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COMMUNICATION FROM THE OBSERVATORY AT LEIDEN.

The variation of the period of RZ Cassiopeiae, by *A. de Sitter*.

The variability of the period of the Algol star RZ Cassiopeiae was announced by K. GRAFF (1) [1]), and confirmed by R. S. DUGAN, who discussed all the material available in 1916 including observations of his own (2). DUGAN's ephemeris contains a sine term having a period of about 17 years. In the present paper a discussion is given of all observations now accessible, including plates taken at Leiden in the years 1925 to 1932.

An application is given of the hypothesis mentioned by HERTZSPRUNG (30) and further worked out by WOLTJER (26) where the change in the period is interpreted as a change of light time caused by the motion of the eclipsing binary round a third body. As shown in fig. 3 the suggestion of periodicity is now much strengthened, and the case is the more interesting as the curve is asymmetrical and thus gives evidence of a sensible excentricity of the hypothetical orbit. Elements are derived by a least squares solution and they are in satisfactory agreement with the observations (see fig. 3). The mass function is reasonably small. The excentricity is 0·40 ± ·12 (m.e.) the projection of the semi-major axis on the line of sight 0·98 a.u. ± ·08 and the period 18·00 years ± ·22.

1. Before giving a general discussion of all available material we will describe in some detail the following hitherto unpublished Leiden photographic observations:

Interval	Instrument	number of minima	number of observations	plates taken by	plates measured by
a) 1925 Sept. 14—1925 Dec. 7	33 cm refr.	4	168	H. VAN GENT	C. J. KOOREMAN
b) 1924 Sept. 10—1932 Aug. 9	Double camera	10	403	A. DE SITTER	C. J. KOOREMAN

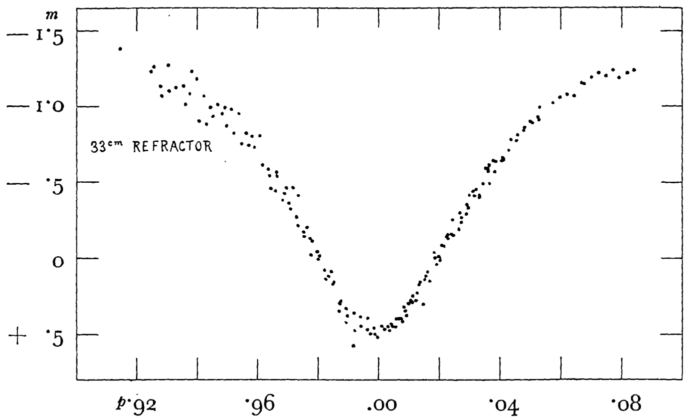
Eastman 40 plates have been used throughout.

a) *The plates of the* 33-cm *refractor* were taken with a grating $\Delta m = {}^{m}\!\cdot 93$ in front of the objective. The variable and the comparison star BD + 69°171 were alternately placed next to each other in a row on the plate, the distance between two consecutive exposures being 0·8 mm or 30″. The time of exposure was 80 sec. with 40 sec. in between. The plates have been measured in a Schilt microphotometer. The individual observations are given in table 5. The difference variable minus comparison star is there given in magnitudes, taking for the provisional constant of the grating $\Delta m = 1^{m}\!\cdot 00$. The difference between this value and the one mentioned above (${}^{m}\!\cdot 93$) does not affect the epochs of minimum considered in this paper. In fig. 1 all the individual observations are plotted together in one diagram against phase, corrected for the computed change of period (see section 2).

b) *The plates of the double camera.* Gratings $\Delta m = {}^{m}\!\cdot 96$ were used in front of the objectives. For the plates taken before J. D. 2426500 the device for moving the plateholders during the exposure was not used. For the plates taken after that date the movement was arranged in such a way that the stars gave square images of ·2 × ·2 mm. The accuracy proved to be increased (see fig. 2). Exposure times from 288 to 400 sec. were used. Special plateholders have been used throughout in which the position of the plates could be changed between the exposures, each exposure taking place on an otherwise unexposed part

[1]) The numbers in parentheses in the text refer to the list of literature at the end of this paper.

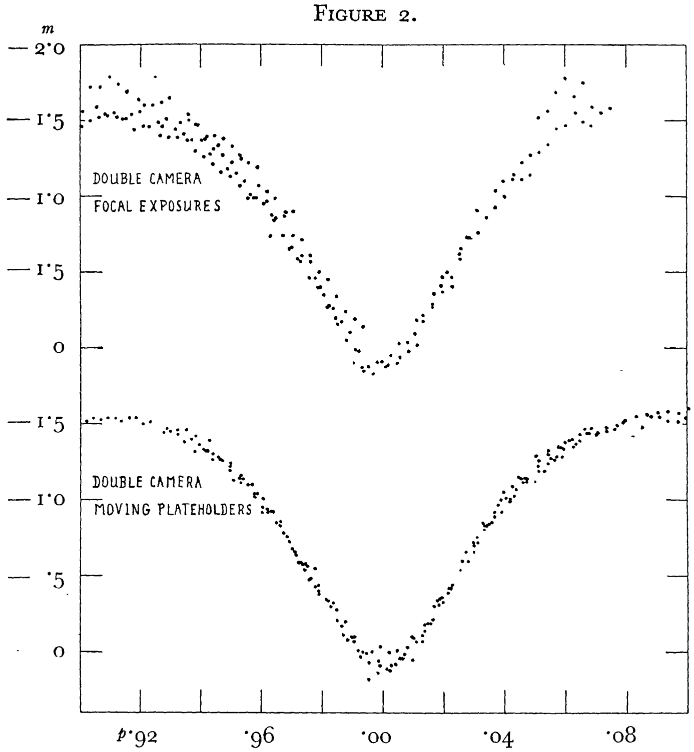
FIGURE 1.



