Scoville *et al.* calculate $L_{\rm CO}$ from D, $\bar{T}_A$, and σ_v, rather than integrating the observed intensities over the clouds.

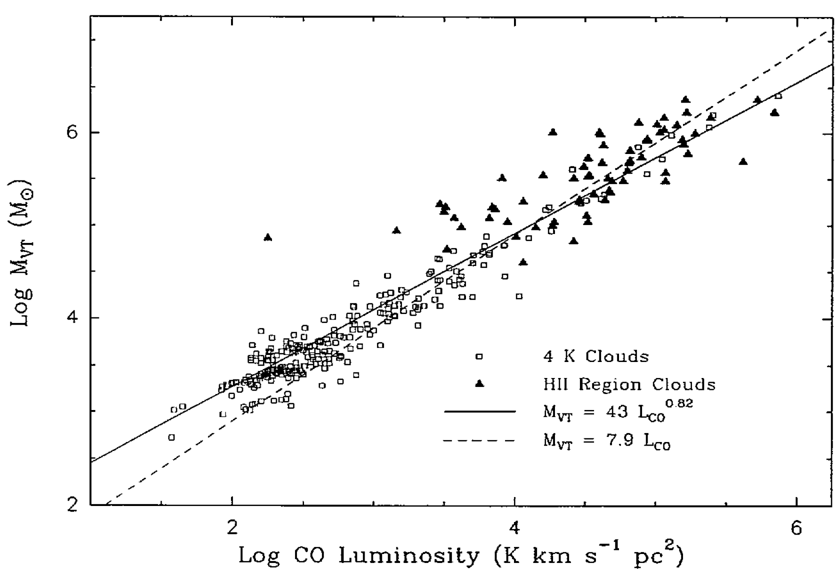

FIG. 1.—Virial theorem mass $M_{\rm VT}$ plotted against CO luminosity $L_{\rm CO}$ for clouds in the sample of Scoville *et al.* (1987). See text for details.

lar clouds are in gravitational virial equilibrium, nor do they provide an estimate of $N(H_2)/I_{CO}$. What then can be said about the masses of molecular clouds?

Although the correlation between the two combinations of observable parameters symbolized by M_{VT} and L_{CO} must be interpreted with caution, it is useful to compare the implied value of $N(H_2)/I_{CO}$, assuming that the gravitational virial masses are correct, with the large-scale average value obtained from comparison of the *COS B* γ-ray data with CO surveys (Bloemen *et al.* 1986; Bhat, Mayer, and Wolfendale 1986, hereafter BMW). Because M_{VT} scales as $L_{CO}^{0.8}$, the derived value of $N(H_2)/I_{CO}$ (corrected for helium) is a function of cloud virial mass: for $M_{VT} = 10^6\ M_\odot$, $N(H_2)/I_{CO} = 1.8 \times 10^{20}$, while for $M_{VT} = 10^5\ M_\odot$, $N(H_2)/I_{CO} = 3.0 \times 10^{20}$. The current best estimates of the γ-ray conversion factor are $N(N_2)/I_{CO} \lesssim 2.3 \times 10^{20}$ (Bloemen 1989) and $N(H_2)/I_{CO} \simeq 1.5 \times 10^{20}$ (BMW). For various reasons the γ-ray values are upper limits (see Bloemen 1989). It is apparent that the virial theorem–derived value of $N(H_2)/I_{CO}$ agrees well with the γ-ray values for clouds with $M_{VT} \gtrsim 10^5\ M_\odot$, which supports the hypothesis of gravitational virial equilibrium for these clouds; this argument was advanced by Solomon *et al.* (1987). In this usage "gravitational virial equilibrium" and "self-gravitating" are indistinguishable; the largest clouds almost certainly do not survive long enough to virialize.

For the large clouds, however, even if they are self-gravitating, there may be difficulties in deriving masses from the virial theorem: since the large clouds tend to be actively star-forming, the injection of energy from stellar winds and radiation (and possibly from supernovae) may result in observed line widths that do not accurately reflect the cloud mass (see also MacLaren, Richardson, and Wolfendale 1988). Scoville *et al.* (1987) find no difference in the ratio M_{VT}/L_{CO} between cold clouds and clouds associated with H II regions in their sample, even though the H II region clouds tend to have higher CO temperatures. This is probably a result of the effect of star formation on the H II region clouds: they have systematically higher velocity dispersions at a given size for $D \lesssim 30$ pc, and so both L_{CO} and M_{VT} (which overestimate the cloud mass) are larger for these clouds. The energy injected into molecular clouds by star formation may be substantial. The power input from OB stars in the form of stellar winds, photons, and supernovae is approximately $6 \times 10^{35} N_*$ ergs s^{-1} (McCray and Kafatos 1987), where N_* is the number of stars in an association more massive than $7\ M_\odot$. For a Salt-peter initial mass function with upper and lower mass limits of 60 and 0.3 $M_\odot$, the total energy input is then $E_{OB} \sim 3 \times 10^{51}(\epsilon/10^{-2})(M_c/10^5\ M_\odot)(\Delta t/10^7\ \text{yr})$, where ϵ is the efficiency of star formation, M_c is the cloud mass, and Δt is the duration of star formation in the cloud. Comparing this with the cloud binding energy, $E_B \sim 7 \times 10^{49}(M_c/10^5\ M_\odot)^{5/3}$ for a cloud with mean H_2 number density of 200 cm^{-3}, we see that $P_{OB}/E_B \sim 43(\epsilon/10^{-2})(M_c/10^5\ M_\odot)^{-2/3}(\Delta t/10^7\ \text{yr})$; only a few percent of the available power needs to be transmitted to a star-forming cloud to disperse it. Thus it is quite likely that many clouds have been substantially affected by star formation and are not in virial equilibrium. It is also possible that what are identified as the most massive clouds in the CO surveys, the spiral arm clouds (e.g., Stark 1979), may not be self-gravitating objects. Roberts and Stewart (1987) have pointed out that the effects of orbit crowding in a spiral potential may produce assemblages of clouds which are not gravitationally bound but which are observationally indistinguishable from a single giant molecular cloud (see also Adler and Roberts 1988).

