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Note on the secondary periods of Cepheid variables, by *Miss H. A. Kluyver*.

The theoretical importance of the phenomenon of commensurability between two of the fundamental frequencies of pulsating stars is pointed out. It should be possible to find the long secondary period, which occurs in the light-changes of a number of cluster-type variables, in the radial velocities too. In this connection the desirability of accurate determination of radial velocities for a few selected Cepheid variables, in the first place for RR Lyrae, is stressed.

In the pulsation theory of Cepheid variables the case of commensurability between two of the fundamental frequencies, evidence of which is found in the occurrence of long secondary periods in the light-variations of cluster-type variables, as well as in the shape of the radial-velocity curves of Cepheids, is especially interesting; it has an important bearing upon the theory of the maintenance of the pulsations[1]).

The observed long period has been ascribed to interference between two frequencies, one of which is approximately equal to a multiple of the other. Computations of the lower fundamental frequencies, based on plausible stellar models, have shown this to be very well possible. However, various authors[2]) have raised the objection that the observations seem to indicate that the two frequencies concerned are about equal, which is not to be expected from the above-mentioned computations. With regard to this apparent contradiction it should be pointed out that the observed long period need not be due to interference between the fundamental frequencies n and n'. It might as well be produced by n and $n' - n$, which are approximately equal if n' is about two times n. Moreover, it should be kept in mind that, if the secondary period is very long compared to the primary period, it does not correspond to the frequency $|2n - n'|$, but to the libration frequency ν[3]), which may, for close commensurability, differ considerably from $|2n - n'|$; hence, in that case, the frequency n' cannot be computed from the observed long period and the primary period.

Our knowledge of the long secondary periods is chiefly based on the extensive series of accurate observations of RW Draconis, AR Herculis and RR Lyrae by Miss J. Balázs and L. Detre[1]), and of RS Bootis by P. Th. Oosterhoff[2]), as well as on the thorough analyses of the light-variations given by these authors.

The star RR Lyrae itself was known already for a long time to exhibit in its light-variation a secondary period of about seventy times the primary period of $^{d}\cdot 567$. In 1943 Detre[3]) published an extensive analysis of 6512 photographic observations of RR Lyrae made at Budapest during the years 1935 to 1941, as well as of many older series of visual and photographic observations by other astronomers, the whole material covering an interval of more than 27000 times the primary period. Detre succeeded in representing the epochs of maximum and of the middle of the rising branch of the light-curve with the aid of two arguments, of the form $nt + \omega$ and $\nu t + \varepsilon$; n corresponds to a period of $^{d}\cdot 56683500$ and ν to a period of $40^{d}\cdot 71$, or $71\cdot 83$ times the first period. The quantities ω and ε change slowly with time; Detre's graphs[4]) showing these changes during the interval considered, are very convincing. The analytical representations of these curves are still uncertain, and the future behaviour of ω and ε cannot yet be predicted.

For many other stars the primary argument is known to contain a slowly changing term, but, so far as I know, RR Lyrae is the first star for which both the linear terms and the slowly changing parts of the two arguments have been accurately determined.

Although the spectroscopic observations made in America during recent years have shown that the

[1]) Dr J. Woltjer Jr, *B.A.N.* No. 303, 193, 1937; *B.A.N.* No. 359, 435 and 441, 1943.

[2]) T. E. Sterne, *Ap. J.* **87**, 133, 1938.

J. Balázs and L. Detre, *Mitteilungen der Sternwarte Budapest-Svábhegy*, Nr. 8, 1939; Nr. 18, 1943.

[3]) See e.g. *B.A.N.* No. 276, p. 320 etc., 1936.

[1]) *Mitteilungen der Sternwarte Budapest-Svábhegy*, Nr. 5, 1938; Nr. 8, 1939; Nr. 18, 1943.

[2]) *B.A.N.* No. 369, 101, 1946.

[3]) *Mitteilungen der Sternwarte Budapest-Svábhegy*, Nr. 17, 1943.

[4]) *L.c.* Figures 19 and 21.