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Observational evidence confirming Lindblad's hypothesis of a rotation of the galactic system, by *J. H. Oort.*

1. *Introduction.*

It is well known that the motions of the globular clusters and RR Lyrae variables differ considerably from those of the brighter stars in our neighbourhood. The former give evidence of a systematic drift of some 200 or 300 *km/sec* with respect to the bright stars, while their peculiar velocity averages about 80 *km/sec* in one component, which is nearly six times higher than the average velocity of the bright stars.

Because the globular clusters and the bright stars seem to possess rather accurately the same plane of symmetry, we are easily led to the assumption that there exists a connection between the two. But what is the nature of the connection?

It is clear that we must not arrange the hypothetical universe in such a way that it is very far from dynamical equilibrium. Following KAPTEYN *) and JEANS **) let us for a moment suppose that the bulk of the stars are arranged in an ellipsoidal space whose dimensions are small compared to those of the system of globular clusters as outlined by SHAPLEY ***). From the observed motions of the stars we can then obtain an estimate of the gravitational force and of the velocity of escape. An arrangement as supposed by KAPTEYN and JEANS, which ensures a state of dynamical equilibrium for the bright stars, implies, however, that the velocities of the clusters and RR Lyrae variables are very much too high. A majority of these would be escaping from the system. As we do not notice the consequent velocity of recession it seems that this arrangement fails to represent the facts.

As a possible way out of the difficulty we might suppose ****) that the brighter stars around us are members of a local cloud which is moving at fairly high speed inside a larger galactic system, of dimensions comparable to those of the globular cluster system. We must then postulate the existence of a number of similar clouds, in order to provide a gravitational potential which is sufficiently large to keep the globular clusters from dispersing into space too rapidly. The argument that we cannot observe these large masses outside the Kapteyn-system is not at all conclusive against the supposition. There are indications that enough dark matter exists to blot out all galactic starclouds beyond the limits of the Kapteyn-system *).

LINDBLAD **) has recently put forward an extremely suggestive hypothesis, giving a beautiful explanation of the general character of the systematic motions of the stars of high velocity. He supposes that the greater galactic system as outlined above may be divided up into sub-systems, each of which is symmetrical around the axis of symmetry of the greater system and each of which is approximately in a state of dynamical equilibrium. The sub-systems rotate ***) around their common axis, but each one has a different speed of rotation. One of these sub-systems is defined by the globular clusters for instance; this one has a very low speed of rotation. The stars of low velocity observed in our neighbourhood form part of another sub-system. As the rotational velocity of the slow moving stars is about 300 *km/sec* and the average random velocity only 30 *km/sec*, these stars can be considered as moving very nearly in circular orbits around the centre.

We may now apply an analysis similar to that

*) *Astrophysical Journal*, 55, 302, 1922; *Mt Wilson Contr.* N°. 230.

**) *Monthly Notices R.A.S.*, 82, 122, 1922.

***) *Astrophysical Journal*, 48, 154; 49, 311 and 50, 107, 1919; *Mt Wilson Contributions* Nos 152, 157 and 161.

****) OORT, *Groningen Publications* N°. 40, pag. 63, 1926.

*) *Cf. Hemel en Dampkring*, Jan. and Feb. 1927.

**) *Arkiv. f. Mat., Astr. o. Fysik*, Bd 19 A, Nos. 21, 27, 35 and Bd 19 B, N°. 7 (*Upsala Meddelanden* Nos 3, 4, 6 and 13); also: *Vierteljahrschrift*, 61ter Jahrgang, p. 265.

***) Of course the rotation considered is not generally one of constant angular velocity throughout the sub-system. In the following comparisons between the speeds of rotation these speeds are taken for stars at the same distance from the axis.

used by JEANS in his discussion of the motions of the stars in a "Kapteyn-universe" *), the only difference being that in the present analysis we do not introduce a second system rotating in the opposite direction. Adopting some probable formula for the gravitational potential we can derive the rotational velocities for each of the sub-systems from our knowledge of the distribution of the peculiar velocities (defined as the velocities remaining after correction for the effects of rotation). The higher the average peculiar velocity in a certain sub-system the slower its rotation will be, and the less flattened it will appear in a direction perpendicular to the galactic plane. If we refer our motions to the centre of the slow moving stars in our neighbourhood, the members of a sub-system with higher internal velocities will appear to lag behind, and LINDBLAD has shown that in this way we can arrive at a connection between average peculiar velocity and systematic motion of the same form as that computed from observation. **)

