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Letter to the Editor

A new derivation of the tensile strength of cometary nuclei: application to comet Shoemaker–Levy 9

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Abstract. The splitting of comets as exemplified by comet Shoemaker-Levy 9 (hereafter SL9), when it passed near Jupiter, is a common phenomenon. Multiple splitting is also not an uncommon occurrence. It is clear that the comet nucleus is fragile; i.e., its tensile strength is small compared with that of solid materials. We show that aggregates of sub-micron interstellar dust particles presumed to consist of a silicate core, an inner mantle of complex organic refractory molecules, and an outer mantle dominated by H_2O ice (Greenberg, 1982) provide the basis for a *quantitative* derivation of the tensile strength of comet SL9 using molecular interactions at the contact interfaces. In fact, using a mean particle size representing interstellar dust as it would appear in its final presolar state one *derives* a tensile strength which describes remarkably well the multiple splitting phenomenon. This derivation of the tensile strength of a particle aggregate resulting from molecular interactions is quite general and can be applied to physical situations involving any sorts of aggregates as well as those representing comet nuclei.

Key words: comets:- tensile strength– comet P/Shoemaker-Levy 9– tidal disruption– secondary fragmentation, Interstellar dust– aggregate

1. Tensile strength of an aggregate

Clearly the tensile strength of an aggregated material is smaller than that of the solid. We quantify this by regarding the nucleus, for simplicity, as a homogeneous distribution of small spherical particles *all* covered with ice mantles as would be true for protostellar dust so that ice is the dominant material at the *interfaces* where binding forces act between molecules. The tensile strength of the aggregate material will then depend on the number of interacting molecules at the contact surfaces. Since this number is substantially less per unit volume than that in solid ice the tensile strength is reduced accordingly. The tensile strength of bulk material can be used to depict a mean molecular interaction energy $E = \alpha\, 10^{-2}\,\mathrm{eV\,molecule^{-1}} = 1.6 \times 10^{-14}\alpha\,\mathrm{erg\,molecule^{-1}}$ for hydrogen or van der Waals bonding (depending on the material) where α is to be determined. Assuming an ice density of $0.9\,\mathrm{cm^{-3}}$ gives a mean molecular volume $\Omega = 30 \times 10^{-24}\,\mathrm{cm^{-3}}$ so that the energy per unit volume (or equivalently force per unit area) is $E/\Omega = 5.3 \times 10^{8}\alpha\,\mathrm{dyne\,cm^{-2}}$. If we compare this with the tensile strength of bulk ice of 20 bar $= 2 \times 10^{7}\mathrm{dyne\,cm^{-2}}$ (Hobbs, 1974) we arrive at $\alpha = 0.038$. However the tensile strength of normally prepared materials is reduced by the presence of imperfections and microcracks so that the derived value of the inter-molecular energy is an underestimate. In specially prepared more perfect materials the tensile strength increases by orders of magnitude. In the case of water molecules a more realistic estimate of the molecular bonding strength would be the dipole-dipole bond (hydrogen bond) energy which has been deduced to be $732\,\mathrm{J\,mol^{-1}}$ at a distance of $5 \times 10^{-8}\,\mathrm{cm}$; which is somewhat greater than the mean molecular diameter of about 3×10^{-8}. Consequently a more realistic value of α than the one derived directly from the tensile strength is at least 20 times larger; i.e., $\alpha \gtrsim 0.76$. We let the comet nucleus consist of a porous aggregate of spherical ice mantled particles of radius a. In the loosest possible aggregate the number of contact points per particle is 1. We define the number of contact points per particle as β where $1 \leq \beta \leq 12$ where $\beta = 12$ corresponds to a cubic lattice. Two spheres in contact are shown in Figure 1. We assume that only those molecules around the point of contact and at a distance of the order of the molecular diameter $\sim 3 \times 10^{-8}\,\mathrm{cm}$ will contribute to the tensile strength. Thus all those molecules within

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the sagitta defined by $h = 3 \times 10^{-8}$ cm are included in the contact area $\Delta A = \pi x^2 \simeq 2\pi a h$ where $h \ll a$ (in the final analysis it will turn out that $h/a \leq 10^{-3}$). Taking into account the number of contact points per particle, the total number of interacting molecules per particle is $\sim \beta \Delta A / h^2 \simeq 2\pi\beta a/h$.

