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COMMUNICATIONS FROM THE OBSERVATORY AT LEIDEN

POLARIZATION AND COMPOSITION OF THE CRAB NEBULA

BY J. H. OORT AND TH. WALRAVEN

Observations were made of the polarization of the continuous light of the Crab nebula. These confirmed the discovery by VASHAKIDZE and by DOMBROVSKY that very strong polarization occurs in this nebula. The present, much more detailed, observations show a mean polarization of 17.2% for the central part within a mean radius of 0′.8. Much higher local polarizations were observed; indications were found that, where a limited volume of space could be isolated, the polarization might be complete. As may be seen from Figures 1 and 2, the resultant polarizations are of the same order and more or less parallel over a considerable part of the nebula. In the more outlying parts the polarizations are often smaller and directed more at random. Due to this random distribution the mean polarization of an outer ring, between 1′.2 and 2′.1 radius, is practically zero.

Provisional measures were made of the surface brightness of the „amorphous'' nebulosity. A plot against distance from the centre (Figure 9) shows that half intensity is reached at $r = 1'.2$. This is 2.15 times smaller than the half-power radius of the radio source as measured by BALDWIN.

The polarization may be explained on the basis of the theory, advanced by SHKLOVSKY, that the continuous radiation of the nebula is due to electrons of very high energy moving in a magnetic field (the „synchrotron'' mechanism). This mechanism was for the first time suggested by ALFVÉN and HERLOFSON to explain the radiation of radio sources. Serious difficulties with which former theories of the continuous radiation of the Crab nebula were faced, disappear with the synchrotron mechanism. It is shown that the mean absolute value of the magnetic induction in the central part must be close to 10^{-3} gauss. The median energy of the electrons responsible for the optical radiation is 2×10^{11} ev. The energy distribution of the fast particles may show some resemblance to that of cosmic rays. It was chosen such as to reproduce the observed ratio of radio-frequency to optical radiation. The total energy in the form of particles with average energies of the order of 10^{11} ev is about 1/10 000th of the total energy that could be released from the sun if it transformed all its H into He. The average half-life of the „optical'' electrons is of the order of 200 years. They cannot therefore be remains of the supernova explosion, but may possibly be continuously ejected by the remnant of the supernova. Remarkable fast-moving light-ripples observed by BAADE in 1942–45 might conceivably represent large-scale injections of relativistic particles into the nebula. Their brightness and frequency could be of the right order to compensate for the loss of energy through radiation. Slower and larger-scale variations in the nebula, discovered by LAMPLAND in 1921, were confirmed by a new inspection of Mt Wilson, Flagstaff and Lick plates.

The magnetic field probably has its seat in the expanding shell of filaments. This may have a mass of between a hundredth and a tenth of that of the sun. The mass of the high-energy particles of the amorphous nebula is only about one millionth of this. The excitation of the filaments is probably due to the radiation of the amorphous mass.

The radiation at radio wavelengths is discussed in section 10. The larger size of the radio source may be interpreted as due to a difference between the energy spectrum in the optical and radio domains, combined with a decrease of the average magnetic field strength to about half its central value at $r = 1'.5$. If the radio-frequency radiation could be measured in the same part of the nebula as the light, it should show the same polarization. However, the outer regions, where the light-polarization was found to be negligible, contribute so much to the total radio radiation that the resultant polarization of the entire source will not exceed 1%. It has not yet been possible to measure this amount.

In section 11 we have very briefly discussed the possibility that the radiation of other radio sources might likewise be due to the synchrotron mechanism. In section 12 we discuss Shklovsky's theory of the production of cosmic rays by supernovae. It appears possible that a large, perhaps even the major, fraction of cosmic rays comes from old supernovae.

In the course of observations aimed at measuring with great precision the integrated as well as the surface brightness of the amorphous part of the Crab nebula to determine possible changes due to expansion, the polarization was also measured. It was found that the light was polarized to an unusually high degree.

Earlier indications of polarization had been briefly reported on by VASHAKIDZE [1]). In fact it was this report that instigated us to include polarization measures in our observations. At the time we did not

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[1]) 'On the degree of polarization of the radiation of near-by extra-galactic nebulae and of the Crab nebula', *Astr. Circ.* No 147 (1954).

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know of the photoelectric measures made by DOMBROVSKY at the Burakan Observatory in the summer of 1953, which were the first to establish with certainty the fact that the Crab nebula showed a large polarization [1]).

1. *The observations.*

The instrument used for the observations of the polarization of the Crab nebula was originally designed for measuring the integrated brightness of the nebula, in the hope of detecting changes in the course of a few years. As such it was still in a preliminary stage because extremely bad weather conditions prevented us from studying the performance of the instrument.

The plan of measuring the integrated brightness was temporarily abandoned when it was decided to check the discovery by the Soviet astronomers of the polarization of the light of the Crab nebula. The photometer was quickly adapted for polarization measurements by mounting a piece of polaroid on the colour-filter disc.

The most essential part of the photometer is a photomultiplier tube made by LALLEMAND at the Paris Observatory. No previous observations had been made with this particular tube. During the polarization measurements it proved to be of excellent quality.

The tube has 19 stages of multiplication which, at 90 volts per stage, requires a supply of 1800 volts. A special, stable voltage supply was therefore constructed, which consisted of 20 condensors of 2μF each, selected for small size and high leakage resistance. These are all connected in series, while the electrodes of the phototube are connected to the junctions. A small hearing-aid battery of 90 volts is mounted on the axis of a rotating switch, which connects it momentarily to each condensor in turn. Every condensor is thus charged to the same voltage as the battery, and after a few rotations a total potential of 1800 volts builds up. This is maintained, in spite of the load imposed by the phototube, by the continued rotation of the switch. A miniature motor provides the motion. The total power supply is small and light, and can therefore be mounted on the telescope.

The phototube had been mounted by LALLEMAND in an airtight metal container provided with a window and electrical leads. This container was fixed on a frame which could be attached to the plateholder of the 13″ photographic telescope of the Leiden Observatory. On the same frame the Fabry lens, the diaphragm and the colour-filter disc were mounted. The anode of the phototube was connected to a d.c. amplifier of which the output current was fed into a Brown recorder.

The filter disc of the photometer contains six holes which, by rotation of the disc, can be shifted in succession into the light-beam. The positions are fixed by notches placed at intervals of 60° on the circumference of the disc. A piece of polaroid was cut and cemented to the disc so that three successive holes are covered by it. The other holes are used for colour filters. With this arrangement, measurements can be made with the polaroid oriented in three position angles differing by 60°. One set of three measurements is sufficient to determine the degree of polarization and the position angle of the plane of vibration of maximum intensity, provided the orientation of the polaroid and the sensitivity of the photometer for the different planes of vibration are exactly known.

The orientation of the polaroid was determined with the photometer in a laboratory arrangement. The sensitivity of the photometer as a function of the position angle of the plane of polarization was found by measuring comparison stars near the Crab nebula, which can reasonably be assumed to have little or no polarized light. The absence of circular polarization in the photometer was checked by measuring star light through an additional polaroid in front of the photometer. In Table 1 the position angles of the plane of electrical vibration of the transmitted light are given for the three orientations of the polaroid. The third column shows the correction factors by which the measurement had to be multiplied in order to remove the instrumental effect.

TABLE 1

Polaroid orientation	Position angle	Correction factor
I	151°.0 ± 0°.5	1.021
II	31.0 ± 0.5	1.003
III	91.0 ± 0.5	0.976

No perceptible change of the correction factors was found during the period of observations, although the photometer was taken off the telescope after every night of observations and even the phototube had occasionally been taken out of its frame.

A few occasional measurements on other nebulae in high galactic latitude were in accordance with the correction factors found for star light. This satisfied us that the distribution of the light in the diaphragm played no role.

In order to save light, the measurements of polarization were made without additional colour filters. As a consequence, the effective wavelength of the light used in our measurements is very uncertain.

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[1]) 'On the nature of the radiation of the Crab nebula', *Doklady Akad. Nauk USSR* **94**, 1021 (1954).

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Toward the long-wavelength end of the spectrum the sensitivity of the phototube drops rapidly at about 5500 A. Toward the blue the sensitivity decreases very gradually due to the absorption by the polaroid. A rough idea about the effective wavelength of our measures was obtained by observing a number of stars of widely differing spectral classes. From these measurements it followed that the magnitude, P, of a star observed through the polaroid, depends on the blue and yellow magnitudes, B and Y, as follows:

$$P = \text{Const.} + 0.67\,Y + 0.33\,B.$$

The magnitudes B were obtained by observing through the Schott filters BG25 + GG13, and the magnitudes Y by observing through GG11.

The Crab nebula measured with these filters shows the same colour as a star of type G2.

In the wide spectral range involved in the measurements of polarization lie several emission lines of the filaments surrounding the amorphous nebula. Of these, the lines at λ 3727 and λ 6560 probably do not contribute, but the $N_{1,2}$ lines are well within the spectral range recorded. How much they contribute to the observed light is not very well known. MINKOWSKI estimates that the emission lines do not contribute more than a few percent to the total brightness [1]).

Usable measurements of polarization were made with the equipment just described on one night in January, three nights in February and eight nights in March 1955 by both authors. On several nights Mr G. WESTERHOUT replaced one of us. Three different diaphragms were used. The integrated light of the nebula was observed with a big diaphragm with an area of 13.77 square minutes of arc, while the nebula was investigated in detail with two small circular diaphragms with areas of 0.651 and 0.174 square minutes of arc, respectively.

The large diaphragm was made by photographic reproduction of a large-scale drawing. It has a special shape, shown in Figure 3, which closely follows the outlines of the nebula as seen on a well-exposed photograph with the 200″ telescope. In this way it was made certain that the total light of the nebula is captured, while the contribution of the sky background is kept down to a minimum. The brightest stars in the field of the nebula are blotted out by small black spots. On both sides of the diaphragm for the nebula there is a diaphragm of exactly the same area, which served for measuring the brightness of the sky; in these the bright stars are also masked. In the corners of the diaphragm plate there are two small circular transparent spots corresponding to two reference stars. By means of these the diaphragm can always be set in exactly the same position relative to the nebula. The photographic reproduction is protected by a cover glass cemented with Canada balsam. Measurements showed that the transparency of the clear parts is quite satisfactory, i.e. 89%, and independent of the wave length.

The most serious difficulty encountered in the observations is the brightness of the sky background. Even on the darkest nights the brightness of the nebula was less than that of the background light entering the diaphragm. The background light, which is mostly reflected street light, is rather unstable. Changes in city illumination were quite perceptible; e.g. every night at a fixed time it dropped by 30%.

The degree of polarization of the Crab nebula observed in integrated light appeared to be 9.3% ± 0.3% m.e. The position angle of the electric vector with maximum amplitude was found to be 160°.0 ± 0°.8 m.e.

By comparison with the stars AG No. 1793, F5, $m_{pg} = 8.96$ and AG No. 1802, F8, $m_{pg} = 8.49$ we found for the Crab nebula $m_{pg} = 9.14$. We had no opportunity to make a comparison with photometric standard stars.

A more detailed study of the polarization of the nebula was made with the small diaphragms, which were circular holes drilled in a metal plate. After a series of observations with the diaphragm of nearly 1′ diameter, which revealed striking differences of the degree of polarization in different parts of the nebula, an attempt was made to observe still finer details by using a smaller diaphragm, of about $^1/_2$′ diameter. Although such a small diaphragm transmits a quantity of light which is barely measurable with our equipment, its use was justified by the results. That significant measures were possible, was due to the large degree of polarization found.