LINDBLAD's hypothesis conforms beautifully with the well-established fact that the average direction of the systematic motion of the high velocity stars is perpendicular to the direction in which the globular clusters are concentrated (galactic longitude 325°, latitude 0°). At first sight it might be hard to imagine how such a mixture of sub-systems of different angular speeds could ever come into existence; but the *possibility* cannot be denied, as is apparent from a comparison with spiral nebulae. ***)

If somewhere there existed a rapidly rotating system of stars and by some cause the internal velocities in this star-system were increased, an asymmetry in the stellar motions would necessarily result in the long run. It must be admitted, however, that the part played by the globular clusters cannot be so easily understood.

The following paper is an attempt to verify in a direct way the fundamental hypothesis underlying LINDBLAD's theory, namely that of the rotation of the galactic system around a point near the centre of the system of globular clusters. In a subsequent paper I hope to be able to make a more detailed comparison of the theory with the observational facts concerning the stars of high velocity collected in *Groningen Publications* N°. 40.

2. *Theoretical effects of the rotation.*

In the present discussion I shall altogether disregard the idea of a number of separate galactic clouds and take into consideration only the forces arising from the greater galactic system as a whole. The gravitational force, K, is consequently directed to the centre of this system and is only a function of the distance, R, from this centre.

Let us now consider a group of stars at a distance r from the sun and let us suppose that r/R is so small that all terms of second or higher order in r/R can be neglected, then it is easily seen that the residual velocity caused by the rotation is equal to

$$rA \sin 2\,(l-l_0)$$

in radial direction, and to

$$rA \cos 2\,(l-l_0) + rB$$

in transverse direction, if l_0 represents the galactic longitude of the centre (about 325°), l the longitude of the stars considered, R the distance of the sun from the centre,

$$V = \sqrt{RK}$$

the circular velocity near the sun,

$$A = \frac{V}{4R}\left(1 - \frac{R}{K}\frac{\partial K}{\partial R}\right)$$

and

$$B = A - \frac{V}{R} \quad . \, .$$

The rotation is supposed to take place in right-hand direction as observed from a point North of the galactic plane.

If, as LINDBLAD tentatively supposed, the principal part of the greater galactic system is formed by an ellipsoid of constant density, the force K will be proportional to R. In this case

$$A = 0 \text{ and } B = -\frac{V}{R}\ ,$$

the system rotates as a solid body and we shall not find any indications of rotation in the radial velocities, but the proper motions in galactic longitude should be systematically negative for stars in all longitudes.

As another extreme case we might suppose that the whole mass is concentrated in the centre and that K is inversely proportional to the square of R. We get

$$A = +\frac{3}{4}\frac{V}{R} \qquad B = -\frac{1}{4}\frac{V}{R}.$$

We shall see below that observations seem to prove that the second alternative is nearly correct. We shall then have to expect a systematic effect in the radial velocities showing maxima at 10° and 190°

*) *Monthly Notices R. A. S.*, 82, 122.
**) STRÖMBERG, *Astrophysical Journal*, 59, 228, 1924; *Mt Wilson Contr.* N°. 275.
***) LINDBLAD, *Upsala Meddel.* N°. 13.

longitude, and minima at 100° and 280°, with a semi-amplitude of $\frac{3}{4}\frac{r}{R}V$. Now the most distant objects observed for radial velocity are at distances of about 1000 parsecs; with $R = 10\,000$ parsecs and $V = 300$ *km/sec* this gives a semi-amplitude of over 20 *km/sec*, which might well be verifiable. With the same assumptions the maximum effect in the proper motions in galactic longitude would be equal to $-0.''005$ per annum. The maximum will occur 90° from the direction towards the centre. In the direction of the centre and in the opposite direction the average proper motion in longitude should be equal to about $+0''.002$. The proper motion effects are, of course, independent of the distance of the objects considered.

3. *Discussion of the radial velocities.*

Several astronomers have remarked upon instances in which the stars in different parts of the sky appeared to move differently.

The hypothesis of a rotation around a distant centre has also been put forward by STRÖMBERG *) on the basis of an investigation of the preferential motions of the stars. He found that the maximum peculiar radial velocity did not occur in two exactly opposite points of the sky but in directions inclined to each other. In explanation of this he suggested a rotation around a centre near 256° longitude. Later on it has become evident, however, that these results were caused by the influence of the stars of high velocity.

