that for these two stars the absorptions are small.

The region of the great Cygnus cloud investigated by BAADE appears to be one of abnormally high transparency. It does not seem unlikely that the brilliance of the cloud is due in larger measure to the absence of absorption than to a great conglomeration of stars. It is especially noteworthy that in the whole interval of about 10 kps between the three nearer stars and V336 Cygni the absorption shows no measurable increase; the difference in absorption between V336 and QY Cygni should be fairly well established, as the colours were measured from the same plates. It is true that the three nearer stars are at about 500 ps from the galactic plane, so that we might be expected to be already outside the absorbing layer; but this expectation is contradicted by the fact that hardly any extra-galactic nebulae have been seen in this neighbourhood. In HUBBLE's survey [1]) there are four fields within a few degrees from the Cepheids considered, at galactic co-ordinates $40° + 5°$, $45° + 3°$, $46° + 3°$, and $43° + 2°$; of these only the first contains 2 nebulae, the others none. BAADE writes that he has searched a 100-inch plate with one-hour exposure on the field of GL and V343 Cygni for nebulae, but has found none. The same negative result was obtained from two one-hour exposures with the 60-inch, centred on QY and V336 Cygni respectively, only one doubtful object being seen in the last mentioned field. All three plates were described as of excellent quality. With a total absorption of only $1^m{\cdot}5$, as estimated above, there should have been some 10 nebulae per plate. The contrast between the negative result of the search for nebulae and the smallness of the absorption found from the colour-excesses as well as from the motions of the Cepheids is remarkable. If the large total absorption would be substantiated by counts of still fainter nebulae, we should have to assume the existence of a large amount of dark matter beyond 500, or possibly even 1000 ps, from the galactic plane. It is well to recall in this connection that previous investigations [1]), in which it was tentatively assumed that the absorption shown by the nebular counts is caused by relatively near-by clouds, led to rather curious results for the density distribution. It was pointed out that these abnormal results (which appear to some extent to conflict with recent investigations of star density near the galactic plane) might alternatively be interpreted as indicating that part of the absorption takes place far from the galactic plane in an enormous elliptical cloud surrounding our system.

It is evident from this how very important a further elucidation of the absorption in the Cygnus region would be for our knowledge of the structure of the galaxy.

[1]) *Ap. J.* **79**, 8, 1934; *Mt Wilson Contr.* No. 485.

[1]) *Ann. d'Astroph.* **1**, 71, 1938; *B.A.N.* **8**, 233, 1938.

On the relation between velocity- and density-distribution of long-period variables,

by *J. H. Oort* and *J. J. M. van Tulder*.

Summary.

The increase of the density of long-period variables in the direction of the centre of the galactic system is determined from the observed velocity distribution and, independently, also from the distribution of these stars over the sky. From the dynamical data the increase of the logarithm of the density per kps is found to be $\cdot 278 \pm \cdot 053$ m.e., while the direct data on the distribution of the variables give a value of $\cdot 254 \pm \cdot 033$ m.e. The mean gradient of $\cdot 27$ corresponds to a decrease of the density with very nearly the 5th power of the distance from the centre.

The axes of the velocity ellipsoid have been calculated for four intervals of the period of light-variation; they are shown in Table 1; *a* is the average peculiar velocity in the direction of the centre, *b* in that of 55° longitude, *c* in that perpendicular to the galactic plane. The coefficients of differential galactic rotation have also been computed (Table 2); these come out smaller than for low-velocity stars of the same distance; it is pointed out that, for stars displaying asymmetry in their motions the constant *A* may be much smaller than the value found for circular motions.

The discussion indicates that, though the variables up to $8^m{\cdot}0$ visual magnitude at maximum are practically complete, only 61% of those between $8^m{\cdot}1$ and $9^m{\cdot}0$, and 26% of those between $9^m{\cdot}1$ and $10^m{\cdot}0$ have been discovered (pp. 333 and 331). The incompleteness is much stronger in the southern than in the northern hemisphere.

Introduction.

In a system which is in dynamical equilibrium a relation should exist between the so-called asymmetrical drift, the average peculiar velocity and the increase of the star density in the direction of the centre of the system. For brevity we shall call this latter the density gradient. It is clear that if there is a considerable density gradient in the galactic system, the majority of the stars in our neighbourhood will come from regions nearer to the centre; these stars are at present in the outer halves of their respective orbits, so that their velocities in the direction of the rotation will be less than the circular velocity; they will thus give rise to the phenomenon of the asymmetrical drift. The amount of the asymmetry will be a measure of the density gradient.

