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Summary. I present some elementary calculations of the acceleration of gaseous bullets by radiation pressure. For simplicity, the radiating surface is assumed to have a blackbody spectrum, and to be either flat, or a hollow disk, or a narrow cone. The temperature of the disk is either constant, or decreases as a power of the height above the disk midplane; the surface is either a Lambert radiator, or has a cosine limb darkening.

In all these cases, it is seen that the combined effect of acceleration by photons that are emitted near the cloud, and deceleration by aberrated blueshifted photons that were emitted far away, produces a finite "magical" terminal speed $v/c=\beta_m$ that is rather less than the speed of light. Besides finding that $\beta_m=\frac{1}{3}(4-\sqrt{7})$ for acceleration above an infinite flat radiator, I show that $\beta_m \lesssim 0.8$ in cases where the temperature of the driving surface decreases upward and outward, and that $\beta_m \lesssim 0.4$ in conical funnels with an opening half-angle of 40° or less.

In order to try and obtain higher final speeds, as are implied in superluminal radio sources, I investigate the various physical effects that influence β_m. Among these is the effect of making the driving radiation vary in time. Although β_m does go up, I show that the required value $\beta_m \approx 0.99$ can be obtained only for huge temperature increases, and corresponding luminosity bursts exceeding a factor 10^6, which are almost certainly unacceptable. The situation can be saved, however, if the scattering of the radiation is "coloured" according to $\kappa \propto \nu^{-n}$ instead of the colourless Thomson scattering. For $n=3$, it is shown that $\beta_m=0.99$ can be achieved with a photon burst from a disk in which the temperature increases by a factor 10.

Key words: relativity – acceleration mechanisms – galaxies: nuclei of – hydrodynamics

1. Jets versus clouds

It is clear from the synchrotron lifetimes of fast electrons in nonthermal radio sources that said sources must have their energy replenished by means of matter that contains very few highly relativistic electrons, if indeed any at all. Models for such replenishment were proposed long ago (Scheuer, 1974; Blandford and Rees, 1974.) The striking linear morphology of the inner parts of radio galaxies (Bridle, 1984) provided *prima facie* evidence for the correctness of these hydrodynamic jet models, and a thriving industry of supersonic flow calculations has sprung up (Ferrari and Pacholczyk, 1983; Bridle and Eilek, 1984; Begelman et al., 1984; Centrella et al., 1985; Winkler et al., 1987, and references therein.)

It seems certain that bulk kinetic energy feeds large scale radio lobes, but it is not necessary that this energy is supplied in the form of continuous streams. The first unambiguous case against continuous jets was put forward by Rudnick (1982) and by Rudnick and Edgar (1984), who noted that many radio sources show clear evidence of interruptions in the flow that feeds the outer lobes of these sources. The interruptions occasionally show some regularity, maybe indicative of "flip-flop" ejection of gas alternating between the two sides of radio nuclei. Some shy attempts at bucking the trend have been made (Wiita and Siah 1981; Saikia and Wiita 1982; Icke 1983), but continuous jets are still the industry standard for models of radio galaxies (Norman et al., 1982, 1983; Winkler et al., 1987.)

The observations need not imply the existence of continuous flow. Whenever we actually see something move, what moves is a blob. This is true for diminutive radio doubles like the galactic source SS 433 (Margon, 1984; Schilizzi et al., 1984; Vermeulen et al., 1987) as well as VLBI nuclei of radio galaxies (e.g. Fanti et al., 1984, *passim.*) Even thermal sources, like the galactic bipolars (e.g. Mundt, 1985; Lada, 1985) and the spindle-type planetary nebulae (Balick et al., 1987) show unmistakable signs of propagating blobs. Moreover, there is now ample evidence that the broad line regions of active galactic nuclei, which are thought to be very close to the central engine, are chock full of clouds; these might well serve as ammunition, given proper circumstances. Indeed, the parameters of gaseous bullets which I will derive below are quite comparable to what is needed for broad-line clouds.

A prime example of moving blobs is provided by the moving optical features in SS 433 (Margon, 1984), and the corresponding VLBI blobs (Vermeulen et al., 1987) show unambiguously that the density in the SS 433 "beams" (whatever they are) must vary by many orders of magnitude as a function of time and position. The trajectories of the moving features can be fitted with an expulsion speed of 0.26 c to a remarkable precision; this excludes the possibility that the blobs observed in SS 433 are due to moving internal shocks, except under extremely contrived circumstances.

It seems very unlikely that the acceleration and collimation mechanism in SS 433 is purely hydrodynamic. The reason is again that the density in the beams is variable over many powers of 10, whereas the speed remains virtually constant. Hydrodynamic acceleration processes have a manifest tendency to keep the momentum density constant, so that one expects high-density flow to go more slowly than low-density gas. In the following I will present an acceleration mechanism that produces a constant terminal speed rather than a fixed momentum density.

The existence of nuclear jets in radio galaxies is unproven, at least if "jet" has its usual meaning of continuous, collimated hydrodynamic flow. I shall instead adopt the hypothesis that the activity near compact accreting objects produces gaseous cannonballs that are shot out of the central engine assembled around the accretion source. This hypothesis has a venerable history (e.g. De Young and Axford, 1967; Christiansen, 1971). I will argue, though, that the most easily accelerated clouds are not "plasmons", but relatively cool clouds (at least initially,) with parameters rather reminiscent of clouds in the interstellar medium.

Some of the severe problems that jets encounter when they must propagate in an external medium, and especially when they must bend (Icke, 1989) can be avoided in the case of propagating blobs, which are not necessarily harmed by the re-entrant flow that wreaks havoc in jets. Also, it is much easier to radiatively accelerate a blob than a jet, because a blob has a backside that can be kicked, and the optical-depth problems that plague radiatively accelerated jets do not occur. On the other hand, blobs also have a front side, which may cause too rapid a deceleration as the cloud interacts (almost certainly via a bow shock) with the surrounding medium. Among the possible solutions to this objection are either a surrounding tenuous medium that moves with the cloud ("gunsmoke"), or having clouds that are fairly dense. As will be seen below, the latter situation is favourable to radiative cloud acceleration.

There is indirect evidence that what some think is a jet is really a stream of blobs: the best-fitting models of radio sources on a large scale, often showing great complexity, are all basically ballistic-blob models (Blandford and Icke, 1978; Icke, 1981; Gower et al., 1982; Hjellming and Johnston, 1982; Owen et al., 1985; Yokosawa and Inoue, 1985.) Some attempts at three-dimensional hydrodynamics of the interaction between a jet and an intergalactic wind have been made (Williams and Gull, 1984); this might prove to be interesting, but the numerical resolution is still quite small. Finally, there is no reason why a continuous jet should end up with a precisely determined speed, as in SS 433; I will show below that photon-driven gas clouds *do* possess a "magical speed".

Once one has supposed that gaseous bullets are responsible for the observed behaviour of nonthermal radio sources, several main lines need to be pursued, among which are (1) how does a supersonically moving gasball behave, (2) how does it get collimated, and (3) how is it accelerated? Subjects (1) and (2) are being investigated separately; here, I wish to present some calculations on the acceleration of gaseous bullets, in the case that radiation pressure provides the requisite push. It will be seen, among other things, that there is a "magical" speed associated with such a mechanism; that the speed of 0.26 c in SS 433 can be produced by this mechanism; that the photon pressure may help confine blobs when they are being accelerated; and that one must expect a correlation between the variability of a source and the speed to which gaseous bullets are accelerated by it. These conclusions follow from exact solutions to somewhat idealized acceleration geometries. The equations given do allow application to more complex situations.

2. Photon surfing above a steady blackbody

2.1. *The equation of motion*

Consider an infinite flat surface that radiates a blackbody spectrum with temperature T. Fix a right-handed Cartesian coordinate system K such that the x- and y-axes lie in the radiating surface. Let an object with rest mass m, which absorbs all incident radiation equally, be allowed to move along the z-axis. Attach a coordinate system K' to m, such that the z'-axis coincides with the z-axis, (x', y') are parallel to (x, y), and m occupies the origin of K', as usual.

