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COULOMB AND INTERFERENCE EFFECTS IN SMALL ELECTRONIC STRUCTURES

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**Coulomb And Interference Effects In Small
 Electronic Structures**

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presence of such a ratio in the current response is the hallmark of a non-self-averaging system. For a small number of channels M_m the resistance R_{qm} scales usually as $1/M_m$. For large M_m , on the other hand, Pendry et al. [9] have shown that in the diffusive regime the probability distribution of the conductance makes extreme excursions, or "maximal fluctuations". We might expect this non-self-averaging system to react to an energy change with maximal changes of the phase ϕ for a minimum number of eigenchannels, whereas in the other eigenchannels no phase changes take place. Thus a non-self-averaging system is likely to exhibit a larger charge relaxation resistance than a usual system. The charge distribution of localized states in the insulator between the capacitor plates is likely to be another important source of mesoscopic fluctuations [10]. To study this effect an approach is required which treats the microscopic potential landscape.

IV. FROM THE DISCRETE-PARAMETER MODEL TO SPATIAL DEPENDENCE

For a system as in Fig. 1a, capacitances C_{ki} can be more accurately found by means of space-dependent induced potentials: this requires the tools of the spatially resolved response, like the characteristic potential functions $u_i(\vec{r})$ and the Lindhard response functions $\Pi_k(\vec{r}, \vec{r}')$ appearing in the table below. In principle, however, the derivation proceeds in full analogy to the scheme of Sect. 2. Crucial are the roles of the surface S and of the invariance under an overall potential shift (OPS) of the electro-chemical and the induced potentials. The following table compares the differing approaches:

<i>Discrete-parameter model</i>	<i>Microscopic model</i>
$edU_k = \sum_i \tilde{u}_{ki} d\mu_i$	$edU([\mu_i], \vec{r}) = \sum_i u_i(\vec{r}) d\mu_i$
$\xrightarrow{\text{OPS}} \sum_i \tilde{u}_{ki} = 1$	$\xrightarrow{\text{OPS}} \sum_i u_i(\vec{r}) = 1$
$dQ_k = \sum_i C_{ki} dU_i$	$-\nabla^2 dU(\vec{r}) = 4\pi e \sum_i dn_k(\vec{r})$
$dQ_k = e(dN_k/dE)d\mu_k - \tilde{\Pi}_k edU_k$	$dn_k(\vec{r}) = (dn(\vec{r}, k)/dE)d\mu_k + dn_{ind,k}(\vec{r}) =$ $= (dn(\vec{r}, k)/dE)d\mu_k - \int d^3r' \Pi_k(\vec{r}, \vec{r}') edU(\vec{r}')$
$\xrightarrow{\text{OPS}} \tilde{\Pi}_k = edN_k/dE$	$\xrightarrow{\text{OPS}} \int d^3r' \Pi_k(\vec{r}, \vec{r}') = dn(\vec{r}, k)/dE$

V. CONCLUSION

A mesoscopic capacitance has been introduced for systems interacting via long-range Coulomb forces and the calculation of the dynamic admittance has been outlined.

Focussing on a simple system for the results, we find that the "capacitance" and "resistance" governing the ac admittance of a nanostructured capacitor exhibit mesoscopic signatures: they are quantities which reflect the behavior of the system as a whole indivisible unit. In fact the charge relaxation resistance in each arm and the corrections to the standard classical capacitance are determined by the scattering properties along either side of the capacitor averaged over the grand-canonical ensemble of the corresponding reservoir.

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SUB-POISSONIAN SHOT NOISE IN A DIFFUSIVE CONDUCTOR

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Abstract. — A review is given of the shot-noise properties of metallic, diffusive conductors. The shot noise is one third of the Poisson noise, due to the bimodal distribution of transmission eigenvalues. The same result can be obtained from a semiclassical calculation. Starting from Oseledec's theorem it is shown that the bimodal distribution is required by Ohm's law.