The diaphragm was centred on a number of places in the nebula by means of a reference star which was set on the cross wires of the adjustable micrometer of the guiding telescope. The rectangular coordinates, x in the direction of right-ascension, and y in the direction of declination, of the centres of the observed regions are given in Table 2. They are expressed in minutes of arc, and are measured from the central double star as origin.

In each orientation of the polaroid regions in the nebula and comparison regions for sky background were measured alternately during a period of about a minute. Often the measurement was repeated, for example if it was doubtful due to passing clouds, or if it showed unusual polarization. As comparison stars and a few standard regions in the nebula also had to be measured, only a limited portion of the nebula could be covered during a night.

The quality of the sky was generally poor; the

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[1]) *Ap. J.* **96**, 202 (1942); *Mt Wilson Contr.* No. 666.

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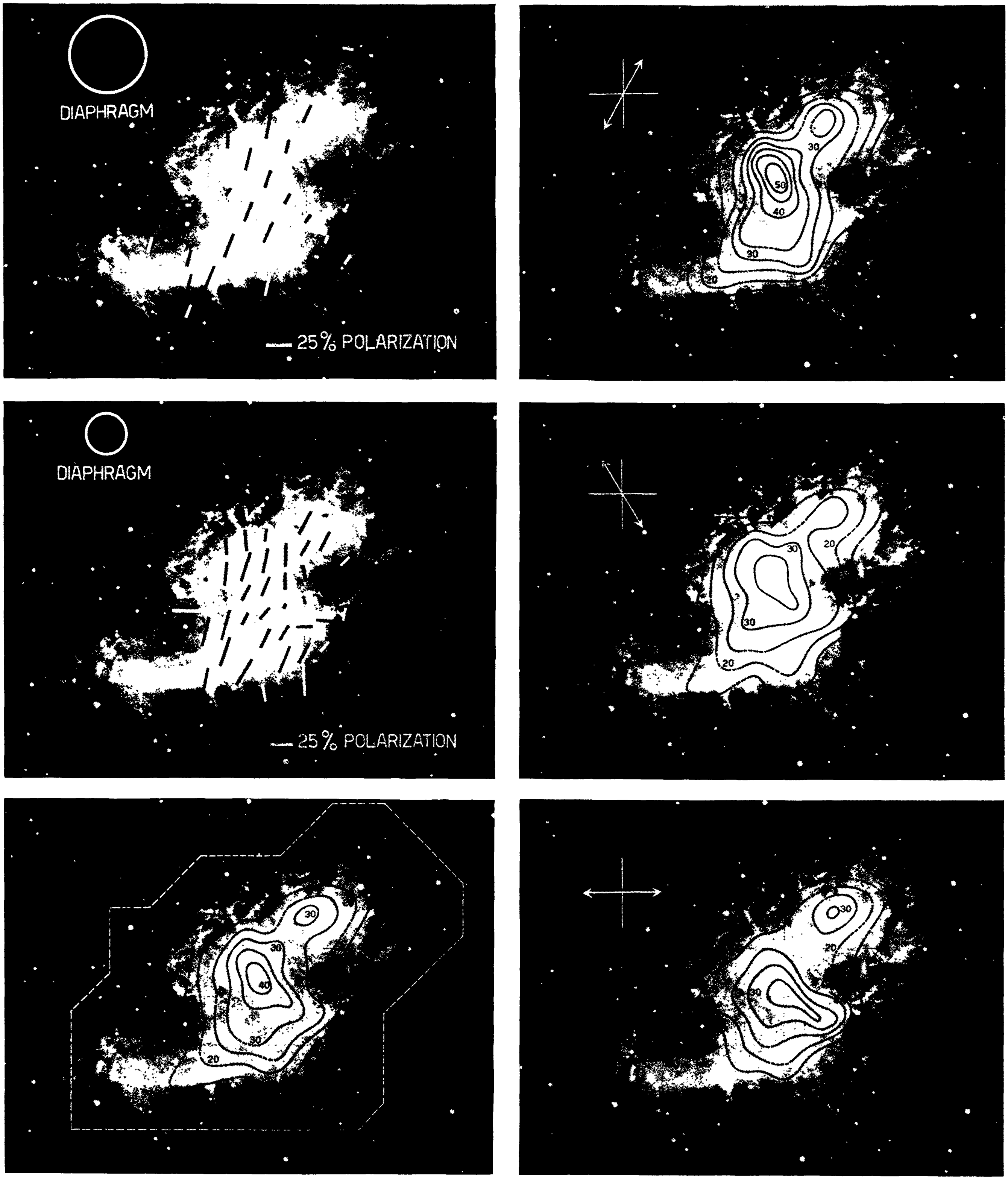

Figures 1 and 2: Polarizations measured with 1′ diaphragm and $^1/_2$′ diaphragm respectively. The polarizations have been indicated by black or white lines according to whether the background was light or dark. The length of each line indicates the percentage P of the light that is polarized, its direction gives the direction of the electric vector of the polarized light. The ratio of intensities measured with the analyzer parallel and perpendicular to this direction is $\frac{1 + P/100}{1 - P/100}$. Figure 3: Contours of equal surface brightness at intervals of 5 units. The broken white line shows the contour of the large diaphragm used in measuring the integrated brightness and polarization. Figures 4, 5, 6: Isophotes for the light measured with the polaroid oriented in the directions I, II and III, respectively. These directions are indicated by the white arrows. North is above, east to the left. Scale 1 mm = $4''.28$.

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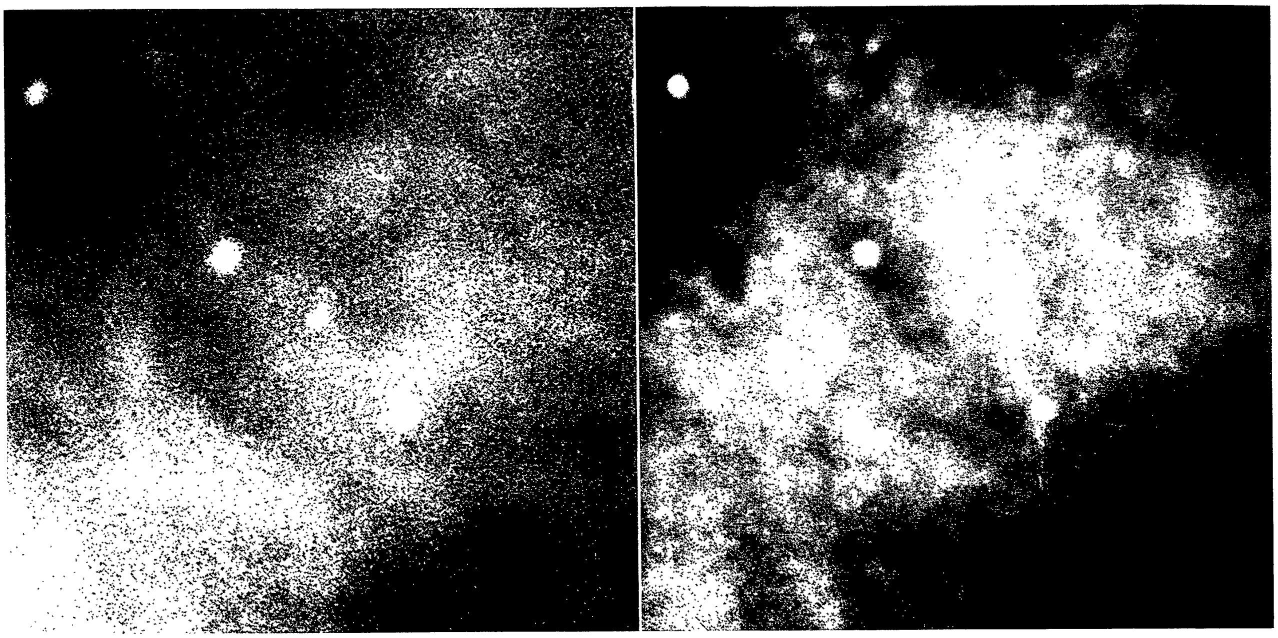
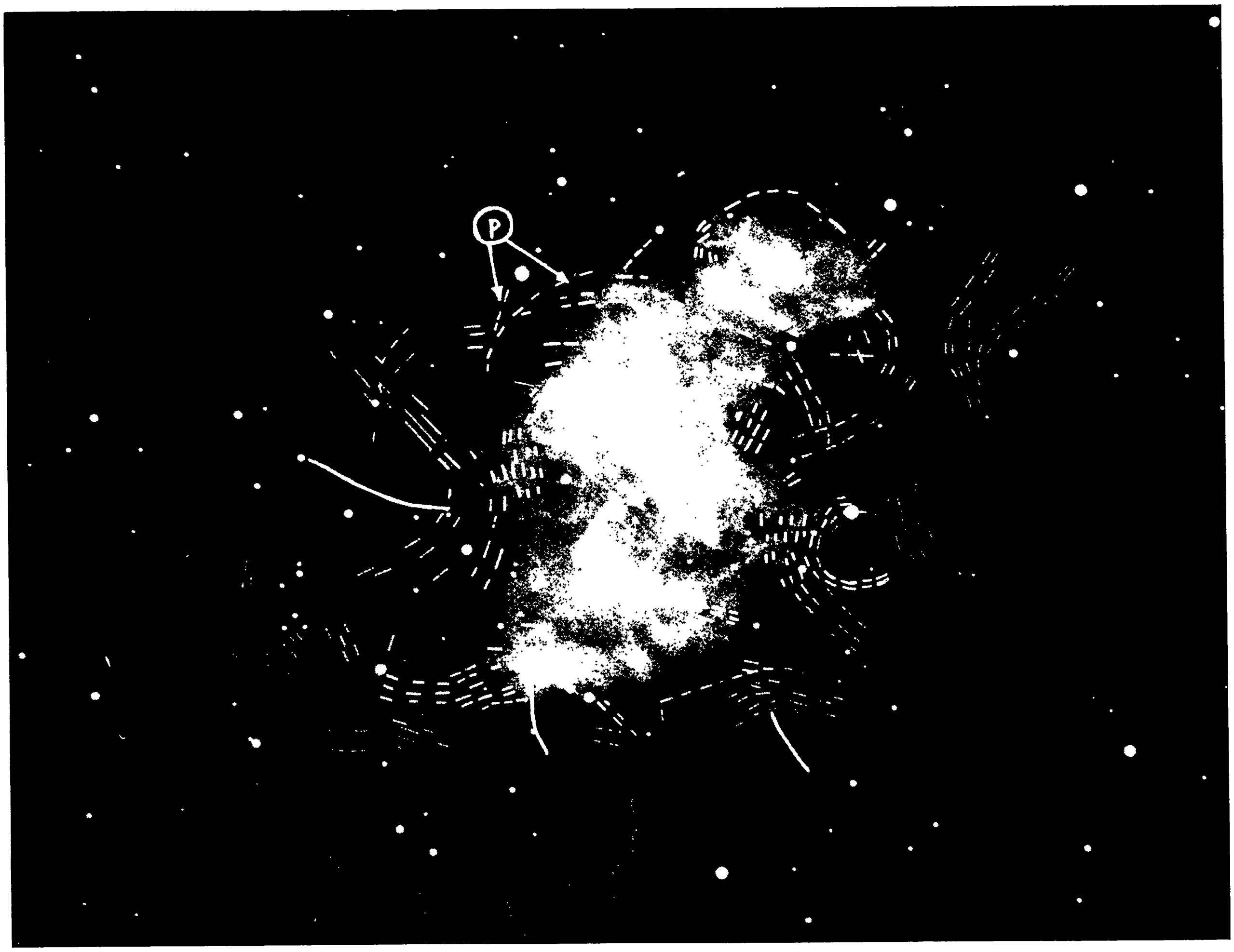
FIGURE 11



A region in the north-western part of the nebula, showing changes in brightness. Enlarged from plates taken with the 100-inch telescope by HUBBLE on November 19, 1924 (left) and by BAADE on October 25, 1938 (right). Scale 1 mm = 0″.80.