Suppose that m has reached point z, with speed v at time t, in K. The radiation it instantaneously receives in K' at its corresponding proper time t', coming in at angle θ' with the positive z'-axis, left the $z=0$ plane at radius R and at time $t-\frac{1}{c}\sqrt{R^2+z^2}$, under an angle θ with the positive z-axis. Evidently, $\tan\theta=R/z$, and Lorentz symmetry produces the familiar aberration

$$\tan\theta = R/z = \frac{\sin\theta'}{\gamma(\cos\theta'+\beta)}. \tag{2.1}$$

The incoming radiation left the emitting surface at time

$$t_{\rm ret} = t - \frac{z}{c}\left(\frac{\beta\cos\theta'+1}{\cos\theta'+\beta}\right). \tag{2.2}$$

If the light was emitted at frequency ν, it is received at ν', where

$$\nu = \gamma\nu'(1+\beta\cos\theta'), \tag{2.3}$$

(e.g. Rybicki and Lightman, 1979.) Now if B_ν is the specific intensity of the radiating plane, the contribution dA' of the photons in the frequency band $d\nu'$ to the acceleration A' is given by

$$dA'(\nu') = \frac{D}{mc} B_{\nu'}\, d\nu', \tag{2.4}$$

in the direction θ' of the incoming radiation; here D is the cross section of m. Assuming D to be independent of frequency, and assuming that m is symmetric about the z'-axis, one finds that the total acceleration A' is directed along the z-axis, and has a magnitude

$$A' = \frac{D}{mc}\int_0^\infty d\nu' \int_0^{\theta'_m} d\theta'\, B_{\nu'}\cos\theta'\,(2\pi\sin\theta'). \tag{2.5}$$

Using the Lorentz invariance of $\nu^{-3}B_\nu$, this can be written as

$$A' = \frac{2\pi D}{mc}\int_0^\infty d\nu \int_{-\beta}^{1} du\, \frac{uB_\nu}{\gamma^4(1+\beta u)^4}, \tag{2.6}$$

Performing the usual integration of B_ν over frequencies, one obtains

$$A' = \frac{aD}{2m}\int_{-\beta}^{1} \left(\frac{T}{\gamma(1+\beta u)}\right)^4 u\, du, \tag{2.8}$$

in which $a=7.565\,10^{-16}\ {\rm J\,m^{-3}\,K^{-4}}$ is the blackbody radiation constant, and $T=T(u)$ is the temperature seen in the direction u, and emitted at the retarded time specified by Eq. (2.2):

$$t_{\rm ret} = t - \frac{z}{c}\left(\frac{\beta u+1}{\beta+u}\right). \tag{2.9}$$

Note, for future applications, that one may extend the functional dependence of T to $T(u, t, R)$, connecting R to t and u by Eq. (2.1):

$$R = \frac{z\sqrt{1+u^2}}{\gamma(u+\beta)}. \tag{2.10}$$

To derive the equation of motion of m, assume that A' acts during an infinitesimal time interval $\Delta t'$, thereby increasing the speed of m in K' from zero to $\Delta v'$. By using the relativistic addition of velocities one easily shows that, seen in K, the acceleration of m is

$$\frac{dv}{dt} = \gamma^{-3} A'. \tag{2.11}$$

Inserting A' from Eq. (2.8), one obtains

$$\frac{d\beta}{dt} = \frac{aD}{2mc}\gamma^{-7}\int_{-\beta}^{1}\left(\frac{T}{1+\beta u}\right)^4 u\,du. \tag{2.12}$$

For future reference, I note that the radiation pressure on each unit of the surface of m is proportional to

$$P = \left(\frac{T(u)}{\gamma(1+\beta u)}\right)^4. \tag{2.13}$$

Now I will make Eq. (2.12) dimensionless by introducing a temperature unit T_0 and a time unit t_0; then if $T \to T/T_0$ and $t \to t/t_0$, one obtains

$$\frac{d\beta}{dt} = Q\gamma^{-7}\int_{-\beta}^{1}\left(\frac{T}{1+\beta u}\right)^4 u\,du, \tag{2.14}$$

$$Q \equiv \frac{acDT_0^4 t_0}{2mc^2} = \frac{2\pi L t_0}{E} \tag{2.15}$$

Here L is the luminosity intercepted by m, and E is the rest mass energy of m.

2.2. *Estimates of parameter values*

Let us consider some plausible values for Q, and see if clouds can be expected to have the parameters necessary to make $Q > 1$ (i.e. rapid acceleration.) If the cloud has radius r, then $D = \pi r^2$ and

$$Q = 2\pi\sigma r^2 T_0^4 t_0/mc^2, \tag{2.16}$$

in which $\sigma = 5.670\,10^{-8}$ W m^{-2} K^{-4} is Stefan-Boltzmann's constant. If we adopt the light crossing time $t_0 = r/c$ as the characteristic time scale, then

$$Q = \frac{2\pi\sigma T_0^4 r^3}{mc^3} = \frac{3\sigma}{2c^3}\frac{T_0^4}{\rho} = 3.152\,10^{-33}\,T_0^4/\rho, \tag{2.17}$$

where ρ is the mean mass density of the cloud. Assuming that the cloud is pure hydrogen, this can be expressed by means of the particle density n as

$$Q = 1.882\,10^{-6}\,T_0^4/n. \tag{2.18}$$

For any given temperature T_0 of the driving radiation, this imposes an upper limit to the density of the gaseous bullet; above this limit, the acceleration occurs on a time scale much larger than the photon crossing time of the cloud.

There is also a lower limit on the density, because if n is too small the cloud becomes transparent to the driving photons, and not enough momentum would be transferred. Thus, the mean free path λ of the photons in the cloud must be much less than the cloud radius r. Together with Eq. (2.18) I obtain the limits

$$\lambda = 1/n\sigma_T \ll r; \quad n \gg 1/r\sigma_T = \frac{1.503\,10^{28}}{r}\,\mathrm{m}^{-3}, \tag{2.19}$$

$$n \lesssim 1.9\,10^{-6}\,T_0^4\,\mathrm{m}^{-3}, \tag{2.20}$$

in which σ_T is the Thomson cross section (other scattering cross sections could of course be substituted, but give roughly the same limits on n.) Together, these equations establish limitations for T_0 and r:

$$T_0 \gg 2.989\,10^8\,r^{-1/4}. \tag{2.21}$$

For example, gaseous bullets with a caliber of 10^{17} mm (i.e. 3.2 milliparsec or 670 A.U.) are efficiently accelerated at temperatures above $9\,10^4$ K; the cloud density must not be less than about 10^{12} m^{-3} (see Fig. 1). The bullet parameters also apply at much smaller scales; for example, in the X-ray emitting zone of SS 433 one has roughly $r = 10^9$ m (Watson et al. 1986). Thus, we must have $n > 1.5\,10^{19}$ m^{-3} and $T > 1.7\,10^6$ K. Assuming that the emission comes from a region with volume r^3, one finds an X-ray luminosity $L_X = 1.4\,10^{-40}\,T^{1/2} n^2$ W (Rybicki and Lightman