I. Introduction

Time-dependent fluctuations in the electrical current caused by the discreteness of the charge carriers are known as shot noise. These fluctuations are characterized by a white noise spectrum and persist down to zero temperature. The noise spectral density P (per unit frequency bandwidth) is a measure for the magnitude of these fluctuations. A well-known example is a saturated vacuum diode, for which Schottky found that $P = 2eI \equiv P_{\text{Poisson}}$, with I the average current.¹⁾ This indicates that the electrons traverse the conductor as uncorrelated current pulses, i.e. are transmitted in time according to Poisson statistics. It is also known that a metal wire, of macroscopic length L , does not exhibit shot noise, because inelastic scattering reduces P by a factor l/L , which is much smaller than 1 in a macroscopic conductor (l is the inelastic scattering length). In the last decade, the investigation of transport on smaller length scales has become accessible through the progress in microfabrication techniques. The physics on this mesoscopic scale displays a wealth of new phenomena.^{2,3)} Theoretical analysis⁴⁻⁷⁾ shows that the shot noise in mesoscopic conductors may be suppressed below P_{Poisson} , due to correlated electron transmission as a consequence of the Pauli principle. This raises the question how large P is in a metallic, diffusive conductor of length $L < l$, but still longer than the elastic mean free path ℓ . It has been predicted theoretically⁸⁻¹⁰⁾ that $P = \frac{1}{3} P_{\text{Poisson}}$. This suppression of the

shot noise by a factor one third is *universal*, in the sense that it does not depend on the specific geometry nor on any intrinsic material parameter (such as ℓ). The purpose of this paper is to discuss the origin of the one-third suppression. First, we review the fully quantum-mechanical calculation, where the suppression originates from the bimodal distribution of transmission eigenvalues. Then, a semiclassical calculation is presented, which surprisingly yields the same suppression by one third. One might therefore ask whether there exists a semiclassical explanation for the bimodal eigenvalue distribution. Indeed, we find that this distribution is required by Ohm's law. We conclude with a brief discussion of an experimental observation of suppressed shot noise in a disordered wire, which has recently been reported.¹¹⁾

II. Quantum-mechanical theory

A scattering formula for the shot noise in a phase-coherent conductor has been derived by Büttiker.⁷⁾ It relates the zero-temperature, zero-frequency shot-noise power P of a spin-degenerate, two-probe conductor to the transmission matrix t :

$$P = P_0 \text{Tr} [tt^\dagger(1 - tt^\dagger)] = P_0 \sum_{n=1}^N T_n(1 - T_n). \quad (1)$$

Here $P_0 \equiv 2eV(2e^2/h)$, with V the applied voltage, T_n denotes an eigenvalue of tt^\dagger , and N is the number of transverse modes at the Fermi energy E_F . It follows from current conservation that the transmission eigenvalues $T_n \in [0, 1]$. Equation (1) is the multi-channel generalization of single-channel formulas found earlier.⁴⁻⁶⁾ Levitov and Lesovik have shown¹²⁾ that Eq. (1) follows from the fact that the electrons in each separate scattering channel are transmitted in time according to a binomial (Bernoulli) distribution (depending on T_n). The Poisson noise is then just a result of the limiting distribution for small T_n . Using the Landauer formula for the conductance

$$G = G_0 \text{Tr} tt^\dagger = G_0 \sum_{n=1}^N T_n, \quad (2)$$

with $G_0 \equiv 2e^2/h$, one finds from Eq. (1) that indeed $P = 2eVG = 2eI = P_{\text{Poisson}}$ if $T_n \ll 1$ for all n . However, if the transmission eigenvalues are not much smaller than 1, the shot noise is suppressed below P_{Poisson} . As mentioned above, this suppression is a consequence of the electrons being fermions. In a scattering channel with $T_n \ll 1$ the electrons are transmitted in time in uncorrelated fashion. As T_n increases the electron transmission becomes more correlated because of the Pauli principle. In a scattering channel with $T_n = 1$ a constant current is flowing, so that its contribution to the shot noise is zero.