If the nucleus has a porosity P, the number of particles per unit volume is

$$n = (1 - P)/[(4/3)\pi a^3] \quad (1)$$

so that the number of interacting molecules per unit volume is

$$N = 3\beta(1 - P)/2a^2 h, \quad (2)$$

and the tensile strength of the aggregate is

$$T = NE = 3\beta(1 - P)E/2a^2 h, \quad (3)$$

where E is the *average* molecular interaction energy at the particle interfaces. If we assume an r^{-6} dependence (dipole-dipole) the mean value of the interaction energy turns out to be about 4 times larger than that which is given by assuming all the molecules in the sagitta to have the interaction energy corresponding to a separation of 5×10^{-8} cm. But, since impurities in the ice will reduce the dipole-dipole interaction we retain the nominal value $E = \alpha 10^{-2}$ eV $= 0.76 \times 10^{-2}$ eV to describe the mean molecular interaction energy. We then obtain the tensile strength of the aggregate to be

$$T = 6.1 \times 10^3 (1 - P)\beta (a/0.1\mu\text{m})^{-2} \,\text{dyne cm}^{-2}. \quad (4)$$

Note that the tensile strength is inversely proportional to the square of the particle radius.

We use the aggregated dust model shown in Fig. 2 as the basis for ascribing a value to the mean number of contact points per interstellar grain as $\beta = 5$. The porosity designed into the model is $P = 0.8$ which is consistent with the comet dust porosities derived from the infrared emission properties of comet Halley dust (Greenberg and Hage, 1990) and the comet nucleus density for this porosity is $\rho = 0.28\,\text{g cm}^{-3}$ which is consistent with that deduced by Rickman from non-gravitational forces (Rickman, 1986). The density assumed by Sekanina et al. (1994) is even lower ($\sim 0.2\,\text{g cm}^{-3}$). The maximum mean interstellar dust grain radius before aggregation to form comets is $a \simeq 0.15\,\mu$m as derived from the complete accretion of all cosmically abundant elements on the typical (or average) large diffuse cloud dust grain of radius $a \simeq 0.1\,\mu$m (Greenberg 1984, 1991; Xing, 1993). The $0.1\,\mu$m diffuse cloud dust depletes about 20% of the cosmically available oxygen and about 50% of the carbon leaving 80% O and 50% C which can lead to an increase in volume of about 150%, when fully accreted, and a corresponding increase in radius of 50%, i.e., $a_{\text{total}} \simeq 0.15\,\mu$m. We should point out that the interstellar dust chemical as well as morphological

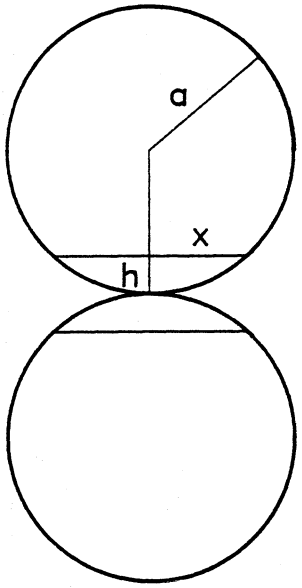

Fig. 1. Two contacting spherical submicron particles of radius a. The height $h \ll a$ is of the order of the diameter of the molecules at the surface. Diagram not to scale.

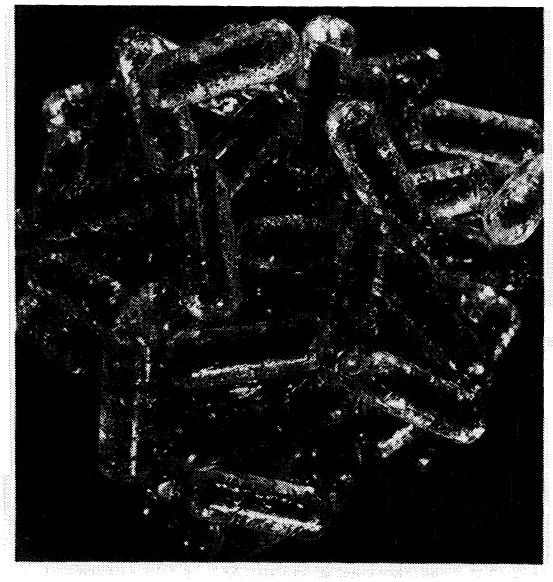

Fig. 2. "Bird's nest" model of a porous aggregate of 100 submicron presolar nebula interstellar particles representing a 3 micron diameter piece of a comet nucleus. Each particle consists of a silicate core, an inner mantle of organic refractory molecules and an outer mantle of ices (predominantly water ice) in which are embedded very small carbonaceous particles. The porosity is 0.8 and the density is $0.28\,\text{g cm}^{-3}$. The scale of this figure conforms with the most recent interstellar dust model sizes (Xing, 1993) and the comet dust model of Greenberg & Hage (1990).

structure of this size and type provided a quantitative basis for explaining the physical and radiative properties of comet Halley dust (Greenberg and Hage, 1990; Chapman 1990) as well as having predicted many of the so-called "surprising" properties of the nucleus (Greenberg, 1986).