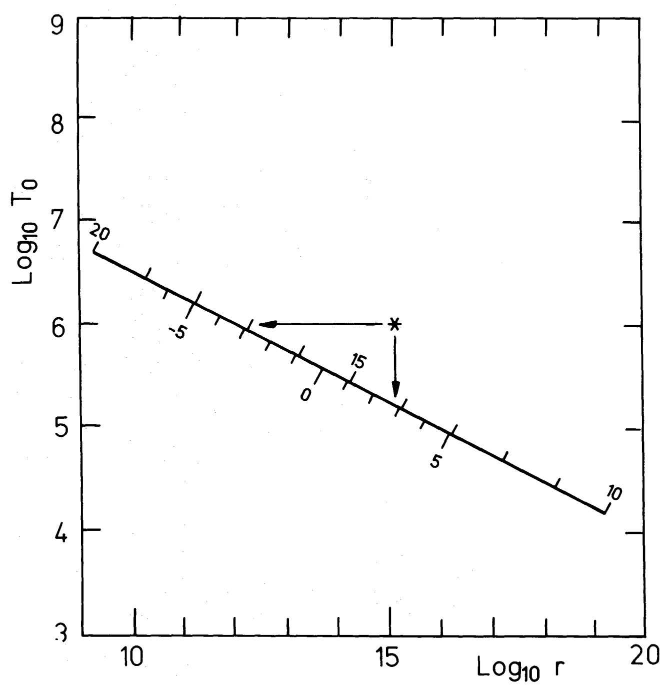

Fig. 1. Temperature T_0 of the driving blackbody versus radius r of the driven cloud. Combinations of T_0 and r above the diagonal line provide a good match between a galactic howitzer's charge and the caliber of its bullet. An allowed combination of parameters (for example, at the asterisk) admits bullets with parameters spanned by the horizontal and vertical intersections from the point (r, T_0), as shown by the arrows. Numbers above the diagonal line show $\log_{10} n$(m^{-3}), below it $\log_{10} m(M_\odot)$. The example at the asterisk shows that a howitzer with a driving temperature of 10^6 K can accelerate dense low-mass gasballs with $n = 1.5\,10^{17}$ m^{-3}, $m = 4\,10^{-4}\,M_\odot$, which are small (10^{12} m radius grapeshot), or less dense and more massive ones, up to $n = 1.5\,10^{14}$ m^{-3}, $m = 526\,M_\odot$ (10^{15} m balls.) The clouds in broad line regions have parameters within this range (Katysheva, 1984). To obtain a given mechanical luminosity, the required firing rate at the "magical β" muzzle speed is inversely proportional to m

1979, p. 162), or $L_X \approx 4.1\,10^{28}$ W. From the Exosat observations, Watson et al. (1986) derive $L_X = 10^{29}$ W. The bullet mass is $2.5\,10^{19}$ kg, its kinetic energy is $7.6\,10^{34}$ J, and the kinetic luminosity of a single bullet is $\frac{1}{2}mv^3/r = 5.9\,10^{33}$ W. The latter is about 20 times larger than is commonly accepted for the mean kinetic luminosity of SS 433 (Margon 1984), so the firing rate of that howitzer must correspond to a duty cycle of 5%; in other words, one 1-day event per 20 days. This is compatible with the observed repeating rate of the VLBI blobs in SS 433 (Vermeulen et al., 1987).

The derived constraints on n indicate that one should expect gaseous bullets to be comparatively dense. They surely can account for the energies observed: a 10^{17} mm bullet with density $10^{14}\,\mathrm{m}^{-3}$ and mass $0.3\,M_\odot$ can be accelerated to $\beta \approx 0.5$ by a radiation field with $T_0 \approx 1.6\,10^5$ K; even an extremely powerful radio source of 10^{39} W would need only one such blast every year or so to keep up its extended emission.

If $Q \approx 1$, then Eq. (2.15) suggests that, during the acceleration phase, the cloud can attain an internal thermal energy on the order of $kT \approx mc^2$. This may be responsible for the appearance of polarized compact radio sources: Jones and O'Dell (1977; see also Jones et al. 1985) show that relativistic electrons may well outnumber less energetic ones by a large margin. Of course this can lead to expansion of the accelerated cloud, but it will be shown below that the radiation pressure distribution on the surface of the cloud is such that the photons help to confine the cloud. After the acceleration phase, expansion to the low densities implied by the Compton optical depth of about 10^{-5} (Jones, 1979) is easily attained. If $n_e \approx 10^6\,\mathrm{m}^{-3}$ in clouds of a few parsec in size (Jones 1979; Katysheva 1984), then these have a mass of about one solar mass at most, and would have had the required optical depth of about unity at the time their radius was 10^{14} m. This is an allowed combination for quick acceleration (see Fig. 1).

Values of the acceleration parameter Q much in excess of unity are not excluded *per se*; as far as the equation of motion (2.14) is concerned, a change in Q only means a change of the timescale. In particular, nothing of what follows below regarding the behaviour of β is influenced by Q. But a very large value of Q implies an influx of energy well in excess of mc^2; thus, it would appear that a very large Q can be maintained only if the cloud can lower its temperature by radiation rather than by adiabatic expansion.

2.3. *Solutions of the equations of motion*

Let us now assume that the bullet and its howitzer are matched such that $Q \approx 1$. If the dimensionless temperature T is constant with time (and, without loss of generality, we may set $T = 1$), the integral over angles in the equation of motion can be found analytically:

$$\frac{d\beta}{dt} = Q\beta^{-2}\gamma^{-7}\left[\frac{1}{3(1+\beta u)^3} - \frac{1}{2(1+\beta u)^2}\right]_{-\beta}^{1} = Q\left(\frac{1}{2} - \frac{4}{3}\beta + \frac{1}{2}\beta^2\right)\sqrt{1-\beta^2}. \tag{2.22}$$

Evidently, the right hand side is zero when

$$\beta = \beta_m = \tfrac{1}{3}(4 - \sqrt{7}) = 0.451. \tag{2.23}$$

At this "magical" speed, the force due to the redshifted photons that would accelerate the cloud from behind is equal and opposite to the force due to the blueshifted photons that are so strongly aberrated that they hit the cloud from the front (cf. Sikora and Wilson, 1981). Several remarks about this magical speed are in order:

1) the root cause of the occurrence of a finite $\beta_m < 1$ is the combination of aberration and redshift, as is evident from the factor γ^3 in Eq. (2.11);
2) the decelerating photons that are responsible for establishing β_m come from far away on the driving surface, because they were emitted at $\theta \lesssim 90°$;
3) therefore, a finite or limb-darkened disk would produce a larger value of β_m;
4) contrariwise, a funnel in a thick disk, surrounding the cloud, would produce a smaller β_m inside the funnel; once the cloud leaves the funnel, a higher β is possible;
5) the value of β_m is a little above the speed $0.26\,c$ observed in SS 433 (Margon, 1984) and the speed $0.19\,c$ inferred for Cygnus X-3 (Icke 1973), but the geometry and the brightness distribution of the disk might take care of that;
6) however, the values of γ up to about 10 inferred for superluminal VLBI cores (e.g. Pearson et al. 1981; Fanti, Kellermann and Setti 1984, *passim*) are grossly incompatible with the β_m in Eq. (2.23), which corresponds to $\gamma_m = 1.121$.

In order to get some idea what magical β's could occur in a funnel or above a limb-darkened driving surface, consider a surface of revolution with the cloud moving on the axis thereof. We cannot expect to find an analytic expression for $\beta(t \to \infty)$ if the section of the surface with a plane through its axis is described by a function $R(h)$ that has an intrinsic length scale; for if that were the case, the height z would occur in the expression for T in a non-separable way. Curved cross-section profiles $R(h)$ are excluded, for these contain a length scale in the form of their radius of curvature. Consequently, analytic solutions for $\beta(\infty)$ are expected only if the driving surface is a cone. Suppose it has a half-apex angle α. Then the integral in the equation of motion is as before, but with a different lower bound:

$$\int_{-u_0}^{1}\left(\frac{T}{1+\beta u}\right)^4 u\,du = T^4\beta^{-2}\left[\frac{1}{3(1+\beta u)^3} - \frac{1}{2(1+\beta u)^2}\right]_{-u_0}^{1},$$
$$u_0 \equiv (\cos\alpha + \beta)/(1 + \beta\cos\alpha). \tag{2.24}$$