Let us now turn to transport through a diffusive conductor ($L \gg \ell$), in the metallic regime ($L \ll$ localization length). To compute the ensemble averages $\langle \dots \rangle$ of Eqs. (1) and (2) we need the density of transmission eigenvalues $p(T) = \langle \sum_n \delta(T - T_n) \rangle$. The first moment of $p(T)$ determines the conductance,

$$\langle G \rangle = G_0 \int_0^1 dT p(T) T, \quad (3)$$

whereas the shot-noise power contains also the second moment

$$\langle P \rangle = P_0 \int_0^1 dT p(T) T(1 - T). \quad (4)$$

In the metallic regime, Ohm's law for the conductance holds to a good approximation, which implies that $\langle G \rangle \propto 1/L$, up to small corrections of order e^2/h (due to weak localization). The Drude formula gives

$$\langle G \rangle = G_0 \frac{N\bar{\ell}}{L}, \quad (5)$$

where $\bar{\ell}$ equals the mean free path ℓ times a numerical coefficient.¹³⁾ From Eqs. (3) and (5) one might surmise that for a diffusive conductor all the transmission eigenvalues are of order $\bar{\ell}/L$, and hence much smaller than 1. This would imply the shot-noise power $P = P_{\text{Poisson}}$ of a Poisson process.

However, the surmise $T_n \approx \bar{\ell}/L$ for all n is completely incorrect for a metallic, diffusive conductor. This was first pointed out by Dorokhov,¹⁴⁾ and later by Imry¹⁵⁾ and by Pendry *et al.*¹⁶⁾ In reality, a fraction $\bar{\ell}/L$ of the transmission eigenvalues is of order unity (open channels), the others being exponentially small (closed channels). The full distribution function is

$$p(T) = \frac{N\bar{\ell}}{2L} \frac{1}{T\sqrt{1-T}} \Theta(T - T_0), \quad (6)$$

where $T_0 \simeq 4 \exp(-2L/\bar{\ell}) \ll 1$ is a cutoff at small T such that $\int_0^1 dT p(T) = N$ (the function $\Theta(x)$ is the unit step function). One easily checks that Eq. (6) leads to the Drude conductance (5). The function $p(T)$ is plotted in Fig. 1. It is *bimodal* with peaks near unit and zero transmission. The distribution (6) follows from a scaling equation, which describes the evolution of $p(T)$ on increasing L .¹⁷⁻¹⁹⁾ A microscopic derivation of Eq. (6) has recently been given by Nazarov.²⁰⁾

The bimodal distribution (6) implies for the shot-noise power (4) the unexpected result⁸⁾

$$\langle P \rangle = \frac{1}{3} P_0 \frac{N\bar{\ell}}{L} = \frac{1}{3} P_{\text{Poisson}}. \quad (7)$$

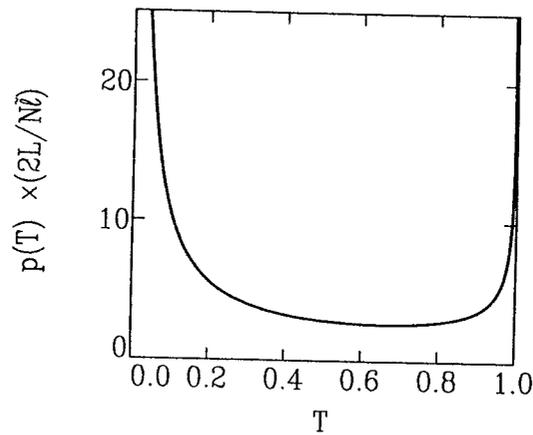


Figure 1. The bimodal distribution of transmission eigenvalues according to Eq. (6). The cutoff for $T \lesssim 4 \exp(-2L/\ell)$ is not shown.

Corrections to Eq. (7) due to weak localization have also been computed,¹⁰⁾ and are smaller by a factor $L/N\bar{\ell}$ (which is $\ll 1$ in the metallic regime).