In a paper on the distribution of stellar velocities GYLLENBERG pointed out that the so-called K-term in the radial velocities of the B type stars depended upon the galactic longitude **). From his drawing it is apparent that the K-term has distinct maxima somewhere around 0° and 180° longitude and minima at 90° and 270°. It is evident from the foregoing that this variation can be explained as the effect of rotation around a centre in 325° longitude, for the longitudes of the maxima are very near those expected in the case of rotation.

In 1922 FREUNDLICH and VON DER PAHLEN ***) have extended GYLLENBERG's investigation. According to their statements they do not doubt the reality of the variation of the K-term with galactic longitude, and they propose several dynamical explanations; but none of these was considered to be very satisfactory.

It is not only the velocities of the B stars which have given evidence of systematic motions. In a statistical study of the c-stars SCHILT has remarked upon the deviations from zero of the mean peculiar velocities of stars in different longitudes *). His table of residuals is reproduced below.

TABLE 1.

Average longitude	Average peculiar velocity	mean error	sin 2 (l—325°)
30°	+ 8 *km/sec*	± 3.5 *km/sec*	+ .77
90	— 8	± 2.7	— .94
150	0	± 3.6	+ .17
210	+ 10	± 3.9	+ .77
270	— 7	± 4.3	— .94
330	0	± 3.5	+ .17

In the last column I have added the coefficient of the rotation term for which we are looking; it varies in very nearly the same manner as the average residual velocity.

A somewhat analogous variation has been found by HENROTEAU in a recent paper on pseudo-cepheids **).

The O-type stars have also been under suspicion of giving different systematic motions in different parts of the sky ***), and in this case too the general character of the residuals is what we must expect if the system of stars is rotating in the way described.

In the following table are summarized the results of a re-discussion of the radial velocities of all objects of which it might be hoped that they would show the effects of the rotation, if it exists. The second column gives the average apparent magnitude, for the Md variables the average maximum magnitude; in the case of the planetary nebulae it gives the limits of the apparent magnitude of the central stars. The third column shows the number of stars used, the fourth their average parallax and its mean error. Excepting the Md variables for which the mean parallaxes were estimated directly from R. E. WILSON's results ****), all the parallaxes were computed anew from all proper motion data avaible, in such a way as to be uninfluenced by possible rotation terms in the proper motions (see section 4). The fifth column shows the semi-amplitude of the rotational term and its mean error. In general the stars were divided into intervals of 15° or 30° galactic longitude and stars of higher galactic latitudes were excluded as mentioned in the remarks. If the longitude of the centre of a

*) *Astrophysical Journal*, 47, 32—34, 1918; *Mt Wilson Contr.* N°. 144.
**) *Lund Meddelanden* Ser. II N°. 13, pp. 22—26, 1915.
***) *Astr. Nachr.*, 218, 369—400.

*) *Bull. Astr. Inst. Netherlands*, Vol. 2, p. 50.
**) *Journal R.A.S. Canada*, 21, 1, 1927.
***) *Groningen Publications* N°. 40, pp. 52—53.
****) *Astronomical Journal*, 35, 129, 1923.

group is called l and the average peculiar radial velocity $\bar{\rho}'$ the equations of condition are

$$\bar{r}A \sin 2\,(l - 325^\circ) = \bar{\rho}'$$

the centre of rotation being assumed to lie near the centre of the system of globular clusters at 325° longitude. A positive sign of $\bar{r}A$ means that the systematic term indicates a rotation in the direction expected.

In several groups where the velocities were sufficiently rich in number or in other respects favourable for an independent determination of the longitude, l_0, of the centre of rotation, two unknowns were introduced. The computed value of l_0 is then shown in the sixth column together with its mean error. The solution gives us two opposite points; only one of these has been inserted. Unless stated otherwise in the remarks the radial velocities have been corrected for the usual value of the solar motion (20 *km/sec*). Except perhaps for the *O*-stars and planetary nebulae this value of the solar motion is very nearly equal to that found for each of the special groups of stars separately.

TABLE 2.

