Galaxy Collisions and Their Influence on the Dynamics and Evolution of Groups and Clusters of Galaxies

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Summary. We investigate the effect of collisions on the dynamics and evolution of groups and clusters of galaxies. It is assumed that galaxies are formed initially with massive spherical halos and their subsequent evolution is studied using N-body simulations. Firstly, individual galaxy-galaxy collisions are simulated for a large number of impact parameters, velocities and masses with approximately 30 point masses in each colliding galaxy. Loss of orbital kinetic energy, merging criteria, increase of internal energy and mass-loss from the galaxies are discussed. Secondly, the results of these calculations are used in calculating the energy change and mass loss of the galaxies in a cluster of 10–60 members. It is shown that, in particular during the collapse of a group or cluster of galaxies, a considerable fraction of the galaxies merge and that elliptical galaxies are good candidates for these merged systems. Observational consequences are discussed for a number of properties of groups of galaxies such as the occurrence of interacting galaxies, the relative number and spatial distribution of elliptical and disk galaxies, the formation of $cD$-galaxies, and the mass-to-light ratio of groups.

Key words: galaxy collisions – clusters – elliptical galaxies

1. Introduction

As early as 1951 Spitzer and Baade pointed out that collisions between galaxies must occur frequently in the central regions of rich clusters. Since then the possibility that galaxies are much larger than their visible component has revived interest in the galaxy collisions. Indications that galaxies have or have had massive halos are fairly convincing. Studies of individual galaxies (Roberts and Rots, 1973; Hartwick and Sargent, 1977; Bosma, 1978), binaries (Turner and Ostriker, 1977), instabilities in disk galaxies (Ostriker and Peebles, 1973) and of galaxy formation (Rees and Ostriker, 1977) give indirect evidence for the existence of dark massive halos surrounding the visible parts of galaxies. We can estimate an upper limit to the mass of these halos from the mass of rich clusters of galaxies derived by the virial theorem. This mass is about ten times larger than the mass present in the visible part of the galaxies. Recent studies (reviewed by Bahcall, 1977) indicate that this "missing mass" does not consist of neutral or ionized gas, but probably of some kind of clumpy material with a very high $M/L$-value (White, 1977). Another indication of the stellar nature of the missing mass comes from recent observations that the hot intracluster medium has approximately the solar abundance and a total mass equal to that present in the visible part of the galaxies (Malina et al., 1978). This suggests that not only the visible mass of the cluster, but the total virial mass was involved in producing this gas. Therefore we feel justified in calculating collisions of galaxies with massive halos and studying their effect on the dynamics and evolution of clusters of galaxies.

2. Galaxy Collisions

2.1. Review of Previous Work

Both numerical and analytical approaches to the problem of collisions between galaxies consisting of a number of point masses have been utilized, but it was only a few years ago that the first selfconsistent $N$-body calculations of collisions between two spherical galaxies were reported. In earlier calculations it was assumed that the over-all structure of the galaxies is not changed during a collision thus restricting the applicability of these calculations to velocities that are much larger than the velocity dispersion of the stars in the galaxies. Analytic calculations of this kind were done by Alladin in 1965 (see also Sastry and Alladin, 1977) assuming that the internal motions of the stars within the galaxies may be neglected during a collision (the "impulsive approximation").

In the numerical work by Richstone (1977) collisions between truncated isothermal spheres as described by King (1966) were calculated. He calculated the orbits of the stars under the influence of the gravitational field of the colliding galaxies, which were assumed to be rigid. From his work, and also from the work of Biermann and Silk (1977), it is evident that collisions must be penetrating in order to give any significant changes in the internal energy. They find mass losses and energy changes less than about 10% in the velocity region for which their assumptions are valid. Structural changes of the galaxies during low impact velocity encounters make a self-consistent calculation of the stellar orbits necessary. Such calculations (Toomre, 1974, 1977; Lauberts, 1974) indicated that collisions can become very inelastic at low velocities.

An axially symmetric model for the head-on collisions of up to 2000 particle galaxies with a density distribution of an $n = 3$ polytrope has been discussed by van Albada and van Gorkom (1976). For low velocities highly inelastic behaviour is found and in particular merging of the galaxies occurs at relative velocities below 3.2 $\sigma$ where $\sigma$ is the internal velocity dispersion of the galaxies (van Albada and van Gorkom, private communication). They find that loss of orbital kinetic energy decreases rapidly with increasing relative velocity. Their code cannot accurately describe...
non-zero impact parameter collisions because both galaxies are assumed to remain symmetric about the line connecting the two centres of gravity.

White's calculations (1978) show that collisions between two spherically symmetric galaxies in a parabolic orbit are strongly inelastic as long as the mass distributions of the galaxies overlap significantly at closest approach.