III. Semiclassical calculation

Since the Drude conductance (5) can be obtained semiclassically (without taking quantum-interference effects into account), one may wonder whether the sub-Poissonian shot noise (7) — which follows from the same $p(T)$ — might also be obtained from a semiclassical calculation. Such a calculation was presented by Nagaev,⁹⁾ who independently from Refs. 8, 10 arrived at the result (7). Nagaev uses a Boltzmann-Langevin approach,^{21,22)} which is a classical kinetic theory for the non-equilibrium fluctuations in a degenerate electron gas. We refer to this method as semiclassical, because the motion of the electrons is treated classically — without quantum-interference effects — whereas the Pauli principle is accounted for, through the use of Fermi-Dirac statistics. Nagaev's approach does not yield a formula with the same generality as Büttiker's formula (1), but is only applicable for diffusive transport.

To put the quantum-mechanical and the semiclassical theories of shot noise on equal terms, we have recently derived a scattering formula for P from the Boltzmann-Langevin approach. This formula is valid from the ballistic to the diffusive transport regime. A detailed description will be the subject of a forthcoming publication. Here, we merely present the result. For simplicity, we consider a two-dimensional wire (length L and width W), with a circular Fermi

surface. The geometry is shown in Fig. 2 (inset). The scattering formula relates P to the classical transmission probabilities $T(\mathbf{r}, \varphi)$, which denote the probability that an electron at position $\mathbf{r} \equiv (x, y)$ with velocity $\mathbf{v} \equiv v_F(\cos \varphi, \sin \varphi)$ (with v_F the Fermi velocity) is transmitted into lead number 2. The result is

$$P = \frac{NP_0}{4\pi W v_F} \int_0^L dx \int_0^W dy \int_0^{2\pi} d\varphi \int_0^{2\pi} d\varphi' W_{\varphi\varphi'}(\mathbf{r}) [T(\mathbf{r}, \varphi) - T(\mathbf{r}, \varphi')]^2 \bar{T}(\mathbf{r}, \varphi) [1 - \bar{T}(\mathbf{r}, \varphi')], \quad (8)$$

where the number of channels $N = W m v_F / \hbar \pi$, and $W_{\varphi\varphi'}(\mathbf{r})$ is the transition rate for (elastic) impurity-scattering from φ to φ' , which may in principle depend also on \mathbf{r} . The time-reversed probability $\bar{T}(\mathbf{r}, \varphi)$ gives the probability that an electron at (\mathbf{r}, φ) has originated from lead 2. From now on we assume time-reversal symmetry (zero magnetic field), so that $\bar{T}(\mathbf{r}, \varphi) = T(\mathbf{r}, \varphi + \pi)$. Equation (8) corrects a previous result.²³⁾ In this notation, the conductance is given by

$$G = \frac{NG_0}{2W} \int_0^W dy \int_0^{2\pi} d\varphi \cos \varphi T(\mathbf{r}, \varphi). \quad (9)$$

Eq. (9) is independent of x because of current conservation. The transmission probabilities obey a Boltzmann type of equation²⁴⁾

$$\mathbf{v} \cdot \nabla T(\mathbf{r}, \varphi) = \int_0^{2\pi} \frac{d\varphi'}{2\pi} W_{\varphi\varphi'}(\mathbf{r}) [T(\mathbf{r}, \varphi) - T(\mathbf{r}, \varphi')], \quad (10)$$

where $\nabla \equiv (\partial/\partial x, \partial/\partial y)$.

We now apply Eq. (8) to the case $W_{\varphi\varphi'}(\mathbf{r}) = v_F/\ell$ of isotropic impurity scattering. Since the scattering is modeled by one parameter, the resulting P is the ensemble average. We assume specular boundary scattering, so that the transverse coordinate (y) becomes irrelevant. Let us first show that in the diffusive limit ($\ell \ll L$) the result of Nagaev⁹⁾ is recovered. For a diffusive wire the solution of Eq. (10) can be approximated by

$$T(\mathbf{r}, \varphi) = \frac{x + \ell \cos \varphi}{L}. \quad (11)$$

Substitution into Eq. (9) yields the Drude conductance $\langle G \rangle = NG_0 \pi \ell / 2L$ in accordance with Eq. (5). For the shot-noise power one obtains, neglecting terms of order $(\ell/L)^2$,

$$\langle P \rangle = NP_0 \frac{\pi \ell}{L} \int_0^L \frac{dx}{L} \frac{x}{L} \left(1 - \frac{x}{L}\right) = \frac{1}{3} P_{\text{Poisson}}, \quad (12)$$

in agreement with Eq. (7).