It is very important to note that the value of $N(H_2)/I_{CO}$ implied by the virial theorem mass estimates increases with decreasing mass (as $M_{VT}^{-0.25}$): the inferred value of $N(H_2)/I_{CO}$ is larger than the γ-ray values by a factor of 2–3 at $M_{VT} = 10^4\ M_\odot$ and by a factor of 6–10 at $M_{VT} = 10^2\ M_\odot$. These numbers disagree strongly with the actual values of $N(H_2)/I_{CO}$ that have been estimated for dark clouds in this mass range from ^{13}CO–derived mass estimates for these objects; the values of $N(H_2)/I_{CO}$ for dark clouds are typically equal to or smaller than the γ-ray conversion factor (Dickman 1978; Maloney 1988). Furthermore, virial theorem mass estimates give grossly erroneous results for the high-latitude clouds (Magnani, Blitz, and Mundy 1985; Keto and Myers 1986). The actual masses of these clouds as estimated from LTE analysis of ^{13}CO emission or from extinction maps are one or two orders of magnitude smaller than the masses calculated assuming gravitational virial equilibrium. These clouds are not dominated by self-gravity and are either confined by the pressure of the interstellar medium (Keto and Myers 1986; Maloney 1988) or else produced by interstellar shocks and short-lived (Blitz 1988).

Direct estimate of the contribution from such a diffuse, pressure-confined molecular cloud component to the observed Galactic CO emission is difficult to make. Polk *et al.* (1988), using the Bell Labs CO survey, have recently compared the large-scale average value of the ratio of ^{12}CO to ^{13}CO integrated intensities (I_{12}/I_{13}) in the Galactic plane (≈ 6.7) with the values found for the dense cores of molecular clouds (≈ 3); this result suggests that a component of molecular gas exists which has $I_{12}/I_{13} \gtrsim 10$. Polk *et al.* identify this component with small, diffuse molecular clouds and suggest that half of the observed Galactic CO emission may be produced by these objects.

The systematic overestimate of cloud masses with decreasing cloud mass that results from the gravitational virial equilibrium analyses strongly suggests that the lower mass molecular clouds ($M_{VT} \lesssim 10^4\ M_\odot$) are not generally in gravitational virial equilibrium, but instead are strongly affected by non-gravitational forces, such as the pressure in the interstellar medium and magnetic fields. Such clouds may contribute a significant fraction of the Galactic CO emission. This is in sharp contrast to the conclusions of Solomon *et al.* (1987), who conclude that molecular clouds are completely dominated by self-gravity.

b) Interpreting the Size–line Width Relation

A relation between velocity dispersion and cloud size of the form $\sigma_v \propto S^{0.5}$ is *not* a necessary consequence of gravitational virial equilibrium: it is expected for virialized clouds only if they have constant mass surface densities (i.e., mean density $\bar{n} \propto S^{-1}$). There is no *a priori* reason to expect molecular clouds to show such a scaling. It has been suggested observationally that a density-size relation of the form $\bar{n} \propto S^{-1}$ does hold for molecular clouds (Myers 1983; Dame *et al.* 1986), but this result may be seriously influenced by observational effects (Maloney and Burton 1990). If gravitational virial equilibrium is assumed, then one of these relations follows from the other, but in that case either the size–line width or the size-density relation must be adopted without any physical basis.

In contrast, scaling relations of the form $\sigma_v \propto S^{0.5}$ and $\bar{n} \propto S^{-1}$ are predicted by models of clouds which are in pressure equilibrium with the intercloud medium (Chieze 1987; Elmegreen 1985; Maloney 1988). The fact that this same size–line width relation is seen over the entire size range of clouds in the samples of Solomon *et al.* (1987) and Scoville *et al.* (1987) suggests that the pressure of the intercloud medium (which may be dominated by magnetic fields) is important, even for the most massive molecular clouds.

In summary, the observed correlation between CO luminosities and virial theorem masses is an artifact of the size–line width relation found for molecular clouds. Comparison of the implied value of $N(H_2)/I_{CO}$ from the virial theorem masses with the γ-ray–derived value suggests that while the more massive molecular clouds are probably self-gravitating, clouds with $M_{VT} \lesssim 10^4\ M_\odot$ are strongly influenced by nongravitational forces, and gravitational virial equilibrium analyses will overestimate their masses considerably. These clouds may contribute a significant fraction of the CO emission in the Galaxy. At all mass scales, energy injection by OB stars may rapidly produce nonvirialized clouds.

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