Some straightforward algebra shows that the integral is equal to

$$I = (1-\beta^2)^{-3}\left[\frac{1}{2}\sin^2\alpha - \beta\left(\frac{4}{3} + \cos\alpha + \frac{1}{3}\cos^3\alpha\right) + \frac{1}{2}\beta^2\sin^2\alpha\right], \tag{2.25}$$

which is zero for the magical value

$$\beta_m = w - \sqrt{w^2 - 1},$$
$$w \equiv \frac{1}{\sin\alpha}\left(\frac{4}{3} + \cos\alpha + \frac{1}{3}\cos^3\alpha\right). \tag{2.26}$$

A graph of β_m as a function of α is plotted in Fig. 2; for amusement, values of $\alpha > 90°$ (i.e. convex cones) have been included. From this we see, for example, that the value $\beta = 0.26$, observed in SS 433, is equal to the magical β of a comparatively shallow cone with half-apex angle 67°; the value $\beta = 0.19$ that is expected in Cygnus X-3 (Icke 1973) is the magical value for a disk

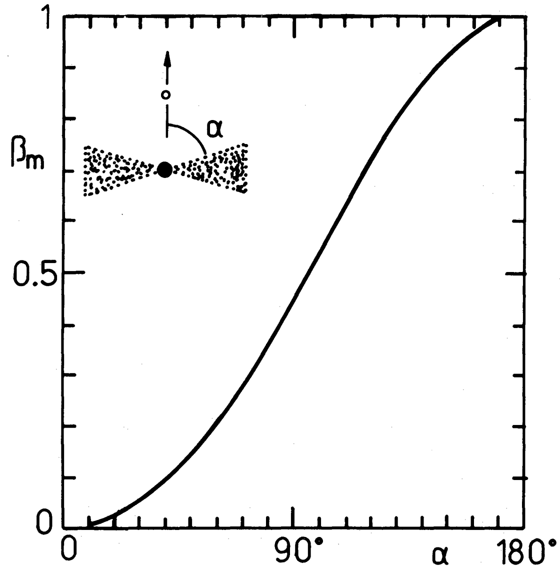

Fig. 2. Equilibrium "magical" speed β_m as a function of the opening half-angle a of the driving cone (see sketch.) The value for a flat disk, $\alpha = 90°$ is $\beta_m = (4 - \sqrt{7})/3$. Only extremely unrealistic convex surfaces ($\alpha > 160°$) reach really high β_m

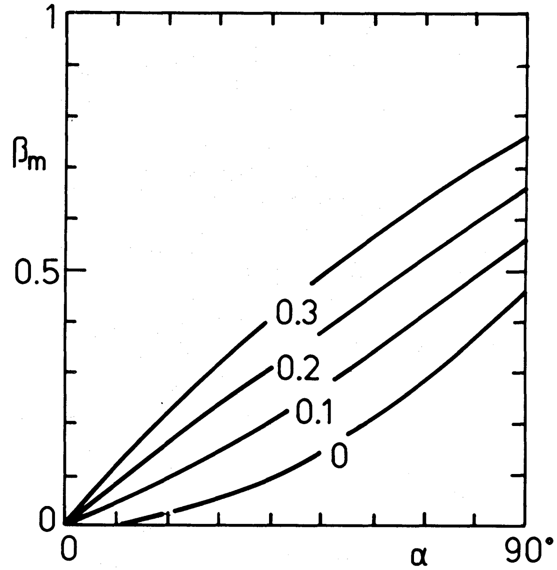

Fig. 3. Magical speed above driving cones with opening half-angle α. Numbers on the curves indicate the power k with which the temperature T of the driving surface decreases as a function of the height h above the $z=0$ plane: $T \propto h^{-k}$

with half-apex angle 58°. A conical shape of the driving surface always depresses β_m. From the point of view of acceleration by radiation pressure, there is no virtue in a narrow funnel, such as the superthin tube of Abramowicz et al. (1980).

Only after the cloud has left the funnel, can appreciable acceleration take place. This sequence is not analysed below, because I have restricted myself to self-similar situations; the case of finite funnels, which of necessity contain an intrinsic length scale, will be considered later (Eulderink and Icke, in preparation.) The introduction of a length scale implies, among other things, that the parameter Q can no longer be absorbed into the time scale, so that $\beta_m = \beta(Q)$. Basically, this happens because at a low value of Q the effective acceleration is small, so that the funnel has been left behind before the terminal value of β has been reached.

Next, suppose that T depends on the distance h above the $z=0$ plane. Again, $T(h)$ must be self-similar in order to obtain a β_m that is independent of z; therefore,

$$T = B h^{-k}, \tag{2.27}$$

$$h = z(1 + \tan\alpha/\tan\theta)^{-1}. \tag{2.28}$$

A magical value of β then must fulfill the condition

$$\int_{-u_0}^{1} u(1+\beta u)^{-4}(1+\gamma(u+\beta)(1-u^2)^{-1/2}\tan\alpha)^{4k}\,du = 0. \tag{2.29}$$

Convergence for $u \to 1$ is obtained only if T increases sufficiently slowly when $z \to 0$, namely

$$0 < k < \tfrac{1}{2}. \tag{2.30}$$

In general, Eq. (2.29) does not have simple analytic solutions; some numerical solutions for various values of k are shown in Fig. 3. Clearly, even a modest decrease of temperature with height produces a significant increase of β_m; for example, the value of $\beta_m = 0.26$ is attained if $T \propto h^{-0.2}$ in a funnel having $\alpha = 34°$.

Finally, consider the consequences of limb darkening. To a first approximation, we can take this into account by adding a factor $\cos\theta$ to B_ν in frame K. In frame K', this gives a factor $(\beta + \cos\theta')/(1 + \beta\cos\theta')$, so that instead of Eqs. (2.22, 23) I obtain

$$\begin{aligned}\frac{d\beta}{dt} &= Q\gamma^{-7}\int_{-\beta}^{1}\frac{u(\beta+u)}{(1+\beta u)^5}\,du = \frac{Q}{4\gamma^7\beta^3}\\ &\times\left[\frac{x(x+\beta^2)}{(1+x)^4} + \frac{2x+\beta^2}{3(1+x)^3} + \frac{1}{3(1+x)^2}\right]_1^{-\beta}\\ &= \frac{Q}{12}(4 - 9\beta + 4\beta^2)\sqrt{1-\beta^2},\end{aligned} \tag{2.31}$$

$$\beta_m = (9 - \sqrt{17})/8 = 0.610. \tag{2.32}$$

Some gain is made over Eq. (2.23), but it is not especially impressive, and certainly not good enough for superluminal sources.

3. Time-dependent surfing

3.1. Instantaneous temperature jump: catching a wave

The above estimates demonstrate that it is rather difficult to achieve the large values of γ that are implied by the VLBI observations of radio cores, even if we consider radiative driving by disks whose temperature drops with height. As we had seen, the magical value of β is determined by the blueshifted, aberrated photons which brake the cloud as β increases. Now these photons were emitted at grazing angles, $\theta \lesssim 90°$, and therefore must have come from far away on the driving surface. Consequently, the accelerated cloud receives these photons from a retarded time that is much larger than the time lag with which it receives the photons at $\theta \approx 0°$, which are responsible for the acceleration. This immediately suggests trying to achieve values of β well above the stationary β_m by using a time-dependent driving surface.

The time scale on which the cloud responds to a change in driving pressure can be obtained by calculating how quickly it approaches $\beta=\beta_m$ at constant T. The equation of motion (2.22) can be written, to good approximation, as

$$\frac{d\beta}{dt}=\frac{1}{2}Q(1-\beta/\beta_m), \tag{3.1}$$

in which β_m is given by Eq. (2.23). The solution is clearly

$$\beta=\beta_m+(\beta_0-\beta_m)\,e^{-Qt/2\beta_m}. \tag{3.2}$$

As expected, the time scale for the adjustment of β is $2\beta_m/Q$; since $\beta_m\approx 0.5$ in many cases, this justifies the requirement $Q\approx 1$ for good acceleration.