Using $\beta = 5$, $P = 0.8$, $a = 0.15\,\mu$m in Eq. 4 gives

$$T = 2.7 \times 10^3\,\mathrm{dyne\,cm^{-2}}. \quad (5)$$

This is about 10^4 times smaller than the tensile strength of solid ice (Hobbs, 1974) which one recalls is about $2 \times 10^7\,\mathrm{dyne\,cm^{-2}}$. Considering a more realistic size distribution of core-mantle interstellar particles which includes smaller as well as larger sizes ($0.07\,\mu\mathrm{m} \leq a \leq 0.2\,\mu\mathrm{m}$) would lead to a larger tensile strength because of more interacting molecules per unit volume. As already noted, the tensile strength of "normal" ice underestimates the true intermolecular bonding. In view of the positive as well as negative effects we shall adopt the value of T in Eq. 5 as a reasonable intermediate representation. In any case the above method of derivation of T appears to be quite general and its value is clearly low.

2. Tidal disruption of a comet

The disruption of comet SL9 has been treated by many authors. It has been suggested by Scotti and Melosh (1994) that the parent body of comet SL9 was about 2 km in radius and split by tidal forces into many pieces at or around perijove. This would imply a very high tensile strength compared with other estimates (Sekanina et al., 1994) which are rather like that given in Eq. 5 (see Sec. 3). On the other hand, Boss (1994) has proposed a tidal fracture well *before* closest approach leading to the production of a rubble pile structure which subsequently breaks up without appreciable shear forces into many fragments. Based on the results of Weaver et al (1994) the effective radii of most of the fragments are at least 1 – 2 km which implies an initial radius of the order of 10 km which is 5 times larger than that used by Scotti and Melosh.

It is likely that the fragmentation of comets is more complex than can be described by a single model so that it is useful to consider a number of possibilities. One of these possibilities may be described *quantitatively* based on the tensile strength derived above for an aggregate of small particles. We shall base our theoretical arguments on the formulation given in Dobrovolskis (1990) and apply them to the passage of SL9 when it split passing by Jupiter. For simplicity we first consider the nucleus as a homogeneous sphere. Brittle bodies may fail by either tensile or shear fracture. As noted by Dobrovolskis the central shear $(\sigma_{xx} - \sigma_{zz}) = (\sigma_{yy} - \sigma_{zz})$, where the z-axis points in the comet-Jupiter direction, is always greater than the maximum tension and weak bodies fail by shear fracture starting at their centers. The greatest shear stress, which occurs at the center, is given by

$$S = [(24\lambda + 18\mu)/(19\lambda + 14\mu)]G\rho_c M_J b^2/d^3, \quad (6)$$

where μ = rigidity (shear modulus), λ = Lamé parameter (related to compressibility), ρ_c = comet nucleus density, G = gravitational constant, M_J = Jupiter mass, d = distance from Jupiter at time of splitting, b = comet radius. The material-dependent ratio $c \equiv (24\lambda+18\mu)/(19\lambda+14\mu)$ may be expressed as $c = (9 + 6\nu)/(7 + 5\nu)$ with ν being the Poisson ratio, and for all reasonable values of $0 < \nu < 1/2$ the coefficient is limited to the range $1.26 \leq c \leq 1.286$. Furthermore if the body is ultracompressible, which is characteristic of underdense materials, the condition $\lambda \ll \mu$ applies and the coefficient c is $9/7 = 1.286$ corresponding to $\nu \to 0$. The evidence is overwhelming that comets have densities less than one gram per cc and this fact will, indeed, be needed to provide the low tensile strength inferred. A somewhat conservatively "high" estimate of density of $0.6\,\mathrm{g\,cm^{-3}}$ (Sagdeev, 1987) implies at least 60% empty space but it is strongly suggested that comets probably have densities as low or lower than $0.3\,\mathrm{g\,cm^{-3}}$ (Greenberg & Hage, 1990; Rickman, 1986; Sekanina et al., 1994) implying at least 80% empty space. Consequently the ultracompressible condition is reasonable and the stress in Eq. 6 may be approximated as

$$S = (9/7)\pi G\rho_c\rho_J b^2 (R_J/d)^3, \quad (7)$$

where ρ_J = Jupiter's bulk density, R_J = Jupiter radius.