However systematic information on the effects of galaxy collisions, especially for low collision velocities, is still lacking. Clearly this information is necessary if one wants to estimate the effects of collisions on the dynamics and evolution of clusters of galaxies.

2.2. Description of the N-body Simulation

The numerical code is a three dimensional N-body code in which the integration of the orbits is done with the same predictor-corrector method as used by an Albada and van Gorkom (1977). As usual in this kind of calculation the Newtonian potential is modified from \( r^{-1} \) to \( (r^2 + \epsilon^2)^{-1/2} \), where \( \epsilon \) is called the softness parameter, in order to suppress large accelerations in close encounters. The time-step is adjusted during a calculation so that the changes in the velocities in one step will not exceed a chosen small value. In our calculations the total energy of the N-body system does not change by more than a few tenths of a percent during 2,000 time-steps. For this type of simulation time reversibility will not be satisfied because the deviations from the "true" orbits grow quickly but the overall statistical properties such as total energy, angular momentum, and total momentum will be conserved as discussed by Smith (1977). The calculation time was found to depend strongly on the value of \( \epsilon \) because short range interactions and thus \( \epsilon \) determine the time step. A PDP 11/45 was used for the calculations of the collisions.

We formed equilibrium galaxies by allowing an initial spherical distribution of about 40 equal mass particles to evolve towards dynamical equilibrium during several crossing times. The \( \epsilon \) used was 10 kpc, about one quarter of the final radius of the smallest galaxy. The trial and error method was used to find initial conditions which formed galaxies with a range of masses from 0.5–2.0 \( 10^{12} M_\odot \) and an internal velocity dispersion of about 250 km s\(^{-1}\). During galaxy formation those mass-points that had enough energy to escape to radii larger than 100 kpc were thrown out of the simulation. The final distribution is similar to a truncated isothermal sphere (King, 1966).

The relaxation time is not very long for a system consisting of only 30 particles. We find no changes in the mean structure of the galaxies during at least 10 crossing times which is about the time we need to calculate a collision. However even after 10 crossing times the galaxies with \( N = 30 \) appeared to have fluctuations about the equilibrium state accounting for roughly 10% of the potential energy with a characteristic time of a crossing time. The amplitude of these apparently radial oscillations tend to decrease with increasing number of particles. In order to trace the effects of these fluctuations we performed collision calculations with an \( N = 94 \) galaxy. The properties of the galaxies we used in the collision calculations are displayed in Table 1. Columns 3, 4, and 5 of this table gives the half-mass radius of the galaxies, the internal velocity dispersion and the number of point masses of the galaxy respectively.

These galaxies are then collided with one another for a large number of impact parameters and relative velocities with an initial separation of about 400 kpc. The calculation is continued after the collision until the galaxies have had time to reach equilibrium again. After the collision, the loss of orbital kinetic energy, the gain of internal energy, the mass loss and the change of internal angular momentum is calculated for each galaxy. We define the mass loss as the percentage of the total mass that can reach a radius >150 kpc, i.e. about half the mean intergalactic distance in a dense group of galaxies.

2.3. Results and Interpretation

In this section we will use the symbols \( U \) and \( \sigma \) for the internal energy and internal velocity dispersion of a galaxy, \( E, p, \) and \( v \) for

Table 1

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<th>( M ) ( 10^{11} M_\odot )</th>
<th>( R_{1/2} ) 100 kpc</th>
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the orbital kinetic energy, the impact parameter and relative velocity of the galaxies at minimum separation and $E_0$, $p_0$, and $v_0$ for these quantities measured at the initial separation of 400 kpc.

2.3.1. Change of Internal Energy of Colliding Galaxies

In a collision the internal energy of the galaxies increases due to mutual tidal interaction. The short range tidal force is most effective during the overlap of the two galaxies in a head-on collision. At maximum overlap of two equal galaxies the inward gravitational force on each particle becomes about twice as large, resulting in a radial contraction followed by an outward splash (Toomre, 1977). The radial oscillation that is excited will be damped out quickly by violent relaxation leaving the galaxy with a higher internal energy than before. The characteristic time scale of the potential variation during a collision is of the order of the collisional crossing time or overlap time. The characteristic response time of the galaxy to an extra gravitational impulse at its center is the free fall time. Thus we expect resonance-like behaviour to occur in head-on collisions at collision velocities of a few times the internal velocity dispersion of a galaxy.