Abscissa : phase corrected for computed change of period.
Ordinate: magnitude on provisional scale.

of the plate. The comparison stars used are BD + 69°·171, 68°208 and 68°212. The individual observations (means of two simultaneous exposures) are given in table 6. The difference between the variable and the mean of the three comparison stars is there given in magnitudes taking for the provisional constant of the grating $\Delta m = 1^m{\cdot}00$. The individual observations, plotted against corrected phase (section **2**), are given in fig. 2, viz. the focal exposures in one diagram, and the exposures with moving plateholders in the other.

FIGURE 2.



Abscissa : phase corrected for computed change of period.
Ordinate: magnitude on provisional scale.

Epochs of minimum from both series of observations were derived by the method described in *B.A.N.* 147 and 166. In the cases where only one branch of the minimum was observed a plot of these observations was shifted, in an analogous way, against a diagram of the whole minimum until coincidence was effected, and thus the epoch of minimum was derived. The epochs derived from the Leiden observations are included in table 1, where L stands for Leiden.

2. *The period.* The individual epochs of the minima observed before 1916 are not reprinted in the present paper as this would give almost a copy of the long list given by DUGAN (2), the only change being the revision and completion of NIJLAND's minima, in accordance with NIJLAND's later publication (4). Thus for the observations before 1916 only combined means are given (first part of table 2). These were computed in the following way. DUGAN's relative weights were adopted and mean epochs of minimum were computed from groups of total weight about 20 each, while the observations on old Harvard plates were combined into one single mean, excluding the first two epochs, which are far isolated from the rest and insufficient for forming a separate normal point of proper weight.

The epochs of the minima observed after 1916 are compiled in table 1, including those derived from the Leiden observations and from the following unpublished observations, accessible through the kindness of the observers, for which I owe them much thank.

R. S. DUGAN 115 observations 1919—1924 polarising photometer.

B. W. KUKARKIN 318 observations 1928—1929 visual estimates.

N. FLORJA 484 observations 1928—1931 visual estimates.

E. LEUTENEGGER 133 observations 1928—1929 visual estimates.

Dr. H. RÜGEMER was so kind as to send me his epochs of minimum (25) before publication. Since I had no access to the individual observations they were not included in the present discussion. RÜGEMER's minima fall in the same time interval as my own, with which they are in good agreement.

The spacing in table 1 indicates the arrangement in groups of total weight about 20 each, from which the combined means, given in table 2, second part, were computed. The relative weights given in column 5 were estimated according to method and number of the observations. There are no means to derive exact

TABLE 1.

E	J. D. Hel. M.T. Grw	phase [1]	phase [8]	wt.	Lit.
	242	p	p		
2859	0772·6420	·2839	·9947	10	(5)
2864	0778·6188	·2843	·9952	3	(5)
2870	0785·7911	·2850	·9958	1	(5)
3007	0949·538	·2826	·9935	5	(2)
3160	1132·422	·2912	·0021	1	(6)
3812	1911·719	·2846	·9954	1	(7)
4128	2289·418	·2838	·9946	1	(7)
4231	2412·5308	·2852	·9961	5	(21)
4445	2668·319	·2886	·9995	1	(7)
4681	2950·3972	·2873	·9982	2	(10)
4696	2968·317	·2798	·9907	1	(11)
4711	2986·251	·2842	·9950	1	(11)
4721	2998·202	·2829	·9937	1	(11)
4724	3001·797	·2906	·0015	1	(7)
4807	3100·9950	·2839	·9948	5	(21)
4927	3244·4272	·2854	·9963	3	(12)
5087	3435·6687	·2863	·9971	5	(21)
5116	3470·3326	·2876	·9984	3	(10)
5399	3808·5907	·2888	·9997	5	(21)
5511	3942·4618	·2912	·0020	2	(10)
5628	4082·3056	·2905	·0013	2	(10)
5803	4291·4792	·2941	·0049	2	(10)
5854	4352·4379	·2948	·0056	2	(10)
5901	4408·6113	·2918	·0027	20	L
5921	4432·5167	·2921	·0029	20	L
5956	4474·3495	·2912	·0021	10	L
5971	4492·2799	·2926	·0034	10	L
6013	4542·466	·2804	·9912	1	(7)
6171	4731·329	·2913	·0021	1	(14)
6240	4813·798	·2884	·9992	1	(7)
6376	4976·3529	·2888	·9996	1	(7)
6397	5001·4515	·2874	·9982	3	(10)
6489	5111·416	·2884	·9992	3	(16)
6519	5147·272	·2871	·9979	1	(14)
6524	5153·251	·2894	·0002	1	(7)
6642	5294·292	·2903	·0011	3	(17)
6652	5306·250	·2949	·0057	1	(7)
6777	5455·650	·2893	·0002	1	(15)
6779	5458·0407	·2895	·0003	5	(22)
6791	5472·387	·2922	·0031	1	(14)
6899	5601·471	·2895	·0003	1	(18)
7047	5778·3707	·2913	·0022	2	(23)
7061	5795·1073	·2939	·0047	5	(24)
7088	5827·380	·2946	·0054	1	(14)
7098	5839·328	·2908	·0016	1	(15)
7101	5842·9132	·2904	·0012	5	(24)
7120	5865·6220	·2895	·0003	10	L
7135	5883·5519	·2904	·0013	10	L
7257	6029·3725	·2902	·0010	20	L
7195	5955·2647	·2884	·9993	2	(13)
7216	5980·3664	·2896	·0004	1	(13)
7247	6017·4201	·2903	·0011	7	L
7262	6035·3490	·2904	·0012	4	(19)
7262	6035·349	·2904	·0012	1	(13)
7411	6213·440	·2890	·9998	2	(20)
7416	6219·4180	·2904	·0012	3	(19)
7769	6641·3372	·2861	·9970	20	L

TABLE 1 (*continued*).