First, let us consider a driving blackbody plane that instantaneously steps up its temperature at $t=t_s$, from $T=1$ to $T=T_\infty$. The equation of motion is

$$\frac{d\beta}{dt}=Q\gamma^{-7}\int_{-\beta}^{1}\left(\frac{T(u,\,t)}{1+\beta u}\right)^4 u\,du. \tag{3.3}$$

The temperature T must be evaluated at the retarded time in the $z=0$ plane. The light travel time from a point (R, 0) to the cloud at (0, z) is, in the dimensionless units used above,

$$t_0=z/\cos\theta=z\left(\frac{1+\beta\cos\theta'}{\beta+\cos\theta'}\right). \tag{3.4}$$

Therefore, the temperature seen by the cloud in the direction θ' is

$$T(u,\,t)=T\left(t-z\frac{1+\beta u}{\beta+u}\right). \tag{3.5}$$

In the case that T is a step-up from $T=1$ to $T=T_\infty$ at $t=t_s$, one has

$$T=\begin{cases}1 & \text{if } t<t_s+z(1+\beta u)/(\beta+u);\\ T_\infty & \text{at later times.}\end{cases} \tag{3.6}$$

Two sample solutions of Eqs. (3.3, 6) for $Q=1$ are shown in Fig. 4. It is immediately clear that $\beta(t)$ overshoots the magical number in Eq. (2.23) and approaches a rather higher value. The reason for this is quite interesting. On the equator of the cloud (i.e. $\theta'=90°$; $u=0$), the incoming photons have a retardation $t_0=z/\beta$. If the speed of the cloud is constant, one has $z=z_0+\beta t$, so that $t_0=t+\text{const}$. In other words, *the distance which the photons must travel from their point of departure to the cloud equator increases with the speed of light!* The distance between the driving surface and the bottom of the cloud ($0\le u<1$) increases more slowly, and the distance from the disk surface to the cloud top ($-\beta<u<0$) increases faster than the speed of light. Consequently, the forward-facing hemisphere of the cloud receives progressively more retarded photons, whereas the hemisphere facing the driving surface sees radiation from increasingly later times. Thus, the angle θ' corresponding to the switch-on time t_s steadily approaches the equatorial value 90°, *but never surpasses it*. Seen in the frame of the cloud, the retardation becomes more noticeable as β increases. The retardation tends to freeze the accelerating field, so that the cloud "catches a wave," riding the crest of the luminosity increase. As time goes by, the integral in Eq. (2.24) approaches the value at which $T=1$ in the interval $-\beta<u<0$, (=forward hemisphere) and $T=T_\infty$ in the interval $0\le u\le 1$ (=back hemisphere). Accordingly, $d\beta/dt=0$ at that value of β which obeys

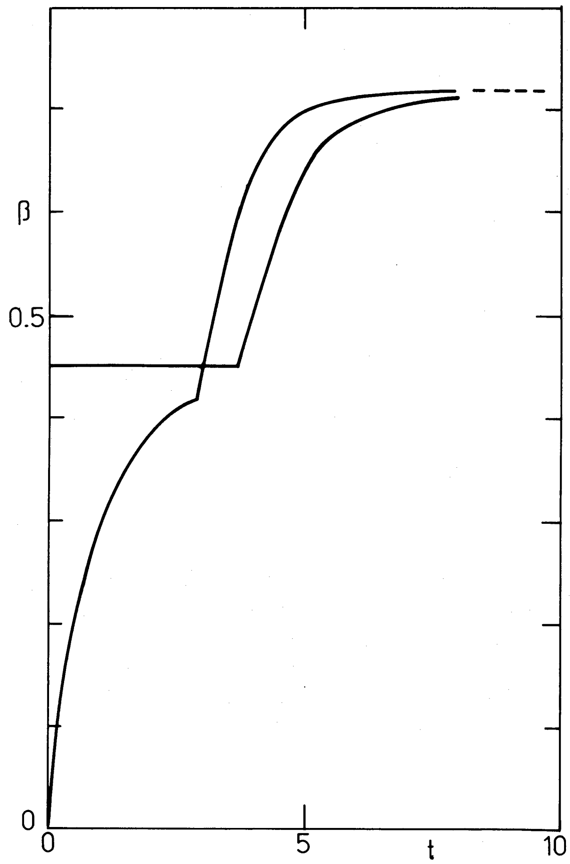

Fig. 4. Speed of a radiatively accelerated bullet above a flat blackbody that increases its temperature everywhere simultaneously (in its rest frame) from $T=1$ to $T_\infty=2$, at time $t_s=1$. Clouds start from rest or from their magical speed $\beta_m=0.451$. The value $\beta=0.72$, towards which the clouds converge as $t\to\infty$, is the solution of Eq. (3.8) for $T_\infty=2$

$$\left[\frac{1}{3(1+\beta u)^3}-\frac{1}{2(1+\beta u)^2}\right]_{-\beta}^{0}+T_\infty^4[\cdots]_0^1=0. \tag{3.7}$$

This condition is fulfilled when

$$T_\infty^4=\beta^2(3-\beta^2)(3+\beta)^{-1}(1-\beta)^{-3}. \tag{3.8}$$

The corresponding graph of the maximally attainable Lorentz factor for a given value of T_∞ is shown in Fig. 5. Apparently, a time-changing T can lift β_m above the value given in Eq. (2.23), even though all photons must ultimately overtake the cloud. Naively, one might expect that this overtaking must always cause the cloud to decelerate towards $\beta_m=0.451$. The reason that this does not happen is, that when a photon emitted after the switch-on reaches the cloud – as of course it must – it does so with a value of u such that $0<u\le 1$; in other words, the "hot" photons always hit the backward facing side of the cloud (yet another advantage of the fact that a cloud has a backside.) Clearly, a value of $\gamma\approx 10$ can be attained only for big temperature jumps, on the order of $T_\infty\approx 40$, or a luminosity increase by a factor of three million.

In the case of a flat surface with a limb darkening factor $\cos\theta$, it follows straightforwardly from Eq. (2.31) that

$$T_\infty = \left(\frac{\beta}{1-\beta}\right)\left(\frac{2-3\beta^2+\beta^4}{4\beta+7\beta^2+4\beta^3+\beta^4}\right)^{1/4}. \tag{3.9}$$

As can be seen from its graph in Fig. 5, this curve differs only 0.15 dex from that of Eq. (3.8).

3.2. *Other temperature changes*

In reality, of course, we do not expect the driving surface to maintain its high state foreover. Therefore, I will consider the response of the cloud to a finite burst of higher luminosity. A simple model of this is given by a temperature that varies as a Lorentz profile in the course of time:

$$T(t) = T_0 + (T_{\max} - T_0)\,[(t-t_{\max})^2 b^{-2} + 1]. \tag{3.10}$$

This is merely an example, chosen for simplicity. For general time-dependent temperatures, the integral on the right hand side of Eq. (3.3) cannot be found analytically, so that numerical integration is called for. Good numerical accuracy then requires proper resolution of $T(t)$ in the retarded time frame. To achieve this, it is preferable to change variables by introducing the new dimensionless quantities

$$q \equiv (t_{\rm ret} - t)/z; \quad t_{\rm ret} = t - z\left(\frac{1+\beta u}{\beta + u}\right), \tag{3.11}$$

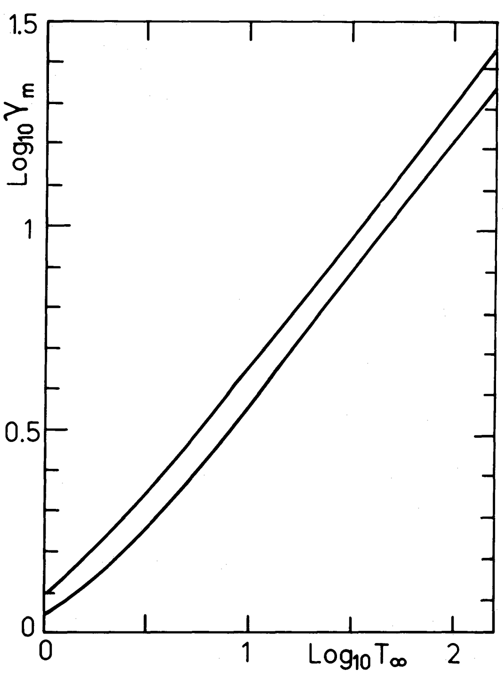

Fig. 5. Magical Lorentz factor γ_m that can be attained by a cloud if the blackbody surface that propels it abruptly increases its temperature from 1 to T_∞. The lower curve corresponds to Eq. (3.8) for an infinite Lambert surface; the upper curve, from Eq. (3.9), is due to driving by an infinite surface with a $\cos\theta$ limb darkening. Notice that the extreme superluminal sources, which are thought to have $\gamma \approx 10$, can be produced by such driving surfaces only if their temperature jumps up by a factor 30–40.