FIGURE 15



Sketch of magnetic lines of force inferred by visual inspection of plates taken by BAADE with

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transparency changed so rapidly and so much that it appeared impossible, with the relatively scarce comparison measurements, to determine reasonably accurate extinction corrections. For this reason we applied night corrections according to the method described by Oosterhoff [1]) in order to bring the intensities of the different observed regions as far as possible into a uniform system. Even so, there is still considerable uncertainty concerning the intensity distribution over the nebula. For the faint regions we expect errors up to ten percent.

The mean intensity of the observed regions, I, is given in Table 2. The intensity 10, which is the lower limit of our observations, corresponds to magnitude 14.0. It may be remarked that through the polaroid this is observed as magnitude 15.3; moreover, this light is measured superimposed on a sky background which is 3 or 4 times stronger; this demonstrates clearly the excellent quality of Lallemand's phototube.

The degree of polarization, P, represents that percentage of the total light-intensity which is completely polarized. If I_1 and I_2 are the intensities measured with the analyzer parallel and perpendicular to the direction of polarization, respectively, we have

$$\frac{I_1}{I_2} = \frac{1 + P/100}{1 - P/100}.$$

The quantity P is determined more accurately than the surface brightness, because it is measured in a relatively short interval of time, and in principle is not affected by extinction. Evidently, the accuracy of the observed degree of polarization depends strongly on the brightness of the region. As an example of the accuracy attained we may refer to the observations of the brightest region of the nebula, situated at $x = -0.19$ and $y = +0.27$. This region was observed eleven times with the 1′ diaphragm. The degree of polarization was found to be 19.0 %, and the mean error ± 1.3 % for one observation. The four observations of the same spot with the $^1/_2$′ diaphragm gave $P = 23$ %, while the individual observations agreed within 1 %. On the other hand, in the weakest regions considerable deviations were found.

Some of the brighter regions showed unexpectedly large scatter in the results. Thus, for example, the region at $x = -0.69$ and $y = -0.23$ was measured four times with the 1′ diaphragm. The degree of polarization was found to be 12.7, 7.6, 8.2 and 9.4 % respectively, with correspondingly large scatter in position angle. This is in all probability due to the fact that the polarization at this place in the nebula varies rapidly, so that small guiding errors become important.

Table 2

Coordinates		1′ diaphragm				$^1/_2$′ diaphragm			
x	y	I	P	q	n	I	P	q	n
+ 1.30	− 0.55	40	16	167	2				
+ 0.80	− 1.37	22	14	156	2				
,,	− 1.04	45	22	164	1				
,,	− 0.71	58	17	166	1				
,,	− 0.05	58	5	88	2	16	30	87	2
,,	+ 0.61	51	2	167:	1	11	4	32:	1
,,	+ 1.26	27	2	32:	2				
+ 0.55	− 0.96	51	26	159	1	17	34	169	1
,,	− 0.30					20	31	164	1
,,	+ 0.36					23	2	167:	1
,,	+ 1.02	52	7	17	2	17	5	152:	1
+ 0.30	− 0.55	94	26	158	2	26	32	163	3
,,	− 0.22					29	32	165	1
,,	+ 0.11	115	9	141	1	28	6	137	2
,,	+ 0.44					29	25	173	1
,,	+ 0.77	93	17	176	1	23	19	176	2
,,	+ 1.43	43	0	—	1				
+ 0.05	− 0.80					19	30	152	1
,,	− 0.14	124	20	156	1	34	16	152	3
,,	+ 0.19					37	20	152	1
,,	+ 0.52	121	23	165	1	38	32	162	2
,,	+ 0.85					28	22	4	1
,,	+ 1.18					14	12	9	1
− 0.19	− 1.05	49	22	168	2	14	19	13	1
,,	− 0.39	115	22	156	2	31	22	153	1
,,	− 0.06					35	16	140	1
,,	+ 0.27	138	19.6	161.6	11	42	23	158	4
,,	+ 0.60					35	27	168	1
,,	+ 0.93	106	24	169	2	23	15	176	1
,,	+ 1.59	42	4	47:	3				
− 0.44	− 0.64					25	30	159	1
,,	− 0.30					29	19	147	2
,,	+ 0.02	131	11	150	1	37	6	158	2
,,	+ 0.35					29	17	0	1
,,	+ 0.68	127	21	165	1	29	24	0	2
,,	+ 1.34					20	15	158	2
− 0.69	− 0.89					13	33	3	1
,,	− 0.56					18	23	156	1
,,	− 0.23	105	9	144	4	29	16	91	2
,,	+ 0.10					27	7	8	1
,,	+ 0.43	101	14	19	1	26	15	10	1
,,	+ 0.76					25	20	152	1
,,	+ 1.09	100	19	157	2	31	27	154	3
,,	+ 1.75	36	8	51	1				
− 0.94	− 0.47	67	9	155	1	13	12	140	1
,,	− 0.14					25	16	85	1
,,	+ 0.19					21	13	40	3
,,	+ 0.85					23	21	152	2
,,	+ 1.18					29	4	97	1
,,	+ 1.50	70	4	150:	1	19	6	31	2
− 1.19	− 0.72	34	11	145	2				
,,	− 0.06	62	6	158	1	18	14	170	1
,,	+ 0.60	66	5	172:	2	15	12	137	2
,,	+ 1.26	79	13	129	2	23	6	85	2
,,	+ 1.92	32	11	78	2				
− 1.44	+ 1.00					14	6	4:	2

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[1]) *B.A.N.* **11**, 299 (1951).

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The position angle, q, indicates the direction of maximum amplitude of electric vibrations. The uncertainty in q likewise varies considerably with the intensity of the observed region, and depends moreover on the degree of polarization. For the brightest observed region, measured with the 1′ diaphragm, the mean error of q appeared to be ± 1°.7 for one observation.

The values of P and q are given in Table 2, the number of observations is shown under n.

2. *Discussion of the results of the observations.*

Figures 1 and 2 show the direction and degree of polarization in the Crab nebula. The direction of a line is that of the maximum electric amplitude; the length is proportional to the degree of polarization. The length of the white line in the lower right-hand corner represents 25 % polarization, while the circles indicate the sizes of the diaphragms. We are indebted to Dr Baade for the picture of the nebula on which Figures 1–6 have been drawn. It was reproduced from an enlargement of a plate taken by Baade with the 100-inch reflector in the photographic region of the spectrum on October 25, 1938.

The lines show a generally prevailing direction with position angle of about 160°. This confirms the observations by Dombrovsky [1]), who measured the polarization with a diaphragm of 3 square minutes of arc. He found polarizations ranging from 9 to 15 % and position angles between 168° and 175°. Our observations, however, being made with much smaller diaphragms, indicate that locally the degree and amount of polarization may vary considerably. It is quite understandable that this could not have been observed by Dombrovsky, because even the 1′ diaphragm shows much less variation than the $^1/_2{}'$ diaphragm, as can be seen by comparing Figure 1 with Figure 2.

The smoothing action of the larger diaphragm is illustrated in Table 3, where the measurements with the 1′ diaphragm of some representative regions are compared with the mean of groups of regions measured with the $^1/_2{}'$ diaphragm, which together occupy about the same area as the 1′ diaphragm. The agreement, which is already satisfactory, could have been improved by a more adequate combination of the measurements.

Table 3

Region	1′ diaphragm		Mean of $^1/_2{}'$ obs.	
	P	q	P	q
$x = -0.19$ $y = +0.27$	20 %	162°	19 %	162°
$x = +0.30$ $y = -0.55$	26	158	27	161
$x = +0.30$ $y = +0.77$	17	176	18	175
$x = -0.69$ $y = +1.09$	19	157	14	158
$x = -0.69$ $y = -0.23$	9	144	6	151

If the mean of a number of measurements with the small diaphragm is determined, such that the total area covered by them is equal to the area of Dombrovsky's diaphragm, the degree of polarization of this mean becomes about 15 %, and is thus in good agreement with Dombrovsky's results. However, our position angle is systematically about 7° smaller than Dombrovsky's. The cause of this discrepancy is not known. It seems improbable that the present results would contain a systematic error of more than a degree.

Table 4

Field	Equiv. area (squ.min.)	Mean radius (minutes)	I	P(%)	q	Remarks
1	0.174	0.255	42	23	158°	direct obs. with $^1/_2{}'$ diaphr.
2	0.651	0.455	138	20	162	direct obs. with 1′ diaphr.
3	0.742	0.486	150	19	162	mean of 9 regions, $^1/_2{}'$ diaphr.
4	2.062	0.810	358	17.2	162.6	mean of 25 regions, $^1/_2{}'$ diaphr.
5	4.538	1.202	629	13.8	162.8	mean of all obs. with $^1/_2{}'$ diaphr.
6	13.77	2.09	1000	9.2	159.6	integrated neb., obs. directly
5−4	2.476	0.81−1.20	271	10.4	162.4	computed from 5 minus 4
6−5	9.23	1.20−2.09	371	2	128	computed from 6 minus 5

As has been mentioned already, the degree of polarization of the integrated nebula is 9.2 %. This shows that the decrease of polarization with increasing area is a phenomenon which takes place systematically from the smallest to the largest observed areas. In order to study this phenomenon we collected in Table 4 the degree of polarization and position angle for a series of concentric regions with systematically increasing area. The first two regions represent the direct observations, with the $^1/_2{}'$ diaphragm and the 1′ diaphragm, of the bright central spot of the nebula.

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[1]) *Doklady Ak.Nauk USSR* **94**, 1021 (1954).

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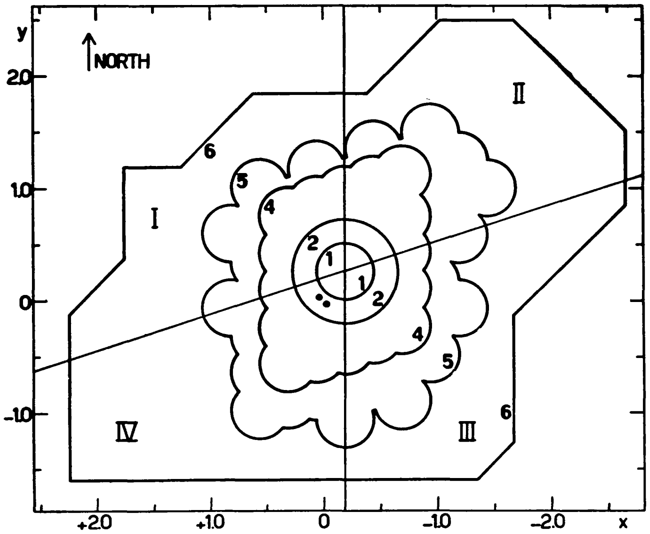
The next two regions have not been observed directly, but represent mean values of a number of measurements with the $^1/_2'$ diaphragm, together covering a field with the indicated area. The shapes of these fields are roughly similar to that of the nebula (see Figure 7). The fifth region of Table 4 is the field covered by all the measurements with the $^1/_2'$ diaphragm. A number of gaps, formed by places not measured directly, were filled in with values interpolated from the measurement of surrounding spots. This field also is concentric with the nebula and has more or less the same elongated shape. The sixth region has been directly observed with the photographic diaphragm which extends roughly to the outer frontier of the nebula. The third column of Table 4 gives the radii, in minutes of arc, of circles which have the same areas as the fields.