E	J. D. Hel. M.T. Grw	phase [1]	phase [8]	wt.	Lit.
	242	p	p		
7856	6745·3246	·2865	·9973	20	L
7734	6599·5040	·2867	·9975	10	L
8010	6929·3914	·2847	·9955	10	L
7873	6765·6430	·2857	·9965	20	L
7887	6782·3770	·2861	·9969	20	L

relative weights since in most cases individual observations have not been published. In the mean the unit of weight will be about the same as in DUGAN's list. The numbers in the last column refer to the list of literature at the end of this paper, while the Leiden observations are indicated by the letter L.

The phases given in column 3 of tables 1 and 2 were computed by the formula

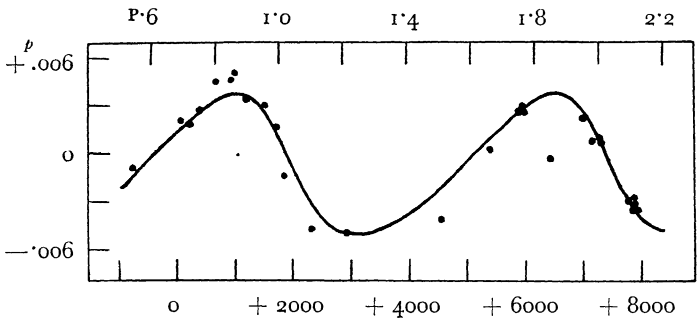
$$\text{Phase} = \overset{d^{-1}}{\cdot 83664292}\,(\text{J.D.} - 2400000) \qquad [1]$$

corresponding to the mean period first adopted viz. $1^{d}\cdot19525305$. E, the number of the epoch, is counted from J. D. 2417355. In column 4 of both tables the phase of minimum, based on formula [8], explained below, is given. In fig. 3 these last phases, for the normal points, are plotted against E. The drawn curve in that figure was computed as explained below.

TABLE 2.

E	J. D. Hel. M. T. Grw.	observed phase [1]	observed phase [8]	computed $t-t_0$	computed phase [8]	$O-C$
	24	p	p	P	p	p
−789	16412·3640	·2881	·9991	·536	·9986	+·0005
+49	7413·9897	·2912	·0021	·688	·0016	+5
195	7588·4963	·2909	·0018	·715	·0020	−2
386	7816·7907	·2918	·0027	·750	·0026	+1
645	8126·3634	·2936	·0045	·797	·0033	+12
937	8475·3774	·2937	·0046	·850	·0038	+8
1011	8563·8267	·2942	·0051	·863	·0038	+13
1241	8838·7329	·2925	·0034	·905	·0036	−2
1511	9161·4509	·2922	·0031	·954	·0026	+5
1691	9376·5947	·2908	·0017	·987	·0015	+2
1950	9686·1616	·2877	·9986	1·034	·9994	−8
2287	20088·9579	·2844	·9953	1·095	·9970	−17
2917	0841·9670	·2841	·9949	1.210	·9951	−2
4526	2765·1302	·2850	·9959	1·503	·9978	−19
5390	3797·8342	·2895	·0003	1·660	·0011	−8
5901	4408·6113	·2918	·0027	1·753	·0027	0
5921	4432·5167	·2921	·0029	1·756	·0028	+1
5964	4483·9123	·2919	·0028	1·764	·0029	−1
6427	5037·3108	·2888	·9995	1·848	·0037	−42
6985	5704·2651	·2914	·0022	1·950	·0027	−5
7128	5875·1846	·2900	·0008	1·976	·0018	−10
7257	6029·3725	·2902	·0010	1·999	·0009	+1
7286	6064·0346	·2900	·0008	2.005	·0007	+1
7769	6641·3372	·2861	·9970	2·092	·9971	−1
7856	6745·3246	·2865	·9973	2·106	·9967	+6
7872	6764·4477	·2857	·9965	2·111	·9966	−1
7873	6765·6430	·2857	·9965	2·111	·9966	−1
7887	6782·3770	·2861	·9969	2·114	·9965	+4

FIGURE 3.



Abscissa: Number of the epoch. Ordinate: phase of mid eclipse (formula [8]) or $z—z_0$. Dots are normalpoints. The curve corresponds with formula [3] when substituting for the constants the values derived in this paper. The scale of abscissae indicated at the upper side of the diagram corresponds with formula [9], giving the phase of the 18*y*. period, counted from passage through periastron.

Let s, the centre of mass of the eclipsing binary, describe an elliptical orbit relative to S, the centre of the triple system. If p'' is the interval between any two consecutive eclipses as seen by an observer on the sun, then

$$p'' = p'\left(1 + \frac{V}{c}\right) + p'\left(1 + \frac{v - V}{c}\right) \qquad [2]$$

where p' is the true period of the eclipsing binary, c is the velocity of light and v and V are the radial velocities of s and S respectively. We put $p'(1 + V/c) = p$ and introduce as unit of length the distance traversed by light in a time interval p. The numerical value z of the phase, based on a period p, of any well defined point on the lightcurve as seen by the observer on the sun, is then, apart from a constant z_0, identical with the distance, in the unit just defined, from s to a plane through S perpendicular to the line of sight. And we have for z the formula, see WOLTJER (26)

$$z = z_0 + a\,(1 - e \cos \mathrm{E}) \sin i \sin u \qquad [3]$$

introducing the following quantities relating to the orbital motion of s relative to S:

a: semi-axis major
e: excentricity
i: inclination
ω: longitude of periastron
P: period, $n = \frac{2\pi}{P}$
t_0: epoch of passage through periastron
E: excentric anomaly
v: true anomaly
u: argument of latitude $= v + \omega$

Identifying z with the phases given in table 2, column 3, provisional values of the unknowns P, z_0, $a \sin i$, e, ω and t_0 were found by trial and error, and then a second approximation was made by a least squares solution. The equation of condition, including a term for a revision of the adopted value of p, is

$$[4] \quad \Delta z = \delta z_0 + \frac{\partial z}{\partial a \sin i}.\,\delta\,(a \sin i) + \frac{\partial z}{\partial e}\,\delta e + + \frac{\partial z}{\partial \omega}\,\delta\omega + \frac{\partial z}{\partial n}\,\delta n + \frac{\partial z}{\partial t_0}\,\delta t_0 + E\,\frac{\delta p}{p}$$

where E is the number of the epoch and the δ's are the corrections to the provisional values of the unknowns.