which leads to the equation of motion

$$\frac{d\beta}{dt} = -\frac{Q}{\gamma}\int_{-\infty}^{-1} [T(zq+t)/q]^4\,(1+q\beta)(q+\beta)\,dq. \tag{3.12}$$

For numerical applications, this can be replaced by an integral with a finite range:

$$\frac{d\beta}{dt} = -\frac{Q}{\gamma}\int_{-q_*}^{-1} \ldots dq - \frac{Q}{\gamma}\left(\frac{\beta}{3q_*^3} - \frac{(1+\beta)^2}{2q_*^2} + \frac{\beta}{q_*}\right), \tag{3.13}$$

in which q_* is chosen large enough that $T(q_*) \approx 1$ (in dimensionless form.) Several examples of cloud accelerations by the temperature burst in Eq. (3.10) are shown in Fig. 6. In order to get some impression of what happens to the cloud during all this, I show in Fig. 7 the retarded temperature (without the relativistic factor $[\gamma(1+\beta u)]^4$) as seen in the K' frame, and in Fig. 8 is shown the radiation pressure

$$P = [T(t_{\rm ret})/\gamma(1+\beta\cos\theta')]^4. \tag{3.14}$$

The duration of the burst in Eq. (3.10) is rather brief (on the order of the photon crossing time of the cloud), so β does not manage to reach β_m. To see how a cloud responds to a more

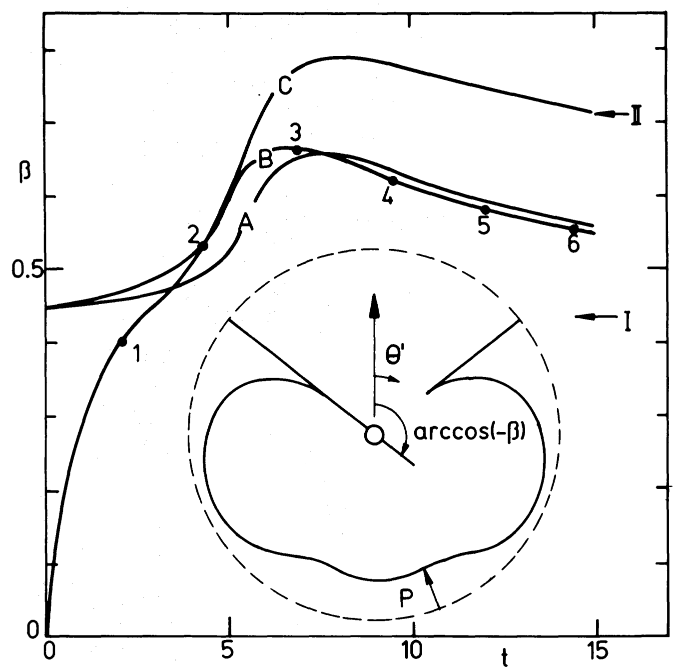

Fig. 6. Speed β versus time t for clouds driven by a Lorentz-profile temperature burst of an infinite flat blackbody. Case A: $T_{\max}=2$, $t_m=2$, $b=0.5$, $\beta(0)=0.451$. Case B: as A, but $\beta(0)=0$. Case C: as A, but with $T_{\max}=3$. Arrow I indicates the magical speed 0.451 for $T_\infty=1$, and II indicates $\beta_m=0.721$ for $T_\infty=2$. The numbered points on curve B correspond to those in Figs. 7 and 8. Twice the number indicated is equal to the dimensionless proper time of the accelerated cloud. The inset shows a vector diagram of the radiation pressure on the cloud at point 4, i.e. proper time $t'=8$. The cloud is moving upward on the page. The distance from the circumference of the circle to the dashed line is proportional to the radiation pressure. Notice that $P=0$ in the forward facing cone for which $\cos\theta' < -\beta$, and that the aberrated photons that come from very far away cause P to peak on the equatorial side of this cone

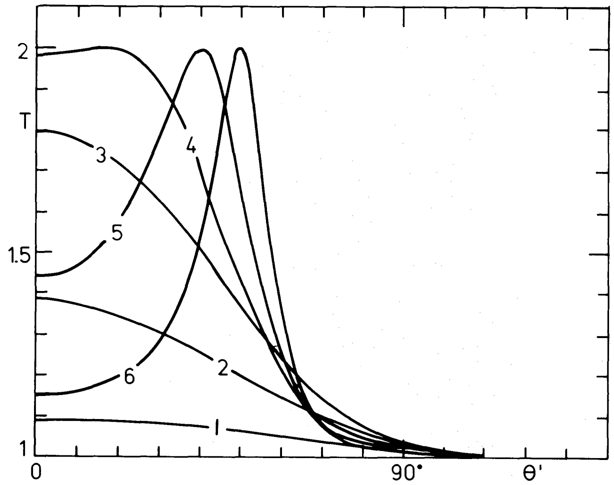

Fig. 7. Temperature T of the driving blackbody of Fig. 6, seen in the direction θ' in the frame K' of the cloud. The numbers on the curves correspond to the points on trajectory B in Fig. 6. Twice the number indicated is equal to the dimensionless proper time of the accelerated cloud

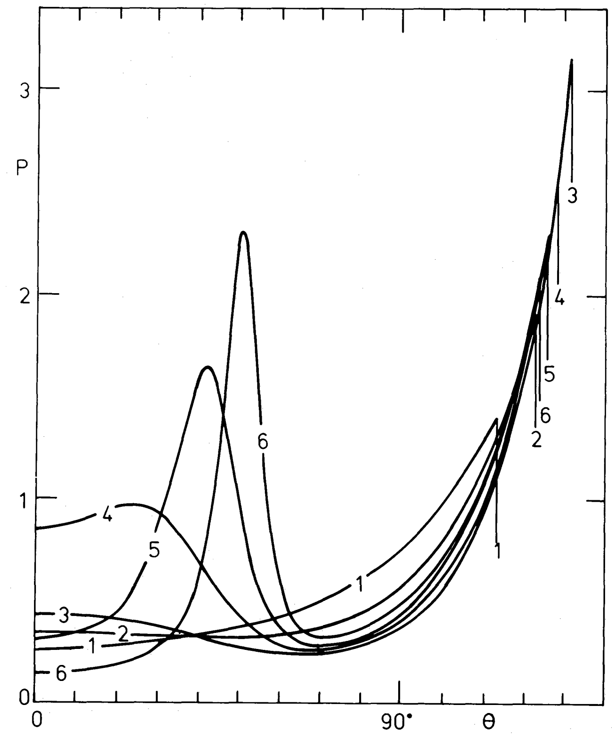

Fig. 8. Radiation pressure P seen in the direction θ' by the cloud on trajectory B in Fig. 6. The numbers on the curves correspond to those on the trajectory. Twice the number indicated is equal to the dimensionless proper time of the accelerating cloud. The pressure distributon of curve 4 is shown in the inset of Fig. 6. Notice how, as time goes by, the perceived peak of the burst creeps towards the equator of the cloud. Simultaneously, the relative importance of P decreases because of the steep drop of γ^{-4}, and because the half-width of T, as seen by the cloud, drops steadily as the peak approaches the equator. Also, it is noteworthy that when the main acceleration stage in frame K is over, the photon pressure in frame K' is still increasing, but acts mostly on the sides of the cloud. This effect, which is due to the aberration and retardation, is quite generic for this acceleration mechanism, and helps to confine the cloud and to collimate it during its initial boost phase

extended burst, consider the temperature profile

$$T = T_0 + \frac{1}{2}(T_\infty - T_0)\left(\frac{t - t_{\max}}{b + |t - t_{\max}|} + 1\right). \tag{3.15}$$

Two examples of the motion of a cloud driven by this switch-on are shown in Fig. 9; the corresponding $T(\theta')$ and radiation pressure, as seen by the cloud at equal intervals of its proper time, are shown in Figs. 10 and 11. The second case, which incorporates limb darkening in the driving surface, shows an approach to the value $\beta_m = 0.815$, corresponding to $T_\infty = 2$; here we begin to see something that might approach the high speeds in active galactic nuclei.