FIGURE 7



Large diaphragm and observed large fields (cf. Table 4).

The lower part of Table 4 shows the data for zones comprised between the successive boundaries of the fields of the upper part of the table. The amount of polarization of these zones was computed from the differences between the integrated intensities of the successive fields as measured in each of the three orientations of the polaroid.

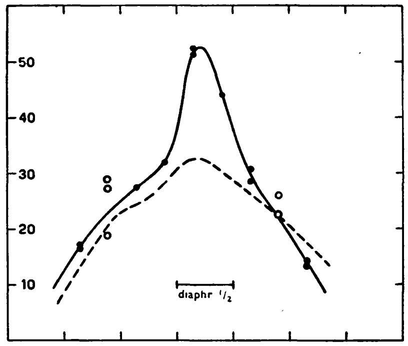
Our observations thus result in the following picture of the polarization of the Crab nebula.

Locally, in regions smaller than $^1/_2'$ diameter, the degree of polarization varies from a few percent up to 34 percent; although there is a prevalence of one particular direction, any direction of polarization may occur. This holds for the main part of the nebula, from the centre up to about a minute of arc distance. In this region no perceptible correlation is found between degree of polarization and distance to the centre, nor between polarization and surface brightness. The observed decrease of polarization with increasing area is mainly a smoothing effect.

However, in the weak outer parts of the nebula, at distances between 1′ and 2′ from the centre, the mean polarization decreases fairly rapidly. As it is only an indirect result of the observations, we cannot decide whether this is due to a real decrease of the degree of polarization or to an increased diversity in the direction of the plane of polarization.

The appreciable increase of polarization found when using the $^1/_2'$ diaphragm instead of the 1′ diaphragm suggests that with smaller diaphragms still greater polarizations may be found. Even now the polarization is quite considerable, for 33 % polarization means that, observed through a rotating analyzer, the intensity of the light varies in a ratio of 1 to 2. Perhaps, with a sufficiently small diaphragm, the polarization of the Crab nebula may be complete in some places.

FIGURE 8



Intensity distribution along a strip through the centre of the nebula with the analyzer in two perpendicular directions, showing large polarization of central region.

Suggestive of this is the following. When we plot the intensity observed through the $^1/_2'$ diaphragm with the polaroid in orientation I (position angle 151°), for the regions along the line running from $x = -1.19$, $y = +0.60$ across the nebula to $x = +0.80$, $y = -0.05$, we get the full-drawn curve shown in Figure 8. The observed intensities have been indicated by dots; the open circles refer to regions on this line which were not observed, but for which the intensities were inferred from the measures in neighbouring regions. The intensity distribution along the same line with the polaroid in perpendicular direction (position angle 61°) has been found by computation, and is indicated by the dashed line in Figure 8. The two curves together suggest strongly that the peak which corresponds to the bright bar of light in the centre of the Crab nebula consists of practically completely polarized light superimposed on a more extended background which is polarized either much less or in different directions.

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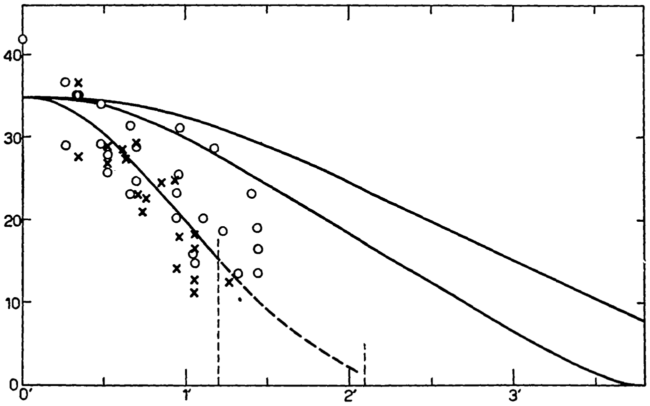
As may be seen in Figures 1 and 2, the direction of polarization varies across the nebula in a more or less systematic way. It may be useful to study this phenomenon from a different point of view. This has been done in Figures 4, 5 and 6, where isophotes are inserted for the intensities observed with the polaroid in different orientations. The shape of the isophotes should not be taken too literally, as, in the first place, appreciable errors may exist in the relative intensities of adjacent regions observed on different nights, due to the uncertainty of the corrections for extinction. In the second place, the intensities of several unobserved places had to be interpolated from surrounding regions; at such places the isophotes have been drawn as interrupted curves. In the third place, the diaphragm is still of appreciable size as compared to the structural details of the nebula. Finally, there must be some distortion due to the emission-line filaments and bright stars in the field, the light of which probably has a different state of polarization.

FIGURE 9



Variation of average surface brightness with distance from the centre. The upper curves show the variation of radio intensity as measured by BALDWIN; the two curves indicate the limits of uncertainty of these measures according to BALDWIN.

But, apart from these disturbing effects, there is no doubt that there is a systematic change in the shape of the nebula when observed with different directions of the polaroid.

It should be remarked that VASHAKIDZE 1) found from photographic observations that „at some places the Crab nebula is so strongly polarized that it changes its shape appreciably for different positions of the analyzer".

For the purpose of comparison we show in Figure 3 the isophotes for the mean intensity, which is equivalent to observation without analyzer. These isophotes should be in accordance with a normal photograph. The isophotes have been drawn at intervals of 5 units, the unit being the same as that used in Table 2.

We conclude the discussion of the observations with a plot of the mean intensity of the observed regions against the distance from the central region. This is shown in Figure 9. Regions lying in the general direction of the major axis (i.e. in the sectors II and IV shown in Figure 7) have been indicated by open circles. The regions situated in the other sectors are represented by crosses. The lower curve gives our estimate of the run of mean surface brightness with distance from the centre. The dashed part of this curve is an extrapolation to $r = 2'.1$, made in such a way that for the outer zone of the nebula, denoted by 6—5 in Table 4, the integrated intensity agrees with that given in Table 4. The average limits of this outer zone have been indicated in Figure 9 by dashed vertical lines.

The two upper curves show the distribution of radio brightness as determined by J. E. BALDWIN 2) at a wavelength of 1.4 metres. The two curves are estimated by BALDWIN to represent the limits within which the true curve almost certainly lies. It is apparent from this plot that the radio source is at least a factor 2 larger in radius than the source of the amorphous light. On the present theory a difference in scale between the optical and radio source is to be expected; a ratio of the order of 2 seems easily possible (see section 10) 3).

3. *Fast-moving light-ripples and other changes in the Crab nebula.*

During a period of intense observation of the Crab nebula in 1942 and '43, BAADE discovered a very

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1) *Astronomical Circular* No. 147, 11, (1954, in Russian).

2) *Observatory* 74, 120 (1954).

3) An accurate comparison of light- and radio distribution has recently been made possible by the observations of the occultation of the Crab nebula by the moon on Nov. 3 and 30, 1955. The radio occultations were observed by Mr SEEGER with the new 25-metre paraboloid at Dwingeloo. They show conclusively that the distribution of the radio-frequency radiation is flatter than that of the light, although the difference is less than that given by BALDWIN. The radio distribution can be approximately obtained from the light-distribution by multiplying the distances from the centre by 1.69. The calculations in section 6 have been adjusted to this scale factor. We are much indebted to Mr SEEGER for allowing us to use these data before publication.

After the present article was written, more detailed photoelectric measures of brightness and polarization were obtained by one of us with the 80-cm reflector of the Observatoire de Haute Provence at St Michel. The results of these measures will be published in one of the forthcoming numbers of these Bulletins.

In September and December 1955 Dr BAADE obtained a number of plates through a polaroid screen with the 200-inch Hale telescope (cf. BAADE's note on p. 312). These plates, which give a very detailed picture of the distribution of brightness and polarization, are now being measured by Mr L. WOLTJER in Leiden.

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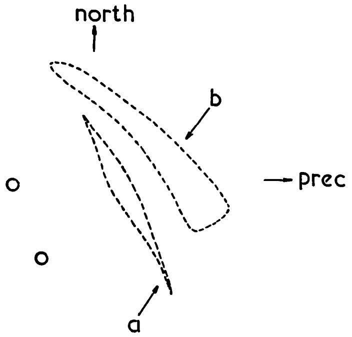
remarkable phenomenon. He found that occasionally wisps and knots appear which move rapidly through the nebula. As at that time his main interest had turned to the Andromeda nebula, he never got around to publishing his discovery, but he told one of us about it in a private conversation in 1948. The phenomenon has ever since intrigued us very much, because it seemed to prove beyond doubt that the nebula must have a central star, and that this star was still very active. At our request BAADE has now re-inspected some of his plates and has kindly permitted us to describe in this article the provisional account of this inspection.

FIGURE 10



Moving light-ripples near the centre.

Figure 10 shows a rough sketch of the central part of the nebula. The two small circles indicate the central double star. BAADE [1]) and MINKOWSKI [2]) have suggested that the south-preceding component of this double is the remnant of the supernova. According to BAADE both components have a photographic magnitude 15.9. The separation is 4″.9; proper motion measures suggest that it is an optical double star. The wisp *b* is a more or less permanent and stationary feature in the amorphous continuum, and forms its brightest part. Its centre lies about 10″ NW of the south-western component of the double star. The other high-intensity ridge in the continuum, marked *a*, represents a moving ridge. It is variable in position and shape, as well as in brightness [3]). BAADE gives the following description of the nebulosities of the type *a*, based on the provisional data he had inspected.

„They make their appearance as bright elongated patches (as shown in the sketch) or, in other cases, as round bright knots, always in the space between the central stars and nebulosity *b*. None, so far, has been observed at a distance less than 7″ from the line joining the central stars. The subsequent motion of the nebulosities *a* is to the west, nearly perpendicular to this line. During this motion their brightness decreases and they usually disappear entirely before reaching the permanent nebulosity *b*. In only one case thus far (1948 Jan. 13) I have seen a wisp *a* reaching *b* and lying on top of it. In the fall of 1944 I made an attempt to follow the development of a nebulosity *a*. Here is the record:

1944 Oct. 11	(plate B1538B) The elongated wisp *a* (like that in the sketch) was located at $d = 7''.17$ from the line joining the central stars.
1944 Nov.	Bad weather prevented observations.
1944 Dec. 18	(plate B1557B). The nebulosity, still of the same form, but considerably fainter, had moved westward and was now located at $d = 8''.18$ from the base line. In the blink comparator the motion between October and December is very striking.
1945 Jan. 15	(plate B1580B). The nebulosity is still recognizable but it has become so faint that no micrometer measures were attempted. However, intercomparisons at the blink comparator show that the westward motion continued in the interval December to January. On the January plate a *new* strong nebulosity *a*, this time round, makes its appearance at $d = 7''.06$ from the base line".

From the motion of 1″.01 in the $67^d.8$ interval between Oct. 11 and Dec. 18, combined with a distance of 1000 pc, a transverse velocity of 2.6×10^9 cm/sec, or roughly one tenth of the velocity of light, is derived.

Only the roughest guess can be made concerning the brightness of the moving ripples. The wisp just described is estimated by BAADE to be about 1^m fainter than the south-preceding component of the central double star, or roughly 1/1500th of the light of the whole nebula. He also estimates that there may be about one ripple per three months.

For the interpretation of these remarkable features the reader is referred to section 9.

Changes in the Crab nebula had already been observed about a quarter of a century earlier by LAMPLAND with the 40-inch Lowell reflector, and had been described in a paper that seems not to have drawn the attention it merited [4]). LAMPLAND draws attention

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[1]) *Ap. J.* **96**, 188 (1942); *Mt Wilson Contr.* No. 665.