The derivatives of z with respect to the elements are:

$$[5] \quad \begin{cases} \dfrac{\partial z}{\partial a \sin i} = (1 - e \cos \mathrm{E}) \sin u \\[2ex] \dfrac{\partial z}{\partial e} = a \sin i \left(- \sin\omega + \dfrac{1 - e \cos \mathrm{E}}{1 - e^2} \cos u \sin v\right) \\[2ex] \dfrac{\partial z}{\partial \omega} = a \sin i\,(1 - \cos \mathrm{E}) \cos u \\[2ex] \dfrac{\partial z}{\partial n} = \dfrac{a \sin i}{\sqrt{1 - c^2}}\,(t - t_0)\,(e \cos\omega + \cos u) \\[2ex] \dfrac{\partial z}{\partial t_0} = - \dfrac{a \sin i}{\sqrt{1 - e^2}}\,u\,(e \cos\omega + \cos u) \end{cases}$$

Taking the provisional value of P, viz. 5625 p, as unit of time and introducing the following symbols:

$$[6] \quad \begin{cases} \varphi \equiv 1 - e \cos \mathrm{E} \\ \tau \equiv t - t_0 \\ \chi \equiv (t - t_0)\,(e \cos\omega + \cos u) \\ A \equiv \delta z_0 - a \sin i \sin\omega\,\delta e - \dfrac{a \sin i}{\sqrt{1 - e^2}}\,ne \cos\omega\,\delta t_0 \\ B \equiv \delta\,(a \sin i) \\ C \equiv a \sin i\,\delta\omega \\ D \equiv \dfrac{a \sin i}{1 - e^2}\,\delta e \\[2ex] X \equiv - \dfrac{a \sin i}{\sqrt{1 - e^2}}\,n\,\delta t_0 \\[2ex] Y \equiv \dfrac{a \sin i}{\sqrt{1 - e^2}}\,\delta n \\[2ex] Z \equiv 5625\,\dfrac{\delta p}{p}, \end{cases}$$

the equation of condition becomes

$$[7] \quad \Delta z = A + \varphi \sin u.\,B + \varphi \cos u.\,C + \varphi \cos u \sin v.\,D + + \cos u.\,X + \chi.\,Y + \tau.\,Z.$$

The functions z, φ and χ for the normals of table 2 were computed with the aid of tables for E and v, see (27) and (28).

From the solution of the normal equations the following values of the unknowns, with their mean errors, were derived:

$$p = 1^{d}\cdot195253065 \pm 12 \qquad e = \cdot40 \pm \cdot12$$
$$P = 5499\,p \pm 66 = 18^{y}\cdot00 \pm \cdot22 \qquad \omega = +161^{\circ}\cdot2 \pm 3^{\circ}\cdot3$$
$$z_0 = \cdot28886\,p/c \pm 23 \qquad t_0 = -\,{}^{P}\cdot680 \pm \cdot046$$
$$a \sin i = 0\cdot00475\,p/c \pm 39 = 0.984 \text{ a.u.} \pm 81$$

As zero of time in this computation was taken $E = 0$.

The residuals $O - C$ given in table 2 were computed by using this solution. The mean error of a normal point as derived from these residuals is $\pm\,{}^{p}\cdot0012$. The computed curve of $z - z_0$ is given in fig. 3 together with the normal values (table 2 column 4). The corrected phases given in column 3 of tables 5 and 6 are phases based on the revised value of the mean period p, computed by the formula

$$[8] \quad \text{phase} = \overset{d^{-1}}{\cdot836642908}\,(\text{J.D.} - 2416886\cdot88058) - 392$$

and corrected by substracting the computed value of $z - z_0$. The corrected phases are also the abscissae of figures 1 and 2. Table 3 gives a new ephemeris of RZ Cas. In the first column the fraction of the 18 year period, reckoned from passage through periastron, is given, as computed by the formula

$$[9] \qquad t - t_0 = \overset{p^{-1}}{\cdot00018185}\,(E - 1762)$$

The second column gives the predicted phase of minimum light corresponding to formula [8], viz. the predicted value of $z - z_0$.

TABLE 3.

P	p	P	p
·019		·538	
	·000		—·001
·030		·580	
	—·001		·000
·052		·627	
	—·002		+·001
·080		·682	
	—·003		+·002
·118		·742	
	—·004		+·003
·175		·813	
	—·005		+·004
·389		·910	
	—·004		+·003
·420		·958	
	—·003		+·002
·479		·983	
	—·002		+·001
·538		·019	

Thus for instance for $E = 9000$ the fraction of the 18-year period is ${}^{p^{-1}}\cdot00018185\,(9000 - 1762) = {}^{P}\cdot316$ and the predicted time of minimum is J.D. Hel. M.T.Grw. $2416886\cdot88058 + 1^{\cdot d}195253065\,(392 + 8999\cdot995)$.

The mass function of the 18 year orbital motion of the eclipsing system is

$$[10] \qquad \frac{M_2^{\,3}\sin^3 i}{(M_1 + M_2)^2} = \frac{(a \sin i)^3}{P^2} = \cdot0029\ \odot$$

where the units are: mass of the sun, astr. unit, year. M_1 and M_2 are the total mass of the eclipsing binary and the mass of the third star respectively.