The solid lines in Fig. 1 give the fractional change of the internal energy of galaxy 1 in head-on collisions with itself (1–1) and with galaxy 3 (1–3) as a function of the collision velocity in units of its internal velocity dispersion. The crosses along the dashed line are results obtained with galaxy 4 ($N=94$). The black squares are results obtained by van Albada and van Gorkom (1978). There are several things to note in this figure. First, there is a very close agreement between the dashed line and the calculations by van Albada and van Gorkom with $N=1000$ galaxies. The agreement between these results and the solid line 1–1 is best for higher velocities. At velocities $\geq 3.5 \sigma$ the dashed curve shows a fall off proportional to $v^{-2}$ as predicted by the impulsive approximation.

The discrepancy between the solid line and the dashed one is caused by the fluctuations around the equilibrium state that galaxy 1 exhibits. Each collision calculation starts at the same initial separation and a galaxy always starts at the same initial phase of its radial oscillation. Hence, the phase at the moment of collision depends only on the initial velocity of the galaxies.

When a galaxy is contracting at the moment of collision this contraction will make the induced contraction stronger and $\Delta U/U$ larger. Similarly, at different velocities a minimum will occur if the collision occurs in an expansion phase. In this way the oscillation of the solid line 1–1 around the dashed line reflects the oscillatory behaviour of galaxy 1.

The increase in the internal energy is accounted for by a loss of orbital kinetic energy of the colliding galaxies, and at velocities lower than $3.1 \sigma$ the galaxies merge. Note that this corresponds to a relative velocity at infinity of about $1.2 \sigma$ (Sect. 2.3.5).

The solid line 1–3 in Fig. 1 shows the change of internal energy that galaxy 1 undergoes as a result of a collision with galaxy 3 which is about half as massive and half as large. $\Delta U/U$ is now about half as large, as expected on the basis of the impulsive approximation. However, galaxy 3 undergoes roughly the same change in internal energy as in a head-on collision with itself. The reason is that during a collision with galaxy 1, galaxy 3 feels the same amount of mass within its radius as in a head-on collision with itself. We adopt the following relation for different masses

$$\frac{\Delta U_1}{U_1} = \frac{M_2}{M_1} \frac{\Delta U_2}{U_2}$$

or, using

$$U = -\frac{1}{2} M \sigma^2,$$

$$\Delta U_1 = \Delta U_2$$

in the case where $\sigma_1 = \sigma_2$ and $M_1 \geq M_2$, while

$$\frac{\Delta U_2}{U_2} \simeq 0.5 \left(\frac{3.5 \sigma}{v}\right)^2$$

for $v \geq 3.5 \sigma$.

We now turn to the $p \neq 0$ collisions. Since we will not have such a strong radial contraction in these cases the differences between the $N=94$ and $N=30$ galaxy collisions will become smaller. It is not simple to isolate the $v$- and $p$-dependences of the energy changes, especially since the variations with velocity are complicated by the radial oscillations of the $N=30$ galaxies. The ratio of the loss of orbital kinetic energy at a particular impact parameter $p$ and velocity $v$ to the loss at the same velocity with $p=0$ is plotted versus $p$ on Fig. 2. The data cover a small range of collision velocities $3 \sigma < v < 4.5 \sigma$, but the collisions with high velocity (filled circles) and low velocity (open circles) do not have systematically different values of $\Delta E$, which we assume has the form

$$\Delta E(p,v) = f(p) g(v).$$

A reasonable linear fit is given by

$$\Delta E(p,v) = \Delta E(p=0,v) (1 - p).$$

The dependence of $\Delta U$ on impact parameter now follows since $\Delta U$ and $\Delta E$ must have the same $p$-dependence (see 2.3.6).

The critical velocity, $v_{\text{crit}}$, the largest velocity for which merging occurs, decreases with increasing $p$. In Fig. 3 values of the velocities and impact parameters for some mergers are given. These points all lie slightly below $v_{\text{crit}}(p)$. In order to account for the small differences in internal velocity dispersion the velocities have been normalised to the average velocity dispersion of the two merging galaxies. The $p$-dependence of the critical velocity can be approximated by the following linear relation:

$$v_{\text{crit}}(p) = 3.1 (1 - 0.4 p), \quad p < R_g$$
In general $\Delta M/M$ and $\Delta U/U$ behave similarly as a function of the collision velocity (Fig. 4). There seems to be no significant dependence of $\langle \Delta M/M \rangle$ on the velocity. Taking averages over different collision velocities it appears that $\langle \Delta M/M \rangle$ is slightly larger than $1/2 \langle \Delta U/U \rangle$ in the cases $1-1$ and $1-2$ and slightly below $1/2 \langle \Delta U/U \rangle$ in collisions between galaxies 2 and 3 which have smaller radii.