For a comet nucleus porosity P the comet density is $\rho_c = (1 - P)\rho_c^{\mathrm{compact}}$ where the density of a fully compact nucleus is about $\rho_c^{\mathrm{compact}} = 1.54\mathrm{g\,cm^{-3}}$ (Greenberg & Hage, 1990). We use $\rho_J = 1.33\,\mathrm{g\,cm^{-3}}$ to find that the stress at the center at the time of SL9 break-up was (ignoring rotation)

$$S = 5.5 \times 10^3(1 - P)(b/\mathrm{km})^2(R_J/d)^3\,\mathrm{dyne\,cm^{-2}}. \quad (8)$$

It is immediately seen that this is similar to the tensile strength derived from the interstellar dust model in Eq. 5. In the next section we will apply this to the splitting phenomenon.

3. Multiple splitting of comet SL9

The evidence for continuing disintegration of comet SL9 has been well documented (see Sekanina et al., 1994). Secondary fragmentation is implied by Weaver et al. (1994) from their observations. Each break up must have been accompanied by many smaller fragments down to the sizes of comet dust. Since comet SL9 broke up into at least 21 pieces, we suggest that sequential splitting (assuming for simplicity equal parts each time) could have occurred as many as 5 times. The mean radius of comet SL9 before it split into many pieces was probably of the order of or greater than 5 km (Weaver et al., 1994; Sekanina et al., 1994). We consider several possible initial comet radii to derive the distance from Jupiter at which the splittings could have occurred by applying Eq. 8 to comet masses (and appropriate radii) resulting from splitting each time

into two equal parts. We first note that given the tensile strength of Eq. 5 and a perijove distance of $d/R_J = 1.32$, the *smallest* comet which will split has a radius of 2.38 km. As already noted, if this were the case, it would confirm the suggestion by Scotti & Melosh (1994). On the other hand, if one requires more than one splitting, the initial nucleus radius must be considerably larger than 2 km. Results are shown in Table 1 with the final splitting leading, in an idealized situation, into 32 pieces which is sufficient (but probably not necessary) to account for the 21 pieces observed.

Table 1. Sequential splitting distances from Jupiter for several possible initial comet Shoemaker-Levy radii.

	Initial radius b(km)		
	5	10	15
order of splitting	d/R_J		
1	2.2	3.4	4.5
2	1.9	2.9	3.9
3	1.6	2.5	3.3
4	1.4	2.2	2.8
5	1.2*	1.9	2.4

* denotes distance smaller than perijove.

The radii of the final "32" pieces for initial radii 10 km and 15 km are, respectively, 3.1 and 4.7 km assuming no loss into smaller fragments as is *most likely* to have occurred. It is certainly not to be expected that all the splittings were into two equal pieces, and, in fact, multiple splittings could also have occurred so that the "need" for 5 splittings to produce 21 "large" pieces is probably an overestimate. Perhaps even three might have been enough if some fragmentations of the comet were three-fold. But the essence of the idea is that *sequential* splittings of the comet fragments are likely to have provided the total number of pieces observed. Thus, because of the unquestionable loss in "undetectable" small fragments one expects that: (1) the identifiable pieces are, on the average, smaller at each stage, and (2) the sequential splittings are on the average at smaller distances from Jupiter than shown in Table 1 because the pieces are smaller. In any case, the range of final sizes are expected to be less than 3-5 km and could be as small as the fragments described by Weaver et al. (1994). We see also that because of the low tensile strength the initial splitting distance probably occurred well before perijove although the final one *could* have occurred close to perijove distance $d/R_J = 1.32$ (Sekanina et al., 1994).

4. Concluding remarks

It appears that the porous aggregate interstellar dust model can provide a *quantitative* basis for explaining the tidal splitting of comet SL9. It is also consistent with the fact that upon impacting on Jupiter it produced clouds of particles of low density which could be providing the long lasting dark areas by floating in the upper atmosphere like volcanic dust in the earth's atmosphere.

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