The spectral type of RZ Cassiopeiae is A2. Let us assume correspondingly for the absolute visual magnitude of the bright componant $m_b + 5 + 5 \log \pi = +0^{m}\cdot5$ which gives for the parallax of RZ Cas $\pi = {}''\cdot006$. For the mass function of the eclipsing binary JORDAN finds $\dfrac{M_f^{\,3}}{(M_b + M_f)^2}\sin^3 i = \cdot041\ \odot$, from spectroscopic observations (29). Substituting for i the value derived by DUGAN from an analysis of the lightcurve (3), and assuming for the mass of the bright component $M_b = 3\odot$, we find for the total mass $M_1 = M_b + M_f = 3\cdot8\ \odot$.

Using these data in combination with formula [10] table 4 was computed. The argument of the table is the assumed mass of the third star, given in the first column; $a \cos i$ is the semi-axis major of the orbit of the point s, as projected on the sky; $(a + a') \cos i$ is the maximum angular distance between the eclipsing binary and the third star; $m_2 - m_1$ is the difference in magnitude between the third star and the bright component of RZ Cas, according to EDDINGTON's mass luminosity relation. Probably this magnitude difference will be considerable, since, as far as I know, the spectrum of RZ Cas has never been suspected of being of composite character. With respect to this point a careful study of spectra taken exactly at the phase of minimum light seems desirable.

TABLE 4.

M_2	$\sin i$	$a \cos i$	$(a + a') \cos i$	$m_2 - m_1$
0·4 ⊙	·93	″·002	″·025	^{m}8·6
0·7	·55	·010	·055	5·9
1·0	·41	·013	·064	4·4
1·5	·29	·020	·070	2·8
2·0	·23	·025	·072	1·3
2·5	·19	·029	·074	0·6
3·0	·17	·035	·078	0·0

Finally it should be noted that the mass assumed above for the bright component of RZ Cas corresponds with a mass proportion $\dfrac{M_b}{M_f} = 3\cdot6$, which is rather in discordance with the light proportion found by DUGAN (3), viz. $\frac{L_b}{L_f} = 7{\cdot}13$. On the other hand, when adopting $\frac{M_b}{M_f} = 2$, as would correspond to DUGAN's data, we find $M_b = 0{\cdot}8\,\odot$, which is not a very probable value for an A star. It remains doubtful if this discordance is real, but if it is, perhaps the light coming from the third star may be made responsible for a false interpretation of the lightcurve.

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TABLE 5. Observations by H. VAN GENT (33 cm refractor).

J. D. Hel. M. T. Grw.	Corr. phase	mag.	J. D. Hel. M. T. Grw.	Corr. phase	mag.	J. D. Hel. M. T. Grw.	Corr. phase	mag.	J. D. Hel. M. T. Grw.	Corr. phase	mag.
d	p	m	d	p	m	d	p	m	d	p	m
4408·5225	·9256	−1·26	4408·6305	·0160	+·09	4432·4957	·9827	+·12	4474·3345	·9867	+·35
·5253	·9279	−1·13	·6333	·0183	−·04	·4985	·9850	+·17	·3373	·9890	+·43
·5280	·9302	−1·27	·6361	·0206	−·09	·5012	·9873	+·29	·3401	·9913	+·58
·5350	·9360	−1·01	·6388	·0229	−·13	·5040	·9896	+·38	·3429	·9937	+·39
·5377	·9383	−1·23	·6430	·0264	−·19	·5068	·9919	+·48	·3456	·9960	+·47
·5405	·9407	−·90	·6458	·0287	−·29	·5206	·0035	+·48	·3484	·9983	+·46
·5433	·9430	−·88	·6485	·0310	−·44	·5234	·0058	+·40	·3512	·0006	+·45
·5460	·9453	−·93	·6513	·0333	−·40	·5262	·0082	+·32	·3539	·0029	+·45
·5488	·9476	−·95	·6541	·0357	−·59	·5289	·0105	+·28	·3567	·0052	+·45
·5516	·9499	−·87	·6569	·0380	−·57	·5317	·0128	+·23	·3595	·0076	+·41
·5544	·9522	−·82	·6596	·0403	−·64	·5345	·0151	+·14	·3622	·0099	+·30
·5571	·9546	−·75	4432·4140	·9143	−1·38	·5400	·0198	−·01	·3650	·0122	+·28
·5599	·9569	−·74	·4264	·9247	−1·23	·5428	·0221	−·14	·3678	·0145	+·30
·5627	·9592	−·73	·4306	·9282	−1·07	·5455	·0244	−·25	·3706	·0168	+·15
·5654	·9615	−·61	·4334	·9305	−1·10	·5483	·0267	−·30	·3733	·0192	+·04
·5682	·9638	−·54	·4361	·9328	−1·12	·5511	·0290	−·35	·3761	·0215	−·08
·5710	·9662	−·56	·4389	·9352	−1·13	·5539	·0313	−·41	·3789	·0238	−·16
·5737	·9684	−·46	·4417	·9375	−1·08	·5566	·0337	−·49	·3830	·0273	−·27
·5765	·9708	−·32	·4444	·9398	−1·18	·5594	·0360	−·57	·3858	·0296	−·41
·5793	·9731	−·21	·4472	·9421	−1·06	·5622	·0383	−·63	·3900	·0331	−·41
·5821	·9754	−·14	·4500	·9444	−·99	·5649	·0406	−·66	·3927	·0354	−·59
·5848	·9777	−·02	·4527	·9467	−1·01	·5677	·0429	−·71	4492·2757	·9970	+·50
·5876	·9800	+·01	·4555	·9491	−·99	·5705	·0452	−·77	·2785	·9993	+·52
·5904	·9824	+·14	·4583	·9514	−·98	·5732	·0476	−·84	·2813	·0016	+·47
·5931	·9847	+·16	·4611	·9537	−·95	·5760	·0499	−·90	·2840	·0040	+·44
·5959	·9870	+·30	·4638	·9560	−·82	·5788	·0522	−·93	·2868	·0063	+·40
·5987	·9893	+·33	·4666	·9583	−·80	4474·3068	·9635	−·58	·2896	·0086	+·35
·6014	·9916	+·36	·4694	·9607	−·80	·3096	·9658	−·44	·2924	·0109	+·29
·6042	·9940	+·45	·4735	·9641	−·46	·3124	·9681	−·38	·2951	·0132	+·18
·6070	·9963	+·40	·4763	·9664	−·54	·3152	·9705	−·36	·2979	·0155	+·12
·6098	·9986	+·50	·4791	·9688	−·42	·3179	·9728	−·27	·3007	·0179	·00
·6167	·0044	+·45	·4818	·9711	−·46	·3207	·9751	−·17	·3034	·0202	+·01
·6195	·0067	+·40	·4846	·9734	−·41	·3235	·9774	−·12	·3062	·0225	−·15
·6222	·0090	+·38	·4874	·9757	−·20	·3262	·9797	−·04	·3090	·0248	−·15
·6250	·0113	+·25	·4901	·9780	−·11	·3290	·9821	+·08	·3117	·0271	−·24
·6278	·0137	+·16	·4929	·9804	−·01	·3318	·9844	+·09	·3145	·0294	−·33

