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COMMUNICATIONS FROM THE OBSERVATORY AT LEIDEN.

Remark on the mean proper motion of stars of a given apparent magnitude,

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In *Groningen Publ.* **30**, Table 21, p. 99, KAPTEYN and VAN RHIJN have given the mean proper motion of stars of different apparent visual magnitude for three zones of different numerical galactic latitude.

If the star density were constant in space, and the distribution of peculiar motions the same everywhere, the relative frequencies of $m + 5 \log \mu$, where m is the apparent magnitude and μ the annual proper motion, would be the same for all magnitudes. The determination of these pseudo relative frequencies is of importance in statistics of proper motions. The real frequencies of different proper motions for stars of a given apparent magnitude are in comparison characterized by the incompleteness of small proper motions, owing to the deficiency of remote stars caused by the falling off of the star density with increasing distance. It will now be of interest to see how far the observed data agree with the simple assumption, that the missing stars are those of smallest possible proper motion.

In *A. N.* 4883, **204**, 187; 1917, Tab. II, column 7 logarithms of numbers of stars in the whole sky between definite limits of proper motion are given for the interval 5^m to 6^m visual, representing a mean magnitude of say $5^m \cdot 55$. These numbers are satisfactorily represented by the formula there given;

$$(1) \quad \log N_{5.55,\,\rho'} = 1{\cdot}1457 - {\cdot}3972\,\rho' - {\cdot}0204\,\rho'^2,$$

where we have put $\rho' = 5 \log \mu$. By the principle stated above, we have*), for arbitrary values of x:

$$(2) \quad \log N_{m,\,\rho'} = 0{\cdot}6\,x + \log N_{m-x,\,\rho'+x},$$

and combining (1) and (2) we find

$$(3) \quad \log N_{m,\,\mu} = -{\cdot}7081 + 0{\cdot}6\,m - {\cdot}1528\,\rho - {\cdot}0204\,\rho^2,$$

where now $\rho = m + 5 \log \mu$, and $N_{m,\,\mu}$ is the number of stars in the whole sky having magnitudes between $m + {\cdot}45$ and $m - {\cdot}55$, and proper motions with logarithms between $\log \mu \pm 0{\cdot}1$. The numbers given in Table III of the quoted paper are equivalent to the formula (3), which can also be written:

$$(3') \quad N_{m,\,\mu}\, d\rho = \frac{A}{\sqrt{\gamma}}\, e^{\beta^2/4\gamma}\, e^{-t^2}\, dt,$$

where

$$t = \sqrt{\gamma}\,(\rho - \frac{\beta}{2\gamma}) = {\cdot}217\,(\rho + 3{\cdot}744)$$

$$\log A = -{\cdot}7081 + 0{\cdot}6\,m$$

$$\beta = -{\cdot}3518, \qquad \gamma = +{\cdot}0470.$$

Further $\mu = e^{\alpha(\rho - m)}, \qquad \alpha = {\cdot}4605.$

From these formulas we can derive the total number of stars between the magnitudes $m + {\cdot}45$ and $m - {\cdot}55$, and the arithmetical mean $\bar{\mu}$ of the proper motions of these stars. If we require the number of stars per square degree, we must replace A by $A' = A/41253$. We find

$$N_m = \frac{A'\sqrt{\pi}}{\sqrt{\gamma}}\, e^{\beta^2/4\gamma}, \qquad \log N_m = -4{\cdot}125 + 0{\cdot}6\,m,$$

$$\bar{\mu} = e^{-\alpha m + \frac{\alpha^2 + 2\alpha\beta}{4\gamma}}, \qquad \log \bar{\mu} = -{\cdot}258 - 0{\cdot}2\,m.$$

It may be mentioned that one half of the arithmetical sum of all proper motions is yielded by $1/15$ (more accurately $1/15{\cdot}06$) of the number of stars, beginning from the largest proper motions.

The actual number of stars is less than N_m, and the actual mean proper motion is larger than $\bar{\mu}$, on account of the deficiency of stars with very small proper motions, as pointed out above. If we make the simple hypothesis, that the formula (3) holds good from the largest proper motions down to a certain value μ_1, and that there are no stars with proper motions smaller than μ_1, then from the observed numbers we can derive μ_1, and from this the mean of all proper motions exceeding μ_1.

*) The form of the following explanation, up to the first six lines on the next page, is due to Prof. DE SITTER, who translated my reasoning into mathematical formulas.