Sastry and Alladin (1977) have noted already that

$$ \langle \Delta M/M \rangle \langle \Delta U/U \rangle$$

might be larger for $p \neq 0$ than for head-on collisions. We find:

$$\langle \Delta M/M \rangle = (0.45 \pm 0.2) \langle \Delta U/U \rangle \quad \text{for} \quad 0 < p < 0.3 \ R_g$$

and

$$\langle \Delta M/M \rangle = (0.7 \pm 0.4) \langle \Delta U/U \rangle \quad \text{for} \quad p > 0.3 \ R_g \quad (6)$$

Of course we have to treat mergers separately (see Fig. 4). For mergers we find roughly (with our definition of mass-loss)

$$\langle M_{\text{merger}} \rangle \approx \frac{M_1}{2} + \frac{1}{2} M_2, \quad \text{where} \quad M_1 \geq M_2. \quad (7)$$

If $M_1 = M_2$, both galaxies will lose $\sim 1/4$ of their mass, while in the case $M_1 \gg M_2$ galaxy 1 will not be perturbed very much, but galaxy 2 will lose about half its mass.

Equation (7) is certainly a very rough estimate because the mass of the merger will depend on the relative velocity at infinity and probably on the impact parameter. The velocity dispersion of the merger is about the same as the velocity dispersion of the merging galaxies. When the energy of the lost mass is about zero we can write:

$$U_{\text{merger}} = U_1 + U_2 + \frac{1}{2} \frac{M_1 M_2}{M_1 + M_2} v^2$$

or

$$-\frac{1}{2} M_{\text{merger}} \sigma^2 = -\frac{1}{2} M_1 \sigma^2 - \frac{1}{2} M_2 \sigma^2 + \frac{1}{2} \frac{M_1 M_2}{M_1 + M_2} v^2,$$

or

$$M_{\text{merger}} = M_1 + M_2 - \frac{M_1 M_2}{M_1 + M_2} \frac{v^2}{\sigma^2} \quad (8)$$

As we will see in Sect. 2.3.4 merging takes place at relative velocities at infinity lower than about $1.2 \sigma$ so whenever a merger takes place the mass of the merger will lie between $M_1$ and $M_1 + M_2$ where $M_1 \geq M_2$. This might explain the average value we find in Eq. (7). We could also use as a crude approximation of (6)

$$\Delta M/M = 0.5 \Delta U/U,$$

which means with Eq. (1)

$$\Delta M = \Delta M_2.$$

In the velocity region around $v \approx 3.5 \sigma$ (and small $p$) this will be about $1/4$ of the mass of the smallest galaxy. This gives for the sum of the masses of the galaxies after the collision, $M_1 + 1/2 M_2$, where $M_1 \geq M_2$.

2.3.3. Structural Changes of the Galaxies

Figure 5 shows the total mass $M(R)$ inside radius $R$ as a function of the radius for galaxy 1 before and after collisions with an identical galaxy. There are indications that the outer parts lose more mass than the central parts. Figure 6 shows the mass distributions for galaxy 1 and some mergers of galaxy 2 with its
twin. Mergers tend to have a higher central density than the unperturbed galaxy, a result that is in agreement with White (1978).

2.3.4. Transfer of Angular Momentum

In collisions where the loss of orbital kinetic energy was not very large the relation

\[ pv = p_\infty v_\infty \]  \hspace{1cm} (9)

was valid to an accuracy better than 5%, which means that the loss of orbital energy and consequently the loss of orbital angular momentum occurs mainly after the two galaxies have reached minimum separation. Our small particle number galaxies do not allow us to make explicit statements on the rotational structure of the collided galaxies, but a large portion of the lost orbital angular momentum is carried away by the particles that are “lost” from the galaxies. Also in the case of merging, a large fraction of the orbital angular momentum is transferred into the outer parts of the galaxies. The central parts are brought together with much lower angular momentum per unit mass so that the angular momentum per unit mass of the final merger will increase with radius.

2.3.5. Relation between \( (p, v) \) and \( (p, v)_\infty \)

We need two relations connecting the collision parameters at infinity and at the moment of closest approach. The first one is given by (9). This relation breaks down when the inelasticity is very large. A second relation can be derived easily for head-on collisions if we assume that the galaxies do not change during the collision. This relation states that the kinetic energy at infinity is equal to the sum of the kinetic energy and the extra potential energy \( \Delta \phi \) that the galaxies feel when they are overlapping. When the masses are equal we get

\[
\frac{1}{2} M \sigma^2 = \frac{1}{2} M v^2_\infty + |\Delta \phi|
\]

\[
|\Delta \phi| = |\Delta \phi_1| + |\Delta \phi_2| = 2 M \sigma^2
\]

so

\[
\nu^2 = 8 \sigma_\infty^2 + 4 \sigma^2.
\]  \hspace{1cm} (10)