[2]) *Ap. J.* **96**, 199 (1942); *Mt Wilson Contr.* No. 666.

[3]) A moving wisp of this type is present on the 200-inch plates of September 21–23, 1955, reproduced in BAADE's note on page 312 of this *Bulletin*. It can be easily seen as a very bright feature bordering the large central mass; on some of the polarization plates it is clearly detached from it.

[4]) 'Observed changes in the structure of the „Crab" nebula (N.G.C. 1952)', *Publ. A.S.P.* **33**, 79 (1921). We are indebted to BAADE for pointing out to us that LAMPLAND had discovered such changes, and to Mr A. G. WILSON for the reference to LAMPLAND's article.

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mainly to differences in various features between his plates of 1921 and earlier plates, the first of which was taken in 1913. He did not definitely note rapid motions such as described above, although he does mention a change in „a small elongated concentration" 10″ NW of the central double star, which must be identical with the ridge *b* in Figure 10. He describes this as „quite narrow and of considerably increased brightness on the last negatives". „This small condensation", he writes, „has developed from a small and more diffuse mass of earlier years, giving the impression that its inner edge has receded slightly from the double star and has been gathered into the narrower, brighter and more sharply defined mass on the recent negatives". He also noted several changes of a much larger scale between the 1921 negatives and those of 1916 and earlier. For instance, on the 1921 negatives the elongated mass of 48″ × 15″ about 45″ NW of the centre stood out prominently in a region where the nebulosity was somewhat diffuse in 1916 and earlier years.

At our request Dr SHANE kindly sent us an enlargement of a negative taken in 1899 with the Crossley reflector (this has been reproduced in *Lick Publ.* VIII). Comparison of this print with prints of plates taken by BAADE with the 100-inch telescope around 1942 shows striking differences in various regions of the nebula. The differences are of such a nature that they cannot be ascribed to differences in gradation or seeing. These changes, as well as those described by LAMPLAND, are mainly, if not wholly, in the amorphous mass, and not in the filaments. The differences seem always to be limited to local changes in brightness and form, the overall shape of the amorphous mass remaining the same.

If, as seems plausible, the fast-moving knots and light-ripples reported by BAADE are due to eruptions from the remnant of the supernova (see the following sections) one would expect this remnant to show large flares. So far, however, no variability of the south-preceding component of the central double star has been reported. It may be that this star is not the remnant of the supernova, and that this latter has not yet been observed. It would seem of interest to look for a faint emission nucleus which might only occasionally become visible.

Since the above was written one of the authors has had the opportunity to inspect during a brief stay in Pasadena, a considerable number of plates taken between 1921 and 1955 with the large reflectors of the Mt Wilson and Palomar Observatories, and to compare these with plates obtained at Flagstaff from 1916 to 1948, as well as with a few Lick plates dating back to 1899. This inspection has fully confirmed LAMPLAND's statements concerning changes in several parts of the nebula. As an example of such changes we have reproduced in Figure 11 (facing p. 289) the north-western part of the nebula as shown on a 100-inch plate taken by HUBBLE on November 19, 1924 and on a 100-inch plate of October 25, 1938, by BAADE. The 1924 exposure shows a dark channel south-east of and parallel with the row of three bright stars near the centre of the reproduction (the star in the middle has a northern companion). On the later plate the northern part of this channel is still quite conspicuous, but it has shifted about a quarter of a minute (i.e. a quarter of a light-year) to the north, so that the most northerly of the three stars is in the middle of it. In 1944 the dark region had become inconspicuous; in 1948 the northern part had filled up entirely. In the article cited LAMPLAND mentions changes between 1916 and 1921 in the neighbourhood of this same star (which lies 70″ from the central star of the nebula, in position angle 330°). Comparison with Figure 2 of BAADE's note on page 312 shows that in 1955 the region had again a different structure. There was then a bright, totally polarized, ridge passing through the upper two stars and slightly to the north of the lowest of the three. Changes of similar kind may be found in many other regions. An example of somewhat different type has been observed between 1918 and 1923. On a plate taken by LAMPLAND on December 4, 1918, a luminous streak may be observed branching off the brightest part of the central bar at about the level of the central double star; it extends in north-north-westerly direction to a distance of about 0′.5 from the central star, where it bends sharply and continues to the north-east over about 0′.3. This boomerang-shaped feature can also be distinguished on later plates, shifting gradually to the north, with a velocity of the order of 1/30ieth of that of light. In Feb. 1922 it has detached itself from the central bar; on later plates it disappears gradually.

Some of the variations noted by LAMPLAND must have proceeded with similar velocities; in most cases, however, the velocities seem to be somewhat lower.

Plates differing less than 5 years show more resemblance with each other than plates with large time intervals. The differences seem particularly striking when the Lick plates of 1899 are compared with recent exposures.

Notwithstanding the conspicuous local variations the general form and appearance of the nebula has remained remarkably the same. This holds also for part of the structural details, such as the dark bay on the eastern side, the bright horseshoe-like ribbon bordering it, and the large dark region shaped like an inverse S that traverses the nebula from north-east to south-west and comprises the dark area surrounding the central double star. On all plates the bright bar forming the north-western border of this dark region is the brightest part of the nebula. But in the

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course of the 56 years of photographic observations it has varied considerably in shape and extent.

We are very much indebted to Drs BOWEN, BAADE, A. G. WILSON and SHANE for their great kindness in furnishing the plates for this general inspection.

In the near future we hope to make photometric measures on some of the plates, in order to obtain quantitative data concerning the relative variations in brightness.

4. *New and old mechanism of radiation.*

In 1950 ALFVÉN and HERLOFSON [1]) suggested that the radiation at radio frequencies emitted by so-called radio stars might come from electrons with energies in the range of cosmic rays, moving in the magnetic field of a star, and trapped in this field. KIEPENHEUER subsequently applied this idea to the general galactic radiation at radio frequencies, suggesting that this might largely be due to high-energy electrons accelerated in the interstellar magnetic fields [2]). The suggestion by ALFVÉN and HERLOFSON has since been taken up by several authors, in particular by SHKLOVSKY. He was the first to suggest that also the optical continuous radiation from the Crab nebula might be due to this mechanism, and would thus be of the same nature as the optical radiation observed in synchrotrons [3]). As SHKLOVSKY points out, this theory does away with the difficulties facing the commonly adopted explanation of the continuous radiation by free-free transitions. The most serious were the great mass required [4]) and the fact that the amorphous mass shows no prominent emission lines, neither those of hydrogen and helium nor any lines of the type observed in the solar corona. Another unsatisfactory feature was the relatively great strength of the emission at radio frequencies, for which no adequate theory was available.

We have long considered the above difficulties as extremely serious. We may point to still another obstacle in the way of the generally assumed model. If practically all of the mass that is observed at present was expelled at the time of the outburst, the density in the past must have been larger by a factor $(900/t)^3$, if t is the time in years elapsed since the outburst. As the amorphous mass is now of the 9th magnitude, it should have been of the 1st magnitude 90 years, and -6^m nine years after the outburst. It is reported, however, that the nova ceased to be visible after about two years.

Two other objections to the theory of emission by free-free transitions must now be added, both of which appear to be still more serious than the ones already mentioned. The first concerns the observed polarization. It seems extremely difficult to explain this if the light were due to transitions on an atomic scale. The second is the fast-moving light-ridges and knots discussed in the preceding section.

None of the above difficulties remain in SHKLOVSKY's model of the Crab nebula, in which all radiation is assumed to come from electrons of cosmic-ray energies moving in weak magnetic fields. The significance of the polarization measures in supporting this model was pointed out by DOMBROVSKY [5]).

The picture that SHKLOVSKY draws of the Crab nebula is roughly as follows: The main part of the nebula is the shell of filaments expanding with a velocity of about 1100 km/sec. No good estimate of the mass of this shell has yet been made. We have provisionally assumed with SHKLOVSKY that it is of the order of 1/100th of the solar mass, but it may well be ten times higher. The shell radiates like a planetary nebula except that the relative abundance of hydrogen may be some ten times lower than normal. The excitation energy for the shell is provided either by a central star, or by far-ultraviolet radiation from the amorphous part.

Compared to this shell the mass of the fast particles in the amorphous part is negligible, of the order of 10^{-8} solar masses. SHKLOVSKY suggests that the magnetic field may be of interstellar origin, but increased by small-scale turbulence in the expanding shell to about 10^{-4} gauss. The electrons emitting optical radiation should then have energies of 10^{12} ev or higher. For radiation at radio frequencies a limiting energy of 10^9 ev is sufficient. Now, as SHKLOVSKY points out, it is very probable that the number of electrons increases with decreasing energy. An energy spectrum such as that of cosmic rays may well account for the ratio of 1000 between the intensity per unit of frequency in the radio observations and the intensity in the optical domain.

5. *Radiation of high-energy electrons in a magnetic field.*

According to SCHWINGER [6]) the power in ergs

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[1]) 'Cosmic Radiation and Radio Stars', *Phys. Rev.* **78**, 616 (1950).

[2]) 'Cosmic Rays as the Source of General Galactic Radio Emission', *Phys. Rev.* **79**, 738 (1950).

[3]) 'On the Nature of the Light from the Crab nebula', *Doklady Akad. Nauk USSR* **90**, 983 (1953).

[4]) The latest calculations by GREENSTEIN and MINKOWSKI (*Ap. J.* **118**, 1, 1953, Table 1) lead to values between 20 and 30 times the sun's mass. The difficulty is enhanced by the fact that supernovae of type I must be considered as Population II objects, for which the upper limit of the mass is supposed to lie between 1½ and 2 times that of the sun (this argument was pointed out to us by BAADE in a private communication).

[5]) *Doklady Akad. Nauk USSR* **94**, 1021 (1954).

[6]) *Phys. Rev.* **75**, 1912 (1949). These phenomena have also been discussed elsewhere, cf. VLADIMIRSKY in *J. Exp. and Techn. Phys.* **18**, 392 (1948) (in Russian), who also gives numerical values. Essentially the same problem has also been treated by G. A. SCHOTT in his classical text book on 'Electromagnetic Radiation', (p. 109 a.f.) (Cambridge Univ. Press, 1912).

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emitted per second by a high-energy electron per c/s is given by

$$P(\nu) = \frac{3^{3/2}}{2}\frac{e^2}{R}\left(\frac{E}{mc^2}\right)^4 \frac{\nu_0 \nu}{\nu_c^2}\int_{\nu/\nu_c}^{\infty} K_{5/3}(\eta)\,d\eta. \qquad (1)$$

In this expression e is the charge of the electron in e.s.u., m its rest mass, c the velocity of light, R the radius of curvature of the orbit, all in cgs units. The integral can be computed from tables of Bessel functions. R is related to the energy E (in ergs) and the magnetic field H (in gauss) by

$$R = \frac{E}{eH_\perp}. \qquad (2)$$

$H_\perp$ stands for $H \sin\theta$, where θ is the angle between the velocity and H. Further, ν_0 is the Larmor frequency

$$\nu_0 = \frac{c}{2\pi R}. \qquad (3)$$

The critical frequency ν_c is

$$\nu_c = \frac{3c}{4\pi R}\left(\frac{E}{mc^2}\right)^3. \qquad (4)$$

The total energy radiated per second is given by

$$\frac{dE}{dt} = -\frac{2e^2 c}{3R^2}\left(\frac{E}{mc^2}\right)^4. \qquad (5)$$

Eliminating R with the aid of (2), we can write

$$P(\nu) = \frac{3^{1/2} e^3}{mc^2} H_\perp \frac{\nu}{\nu_c}\int_{\nu/\nu_c}^{\infty} K_{5/3}(\eta)\,d\eta = CH_\perp \frac{\nu}{\nu_c}\int_{\nu/\nu_c}^{\infty} K_{5/3}(\eta)\,d\eta, \qquad (6)$$

in which $C = 2.343 \times 10^{-22}$;

$$\nu_c = \frac{3e}{4\pi mc} H_\perp \left(\frac{E}{mc^2}\right)^2 = \qquad (7)$$

$$= 6.268 \times 10^{18} H_\perp E^2 = LH_\perp E^2_{\mathrm{Gev}},$$

where $L = 1.608 \times 10^{13}$.