Type	$\bar{m}$	n	$\bar{\pi}$	m.e.	$\bar{r}A$	m.e.	l_0	m.e.	A	m.e.
B3—B5	4.9	182	0.″0058	± .″0004	+ 5.7	± 1.0	322°	± 5°	+ 0.033	± .006
B0—B2	4.6	86	.0037	± .0005	+ 9.3	± 1.5	322	± 5	+ .034	± .008
δ Cep variables	5.4	13	.0036	± .0007	+ 11.	± 5.	—		+ .040	± .020
M6e—M8e, Se	7.0	78	.003	—	+ 2.	± 5.	—		+ .006	± .015
M1e—M5e	7.7	55	.003	—	— 10.	± 14.	—		— .030	± .043
c-stars	4.0	44	.0028	± .0005	+ 9.1	± 3.0	330	± 8	+ .025	± .009
Oa—Oe	7.0	8	.0028	± .0025	+ 25.	± 18.	—		+ .070	± .080
Oe5	6.2	27	.0020	± .0006	+ 19.	± 5.	—		+ .038	± .015
N	—	18	.0017	± .0008	+ 17.	± 8.	—		+ .029	± .017
c-stars	6.2	49	.0015	± .0005	+ 25.1	± 2.7	321	± 4	+ .038	± .014
O-star Ca clouds	—	40	—		+ 5.6	± 2.2	—		—	
Planetary Nebulae	< 14	60	—		+ 10.	± 6.	—		—	
,, ,,	unknown	30	—		+ 25.	± 10.	—		—	
,, ,,	> 14	20	—		+ 41.	± 11.	333	± 10	—	

Remarks to the table.

The *B* stars were taken from the following sources: VOÛTE's *First Catalogue of radial velocities; Astrophysical Journal*, **57**, 149 (*Mt. Wilson Contr.* N°. 258); *Victoria Publications*, **2**, 19, and *Astrophysical Journal*, **64**, 11. A constant correction, K, was introduced as a third unknown in this case. The following values were derived for this constant

B0—B2 $K = +1.4$ *km/sec* ± 1.1 m.e.
B3—B5 $K = +3.4$ *km/sec* ± 0.7 m.e.

All stars with galactic latitudes of 20° and higher were excluded.

Only those δ Cephei-variables have been used for which reliable orbit-determinations could be found in *Lick Bulletins*, **11**, 141. A solar velocity of 20 *km/sec* was assumed for the above solution. As it has been suggested that the solar velocity with respect to these stars is considerably lower than 20 *km/sec*, a second solution has been made in which the solar velocity, V_0, was introduced as a variable. The resulting values were:

$V_0 = 13$ *km/sec* ± 4 m.e. and $\bar{r}A = +8$ *km/sec* ± 4 m.e.

The *Md* variables were taken from MERRILL's list in *Astrophysical Journal*, **58**, 215 (*Mt. Wilson Contr.* N°. 264). The solar velocity was introduced as an unknown, with the following results

M1e—M5e $V_0 = 66$ *km/sec* ± 11 *km/sec* m.e.
M6e—M8e and *Se* $V_0 = 34$ *km/sec* ± 6 *km/sec* m.e.

As many stars in this class have high galactic latitudes the equations of condition were modified into

$$\rho = V_0 \cos\lambda + \bar{r}A \sin 2\,(l - 325^\circ) \cos^2 b,$$

λ being the distance from the star to the solar apex.

The *c*-stars were taken from a manuscript catalogue of *c*-star radial velocities prepared by Dr. SCHILT. I am obliged to him for a copy of his list. Though there are indications of several groupings among these stars, none were excluded on account of this. The fainter class contains a considerable number of stars belonging to the Perseus cluster. All stars with latitudes higher than 19° were excluded.

The *O*-type stars were mainly taken from *Victoria Publications*, **2**, 316. They were divided into two groups according to the Harvard spectrum. To the *Oe*5 group were added the stars classed as *O* by PLASKETT and as *B'* by miss CANNON. Both the *Oe*5 stars and the Wolf-Rayet stars are very unevenly distributed over the different longitudes so that it was found impossible to solve both for the rotation-effect and the solar motion. In view of the moderate residual velocities it seemed fairly safe to assume a value of 20 *km/sec* for the *Oe*5 stars. The *K*-correction was assumed to be negligible; the residuals seem to show however that there might exist a considerable positive *K*-term for the *Oe*5 stars. After appliciation of the rotation term the average residual velocity of the *Oe*5 stars is only 16 *km/sec*, thus of the same order as the average velocity of the ordinary giants.