For the ordinary types of stars, for which abundant velocity data are available, the average differences with the circular velocity are so small that no satisfactory dynamical determination of the density gradient can be made[1]) (moreover, there are good reasons for supposing a considerable deviation from a steady state for these slow-moving stars). For the long-period variables, on the other hand, conditions are different; it is well-known that they have large peculiar velocities and show an appreciable asymmetrical drift. At the same time they have a high intrinsic brightness, about -2^M at their maximum, so that the space distribution can be studied up to considerable distances. This makes it possible to obtain a direct determination of the density gradient which may be compared with the gradient calculated from the motions. It is interesting to investigate whether these two entirely independent determinations agree with each other.

The density gradient determined from the motions.

A good opportunity for such a test seemed to be furnished by the appearance of the extensive radial velocity data recently published by MERRILL [1]). MERRILL's study has substantiated the remarkable variation of average velocity with physical characteristics, such as spectral sub-type and period, which he had discovered in 1923. The highest velocities seem to be obtained for stars with periods between 150 and 200 days. In order to get groups which are reasonably homogeneous the stars used in the present investigation were divided into four classes according to period. Table 1 shows these groups and the numbers of velocities used in each of them. The column V_0 gives the systematic velocity in the direction of 235° longitude, with respect to the ordinary slowly moving stars (V_0 is thus the asymmetrical drift). The columns a, b, c show the average peculiar velocities (remaining after correction for the systematic motion), a in the direction of the centre at 325° longitude, b in that of 55° longitude and 0° latitude, c in the direction of the galactic poles. The estimated mean errors have been added. The material appeared to be too small for a successful determination of the directions of the axes. The

TABLE 1.

Period	n	V_0	m.e.	a	m.e.	b	m.e.	c	m.e.	v	$\frac{\partial \log \nu}{\partial \log \varpi}$	m.e.	$\frac{\partial \log \nu}{\partial \varpi}$	m.e.
^{d}150 — 199^d	27	+128·8	±26·1	91·4	±19·6	45·0	±14·0	35·6	±8·9	62·5	−4·8	±1·8	−·26	±·10
< 150 and 200 — 299	124	+ 42·1	± 7·4	44·0	± 5·1	33·7	± 3·6	32·8	±4·3	36·9	−7·3	±2·0	−·40	±·11
300 — 349	75	+ 13·7	± 8·1	31·1	± 4·3	30·1	± 3·8	22·6	±5·1	29·4	−4·8	±3·0	−·26	±·17
> 349	79	+ 6·6	± 3·1	26·9	± 3·4	11·1	± 1·5	23·3	±4·7	20·5	−3·9	±1·6	−·21	±·09
average	305										−5·12	± ·97	−·278	±·053

column marked v shows the total average velocity.

A previous investigation had failed to indicate any signs of an ellipsoidal distribution of velocities; the present, much larger material appears to show a distinct difference between the two galactic axes.

A solution has also been made for the differential galactic rotation in the last three groups; in the first group such a solution would have been meaningless. In order to have a somewhat homogeneous and distant material the solution was confined to stars whose visual magnitudes at maximum were 8·0 or fainter; the mean maximum apparent magnitude of the remaining stars is 9·2. The results are given in Table 2 (in km/sec).

The coefficients of the rotation terms come out rather smaller than might at first sight be anticipated. As shown in the note following this article, the mean distance of variables of magnitude 9·2 at a median latitude of 20° may be estimated at 1800 ps for stars with periods smaller than 300^d, and at 1200 ps for stars with periods exceeding 300^d [2]), which would correspond with $\bar{r}A = +32·2$ and $+22·5$ respect-

TABLE 2.
Differential galactic rotation

P	n	$\bar{r}A$	m.e.
<150^d 200—299	97	+ 2·2	± 9·4
300—349	57	+ 16·1	± 6·8
>349	50	+ 7·4	± 5·4

[1]) In this connection see, however, p. 335 of this Bulletin.

[1]) *Ap. J.* **94**, 171, 1941; *Mt Wilson Contr.* No. 649.
[2]) Computed with $M_0 = -2·2$ and $-1·4$, respectively, and a dispersion of $\pm 0^{M}·5$.

ively, if a normal value for A (·018 km/sec.ps) be assumed; but it should be noted that if there is a large asymmetrical drift, as for the stars in the first line, the variation of the systematic motion with the distance from the centre may be quite different from the variation of the circular motion, so that the differential rotation would not be comparable with that found in the case of nearly circular orbits (for a numerical estimate of this effect see the last article of this Bulletin). It is quite conceivable that also the last two groups have been influenced by this.