I have calculated the acceleration of clouds under the influence of several more different forms of $T(t)$, using the methods described above. In these cases, temperature waves were assumed to propagate radially outward in the disk with speed $c/\sqrt{3}$. The shapes of the temperature waves that moved outward were, first, an abrupt increase; second, a smooth increase; third, a hot ring. In all these cases, the results were qualitatively the same as those reported above. Quantitatively, the final values of β (in the first two cases) were, as expected, systematically lower than those obtained by switching the disk instantaneously to a hotter state. The difference was no more than 20%. In the third case, the speed went up and down more or less as in the Lorentz-profile burst. I have not tried inward-propagating temperature waves, because these would evidently only lead to deceleration.

4. Coloured clouds

Amusing though it may be to hit clouds with variously tailored photon blasts, Eq. (3.8) represents a soberingly small upper limit to the value of β. As we saw in Sect. 2, playing around with the shape of the driving surface and its temperature distribution won't change β_m much, although excessively narrow funnels are often bad. The velocities achieved are enough for SS 433, Cyg X-3, and the like, but they keep falling short of the values around 0.99 that are needed for superluminal radio sources.

The source of the trouble is the resistance which the cloud encounters from aberrated, blueshifted photons. This immediately suggests that we drop the assumption, made in Sect. 2, that the absorptivity of the cloud is independent of frequency. If we allow the cloud to have a "colour", let's say by assuming that its absorption cross section varies as ν^{-n}, we have, instead of Eq. (2.4),

$$dA'(\nu') = \frac{D}{mc} B_{\nu'} \left(\frac{\nu_0}{\nu'}\right)^{-n} d\nu'. \tag{4.1}$$

This leads, via the same route that produced Eq. (2.12), to an equation of motion of the form

$$\frac{d\beta}{dt} = Q'\gamma^{-3} \int_{-\beta}^{1} \left(\frac{T}{\gamma(1+\beta u)}\right)^{4-n} u\,du, \tag{4.2}$$

with suitable constant Q', depending on the integral over $\nu^{-n} B_\nu$. Following precisely the analysis of Sect. 3, it is a straightforward matter to show that the magical value β_m must be a solution of

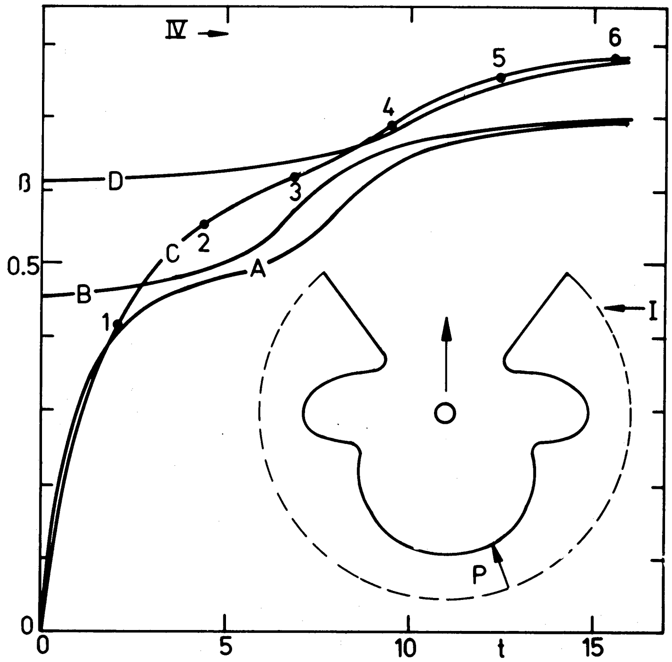

Fig. 9. Speed β versus time t for clouds driven by the smooth step-up in temperature prescribed in Eq. (3.15). Case A: $T_\infty=2$, $t_m=3$, $b=0.5$, $\beta(0)=0$. Case B: as A, but with $t_m=2$ and $\beta(0)=0.45$. Case C: as A, but driven by a blackbody that is limb darkened by a factor $\cos\theta$. Case D: as C, but with $\beta(0)=0.61$ and $t_m=2$. Arrow I indicates the magical speed $\beta_m=0.451$ for $T_\infty=1$, and II indicates $\beta_m=0.721$ for $T_\infty=2$. Arrows III and IV show the corresponding values 0.610 and 0.815 in the limb darkened case. The numbered points on curve C correspond to those in Fig. 10 and 11. Twice the number indicated is equal to the dimensionless proper time of the accelerated cloud. The inset shows a vector diagram of the radiation pressure on the cloud at point 5, i.e. proper time $t'=10$

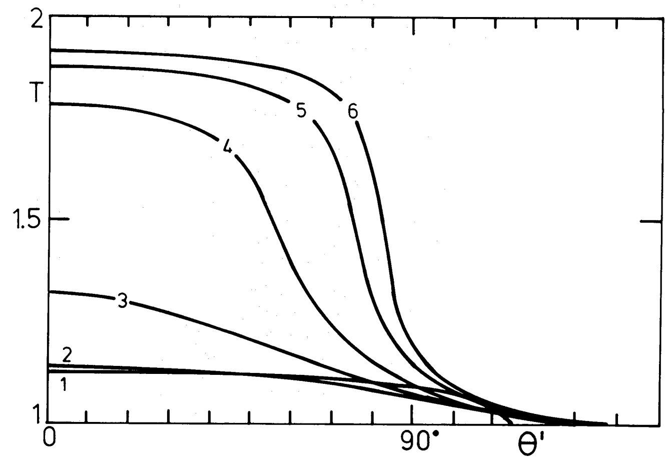

Fig. 10. Temperature T of the limb darkened driving blackbody of Fig. 9, seen in the direction θ' in the frame K' of the cloud. The numbers on the curves correspond to the points on trajectory C in Fig. 9

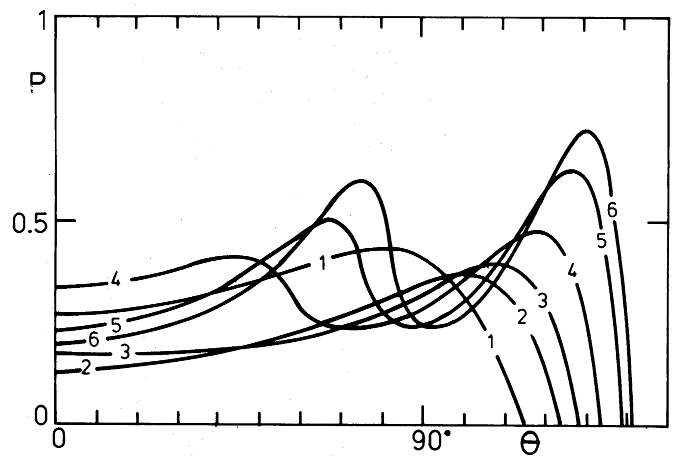

Fig. 11. Radiation pressure P seen in the direction θ' by the cloud on trajectory C in Fig. 9. The numbers on the curves correspond to those on the trajectory. The pressure distribution of curve 5 is shown in the inset of Fig. 9

the equation

$$T_\infty^{4-n}=\frac{(1-\beta^2)^{n-3}[\beta^2(3-n)-1]+1}{(1+\beta)^{n-3}[\beta(n-3)-1]+1}\,, \qquad (4.3)$$

or, in the logarithmic case $n=3$,

$$T_\infty=\frac{\beta^2+\log(1-\beta^2)}{-\beta+\log(1+\beta)}. \qquad (4.4)$$

Curves of the corresponding Lorentz factors γ versus T_∞ are shown in Fig. 12. The extreme case $n=3$ is best, of course. Such a dependence on frequency is expected in free-free scattering, when $\kappa\propto\nu^{-3}\,(1-e^{-h\nu/kT})$; a similar frequency dependence occurs on the blue side of an absorption edge (Rybicki and Lightman, 1979, p. 284.)