For the *N*-type MOORE's radial velocities have been used (*Lick Bull.*, **10**, 160). Only those with latitudes between + 30° and — 30° were considered.

The *Ca*-cloud velocities are from PLASKETT's list in *Monthly Notices R.A.S.*, **84**, 80; they are, therefore, mostly from the *Ca* lines in *O*-type stars.

The planetary nebula velocities are from *Lick Publ.*, **13**, 168, completed by a few more recent results. They were roughly grouped according to the magnitudes of the central stars, as from a study of the average galactic latitudes this criterion seemed to be most strongly correlated with the distance.

Except for the uncertain result from the early *Me* stars the values of $\bar{r}A$ in the table are all positive, indicating a rotation in the same direction as that found from our velocity relative to the globular clusters. Several values are so large as to leave hardly any doubt about the reality of this $\sin 2(l-325°)$-term. It is possible, of course, that the term may be explained by systematic motions arising from another cause than rotation, but these systematic motions must then bear a remarkably close resemblance to a rotation.

In general the velocities appear to be quite satisfactorily represented by the rotation term, the residuals showing no tendency to be systematic. Only in the case of the *B*-stars the average group-residual is larger than what we should have expected from the small peculiar motions of these stars.

The five determinations of the longitude of the centre are in good agreement with each other. The average 323° ± 2°.4 *m.e.* is quite near the direction towards the centre of the globular cluster system as estimated by SHAPLEY.

The objects in the first division of the table have been arranged in order of decreasing mean parallax. It appears that the better determinations give evidence that the value of $\bar{r}A$ increases proportional with the mean distance, as it should do if the term is interpreted as a rotation. The determinations with a relative mean error of less than a third are put together in the following table.

TABLE 3.

Type	$\bar{\pi}$	$\bar{r}A$	m. e.
B3 — B5	″.0058	+ 5.7 *km/sec*	± 1.0
B0 — B2	.0037	+ 9.3	± 1.5
bright *c*-stars	.0028	+ 9.1	± 3.0
Oe5	.0020	+ 19.	± 5.
faint *c*-stars	.0015	+ 25.1	± 2.7

By multiplying $\bar{r}A$ by the average parallax we have made estimates of the absolute value of *A*, as shown in the last column of Table 2. Properly speaking these values should have been computed in a more elaborate way. For if there is considerable spreading in the distances within one group the average value of *r* will be somewhat higher than the reciprocal of the average parallax. For the present I have not tried to derive corrections, as these would of necessity be very uncertain; I do not believe that they would seriously influence the present conclusions. The total average of *A* is found to be + 0.0310 ± 0.0037 (*m.e.*); it represents the semi-amplitude of the rotation term for objects at a distance of one parsec.

We can now use this constant for estimating the average distance of some of the objects of unknown parallax, collected in the second division of the table. The average distance of PLASKETT's Calcium clouds is thus found to be roughly 180 parsecs, which shows rather conclusively that the clouds are not connected with the *O*-stars themselves. The distances of the three groups of planetary nebulae are estimated as 320, 810 and 1300 parsecs respectively. Though the percentage errors are large, the results confirm the serviceability of the magnitude of the central star as a criterion for relative distance. The absolute magnitudes of these central stars would seem to be at least six or seven units larger on the average than those of the *O*-stars, which they resemble in some spectral characteristics.

4. *Proper motion data.*

If the interpretation of the systematic term in the radial velocities as a rotation is right, a similar term should occur in the proper motions. But, as is evident from the formulae given in section 2, the rotation terms in the proper motions cannot be predicted from the radial velocity results unless we make an assumption as to the character of the general gravitational force. Now it will be shown in the next section that the radial velocity results make it very probable that a great part of this force varies inversely proportional to the square of R. We shall suppose that the total gravitational force in this part of the galactic system can be represented as the sum of two forces, K_1 and K_2, the first of which varies inversely proportional to R^2 and the second directly proportional to R. We want to determine what percentage of the total force is made up of K_1, and what of K_2. The residual transverse velocity in *km/sec* is easily seen to be equal to

$$\frac{r}{R} V \left\{ \frac{3}{4} \frac{K_1}{K} \cos 2 (l - 325^\circ) - \frac{1}{4} \frac{K_1 + 4 K_2}{K} \right\}$$