TABLE 5 (*continued*).

J. D. Hel. M. T. Grw.	Corr. phase	mag.	J. D. Hel. M. T. Grw.	Corr. phase	mag.	J. D. Hel. M. T. Grw.	Corr. phase	mag.	J. D. Hel. M. T. Grw.	Corr. phase	mag.
d	p	m	d	p	m	d	p	m	d	p	m
4492·3173	·0318	— ·45	4492·3339	·0457	— ·81	4492·3505	·0596	— 1·06	4492·3658	·0723	— 1·22
·3201	·0341	— ·49	·3367	·0480	— ·86	·3533	·0619	— 1·08	·3685	·0746	— 1·20
·3228	·0364	— ·61	·3395	·0503	— ·89	·3561	·0642	— 1·07	·3713	·0770	— 1·24
·3256	·0387	— ·64	·3422	·0526	— ·91	·3588	·0665	— 1·16	·3741	·0793	— 1·19
·3284	·0410	— ·65	·3450	·0549	— ·99	·3602	·0677	— 1·15	·3768	·0816	— 1·22
·3311	·0433	— ·78	·3478	·0573	— 1·02	·3630	·0700	— 1·19	·3796	·0839	— 1·24

TABLE 6. Observations by A. DE SITTER (double camera).

J. D. Hel. M. T. Grw.	Corr. phase	mag.	J. D. Hel. M. T. Grw.	Corr. phase	mag.	J. D. Hel. M. T. Grw.	Corr. phase	mag.	J. D. Hel. M. T. Grw.	Corr. phase	mag.
d	p	m	d	p	m	d	p	m	d	p	m
5865·4910	·8889	— 1·53	5883·4428	·9084	— 1·54	6017·3775	·9645	— ·85	6029·3614	·9908	+ ·01
·4945	·8918	— 1·47	·4463	·9112	— 1·55	·3816	·9679	— ·89	·3648	·9937	+ ·15
·4979	·8947	— 1·50	·4498	·9141	— 1·51	·3846	·9704	— ·90	·3683	·9966	+ ·17
·5014	·8976	— 1·45	·4532	·9170	— 1·54	·3880	·9733	— ·71	·3718	·9995	+ ·09
·5049	·9005	— 1·46	·4567	·9199	— 1·56	·3915	·9762	— ·61	·3789	·0055	+ ·06
·5083	·9034	— 1·50	·4602	·9228	— 1·46	·3950	·9791	— ·50	·3823	·0083	+ ·02
·5118	·9063	— 1·52	·4636	·9257	— 1·49	·3984	·9820	— ·45	·3856	·0111	— ·02
·5153	·9092	— 1·53	·4671	·9286	— 1·51	·4019	·9849	— ·34	·3925	·0169	— ·27
·5187	·9121	— 1·52	·4706	·9315	— 1·44	·4053	·9878	— ·24	·3960	·0198	— ·37
·5222	·9150	— 1·51	·4740	·9344	— 1·41	·4088	·9907	— ·19	·3995	·0227	— ·41
·5256	·9179	— 1·44	·4775	·9373	— 1·37	·4123	·9936	— ·14	·4030	·0256	— ·66
·5291	·9208	— 1·46	·4809	·9402	— 1·37	·4232	·0027	+ ·05	·4064	·0285	— ·73
·5360	·9266	— 1·40	·4844	·9431	— 1·28	·4261	·0052	— ·03	·4099	·0314	— ·76
·5395	·9295	— 1·39	·4879	·9460	— 1·34	·4296	·0081	— ·04	·4135	·0344	— ·86
·5430	·9324	— 1·39	·4913	·9489	— 1·18	·4330	·0110	— ·19	·4171	·0374	— ·93
·5464	·9353	— 1·37	·4948	·9518	— 1·15	·4365	·0139	— ·22	·4203	·0401	— 1·00
·5499	·9382	— 1·30	·4983	·9547	— 1·10	·4400	·0168	— ·36	·4239	·0431	— 1·14
·5534	·9411	— 1·26	·5017	·9576	— ·99	·4434	·0197	— ·47	·4272	·0458	— 1·22
·5568	·9440	— 1·21	·5054	·9606	— ·95	·4469	·0126	— ·47	·4307	·0487	— 1·27
·5603	·9469	— 1·16	·5089	·9636	— ·88	·4504	·0255	— ·59	·4342	·0516	— 1·29
·5640	·9499	— 1·13	·5121	·9663	— ·74	·4538	·0283	— ·74	·4377	·0545	— 1·34
·5675	·9528	— 1·07	·5156	·9692	— ·65	·4573	·0312	— ·91	·4412	·0574	— 1·45