When two unequal mass galaxies are overlapping \( (\sigma_1 = \sigma_2, M_1 \geq M_2) \) we have (see Sect. 2.3.1)

\[
|\Delta \phi_1| = |\Delta \phi_2| = M_2 \sigma^2.
\]
with $M_1 \geq M_2$, and $R_1$ the radius of galaxy 1. Note that (12) can be used to calculate $v$. At high velocities the dependences on the mass-ratio and impact parameter explicitly shown in (14) give a good explanation of the behaviour of $\Delta E/E_0$ shown in Fig. 6. At low velocities ($v_0 \lesssim 2\sigma$) the first term in (12) becomes important reducing the sensitivity of $\Delta E/E_0$ to $M_2/M_1$ and $p$. Neglecting the second term in (12) and using $p = p_0/2$ we find for $\Delta E/E_0$ at velocities close to the critical velocity

$$\Delta E/E_0 = \frac{3}{5} \left( \frac{1 - 0.4 p_0/R_1}{1 - 0.2 p_0/R_1} \right)^{1/2}$$

This explains why the critical velocity measured at the initial separation is only a very weak function of $M_2/M_1$ and $p$. Galaxies therefore have a large cross-section for merging.

Since $\Delta E/E_0$ will have the same value at the critical velocity, we find

$$v_{\text{crit}}(p) = 1.6 \sigma \frac{1 - 0.4 p_0/R_1}{1 - 0.2 p_0/R_1} \left( \frac{1 + M_2/M_1}{2} \right)^{1/4}$$

This is not a bad representation of the data given in Fig. 3. Calculations with a larger number of particles suggest that a value of $3.2 \sigma$ for the critical collision velocity in head-on collisions between equal galaxies is probably better than $3.1 \sigma$. This value corresponds to $v_0 = 1.6 \sigma$, or $v_\infty = 1.2\sigma$. A smaller softness parameter might give a critical velocity that is even a few percent higher.

### 3. The Evolution of Groups and Clusters of Galaxies

#### 3.1. Introduction

In Sect. 2 we have shown that at low relative velocities, collisions between galaxies will have strong effects. Such effects have not been included in previous work on the dynamics of clusters of galaxies. We will show here that collisions between galaxies might explain several observed features of groups and clusters of galaxies if galaxies are formed with massive halos. Initially all our galaxies have a mass of $1.4 \times 10^{12} M_\odot$ and a radius of 80 kpc. Galaxies in groups and clusters will have about this mass if all the mass of the cluster is bound to the galaxies. We further assume that all galaxies in a cluster are formed before the region of the universe to which they belong reached maximum expansion. When the calculations are started the clusters do not have any systematic radial motion.

#### 3.2. Description of the Calculation

The same code used for $N$-body collisions in Sect. 2 is used here but with a larger softness parameter. This is an important parameter in the code since our galaxies must not behave like point masses at small distances thus reaching high collision velocities. If $\varepsilon$ were chosen to be too large the velocity at the moment of collision would be underestimated and the effects of the collision overestimated. We have adopted $\varepsilon = 25$ kpc. This value will give collision velocities for head-on collisions that are a bit too large, but it is a good approximation for larger impact parameter collisions which are probably the most numerous.

To use the results of the galaxy collisions in a cluster calculation we have made some model fits to the data. It is not necessary to
Fig. 8a–f. Projected distributions of the galaxies after the evolution of the models. The scale is the same for all pictures. (Fig. 12a) Crosses are galaxies, big dots are merged galaxies, big dots in circles are multiple mergers and small dots are background particles stripped from the galaxies. The time unit is $6.6 \times 10^9$ yr.
Table 2

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use exact data for each collision since we are mainly interested
in the over-all effects of collisions in a cluster calculation.

For the critical collision velocity at zero impact parameter we
adopt

$$v_{\text{crit}}(p=0)=700 \text{ km s}^{-1}.$$  

For the dependence of the critical velocity on the impact parameter
we use Eq. (4).

The loss of orbital kinetic energy is approximated by

$$\Delta E/E = 0.1 \left(1 - p/80 \text{ kpc}\right).$$ (17)

If $p > 80$ kpc or $v > 1000$ km s$^{-1}$ the collision is inelastic. To
calculate the mass-loss in the case of a merger we applied (7) and
in the region $700 < v < 1000$ km s$^{-1}$ we used $\Delta M/M = 0.25$. The
calculation is started by choosing random positions for the galaxies
in a sphere of radius $1 \leq R_0 \leq 2.5$ Mpc according to a density
distribution of the form $n(r) \propto r^2 (1 - r/R_0)$. This density
distribution has a maximum in a spherical shell at a distance of $2/3 R_0$
from the cluster centre. During the collapse this shell breaks up
into subclumps (see Fig. 7a) which merge into one cluster. The
initial velocity distribution is Maxwellian with a small velocity
dispersion of $60$ km s$^{-1}$. Initially galaxies are further than $300$ kpc
apart. In every collision the total momentum of the centre of mass
of the two galaxies is conserved. The mass-loss that a galaxy
undergoes is stored until it exceeds one-quarter of the initial mass
of the galaxies. A background particle having this mass is then
created and sent away radially from the parent galaxy in a random
direction starting from a distance of $150$ kpc with a velocity which
we choose to be $60$ km s$^{-1}$.