We can go beyond Ref. 9 and apply our method to quasi-ballistic wires, for which ℓ and L become comparable. In Ref. 24 it is shown how in this case the probabilities $T(\mathbf{r}, \varphi)$ can be

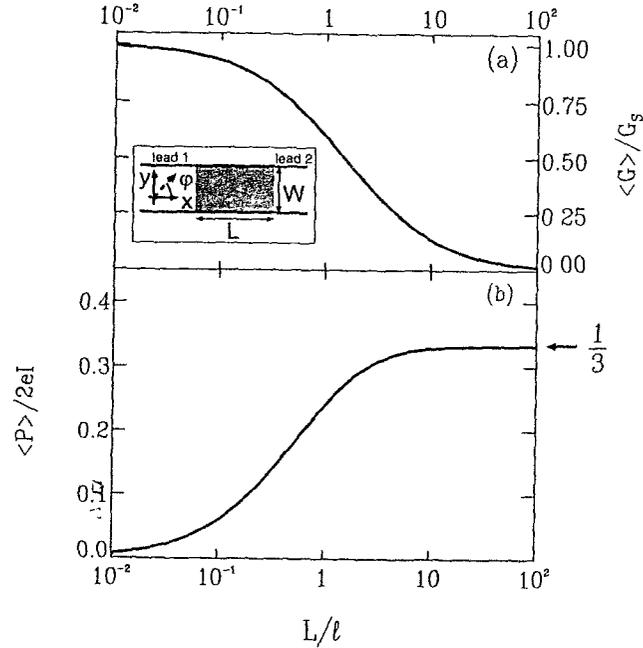


Figure 2. (a) The conductance (normalized by the Sharvin conductance $G_S \equiv NG_0$) and (b) the shot-noise power (in units of $P_{\text{Poisson}} \equiv 2eI$), as a function of the ratio L/ℓ , computed from Eqs. (8) and (9) for isotropic impurity scattering. The inset shows schematically the wire and its coordinates

calculated numerically by solving Eq (10) through Milne's equation. In Fig 2 we show the result for both the conductance and the shot-noise power. The conductance crosses over from the Sharvin conductance ($G_S \equiv NG_0$) to the Drude conductance with increasing wire length.²⁴⁾ This crossover is accompanied by a rise in the shot noise, from zero to $\frac{1}{3}P_{\text{Poisson}}$.

IV. Bimodal eigenvalue distribution from Ohm's law

Now that it is established that the quantum-mechanical calculation (Sec. II) and the semiclassical approach (Sec. III) yield the one-third suppression of the shot noise, we would like to close the circle by showing how the bimodal distribution (6) of the transmission eigenvalues can be obtained semiclassically.

It is convenient to work with the parametrization

$$T_n = \frac{1}{\cosh^2(\alpha_n L)}, \quad n = 1, 2, \dots, N, \quad (13)$$

which relates the eigenvalues T_n of tt^\dagger to the eigenvalues $\exp(\pm 2\alpha_n L)$ of MM^\dagger . Here t is the $N \times N$ transmission matrix, M is the $2N \times 2N$ transfer matrix of the conductor, and $\alpha_n \in [0, \infty)$ for all n . The eigenvalues of MM^\dagger come in inverse pairs as a result of current conservation.¹⁹⁾ The α_n 's are known as the inverse localization lengths of the conductor. Scattering channels for which the localization length is longer than the sample length ($\alpha_n L \ll 1$) are open, if the sample length exceeds the localization length ($\alpha_n L \gg 1$) the scattering channel is closed, as is clear from Eq. (13). The bimodal distribution (6) of the transmission eigenvalues is equivalent to a uniform distribution of the inverse localization lengths,