With the data in the first part of Table 1 the density gradients $\partial \log \nu / \partial \varpi$ were calculated, ϖ being expressed in kps. For this calculation we have made use of formula (31), *B.A.N.* No. 159, after having verified that in each of the groups considered the distribution of peculiar velocities in any one component could be sufficiently well represented by a Gaussian function; for this had been presupposed in deriving the formula.

Introducing the quantities V_0, a and b as defined above, instead of Θ_0, h and k, the formula may be written:

$$\frac{\partial \log \nu}{\partial \log \varpi} = -\frac{2}{\pi}\frac{V_0(2\Theta_c - V_0)}{a^2} - \left\{1 - \frac{b^2}{a^2}\right\}.$$

If we want to determine $\partial \log \nu / \partial \varpi$ the above expression should be multiplied by Mod/ϖ. Assuming the circular velocity Θ_c to be 270 km/sec, and the distance, ϖ, to the centre 8 kps, we obtain the results shown in the last two columns of Table 1. The weighted average value of $\partial \log \nu / \partial \log \varpi$ is $-5{\cdot}12 \pm {\cdot}97$ m.e., corresponding to $\partial \log \nu / \partial \varpi = -{\cdot}278 \pm {\cdot}053$ m.e. In the computation of the mean errors no account has been taken of the uncertainty in Θ_c and ϖ. It should be noted that in $\partial \log \nu / \partial \varpi$ the only important factor is the *ratio* $\Theta_c/\varpi = A - B$; the uncertainty of this ratio will not be much greater than 10%.

The density gradient determined from the distribution of the stars.

Several investigators have in the past commented upon the asymmetrical distribution of long-period variables or of Md stars, and upon the great abundance of faint variables of this type in the Sagittarius regions [1]).

The distribution is complicated by the circumstance that it has apparently been strongly influenced by observational selection as well as by the effects of varying absorption. No attempt has so far been made to estimate the density gradient from the distribution of these stars.

Yet, the fainter long-period variables give distinct evidence of concentration towards the galactic centre. In the accompanying graphs we have plotted the variables with periods of 100 days and longer, and with amplitudes exceeding $2^m{\cdot}4$, contained in the *Katalog und Ephemeriden veränderlicher Sterne* for 1941. The stars have been plotted in galactic co-ordinates on an equal-area Mercator-Sanson projection of the sphere. The curve drawn in each figure represents the equator.