Further,

$$\frac{dE}{dt} = -\frac{2e^4}{3m^2c^3} H_\perp^2 \left(\frac{E}{mc^2}\right)^2 = -2.368 \times 10^{-3} H_\perp^2 E^2,$$

$$\text{or} \quad \frac{dE_{\mathrm{Gev}}}{dt} = -AH_\perp^2 E^2_{\mathrm{Gev}}, \qquad (8)$$

with $A = 3.793 \times 10^{-6}$.

E_{Gev} is the energy of the electron in units of 10^9 ev. The solution of (8) may be written

$$E_{\mathrm{Gev}} = \frac{1}{AH_\perp^2}\left(t + \frac{1}{AH_\perp^2 E_a}\right)^{-1}, \qquad (9)$$

where E_a is the energy in units of 10^9 ev with which the electron was expelled from the central star, and t is the time in seconds elapsed since. The energy will diminish to half of its original value in a time

$$T_{1/2} = \frac{1}{3.156 \times 10^7 AH_\perp^2 E_a} = \frac{0.00835}{H_\perp^2 E_a}\ \text{years}. \qquad (10)$$

The radiation is totally polarized, with the electrical vector parallel to the radius of curvature of the orbit. The radiation is concentrated in a very sharp cone, the axis of which co-incides with the velocity of the electron. The average angle in radians, between the light-rays and the velocity is according to SCHWINGER

$$\bar{\vartheta} = mc^2/E. \qquad (11)$$

$P(\nu)$ has a maximum near $\nu/\nu_c = 0.30$. It varies relatively little over a large range of frequencies, decreasing to half of its maximum value at $\nu/\nu_c = 0.011$ and $\nu/\nu_c = 1.47$. Numerical values for

$$\frac{\nu}{\nu_c}\int_{\nu/\nu_c}^{\infty} K_{5/3}(\eta)\,d\eta = F(\nu/\nu_c)$$

are shown in Table 5 and Figure 12.

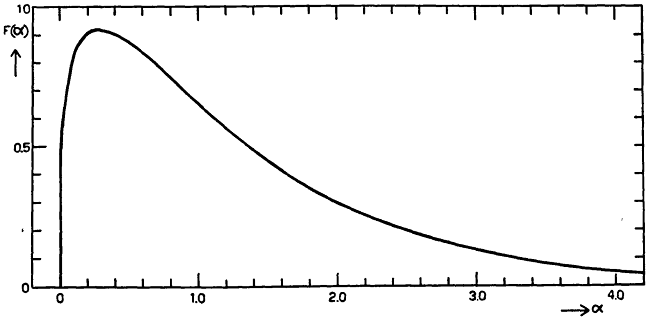
We are much indebted to Mr H. VARMA for calculating the table. After it had been computed, we got access to the publication by VLADIMIRSKY cited

TABLE 5

$$F(\alpha) = \alpha \int_{\alpha}^{\infty} K_{5/3}(\eta)\,d\eta \quad (\alpha = \nu/\nu_c)$$

α	$F(\alpha)$	α	$F(\alpha)$
0.001	.213	1.00	.655
0.005	.358	1.20	.566
0.010	.445	1.40	.486
0.025	.583	1.60	.414
0.050	.702	1.80	.354
0.075	.772	2.00	.301
0.10	.818	2.50	.200
0.15	.874	3.00	.130
0.20	.904	3.50	.0845
0.25	.917	4.00	.0522
0.30	.919	4.50	.0339
0.40	.901	5.00	.0214
0.50	.872	5.59	.0124
0.60	.832	6.25	.0068
0.70	.788	6.93	.0037
0.80	.742	8.73	.00064
0.90	.694	10.67	.00010

FIGURE 12



Radiation of an electron as a function of $\alpha = \nu/\nu_c$.

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above. The values in the table have been further checked by comparison with those given by VLADIMIRSKY. The five last values were taken from this source.

In the following we shall write

$$\nu/\nu_c = \alpha.$$

For small values of α we have approximately

$$P(\nu) = 2 \times 3^{1/6}\Gamma(2/3)\,\frac{e^2}{R}\left(\frac{\nu}{\nu_0}\right)^{1/3} = 5.04 \times 10^{-22}\, H_\perp\, \alpha^{1/3}, \qquad (12)$$

and

$$F(\alpha) = 2.15\, \alpha^{1/3}. \qquad (13)$$

These expressions give an approximation which is sufficient for our purpose when α is less than about 0.01; at $\alpha = 0.01$ they are 4% higher than the rigorous formulae. For $\alpha > 10$, $P(\nu)$ and $F(\alpha)$ may be written

$$P(\nu) = \left(\frac{3\pi}{2}\right)^{1/2} \frac{e^3}{mc^2}\, H_\perp\, \alpha^{1/2} e^{-\alpha} = 2.94 \times 10^{-22}\, H_\perp\, \alpha^{1/2} e^{-\alpha}, \qquad (14)$$

$$F(\alpha) = 1.26\, \alpha^{1/2} e^{-\alpha}. \qquad (15)$$

6. *Numerical estimates for the Crab nebula.*

If we want to estimate the total energy contained in the high-energy particles in the Crab nebula, we must make some assumption about the distribution function of the energies.

In order to get optical radiation, ν_c must be of the order of optical frequencies. For $\nu_c = 10^{15}$ we obtain from (7) $E_{\mathrm{ev}} = 2 \times 10^{11}$, if we take for H the value of 10^{-3} gauss found below. The energies needed lie therefore in the range of cosmic-ray energies. As a matter of fact, as SHKLOVSKY has pointed out, the Crab nebula is likely to be a powerful source of cosmic rays. If we assume that, together with the fast electrons, equal numbers of high-energy protons and other nucleons were formed, it may indeed have provided a fair fraction of the cosmic rays that we observe.

We shall see below that the electrons responsible for the optical radiation probably suffer considerable losses by radiation. If this is true, the electrons emitting the light observed at present cannot have derived their high energies from the explosion in 1054, but must have acquired them during the last few centuries. As a working hypothesis we suppose, therefore, that electrons (and nucleons) are continually accelerated to the required energies by the remnant of the supernova, or by some other mechanism, and that the intensity of this process has not changed appreciably with time (an alternative theory will be mentioned in section 8). We shall further assume that the *energy spectrum* of the electrons accelerated during each time interval is similar to that of primary cosmic rays. The present energy distribution will be different because the energy of all but the slowest and most recently added electrons will be degraded. The precise solution of the present energy spectrum would involve the equation of transfer from one energy interval to another. However, as a rough approximation we assume that the electrons keep their initial energy during $T_{1/2}$ years and are then lost. With this assumption we observe at present only the electrons emitted during the last $T_{1/2}$ years if $T_{1/2} < 900$ years. Here $T_{1/2}$ is the half-life of the electrons considered, i.e., the time elapsed before they have lost half their energy. At energies for which $T_{1/2} > 900$ years, all electrons produced during the 900 years of the nebula's existence are assumed to be present with their original energies.

The data that can be used to verify the energy spectrum, are the ratio of radio to optical emission and the spectrum in the radio domain. The latter is still uncertain, but will ultimately give a precise determination of the low-energy part of the spectrum. The ratio of radio to optical radiation is computed below.

The initial hypothesis that the energy spectrum was the same as that of cosmic rays, for which we assumed the exponents (19), gave too high a ratio of radio to optical emission. We have therefore ultimately adopted somewhat larger values of the exponents, which approximately reproduce the observed ratio. These exponents are given in (29).

Let $n_0(E)$ be the total number of electrons per unit interval of E produced since the birth of the nebula. We suppose that

$$n_0(E) = kE^{\gamma-1}, \qquad (16)$$

or, if $N_0(E)$ is the number with energies greater than E,

$$N_0(E) = \frac{k}{-\gamma} E^{\gamma}. \qquad (17)$$

For cosmic rays with energies between 1.5×10^{10} and 10^{15} ev, γ has been found to be -1.8, while between 2×10^{9} and 1.5×10^{10} ev, γ appears to be about -0.9; for smaller energies the curve becomes still flatter [1]). Following this undoubtedly rough and uncertain representation of the energy spectrum, we have assumed for our first tentative computations that since the birth of the Crab nebula the following numbers of fast electrons per Gev have been formed in it.

$$\begin{array}{rl} E_{\mathrm{Gev}} > E_1 & n_0(E) = k_1\, E_{\mathrm{Gev}}{}^{\gamma_1 - 1} \\ E_1 > E_{\mathrm{Gev}} > E_2 & n_0(E) = k_2\, E_{\mathrm{Gev}}{}^{\gamma_2 - 1} \\ E_2 > E_{\mathrm{Gev}} > E_3 & n_0(E) = k_3\, E_{\mathrm{Gev}}{}^{\gamma_3 - 1} \end{array} \qquad (18)$$

in which

$$\begin{array}{ll} E_1 = 15.5 \ \ \mathrm{Gev} & \gamma_1 = -1.8 \\ E_2 = 1.55 \ \ \mathrm{Gev} & \gamma_2 = -0.8 \\ E_3 = 0.155\ \mathrm{Gev} & \gamma_3 = +0.2 \end{array} \qquad (19)$$

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[1] W. HEISENBERG, 'Kosmische Strahlung' (Springer Verlag, 1953), pp. 20–26.

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For reasons of continuity we must have

$$k_2 = k_1/15.5, \quad k_3 = k_1/(15.5 \times 1.55). \qquad (20)$$

The loss of energy by radiation is taken into account by multiplying the numbers given in (18) by $T_{1/2}/900$. If we had a more or less homogeneous magnetic field, $T_{1/2}$ might be found from (10). Actually, we have used a higher value. We have seen that the major part of the light-emission is confined to a region which is much smaller than the outer limits of the nebula indicated by the filaments. As the radio emission seems to extend up to about these latter limits, the volume occupied by the fast electrons must be at least as large as this. That they emit little light while passing through the outer regions, must be due to a diminution of the average field strength in these regions. As we shall see in section 10, a decrease in the magnetic field has a much greater influence on the optical emission than on the radio emission.

In order to take these factors roughly into account we introduce the following rather drastic schematization. We shall suppose that H is constant throughout the inner part, up to a mean radius of about 1′.0. In the region surrounding this central part and extending to the edge of the filamentary nebula which has a mean outer radius of 2′.5, the field is again supposed to be homogeneous, with an average strength of half that in the inner region. We assume that the electron density is likewise constant in this shell, but is half that in the inner part.

We shall see on p. 306 that the optical radiation of an electron is proportional to $H^{2.15}$. With the numbers just given the total brightness within a circle of 1′ radius around the centre becomes then roughly half the total light of the nebula, in accordance with the data in Table 4. The light emitted by the central *sphere* of radius 1′ is 38% of the total light.

As the volume of the shell is 14.6 times that of the inner part, it will contain 7.3 times as many electrons. If the electrons move through both parts of the nebula, they may be supposed to spend 88% of their time in a region where their rate of radiation is $(0.5)^{2.15}$ times that in the central portion. It follows that their half-life is 2.9 times longer than it would have been if they had spent their whole time in the inner region.