·5706	·9556	— ·99	·5190	·9720	— ·59	·4606	·0340	— 1·12	·4447	·0603	— 1·47
·5741	·9584	— ·99	·5260	·9778	— ·46	·4647	·0374	— 1·04	·4483	·0635	— 1·55
·5809	·9641	— ·84	·5294	·9807	— ·35	·4677	·0399	— 1·10	·4518	·0663	— 1·49
·5845	·9671	— ·74	·5329	·9836	— ·26	·4711	·0428	— 1·11	·4548	·0690	— 1·59
·5880	·9700	— ·66	·5363	·9865	— ·17	·4746	·0457	— 1·11	·4583	·0719	— 1·55
·5915	·9729	— ·57	·5398	·9894	— ·10	·4781	·0486	— 1·14	·4618	·0748	— 1·58
·5950	·9758	— ·46	·5433	·9923	+ ·02	·4815	·0515	— 1·52	6599·3869	·9022	— 1·52
·5985	·9787	— ·40	·5467	·9952	+ ·12	·4850	·0544	— 1·56	·3897	·9045	— 1·54
·6019	·9816	— ·27	·5502	·9981	+ ·09	·4884	·0573	— 1·69	·3924	·9068	— 1·52
·6054	·9845	— ·20	·5537	·0010	+ ·12	·4919	·0602	— 1·78	·3952	·9092	— 1·54
·6122	·9903	+ ·10	6017·2980	·8980	— 1·56	·4954	·0631	— 1·66	·3980	·9115	— 1·54
·6157	·9932	+ ·12	·3015	·9009	— 1·56	·4988	·0660	— 1·75	·4007	·9138	— 1·52
·6257	·0016	+ ·11	·3049	·9038	— 1·72	·5023	·0689	— 1·48	·4035	·9161	— 1·54
·6295	·0048	+ ·10	·3084	·9067	— 1·72	6029·2852	·9271	— 1·46	·4063	·9184	— 1·54
·6330	·0077	— ·07	·3119	·9096	— 1·79	·2956	·9358	— 1·50	·4091	·9207	— 1·50
·6365	·0106	— ·10	·3153	·9125	— 1·74	·2990	·9387	— 1·47	·4118	·9231	— 1·51
·6400	·0135	— ·18	·3188	·9154	— 1·69	·3025	·9416	— 1·39	·4174	·9277	— 1·44
·6435	·0164	— ·29	·3224	·9184	— 1·64	·3059	·9445	— 1·40	·4201	·9300	— 1·45
·6469	·0193	— ·41	·3257	·9212	— 1·60	·3094	·9474	— 1·38	·4229	·9323	— 1·46
·6504	·0222	— ·50	·3292	·9241	— 1·60	·3128	·9503	— 1·33	·4257	·9346	— 1·38
·6537	·0251	— ·62	·3326	·9270	— 1·62	·3162	·9532	— 1·25	·4284	·9370	— 1·40
5883·3977	·8705	— 1·55	·3359	·9297	— 1·65	·3197	·9561	— 1·11	·4312	·9393	— 1·36
·4015	·8737	— 1·52	·3401	·9332	— 1·49	·3232	·9590	— 1·07	·4340	·9416	— 1·32
·4049	·8766	— 1·54	·3430	·9357	— 1·54	·3267	·9619	— 1·03	·4368	·9439	— 1·26
·4083	·8794	— 1·57	·3465	·9386	— 1·47	·3300	·9645	— ·99	·4451	·9509	— 1·13
·4117	·8823	— 1·51	·3499	·9415	— 1·38	·3339	·9678	— ·87	·4478	·9532	— 1·11
·4155	·8854	— 1·55	·3534	·9443	— 1·31	·3371	·9705	— ·74	·4534	·9578	— 1·02
·4195	·8888	— 1·56	·3569	·9472	— 1·27	·3406	·9734	— ·61	·4561	·9601	— ·92
·4226	·8914	— 1·48	·3603	·9501	— 1·22	·3441	·9763	— ·57	·4589	·9624	— ·92
·4256	·8939	— 1·61	·3638	·9530	— 1·27	·3475	·9792	— ·40	·4617	·9648	— ·85
·4290	·8968	— 1·62	·3673	·9559	— 1·22	·3510	·9821	— ·28	·4665	·9688	— ·76
·4325	·8997	— 1·54	·3707	·9588	— 1·19	·3544	·9850	— ·16	·4693	·9712	— ·64
·4394	·9055	— 1·59	·3742	·9617	— ·96	·3579	·9879	— ·05	·4721	·9735	— ·58

TABLE 6 (*continued*).