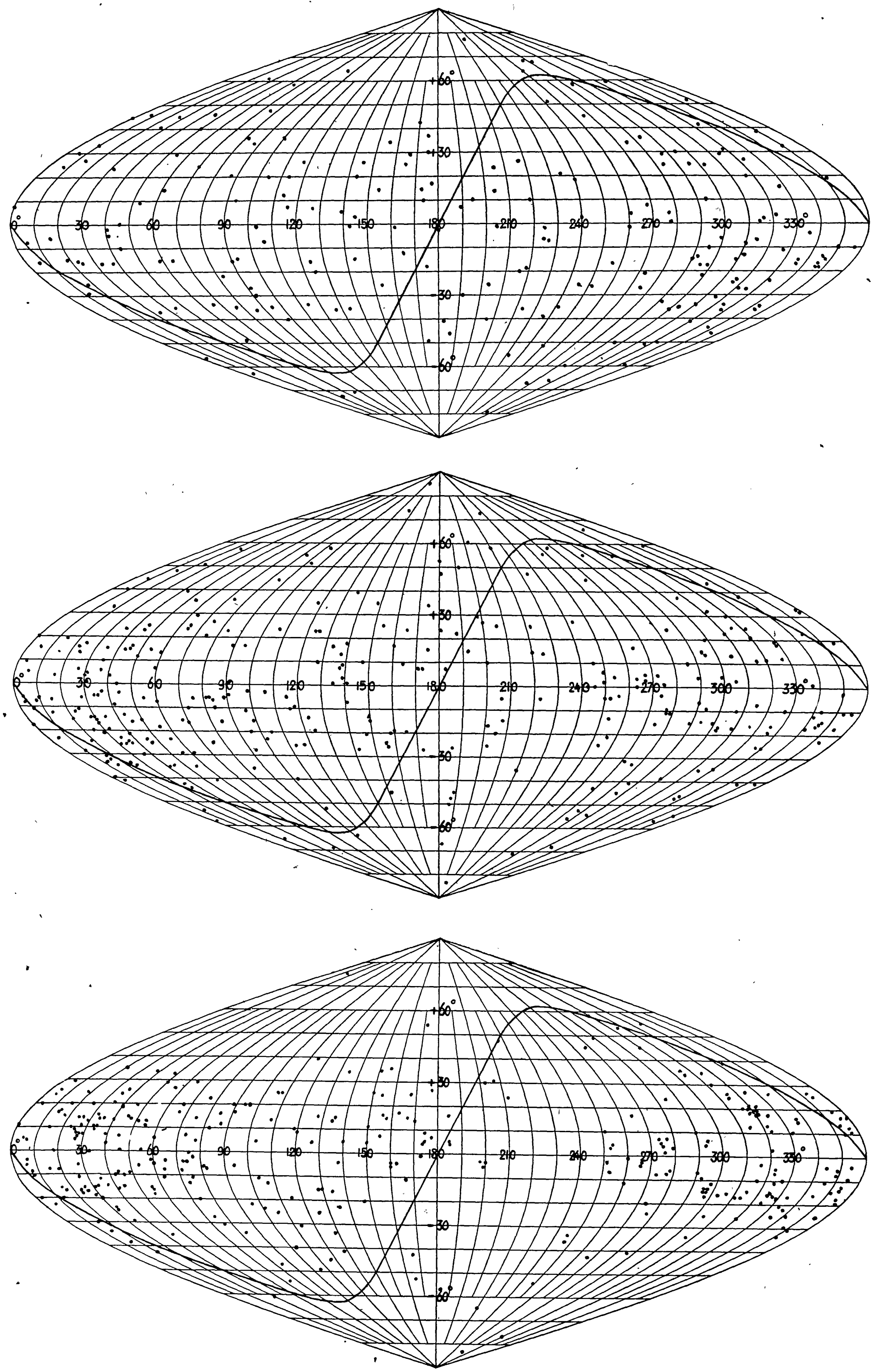
Even a superficial inspection of the figures shows that the material is far from homogeneous. It is evident that the incompleteness of our knowledge of these variables is very different in different parts of the sky. The most striking difference occurs between the two hemispheres: in the third figure there is a large discontinuity in the distribution of variables when passing from the northern to the southern hemisphere, and to a less extent this is also present in the second figure. In studying the density distribution it will evidently be necessary to consider the two hemispheres separately, and to discard the regions near the equator, where northern and southern observations overlap. We cannot expect that all observational selection will be eliminated by this procedure, but we may hope that at least large part of the effect will disappear when the results for the two hemispheres and different zones of latitude are combined. This hope seems to be confirmed by the results (see Table 4).

Table 3 shows the numbers of variables counted in 30°-intervals of longitude for two zones, extending from $-20°$ to $+20°$, and from $\pm 21°$ to $\pm 40°$ galactic latitude, respectively. In the table these zones are denoted as $b = 10°$ and $b = 30°$. The lon-

TABLE 3.
Observed distribution of long-period variables

l \ m, b	7·1—8·0 10°	7·1—8·0 30°	8·1—9·0 10°	8·1—9·0 30°	9·1—10·0 10°	9·1—10·0 30°
125°—165°	9	5	17	8	19	8
85 —125	10	10	17	8	14	8
45 — 85	5	7	19	18	38	8
5 — 45	11	9	39	24	55	15
305 —345	12	14	17	14	33	17
265 —305	11	13	24	3	25	3
225 —265	6	5	12	4	7	2
185 —225	8	7	8	8	5	3
total	204		320		325	
$\bar{r}$ (kps)	·64	·76	·96	1·18	1·48	1·80

1) Especially SHAPLEY and Miss CANNON in *Harvard Circ.* No. 245, 1923. The asymmetrical distribution has been most clearly exhibited in a graph published by GYLLENBERG in *Lund Medd.* Serie II, No. 54, Fig. 2, 1929, which shows the distribution of all long-period variables. Later investigators, like THOMAS (*Babelsberg Veröff.* **9**, Heft 4, 1932) and AHNERT (*A.N.* **269**, 241, 1939) have added little to the problem of the general distribution in longitude.



Distribution of long-period variables in galactic co-ordinates.