There may well have been losses other than by radiation, for instance through escape from the nebula. These have been neglected, because we saw no way to estimate them (cf. page 306).

In the present section, whenever we mention $H_\perp$ or H, these will refer to the average field strength in the inner part. The real half-lives of the electrons will then be those given by (10) multiplied by the factor 2.9 just estimated. We fully realize the roughness of the procedure, but it seems better to apply a tentative factor of 2.9 than to apply no factor at all.

The number of electrons actually present in the whole nebula is thus obtained by multiplying the distribution (18) by

$$f = 2.73 \times 10^{-5}\, H_\perp^{-2}\, E_{\mathrm{Gev}}^{-1}, \qquad (21)$$

for

$$E_{\mathrm{Gev}} > 2.73 \times 10^{-5}\, H_\perp^{-2}. \qquad (22)$$

The numbers in the inner region (with mean radius of 1′), which will be denoted by $n(E)$, are 8.3 times smaller, so that

$$n(E) = \frac{f}{8.3}\, n_0(E), \qquad (23)$$

where $n_0(E)$ is given by (18), and f has the value (21) for all energies exceeding the limit (22), while $f = 1$ for smaller values of E.

We now proceed to calculate the radiation per unit frequency at a frequency ν, produced by a set of electrons with the following energy distribution (the unit of energy being always 10^9 ev):

$$\begin{array}{ll} E > E_1 & n(E) = K_1 E^{\Gamma_1 - 1} \\ E_1 > E > E_2 & n(E) = K_2 E^{\Gamma_2 - 1} \\ E_2 > E > E_3 & n(E) = K_3 E^{\Gamma_3 - 1}. \end{array} \qquad (24)$$

E_1, E_2, E_3 have the same values as in (19). For those energies at which the half-lives of the electrons exceed 900 years, Γ will be equal to γ as given in (19), and $K = k/8.3$. If, on the other hand, the age factor comes in, we have $\Gamma = \gamma - 1$ and $K = 2.73 \times 10^{-5} H_\perp^{-2}\, k/8.3$. If the demarcation between these cases lies in one of the three intervals of (24), this is split up into two intervals with two different values of K and Γ.

Let $\varphi(\nu_c)$ be the number of electrons with ν_c in a unit interval around ν_c, so that

$$\varphi(\nu_c)\, d\nu_c = n(E)\, dE.$$

Using (7) and (24) we obtain

$$\varphi(\nu_c) = \tfrac{1}{2} K_1 (LH_\perp)^{-1/2\,\Gamma_1}\, \nu_c^{-1 + 1/2\,\Gamma_1} \qquad (25)$$

for the range in ν_c corresponding with the interval $E > E_1$, or $\nu_c > LH_\perp E_1^2$. Similar expressions hold in the other intervals of E. Each electron emits a radiation which, in ergs per sec per c/s, is given by (6), for which we write $P(\nu) = CH_\perp F(\alpha)$. We thus find the following expression for the *radiation emitted* by the inner region

$$\left.\begin{aligned} J(\nu) = \int_0^\infty P(\nu)\, \varphi(\nu_c)\, d\nu_c = \\ \frac{CH_\perp}{2}\Bigg\{ K_1 \left(\frac{\nu}{LH_\perp}\right)^{1/2\,\Gamma_1} \int_0^{\alpha_1} \alpha^{-1 - 1/2\,\Gamma_1} F(\alpha)\, d\alpha \\ + K_2 \left(\frac{\nu}{LH_\perp}\right)^{1/2\,\Gamma_2} \int_{\alpha_1}^{\alpha_2} \alpha^{-1 - 1/2\,\Gamma_2} F(\alpha)\, d\alpha \\ + K_3 \left(\frac{\nu}{LH_\perp}\right)^{1/2\,\Gamma_3} \int_{\alpha_2}^{\alpha_3} \alpha^{-1 - 1/2\,\Gamma_3} F(\alpha)\, d\alpha \Bigg\}, \end{aligned}\right\} \qquad (26)$$

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in which

$$\alpha_1 = \frac{\nu}{L H_\perp E_1^2}, \qquad (27)$$

and similarly for the other indexes.

Rather similar computations, based on a constant value of γ for the entire range from optical to radio radiations, have been carried out by GINZBURG 1).

For optical frequencies, α_1 is large for all values of H that are possible on the basis of the present theory. For instance, for $H_\perp = 10^{-3}$, $\nu = 7.06 \times 10^{14}$ ($\lambda = 4250$ A), the value of α corresponding with $E_1 = 15.5$ Gev is $\alpha_1 = 183$. The radiation from electrons with energies below E_1, or α larger than α_1, is completely negligible. It remains negligible even if $H_\perp$ were to be taken 10 times larger. In the optical range we need, therefore, only the first term of (26). Knowing $J(\nu)$ from observations, we can then determine K_1 from (26), and thus also the total number of high-energy electrons in the nebula, as well as their average energy.

Before proceeding in this way, we have tried to check the values of γ given in (18) by computing the radiation at radio frequencies.

For the radio domain the first term in (26) is in general small compared to the two other terms.

For the following calculations we shall assume that the direction of H is distributed roughly at random with respect to the direction of the observer, and that θ shows likewise a random distribution over one hemisphere. We shall accordingly put

$$H_\perp = H\overline{\sin\theta} = \frac{\pi}{4} H. \qquad (28)$$

Inserting (28) into (22) we find that the „age factor" (21) comes in for $E_{\text{Gev}} > 4.42 \times 10^{-5} H^{-2}$, i.e., $E_{\text{Gev}} > 44.2$ if $H = 10^{-3}$, and $E_{\text{Gev}} > 0.442$ if $H = 10^{-2}$. For these energies the relation between K and k becomes then $K = 5.33 \times 10^{-6} H^{-2} k$. For $H = 10^{-4}$ no factor comes in at all, the contribution from electrons with $E > 4420$ being negligible.

We now compute from (26), (27) and (28) the ratio of the radio-frequency radiation per c/s at $\nu = 1.0 \times 10^8$ to the optical radiation per c/s at $\nu = 7.06 \times 10^{14}$. This ratio will be denoted by β. Using the energy spectrum of cosmic rays, (19), and three different values of the average magnetic field H, we obtain the results shown in Table 6 under (19). The value of H indicated in this table, as well as everywhere else in this section, refers to the absolute value of the magnetic induction averaged over the central part of the nebula. The vector average will be considerably smaller.

TABLE 6

	computed values of β			radiation computed from (26) and (29)	
H \ γ	(19)	(19) without age factors	(29)	optical	radio
10^{-4}	14 600	14 600	509	$0.370 \times 10^{-30} k_1$	$1.88 \times 10^{-28} k_1$
10^{-3}	7 600	1 420	434	2.74 ,,	11.9 ,,
10^{-2}	11 700	91	881	3.85 ,,	33.9 ,,

These values of β should be compared with observation. For the optical radiation into 4π solid angle we adopt MINKOWSKI's estimate 2) $J(\nu_l) = 3.4 \times 10^{21}$ erg per c/s per sec at $\lambda = 4250$ A, or $\nu_l = 7.06 \times 10^{14}$. This corresponds to a photographic absolute magnitude -2.2 for the amorphous mass. For the emission at radio frequencies we have used STANLEY and SLEE's result 3) $J(\nu_r) = 2.7 \times 10^{24}$ erg per c/s per sec at $\nu_r = 10^8$. Both values are quite uncertain 4). The distance of the nebula was assumed to be 1100 pc.

From the above numbers the radio brightness per c/s is 790 times the optical brightness at $\nu = 7 \times 10^{14}$. This refers to the entire nebula. We have seen in section 2 that the distribution of the surface intensity of the light differs considerably from that of the radio emission, the difference being probably caused by a decrease of the magnetic field intensity at greater distances from the centre. For the present calculation we need a comparison for an approximately homogeneous volume of space. We shall choose the central sphere of 1′ radius for this purpose. From SEEGER's occultation observations at λ 75 cm we found a provisional value of 1.69 for the ratio of the scales of the radio source and the optical nebula. In order to find the ratio of radio to optical radiation per cm³ near the centre we must divide the ratio of 790 valid for the whole nebula by $(1.69)^3$. This gives 164. Estimating that the light-emission per cm³ averaged over the central sphere is about 0.7 that at the centre, while the radio emission will be practically constant

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1) *Uspekhi phyz. Nauk*, **51**, 343 (1953).

2) *Ap. J.* **96**, 199 (1942); *Mt Wilson Contr.* No. 666.

3) *Australian J. Sci. Res. A.* **3**, 234 (1950).

4) After the present article was practically finished, Dr HAGEN kindly communicated to us the results of recent measures of the flux made at the Naval Research Laboratory. At 21 cm these correspond to $J(\nu) = 1.5 \times 10^{24}$. Had we used this value instead of the one cited above we would have got practically the same result for the energy spectrum.

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over this sphere, we find for the average ratio in the central sphere $\beta = 230$. The uncertainty of the data is such that the true value of β may well differ by a factor 2 from the adopted value of 230.

Evidently, an energy spectrum like that of cosmic rays (column (19)) gives much too high intensities in the radio domain, at least if the age factor is included. Should we assume that all fast particles produced in the past have escaped from the nebula, so that what we observe at present is only the electrons produced in the last century or so, then the age factor should be omitted, at least for $H = 10^{-4}$ and 10^{-3}. With H between 10^{-3} and 10^{-2} an energy distribution corresponding to that of cosmic rays would then approximately produce the observed value of β.

Unless we are prepared to accept such a rapid rate of escape, we must assume an energy spectrum which is less steep than that of the cosmic rays in order to reproduce the observed ratio between radio and optical radiation. We have therefore repeated the calculations with the following exponents for the integrated energy spectrum:

$$\begin{array}{rl} E_{\mathrm{Gev}} > 15{\cdot}5 & \gamma_1 = -1{\cdot}3 \\ 1{\cdot}55 < E_{\mathrm{Gev}} < 15{\cdot}5 & \gamma_2 = -0{\cdot}3 \\ 0{\cdot}155 < E_{\mathrm{Gev}} < 1{\cdot}55 & \gamma_3 = +0{\cdot}7 \end{array} \qquad (29)$$

The results are in column (29) of Table 6. The increase of 0.5 of the exponents γ has brought the results for β close to the observed value 230. The last two columns show the radiations themselves in ergs per c/s per sec. The energies given in the following have all been computed with the energy spectrum (29).

Having fixed the energy spectrum, we can now compute from (26) the optical radiation from the central part, but for the unknown coefficient k_1 defined in (18) (K_1, K_2, K_3 can, of course, readily be expressed in k_1). We find k_1 by equating the radiation so calculated to the observed radiation from the central sphere, which is $0.38 \times 3.4 \times 10^{21}$ erg per c/s per sec.

The total numbers of electrons (Σn) produced since the supernova explosion, and *their total energy* (ΣE) can now be computed from (18) and (29). As before, we have assumed that there have been no losses through escape. Table 7 shows the results for an assumed magnetic field of 10^{-3} gauss, and for various intervals of energy. The corresponding values for $H = 10^{-4}$ may be obtained by multiplication by 7.37, those for $H = 10^{-2}$ by multiplication by 0.710.

TABLE 7

Total numbers of fast electrons and total energies (in ergs) for $H = 10^{-3}$.