In the most favourable case, we can get $\gamma\approx 10$ if the temperature jumps by a factor of about 10; at $n=3$, the β_m for a steady photon source is 0.684. Thus, one would predict that comparatively steady radio nuclei produce blobs at $v/c\approx 0.6$, whereas those that vary violently, with a temperature amplitude of a factor 10, can produce $v/c\approx 0.99$. There is one price to be paid for a higher β_m, though: as can be easily seen from the equation of motion (4.2), the characteristic time scale for the acceleration (i.e. the response time of the cloud) scales as T_0^{n-4}; thus, a higher β_m is accompanied by a more sluggish acceleration (see also Eqs. (2.17) and (3.2).)

It is easier to take advantage of the colour of the cloud, if the latter is only marginally optically thick. This condition is favourable for other reasons as well. In accelerating a cloud, we should like to see it respond to the applied pressure more or less as a whole. This can be achieved in either of two ways: the speed of sound in the cloud is close to the maximally allowed value $c/\sqrt{3}$, so that the blob can quickly communicate external influences throughout its interior; or the radiation acts as a body force, in which case the cloud must be mildly transparent. The former possibility implies a very high temperature, which may cause difficulties for the integrity of the cloud, whereas the latter merely implies that $\lambda\approx r$ in Eq. (2.20). If this is the case, one would predict that actual galactic howitzer bullets lie close to the line in Fig. 1. Having $\lambda\approx r$ would also prevent the cloud from overheating. But the cloud must not be too thin, or else the momentum transferred by the photons will be too small. It is natural to expect that the clouds automatically adjust to the right transparency condition: bullets that are too thick are not efficiently accelerated because they are too inert, and those that are too

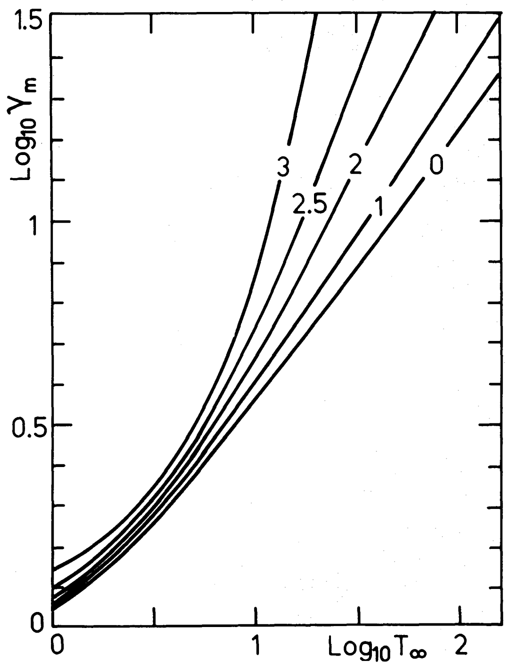

Fig. 12. As Fig. 5, but for clouds that absorb radiation according to the "coloured" opacity law $\kappa \propto \nu^{-n}$. The curves are labeled with the value of n. For $n=0$, we have Eq. (3.8); $n=1$, 2, or 2.5, Eq. (4.3); and $n=3$, Eq. (4.4)

transparent are either evaporated or do not acquire enough momentum.

5. Remarks

5.1. What makes smooth jets?

Many bipolar radio sources show very elongated structures that appear to be quite featureless, showing little, if any, evidence of blobs (e.g. Ferrari and Pacholczyk, 1983; Bridle and Perley, 1984.) What could these be, if not hydrodynamic jets? In the gasblob model, there are two possibilities. First, it can be shown that propagating gasballs must lose mass at their surface, thereby forming a "tracer bullet" (Icke, 1988). I believe that this effect is responsible for the low brightness plateau that is visible or inferred to be present underneath the brightness peaks near the central engine. Second, the gasballs can disintegrate upon impact with an obstacle ahead, for example slower moving clouds, or slow diffuse gas further down the source. I believe that this is the case in sources like M 87, where the radio trail suddently brightens and broadens at "knot A" (Biretta et al., 1983). If this occurrence is generic, it could be responsible for the sudden broadening and brightening of the inner trail in sources like 3C449 and 3C31. Thus, the question is not "what makes smooth jets," but rather "what makes 'jets' smooth."

5.2. Conclusions

I have investigated how photon driving can establish the propulsion of gaseous bullets to a fixed speed. The predominance of radiation as the driving agent allows the aberration and the redshift to produce a finite magical speed that is usully well below the speed of light. This mechanism is in sharp contrast with hydrodynamic mechanisms. In Newtonian dynamics, a force causes the momentum to change; correspondingly, in hydrodynamics, the paramount quantity is the momentum density. Thus, steady hydrodynamic mechanisms typically produce only minor variations in ρv. Therefore, large variations in ρ, which are almost always observed, should be accompanied by comparable variations in v. Such velocity changes are not seen in well-observed sources like SS 433.

In summary, it appears from the above that photon surfing can produce the gaseous bullets shot out of the vicinity of compact objects if:

1) in sources with large β (superluminals) violent variations of the driving luminosity occur;
2) the clouds are almost transparent, so they mustn't be too large for a given density;
3) the clouds must not be too dense, or they would respond too sluggishly;
4) the opacity of the clouds should drop steeply with increasing frequency, at least in those bullets that reach $\beta \lesssim 0.99$.

The quantitative expression of (1) is given in Figs. 5 and 9; of (2) and (3), in Fig. 1; of (4), in Fig. 12.

Consequently, one might envisage a sketch of the ejection process as follows. A cloud detaches itself in the vicinity of an accretion source; radiation impinging on it ablates and expands it, until it has become small enough to begin being transparent, and its density is low enough for the radiation pressure to become effective throughout the cloud (Fig. 1). Then the resulting gaseous bullet is shot out, achieving a speed of roughly $0.6\,c$ at most, and possibly much less, depending on the geometry of the accelerating surface. Higher values of v/c can be reached only if the radiation source is violently variable; in that case, acceleration up to about $0.99\,c$ may occur (Fig. 12).

It would appear from the above, that the radiative transfer in the accelerated cloud is of the essence for determining precisely what happens. As was pointed out by O'Dell (1981) and Cheng and O'Dell (1981), it is even possible to get an extra boost at the expense of the random motions of particles in the cloud (which they called the "Compton rocket" effect.) Coloured clouds have different allowed combinations of density, size, and temperature of the driving surface, because Eq. (2.20) must be modified if $\kappa \propto \nu^{-n}$. Work on this problem is in progress (Icke and Eulderink, in preparation.) The exact solutions, presented above, serve to define the region in parameter space where useful solutions to the acceleration problem are most likely to be found. For low-β (≈ 0.3) sources, a steady luminosity from a thick disk with not too narrow a funnel (30°–60° cone half-angle) will do. High-β ($\lesssim 0.9$) sources are more demanding and require a driving surface with a steep outward decrease of brightness and considerable limb darkening, and bullets whose absorption cross section drops quickly with increasing frequency. Sources with very high β (about 0.99) have the same requirements as high-β ones; in addition, they must have a violently variable driving luminosity.

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