During the evolution of a cluster the total energy must be
conserved i.e. the sum of the potential and kinetic energies of the
point masses and the internal energies of the galaxies. We are
unable to treat mass- and energy-losses in a completely consistent
way because of our lack of knowledge of the structure of the galaxies
after a few collisions. We realize that our choice of
treating mass-loss is still rather crude and that a different choice
might influence the final mass-distribution of the cluster.

The maximum number of point masses in these calculations is
larger than in the collision calculations so the IBM 370/158
machine was used for this part of the work.

3.3. Results and Interpretation

A sequence of 4 models was run with an initial radius lying in the
range 1–2.5 Mpc and with $N$ galaxies, where $N=10, 30, 45,$ and 60,
randomly distributed in the initial volume as described in the
previous section. The results are given at different epochs for
these models in Table 2. The first four columns give the properties
of the models at $t=0$. Columns 6 and 7 give the average distance
to the cluster centre for all the galaxies and the merged galaxies.
Columns 8 and 9 give the velocity dispersion for all the galaxies
and for the merged ones. Columns 10, 11, and 12 give the per-
centage of mergers, the total number of galaxies and the number
of background particles. For comparison, we did the same cal-
culations for the case of elastic collisions. It is clear that in each
simulation collapse has occurred. From previous $N$-body cal-
culations by Peebles (1970) it is known that virialisation occurs
after approximately twice the collapse time starting from maximal
expansion. The collapse and virialisation times are proportional
to $\rho_0^{-1/2}$ where $\rho_0$ is the initial mass density. Consequently model 1
is expected to reach virialisation first. At the end of the calculation
models 1 and 2 have entered the virialised regime. Models 3 and 4
are approaching this virialised state. Projected galaxy distributions
for model 1, 2, and 3 are given at various epochs in Fig. 8.

3.3.1. Number of Mergers and Strongly Interacting Binaries

The percentage of mergers as a function of time is given for the
four models in Fig. 8 with timescales proportional to the collapse
time of model 2. It can be seen that merging occurs predominantly
during the collapse. After the collapse mergers are about 20% of
the total number of galaxies. There is only a slow increase in the
number of mergers after collapse because the velocity dispersion
of the galaxies increases and thus the probability of mergers
which depends on low velocity collisions, decreases. The model
predicts a maximum in the number of interacting binaries in the
collapse stage of a group or cluster while the resulting number of
merged galaxies is still small. It may be possible to check this
observationally, the decay of the orbit of the two galaxies that
will eventually merge can take $\approx 2 \times 10^6$ yr, but the stage in which
the two central parts are merging together will only last a few
be understood on the basis of the following simple estimation. The mergers are formed during collapse of the cluster while the dispersion of the galaxies in the cluster is still smaller than the internal velocity dispersion of the galaxies. Starting with a velocity dispersion of about zero the time needed to accelerate the galaxies to \( \sigma \) in a cluster mass \( M_\text{cl} \), radius \( R_\text{cl} \), density \( \rho_\text{cl} \) is

\[
\Delta t = \frac{R_\text{cl}^2}{GM_\text{cl}} = \frac{3\sigma}{4\pi G R_\text{cl} \rho_\text{cl}}.
\]

The total number of mergers formed in this interval divided by the total number of galaxies is approximately

\[
\frac{N_\text{M}}{N} = \text{collision rate} \times \Delta t.
\]

For the collision rate we can write

\[
\text{collision rate} = \frac{\rho_\text{cl} \pi (2R_\text{e})^2 \sigma}{M_\text{g}}
\]

where \( \pi (2R_\text{e})^2 \) is taken for the cross section of a galaxy near the critical velocity. This is based on the results of 2.3.6. Putting these estimates together and using for galaxies mass \( M_\text{g} \)

\[
M_\text{g} = \frac{\sigma^2 R_\text{e}}{G}
\]

we get

\[
\frac{N_\text{M}}{N} \approx \frac{3R_\text{e}}{R_\text{cl}}.
\]

This is of course a rather crude estimation but it shows that the percentage of mergers will be of the order of 20% after the collapse of a cluster and that this percentage does not depend very much on the density of the cluster. The results presented in this section show that it is certainly possible to explain the overall percentage of elliptical galaxies via the merging picture as first suggested by Toomre and Toomre (1972). The actual percentage we have found probably depends on our initial conditions but it is not possible to prevent a considerable fraction of the galaxies from merging during the formation of groups and clusters if galaxies are formed with massive halos. Elliptical galaxies are the most likely candidates for these merged systems. In a subsequent paper the dependence of our results on the initial conditions will be investigated.