and the residual radial velocity

$$\frac{3}{4} \frac{r}{R} V \frac{K_1}{K} \sin 2 (l - 325^\circ)$$

Theoretically we can determine both $\frac{V}{R}\frac{K_1}{K}$ and $\frac{V}{R}\frac{K_2}{K}$ from the proper motions, but for several reasons a solution of both unknowns is not very likely to yield useful results. Accordingly it was decided to assume the value of $+0.031$ found from the radial velocities for $\frac{3}{4}\frac{V}{R}\frac{K_1}{K}$ and only to use the proper motions for determining the ratio $\frac{K_2}{K_1}$.

For the determination of this ratio I have utilized the proper motions of some 600 stars, all of types that are known to possess very small peculiar proper motions. These proper motions were corrected for the systematic errors published by RAYMOND *) and also for solar motion, the secular parallax being derived separately for a number of groups, each group containing stars of a small range in magnitude and spectral type only. If μ_l' denotes the proper motion in galactic longtitude remaining after these corrections and w its weight, estimated from the peculiar motions and from the errors of the proper motions, the ratio is determined by the following formula:

$$\frac{K_2}{K_1} = \frac{\Sigma w \mu_l' + 0''.0022\, \Sigma w - 0''.0065\, \Sigma w \cos 2 (l - 325^\circ)}{-0''.0087\, \Sigma w}$$

all the constants being determined with the aid of the radial velocity results. In this way we find:

$$\frac{K_2}{K_1} = 0.11$$

*) *Astronomical Journal*, 36, 136, 1926.

which gives a rather satisfactory agreement with the observed average proper motions in the various intervals of galactic longitude.

As it is probable that part of the rotation has been absorbed in the constant of precession which has been empirically determined, the results of an investigation of rotation terms in the proper motions are somewhat uncertain; the details of this discussion will therefore be published in a separate note in one of the next numbers of the *B. A. N.*, as soon as a new determination of the constant of precession from the proper motions in galactic latitude has been completed. However I do not think it very likely that the value of K_2/K_1 derived above will be in error by more than 0.20.

An empirical term in the proper motions, explicable as a rotation of the system of fixed stars, has been discussed by several authors (ANDING, SEELIGER, DE SITTER, WOLTJER, CHARLIER, INNES). In a memoir entitled "The motion and the distribution of the stars" *) CHARLIER derives an average value of $\mu'_l = -0''.0024$, agreeing at least qualitatively with the results found above from the proper motions of distant stars.

5. *Concluding remarks.*

It has been shown from radial velocities that for all distant galactic objects there exist systematic motions varying with the galactic longitudes of the stars considered. The relative systematic motions are always of the same nature and they increase roughly proportional with the distance of the objects. Probably the simplest explanation is that of non-uniform rotation of the galactic system around a very distant centre. This explanation is capable of representing all the observed systematic motions within their range of uncertainty (except perhaps in the case of the B stars). If with this supposition we compute the position of the centre from the radial velocities, we find that it lies in the galactic plane, either at 323° longitude or at the opposite point. The first direction is in remarkably close agreement with the longitude of the centre of the system of globular clusters (325°). The observations would therefore seem to confirm LINDBLAD's hypothesis of a rotation of the entire galactic system around the latter centre.

The proper motions corroborate the above interpretation, at least qualitatively. They were used mainly to determine the character of the non-uniformity of the rotation. This character corresponds to a gravitational force which can sufficiently well be represented by the

*) *Memoirs of the Univ. of California*, 7, 32, 1926.

following formula: $K = \frac{c_1}{R^2} + c_2 R$, if R is the distance of the centre. A provisional solution gave: $\frac{c_2}{c_1} = \frac{0.11}{R^3}$

Such a force would for example result if 9/10th of the total force came from mass concentrated near the centre and 1/10th from an ellipsoid of constant density large enough to contain the sun within its borders. The true character of the force will of course be more complicated.