J. D. Hel. M. T. Grw.	Corr. phase	mag.	J. D. Hel. M. T. Grw.	Corr. phase	mag.	J. D. Hel. M. T. Grw.	Corr. phase	mag.	J. D. Hel. M. T. Grw.	Corr. phase	mag.
d	p	m	d	p	m	d	p	m	d	p	m
6599·4748	·9758	— ·48	6641·4618	·1040	— 1·56	6765·5581	·9289	— 1·47	6782·3215	·9538	— 1·14
·4783	·9787	— ·44	·4660	·1075	— 1·52	·5623	·9324	— 1·44	·3263	·9579	— 1·04
·4811	·9810	— ·35	6745·2767	·9605	— 1·02	·5658	·9353	— 1·46	·3312	·9619	— ·94
·4838	·9833	— ·32	·2802	·9634	— ·93	·5692	·9382	— 1·42	·3360	·9660	— ·83
6641·2693	·9430	— 1·39	·2836	·9663	— ·87	·5727	·9411	— 1·33	·3457	·9741	— ·58
·2734	·9465	— 1·26	·2871	·9692	— ·74	·5762	·9440	— 1·30	·3513	·9788	— ·40
·2776	·9500	— 1·24	·2905	·9721	— ·60	·5796	·9469	— 1·26	·3561	·9828	— ·30
·2817	·9534	— 1·16	·2940	·9750	— ·58	·5831	·9498	— 1·19	·3603	·9863	— ·17
·2859	·9569	— 1·10	·2975	·9779	— ·56	·5865	·9527	— 1·14	·3651	·9903	— ·07
·2901	·9604	— ·95	·3079	·9866	— ·12	·5900	·9556	— 1·10	·3700	·9944	+ ·01
·2942	·9639	— ·92	·3113	·9895	— ·11	·5935	·9585	— 1·04	·3748	·9984	+ ·06
·2984	·9673	— ·78	·3148	·9924	+ ·02	·6073	·9701	— ·68	·3796	·0025	+ ·09
·3025	·9708	— ·66	·3182	·9952	+ ·17	·6108	·9730	— ·59	·3845	·0066	+ ·02
·3067	·9743	— ·54	·3217	·9981	+ ·13	·6142	·9759	— ·48	·3893	·0106	— ·07
·3108	·9778	— ·43	·3252	1·0010	+ ·10	·6177	·9788	— ·38	·3942	·0147	— ·19
·3191	·9847	— ·21	·3286	·0039	+ ·07	·6212	·9817	— ·34	·3990	·0187	— ·35
·3233	·9882	— ·08	·3356	·0097	+ ·04	·6246	·9846	— ·28	·4039	·0228	— ·44
·3275	·9917	— ·01	·3390	·0126	— ·08	·6281	·9875	— ·20	·4136	·0309	— ·72
·3316	·9952	+ ·07	·3425	·0155	— ·20	·6316	·9904	— ·10	·4184	·0349	— ·84
·3358	·9986	+ ·09	·3459	·0184	— ·32	·6350	·9933	·00	·4233	·0390	— ·92
·3399	·0021	+ ·12	·3494	·0213	— ·40	·6385	·9961	·00	·4281	·0431	— 1·03
·3441	·0056	+ ·04	·3563	·0271	— ·67	·6419	·9990	— ·04	·4330	·0471	— 1·14
·3482	·0091	— ·10	·3598	·0300	— ·67	·6454	·0019	·00	·4378	·0512	— 1·19
·3524	·0126	— ·11	·3633	·0329	— ·86	·6489	·0048	— ·01	·4427	·0552	— 1·27
·3565	·0160	— ·21	·3667	·0358	— ·86	·6523	·0077	— ·03	·4475	·0593	— 1·34
·3607	·0195	— ·33	·3702	·0387	— 1·01	·6593	·0135	— ·14	·4524	·0633	— 1·37
·3732	·0299	— ·70	·3736	·0416	— 1·00	·6627	·0164	— ·30	·4572	·0674	— 1·44
·3773	·0334	— ·83	·3771	·0445	— 1·16	·6662	·0193	— ·36	6929·3986	·0056	— ·06
·3815	·0369	— ·89	·3806	·0474	— 1·13	·6696	·0222	— ·42	·4034	·0097	— ·10
·3856	·0404	— 1·01	·3840	·0503	— 1·30	·6731	·0251	— ·54	·4083	·0137	— ·18
·3898	·0439	— 1·07	·3875	·0532	— 1·20	·6766	·0280	— ·60	·4131	·0178	— ·34
·3939	·0473	— 1·14	·3910	·0561	— 1·27	·6800	·0309	— ·76	·4180	·0218	— ·42
·3981	·0508	— 1·22	·3944	·0590	— 1·29	·6835	·0338	— ·80	·4228	·0259	— ·60
·4023	·0543	— 1·30	·4013	·0647	— 1·40	·6870	·0367	— ·92	·4277	·0299	— ·72
·4064	·0578	— 1·35	·4048	·0676	— 1·45	·6904	·0396	— 1·05	·4325	·0340	— ·83
·4106	·0612	— 1·39	·4083	·0705	— 1·45	·6939	·0425	— 1·09	·4374	·0380	— ·97
·4147	·0647	— 1·43	·4117	·0734	— 1·48	·6973	·0454	— 1·14	·4422	·0421	— 1·00
·4189	·0682	— 1·47	·4152	·0763	— 1·48	·7043	·0512	— 1·26	·4471	·0462	— 1·12
·4289	·0766	— 1·49	·4187	·0792	— 1·52	·7077	·0541	— 1·24	·4519	·0502	— 1·12
·4327	·0797	— 1·52	·4221	·0821	— 1·43	·7112	·0570	— 1·28	·4616	·0583	— 1·32
·4369	·0832	— 1·55	·4256	·0850	— 1·49	·7147	·0599	— 1·38	·4665	·0624	— 1·35
·4410	·0867	— 1·56	·4290	·0879	— 1·56	·7178	·0625	— 1·35	·4713	·0664	— 1·40
·4452	·0901	— 1·57	·4325	·0908	— 1·56	6782·3021	·9376	— 1·32	·4762	·0705	— 1·47
·4493	·0936	— 1·58	·4360	·0937	— 1·53	·3069	·9417	— 1·32	·4810	·0745	— 1·43
·4535	·0971	— 1·57	·4394	·0966	— 1·52	·3118	·9457	— 1·28	·4859	·0786	— 1·45
·4577	·1006	— 1·60	·4429	·0995	— 1·55	·3166	·9498	— 1·22			