The upper figure shows all variables in the *Katalog und Ephemeriden* for 1941 with periods in excess of 100 days and amplitudes in excess of $2^{m}\cdot 4$, whose visual magnitudes at maximum are between $7^{m}\cdot 1$ and $8^{m}\cdot 0$; in the second figure those between $8^{m}\cdot 1$ and $9^{m}\cdot 0$ have been plotted, and in the third those between $9^{m}\cdot 1$ and $10^{m}\cdot 0$. The curve indicated in each figure is the equator.

gitudes from 345° to 5° and from 165° to 185° have been left out because of their proximity to the equator. The 117 stars with maximum magnitude 7·0 and brighter were omitted because they are too near to give information on the density gradient. The total numbers of stars over the whole sky in the other magnitude groups are given in the next to last line of the table. The bottom line shows the average distances computed with the data given at the end of the following article ($M_0 = -2{\cdot}1$, $\varepsilon = \pm 0{\cdot}5$).

In each hemisphere considered separately there is clear evidence of a density gradient in the direction of the centre. We have convinced ourselves that there is no conspicuous difference in this respect between stars with periods larger and smaller than 300 days respectively. With the aid of the mean distances just mentioned we may calculate the average co-ordinates corresponding to the stars of various magnitudes in each sector of longitude, and thus derive numerical values for $\partial \log \nu / \partial \varpi$. The results and their mean errors are shown in Table 4; the various magnitudes have been combined, as there appeared to be no systematic variation with the magnitude. The same weight was given to the logarithm of each number in Table 3; this is not accurate, of course, but as the main uncertainty will probably come from systematic differences in absorption and observational incompleteness we have not considered it useful to try a more refined system of weighting.

TABLE 4.

Density gradient per kps from the space distribution of long-period variables

	$b = 10°$	$b = 30°$
northern hemisphere	−·226 ±·065	−·179 ±·059
southern hemisphere	−·351 ±·055	−·192 ±·132
weighted average	−·299 ±·042	−·181 ±·054

The agreement between the two hemispheres and between the two latitude zones is satisfactory; this shows that the gradients are real and have not been excessively influenced by effects of observational selection or absorption. The total average is − ·254 ± ·033 m.e. Comparing with the independent value − ·278 ± ·053 m.e. derived from the motions we see that this latter, indirect, determination finds a remarkable confirmation in the direct evidence from the space distribution; we may conclude that the mean gradient of − ·27 is now fairly well established. This corresponds to a diminution of the density proportional with the 5th power of the distance from the centre. The gradient found for the long-period variables is about twice as steep as that found for the stars in general in the layer between 1000 and 1500 ps from the galactic plane [1]).

Incidentally, these calculations give an impression of the relative incompleteness of the long-period variables in the southern hemisphere as compared to the northern. The ratios of the observed star density in the southern hemisphere to that in the northern, when both are reduced to a point differing 90° in longitude from the centre, come out as follows

$7^m{\cdot}1$	to	$8^m{\cdot}0$	·92
8·1	„	9·0	·38
9·1	„	10·0	·25

It is thus clear that up to $8^m{\cdot}0$ the southern variables have been discovered as completely as the northern ones; this is in agreement with the result found on p. 333 that up to $8^m{\cdot}0$ there are no signs of serious incompleteness anywhere. The incompleteness between $8^m{\cdot}1$ and $9^m{\cdot}0$, where, according to p. 333, 39% of the variables are still undiscovered, appears to be almost wholly confined to the southern hemisphere. For the interval $9^m{\cdot}1$ to $10^m{\cdot}0$ both hemispheres are badly incomplete; in the north 60% of the variables in this range are still undiscovered, and in the south no less than 90%. There is thus no good ground for the supposition made by AHNERT [2]) that the variables would be practically complete down to $10^m{\cdot}4$ vis.

It would appear to be a most useful undertaking to make a systematic search for bright long-period variables over the whole sky. Both on account of their high intrinsic luminosities as well as of their relatively frequent occurrence these variables would also be very suitable for a determination of the distance of the centre of the galactic system, in the same way as this has been tried by VAN GENT [3]) and by SHAPLEY [4]) with the aid of RR Lyrae variables. Of still greater interest seems the possibility of a new determination of the longitude of the centre, from the distribution of faint long-period variables above 20° latitude. So far, our knowledge of the direction of the centre rests wholly on the sparse globular clusters outside the galactic zone. It is a problem of prime importance to settle the question whether or not the longitude derived from the differential galactic rotation co-incides with that of the centre of the system; this could probably be done with the aid of the long-period variables.

1) See *B.A.N.* **8**, 262, 1938.
2) *A.N.* **269**, 241, 1939.
3) *B.A.N.* **7**, 29, 1933.
4) *Proc. Nat. Ac. Washington*, **25**, 117, 1939.