We can derive a numerical result for R as soon as the circular velocity, V, is known. An estimate of this circular velocity may be made from the radial velocities of the globular clusters. According to STRÖMBERG these clusters possess a systematic motion nearly perpendicular to the direction of their centre and equal to 286 *km/sec* ± 67 (*m.e.*) relatively to the sun, or 272 *km/sec* relatively to the centre of the slow moving stars. This would give us an estimate of the circular velocity if we were sure that the system of globular clusters had no rotation. From the ellipsoidal arrangement in space LINDBLAD derives a rotation for these objects, such that the circular velocity would be increased to 426 *km/sec*. *) As, however, the apparent positions in the sky give no indication whatever of the system of globular clusters being flattened towards the galactic plane, it seems better to assume that they possess no rotation. In fact it seems more probable from dynamical consideration that the true circular velocity is below the value found by STRÖMBERG than above it. Assuming $V = 272$ *km/sec* we find $R = 5900$ parsecs. As the longitude of the centre of rotation agreed with that of the system of globular clusters, it is probable that the distance will agree as well. The distance of the centre of the globular cluster system is very uncertain, however, on the one hand by an uncertainty in the scale of the cluster distances and on the other hand by our incomplete knowledge of the more distant clusters. SHAPLEY gives estimates varying from 13000 to 25000 parsecs. The value found above is considerably smaller. Even if we made the extreme supposition that $K_2 = 0$, or that all the mass of the galactic system were concentrated near the centre, the distance R would only be increased to 6600 parsecs.

In order to explain the rotation there must be near the centre an attracting mass of at least 8×10^{10} times the mass of the sun. There remains the difficulty why we do not observe this large mass. Near 6000 parsecs KAPTEYN and VAN RHIJN find an almost negligible density, whereas it *should* be very much greater than in our neighbourhood. Part of the discrepancy may have resulted from the approximative character of their solution, in which all galactic longitudes were combined. Discussing various galactic regions separately KREIKEN finds indications of a centre near 314° longitude, at a distance of 2270 parsecs *) which is in the right direction, but certainly at too small a distance and too little defined. **) The most probable explanation is that the decrease of density in the galactic plane indicated for larger distances is mainly due to obscuration by dark matter. Such a hypothesis receives considerable support from the marked avoidance of the galactic plane by the globular clusters, a phenomenon for which up to the present time no other well defensible explanation has been put forward. ***)

LINDBLAD suggests that the starstreaming is an indirect consequence of the rotation. In so far as his computations result in a starstreaming to and from the centre, in the same way as originally suggested by TURNER, the present data do not entirely confirm the hypothesis. The true vertices according to EDDINGTON lie at 166° and 346° longitude; it does not seem possible to admit an error of over 20°, which would be required to make vertex II coincide with the direction to the centre as derived in the present paper.

It may be remarked that the rotation offers a means of determining average distances from radial velocities, the relative accuracy of the method increasing with the distance and being independent of possible absorption of light in space. We have thus been able to derive a value for the average distance of the most distant planetary nebulae.

The question naturally arises which would be the most valuable observations that could be made in order to check the present results. Probably the most promising results could be derived from the radial velocities of some very faint *c*-stars. If one could get down to the 8th or 9th magnitude the semi-amplitude of the rotation term might become as large as 50 *km/sec*; a small number of stars would then suffice to give reliable results. There is another class of stars which deserves mentioning, viz. very faint δ Cephei variables. Several variables are known whose estimated distances are of the order of 5000 parsecs and larger, thus bringing us quite near the hypothetical centre of the galactic system. If rough values were known for the velocities of a few stars of this type favourably situated for the purpose, it might

*) *Upsala Meddel.* N°. 4, p. 6.

*) *Monthly Notices R.A.S.*, 86, 686, 1926.

**) Indications of a galactic centre near 325° longitude are also obtained from a study of the distribution of planetary nebulae and of novae.

***) *Hemel en Dampkring*, 25, 67—68, 1927.

well become possible to derive reliable absolute values of the distance to this centre as well as of the circular velocity in our neighbourhood.

In conclusion I want to express my gratitude to Mr. PELS who has so ably assisted me in making the rather extensive computations required for the preparation of Table 2.

Note added to proof.

While this paper was going through the press a provisional correction to the constant of precession was derived from proper motions in galactic latitude, and a corresponding correction was applied to the proper motions in longitude. Both direction and amount of the angular velocity of rotation derived from the radial velocities are satisfactorily confirmed by the corrected proper motions.

The ratio K_2/K_1, which in section 4 was found to be 0.11, is changed into 0.29 by the above correction. The corresponding estimate of the distance of the centre changes from 5900 to 5100 parsecs.