One might think of a scenario for the evolution of galaxies in which the gas lost from the stars in the massive halo during stellar evolution falls to the centre, dissipates its energy and settles down in a rotating disk when the halo has just a small amount of rotation. When two such galaxies merge the central luminous disk will be brought together and will also merge. We speculate that the parameter which may most strongly determine the ellipticity of the final merger is the angle between the disks of the two merging galaxies while the rotation of the merged galaxies will be determined by the orbital angular momentum of the colliding galaxies and their spins. Most of the orbital angular momentum will be transferred into the outer parts of the merger so that it might be possible to explain the low rotational velocities of elliptical galaxies in the merging picture (Illingworth, 1977). It seems likely that the central luminous part of the merger has a lower gas content than the disks from which it was formed because the relative velocity of the two disks during a merger is high enough to heat the gas in the disks to temperatures of \( 10^7 \) K if cooling is not important. This hot gas will quickly expand into a much larger volume than it occupied before the collision.

Table 3

<table>
<thead>
<tr>
<th>( N_0 = 60 )</th>
<th>( N_0 = 45 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( &lt;N_{\text{tot}}&gt; )</td>
<td>( &lt;N_{\text{tot}}&gt; )</td>
</tr>
<tr>
<td>0 &lt; ( R ) &lt; 0.7 Mpc</td>
<td>10.5</td>
</tr>
<tr>
<td>0.7 &lt; ( R ) &lt; 1.0 Mpc</td>
<td>8.5</td>
</tr>
<tr>
<td>1 &lt; ( R ) &lt; 3.0 Mpc</td>
<td>30.5</td>
</tr>
<tr>
<td>Total</td>
<td>19%</td>
</tr>
</tbody>
</table>

Times \( 10^8 \) yr, so the percentage of interacting binaries that one can see might be very low. The percentage of mergers after collapse seems to depend only slightly on the density and agrees remarkably well with the percentage of ellipticals that is found in regions of different galaxy density (Bahcall, 1977). The results in Fig. 9 can
3.3.2. Distribution of Galaxies of Different Morphological Type

From Table 2, but also by visual inspection of Fig. 8 we see that the distribution of merged galaxies has a smaller mean radius than the whole system of galaxies. This type of segregation is also observed in clusters of galaxies between elliptical galaxies and disk galaxies, despite the fact that the elliptical and SO populations are often taken together (van den Bergh, 1978). Table 3 shows how the composition of our models 3 and 4 changes with distance to the cluster centre. The averages are taken over a few models that have the same initial parameters but different random numbers for the initial velocities and positions.

Because ellipticals should occur mostly in collapsed regions and because their distribution will have a smaller characteristic radius the amount of clustering of ellipticals must be higher than that for spirals. Davis and Geller (1976) give supporting evidence for such an effect in the two-point correlation function for ellipticals. Besides the spatial segregation between ellipticals and other galaxies there is some indication of velocity segregation in the observations (van den Bergh, 1978; Moss and Dickens, 1977). In agreement with these observations the merged galaxies in our model tend to have a somewhat smaller velocity dispersion than the other galaxies.

We conclude that merged galaxies are more strongly bound in a cluster than the other galaxies. In our model this is not caused by two-body relaxation or dynamical friction. The average mass of the mergers is not much larger than the average mass of the other galaxies (see Fig. 10). Merging occurs among low velocity galaxies in high density regions where the collision rate is high.

Multiple mergers occur preferentially in the central regions of clusters and groups (see Fig. 8) and we associate such cases with giant ellipticals and cD galaxies. It is understandable that cD's can occur in groups (Albert et al., 1977) as well as in rich clusters. Because clusters are probably formed through merging of sub-clumps (White, 1977) which often have a multiple merger in their centre, cD's need not be found right in the centre of rich clusters.

Our prediction that interacting galaxies should occur predominantly outside collapsed regions where the percentage of ellipticals is low is in agreement with Thompson's observation (1977) that ring galaxies lie preferentially outside rich clusters (if one adopts the interpretation of ring galaxies by Lynds and Toomre, 1976).