$$\rho(\alpha) = N\bar{\ell}\Theta(\alpha - 1/\bar{\ell}), \quad (14)$$

where $\rho(\alpha) \equiv \langle \sum_n \delta(\alpha - \alpha_n) \rangle$. Furthermore, the distribution of the α 's implied by Eq. (14) is independent of the sample length L . We will argue that these two properties, L -independence and uniformity, of $\rho(\alpha)$ follow from Oseledec's theorem²⁵⁾ and Ohm's law, respectively.

We recall¹⁹⁾ that the transfer matrix has the multiplicative property that if two pieces of wire with matrices M_1 and M_2 are connected in series, the transfer matrix of the combined system is simply the product $M_1 M_2$. In this way the transfer matrix of a disordered wire can be constructed from the product of N_L individual transfer matrices m_i ,

$$M = \prod_{i=1}^{N_L} m_i, \quad (15)$$

where $N_L \equiv L/\lambda$ is a large number proportional to L . The m_i 's are assumed to be independently and identically distributed random matrices, each representing transport through a slice of conductor of small, but still macroscopic, length λ . In the theory of random matrix products,²⁶⁾ the limits $\lim_{L \rightarrow \infty} \alpha_n$ are known as the Lyapunov exponents. Oseledec's theorem²⁵⁾ is the statement that this limit exists. Numerical simulations¹⁹⁾ indicate that the large- L limit is essentially reached for $L \gg \ell$, and does not require $L \gg N\ell$. This explains the L -independence of the distribution of the inverse localization lengths in the metallic, diffusive regime ($\ell \ll L \ll N\ell$).

Oseledec's theorem tells us that $\rho(\alpha)$ is independent of L , but it does not tell us how it depends on α . To deduce the uniformity of $\rho(\alpha)$ we invoke Ohm's law, $\langle G \rangle \propto 1/L$. This requires

$$L \int_0^\infty d\alpha \rho(\alpha) \frac{1}{\cosh^2(\alpha L)} = C. \quad (16)$$

where C is independent of L . It is clear that Eq. (16) implies the uniform distribution $\rho(\alpha) = C$. A cutoff at large α is allowed, since $1/\cosh^2(\alpha L)$ vanishes anyway for $\alpha L \gg 1$. From Drude's formula (4) we deduce $C = N\bar{\ell}$, and normalization then implies a cutoff at $\alpha \gtrsim 1/\bar{\ell}$, in accordance with Eq. (14).

V. Conclusion

In summary, we have discussed the equivalence of the fully quantum-mechanical and the semiclassical theories of sub-Poissonian shot noise in a metallic, diffusive conductor. Both approaches yield a one-third suppression of P relative to P_{Poisson} . The bimodal distribution, which is at the heart of the quantum-mechanical explanation, can be understood semiclassically as a consequence of a mathematical theorem on eigenvalues (Oseledec) and a law of classical physics (Ohm's law). \sphericalangle

The fact that phase coherence is not essential for the one-third suppression of P suggests that this phenomenon is more robust than other mesoscopic phenomena, such as universal conductance fluctuations. This might explain the success of the recent attempt to measure the shot-noise suppression due to open scattering channels in a disordered wire defined in a 2D electron gas.¹¹⁾ In this experiment a rather large current was necessary to obtain a measurable shot noise, and it seems unlikely that phase coherence was maintained under such conditions.

In both the quantum-mechanical and semiclassical theories discussed in this review, the effects of electron-electron interactions have been ignored. The Coulomb repulsion is known to have a strong effect on the noise in confined geometries with a small capacitance.²⁷⁾ We would expect the interaction effects to be less important in open conductors.²⁸⁾ While a fully quantum-mechanical theory of shot noise with electron-electron interactions seems difficult, the semiclassical Boltzmann-Langevin approach discussed here might well be extended to include electron-electron scattering and screening effects.

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