3.3.3. Background Material

Table 2 shows that at the end of the calculation the stripped mass amounts to only 1/4 of the total mass. This stripped mass has about the same radial distribution as the unmerged galaxies, but is slightly more centrally concentrated. We have already noted in 2.3.1 that we have probably underestimated mass-loss. The distribution of the stripped mass in our model 3 is slightly more centrally concentrated than the distribution of the unmerged galaxies. The total mass within 0.5 Mpc of the centre is about 10% higher in the inelastic case than in the elastic case. This might have consequences for the X-ray morphology of clusters of galaxies.

In Fig. 10 the mass-spectrum at the end of the calculation is shown. The differences in mass have not become large. During the merging process just a small amount of mass is gained, but merged galaxies also lose mass through collisions with other galaxies. The model predicts that merged galaxies are embedded in considerable amounts of stripped mass.

3.3.4. M/L-values

In applying the virial theorem to poor clusters or groups, a large part of the stripped mass might be missed when the radial distribution of this matter is broader than the radial distribution of the galaxies. This would lead to a lower M/L-value for these systems. At present our results only give hints in that direction.

Several authors have noted an increase in the M/L-value going from groups of galaxies to rich clusters. Gott and Turner (1977) studied a sample of 39 groups of galaxies and discuss several important difficulties in extracting M/L-values for them such as: a) low number of galaxies with known redshift, b) contamination by foreground and background galaxies, c) projection effects, and d) non-equilibrium effects. They found a mass-to-light ratio for groups that is about a factor 2–3 smaller than for rich clusters. Rood and Dickel (1978) have completed an extensive study on the M/L-values for groups and they have found correlations of the form

\[ M/L \propto V^{1.6} R. \]

But, as pointed out by Turner and Sargent (1974), such a relation can easily be created in an artificial way when the virial theorem is not applicable. Application of the virial theorem gives of course the wrong results if: spurious galaxies are counted as group members or the group has not yet collapsed or virialised. Turner and Gott (1976) point out that on the average the groups are just entering the virialised regime. Many of these groups might still be in their collapse stage. Application of the virial theorem to groups of galaxies especially to loose groups is therefore very uncertain. Gott and Turner (1977) found that \( \langle M/L \rangle = 200 \) for groups with \( L > 10 L^* \) and \( \langle M/L \rangle = 65 \) for groups with \( L < 10 L^* \), where \( L^* = 3.4 \times 10^{10} L_\odot \). They note that this might correspond to the larger portion of spirals in the low-luminosity groups. This agrees with the difference they found between the M/L of the early type (E, SO) and the late type (S, SB, Irr) population in their groups \( \langle M/L \rangle_{E,SO} = 136 \pm 50 \), \( \langle M/L \rangle_{S,SB,IRR} = 61 \pm 21 \).

This might of course mean that the M/L of individual galaxies in the two classes differs by a factor of about 2, but this would be in contradiction with the most recent information on M/L-values for galaxies of different types from Dickel and Rood (1978). They find no difference in the M/L-values for spirals and for ellipticals. Another interpretation is now possible on the basis of the results presented in this paper, namely, that the low-density spiral-rich regions such as loose groups tend to give low M/L-values because these regions have not yet undergone collapse.

4. Conclusions

Our principal conclusions are as follows:

4.1. Galaxy Collisions

(i) Galaxy collisions tend to be strongly inelastic at collision velocities smaller than 4.5 \( \sigma \). That means that the radius of the largest colliding galaxy.

(ii) The criterion for merging to take place between two galaxies with equal internal velocity dispersion \( \sigma \) is

\[ v \leq 3.1 \sigma \left( 1 - 0.3 \frac{p}{R_\phi} \right) \left( 1 + \frac{M_2/M_1}{2} \right)^{1/4}, \quad M_2 \leq M_1, \quad p/R_\phi < 1 \]
for the collision parameter at the moment of collision, or
\[ v_\infty \leq 1.1 \sigma (1 - 0.25 p_\infty / R_g), \quad p_\infty / R_g < 2 \]
for the collision parameters measured at infinity. Here, \( R_g \) is the radius of the largest galaxy.

(iii) At higher velocities the loss of orbital kinetic energy is given by (14).

### 4.2. Evolution of Groups and Clusters of Galaxies

(i) During the collapse of clusters merging takes place which can result in \( \approx 20\% \) merged galaxies after collapse. After collapse the percentage of merged galaxies rises only slowly.

(ii) Multiple mergers can be found in the subclumps that form before the collapse of the cluster and afterwards in the central region of the cluster.

(iii) There is a clear increase in the percentage of mergers towards the cluster center.

We regard points (i), (ii), and (iii) as strong support for the idea of Toomre and Toomre that elliptical galaxies might be the remnants of merged galaxies. Giant ellipticals and cD's might very well be the products of a multiple merging process.

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