E_{Gev}	Σn	ΣE
>155	$\cdot05 \times 10^{49}$	$5{\cdot}57 \times 10^{47}$
15·5 − 155	$\cdot98 \times$,,	$5{\cdot}57 \times$,,
1·55 − 15·5	$4{\cdot}46 \times$,,	$3{\cdot}81 \times$,,
·155 − 1·55	$3{\cdot}06 \times$,,	$\cdot38 \times$,,
Total	$8{\cdot}55 \times 10^{49}$	$15{\cdot}33 \times 10^{47}$

If we assume that the numbers of protons of the same energies produced in the nebula are equal to those of the electrons, the numbers and energies should be doubled if we want to include all fast particles. The total energies so obtained are shown in Table 8, under ΣE_{fp}.

TABLE 8

Total energy and energy density of fast particles in the Crab nebula.

H	ΣE_{fp} (ergs)	Total number of protons	E_{fp} (erg/cm^3)	E_H (erg/cm^3)	med. $E_{\mathrm{opt.}}$ (ev)	$T_{1/2}$ (opt.) (years)	$R_{\mathrm{opt.}}$ (cm)
10^{-4}	23×10^{48}	$6{\cdot}3 \times 10^{50}$	68×10^{-8}	$0{\cdot}04 \times 10^{-8}$	$7{\cdot}0 \times 10^{11}$	5600	300×10^{11}
10^{-3}	$3{\cdot}1 \times$,,	$0{\cdot}85 \times$,,	$7{\cdot}3 \times$,,	$4{\cdot}0 \times$,,	$2{\cdot}2 \times$,,	180	$9{\cdot}4 \times$,,
10^{-2}	$2{\cdot}2 \times$,,	$0{\cdot}60 \times$,,	$3{\cdot}4 \times$,,	$400 \times$,,	$0{\cdot}70 \times$,,	5·6	$0{\cdot}30 \times$,,

These total energies are surprisingly high when compared to the total energy available in a star. For instance, a complete conversion of all hydrogen in the sun into helium would produce 1.0×10^{52} ergs (assuming a hydrogen content of 82%). If we are dealing with a star that has already used up its hydrogen, the combination of helium into carbon by SALPETER's reaction may produce a comparable amount of energy. As BAADE has pointed out, it is unlikely that the central star of the Crab nebula has a mass much exceeding that of the sun, because supernovae of type I appear to be objects of Population II. We must conclude, then, that in the course of its 900-year life the old supernova has concentrated on to particles with average energies of the order of 10^{11} ev at least 1/10000th of the total energy it could produce by completely burning up all its hydrogen or helium. In our opinion this forms the most serious difficulty for the theory that the amorphous light is due to synchrotron radiation. We return to this problem in section 8.

We now proceed to discuss what is the most likely *strength of the magnetic field* in the Crab nebula. In the first place we shall compare the energy density of the

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fast particles in the inner region with that of the magnetic field. In order to obtain the energy for the central region we applied a factor of 1/8.3 to the values of ΣE_{fp} in Table 8. The loss of energy through radiation has been taken into account by applying to the electrons the age factors (21) in the energy ranges concerned. The volume of the inner sphere was taken 0.40×10^{55} cm^3. The energy densities so derived are shown under E_{fp} in Table 8. The next column gives the energy density of the magnetic field, $E_H = H^2/(8\pi)$.

A comparison of these columns shows that for $H = 10^{-4}$ the energy density of the electrons needed for the observed optical radiation, plus that of the accompanying protons, is about 3 factors of 10 higher than the energy density of the magnetic field. This does not seem possible, as most of the fast particles would in this case have long ago escaped from the nebula. The condition that the density of the kinetic energy of the charged particles probably cannot much exceed the magnetic energy density, limits the magnetic field to values equal to or larger than about 10^{-3} gauss [1]. Fields much lower than this are improbable also because the total energy of the fast particles would seem to become prohibitive.

On the other hand the magnetic field cannot be *much* larger than 10^{-3}, because for higher field strengths the lives of the electrons responsible for the optical radiation become too short.

In order to estimate the mean half-life of these electrons, we first consider the distribution of the energies of the electrons contributing to the optical radiation. This is found from (26) by replacing α by E as independent variable. After division by $J(\nu)$ and filling in the proper values for C and K_1 we get the following normalized distribution for $H = 10^{-3}$:

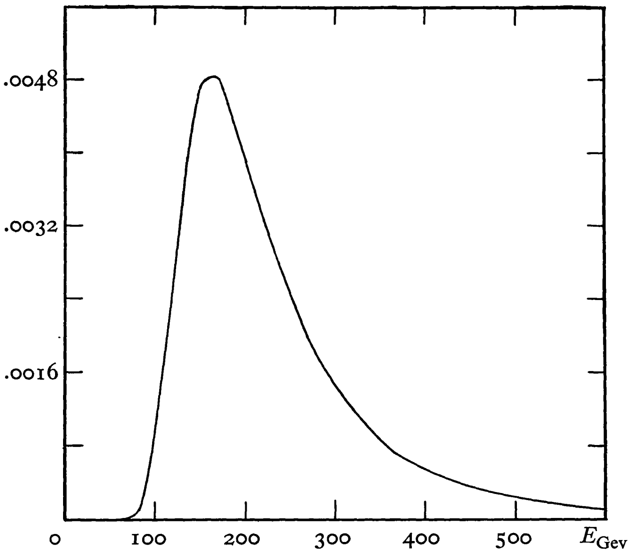
$$f(E)_{\mathrm{opt}} = 3.59 \times 10^5 E^{-3.3} F(5.60 \times 10^4 E^{-2}), \quad (30)$$

E being again measured in units of 10^9 ev, and F being the function tabulated in Table 5. $f(E)_{\mathrm{opt}}$ is shown graphically in Figure 13. The median value of the energy is 221×10^9 ev. The half-life of electrons of this energy moving through the whole nebula is 180 years.

For other values of H the distributions will be similar, except that the abscissae are to be multiplied by $\sqrt{10^{-3}/H}$, and the ordinates to be divided by this factor. The half-lives corresponding to the median energies are proportional to $H^{-3/2}$. These half-lives in years have been entered in the next to last column of Table 8.

As we have already found it to be improbable that the field strength would be smaller than about 10^{-3}, we are led to conclude that the half-lives of the electrons causing the optical radiation are probably at least five times shorter than the time elapsed since the outburst. It follows that the fast electrons we see radiating at the present moment cannot be remains of the outburst, but must have been recently expelled from the remnant of the supernova, or have been accelerated in some other way.

FIGURE 13



Fractions of optical radiation due to electrons of various energies (per Gev) for $H = 10^{-3}$.

The maximum diameter of the Crab nebula is 6 light-years. If the average velocity of the electrons along the lines of force is estimated to be $^1/_2\, c$, the half-lives of the „optical" electrons must be several decades in order for them to fill the entire nebula. For this reason it seems improbable that the average field strength would be much larger than about 3×10^{-3}, at which $T_{1/2}$ would be 34 years.

These considerations appear to limit the average field to the small range from about 10^{-3} to 3×10^{-3}. In the following general considerations we shall suppose $\bar{H}$ to be 1.0×10^{-3}.

It is probably no accident that the energy density of the fast electrons observed is nearly equal to that of the magnetic field. The equality suggests that the nebula contains just so many high-energy particles as it can contain, the surplus having escaped in the form of cosmic rays.

At $H = 10^{-3}$ the whole amorphous nebula must be renewed in about 200 years. Although we do not understand how these enormous quantities of particles of extreme energy are produced, the observations by BAADE described in section 3 suggest that such production may be taking place, and even that the rate of production may be of the right order. Recent measures of polarization indicate that these rapidly moving condensations consist of relativistic particles.

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[1] For a further discussion of this point we may refer to p. 302.

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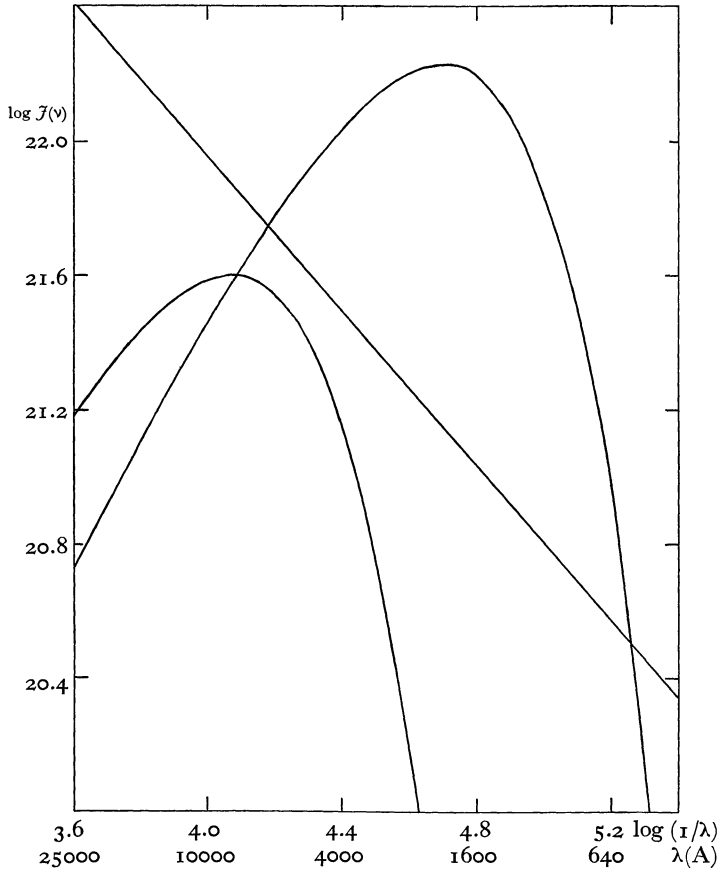
With BAADE's rough estimates of their magnitude and frequency the light-ripples would yield about 1/400th part of the nebula per year. As it is probable that only the largest eruptions have been noted, it does not seem unreasonable to suppose that the total amount of fast particles expelled would be a few times higher.

The last column of Table 8 shows the average radius in cm of the spirals described around the lines of force by particles with an energy equal to the median energy of the electrons giving the optical radiation. For all possible values of H the radii are small compared to the dimension of the nebula.

The total amount of matter responsible for the radiation of the amorphous part of the nebula is presumably small. According to Table 7 the number of high-energy electrons formed since the outburst would be about 0.9×10^{50}. If we assume that there is one proton to each electron we obtain a total rest mass of 1.4×10^{26}, or 10^{-7} times the sun's mass. This is negligible compared to the mass of the shell of filaments, which has been estimated at 10^{-2} solar masses (the estimate may well be wrong by a factor 10).

FIGURE 14



Comparison of the radiation of the continuous part of the Crab nebula with that of an ordinary Bo star and a G dwarf, both being approximated by a Planck curve. The radiation of the G dwarf has been multiplied by 1000.

The optical spectrum of the amorphous nebula is directly related to the energy distribution. We assume that $H = 10^{-3}$. The light-emission is determined by that part of the first term of (26) in which the age factor had to be introduced, the energies below 44.2 Gev being of no importance for the production of light. Assuming the energy spectrum (29) we have then $\Gamma_1 = \gamma_1 - 1 = -2.3$. It follows that for all wavelengths smaller than about 15000 A:

$$J(\nu) \sim \nu^{-1.15}. \tag{31}$$

The spectrum extends over a much wider range in frequency than the radiation from a black body. This is illustrated in Figure 14, where the Crab-nebula radiation (straight line) is compared with the radiation of a Bo star of -3.4 photographic absolute magnitude supposed to radiate like a black body of 25 000° temperature (upper curve). The lower curve gives the radiation of a black body at 6000°, adjusted in height so as to fit in the graph; it corresponds roughly to 1000 times the radiation of a G dwarf.

The large range of the spectrum explains partly why the total energy gone into the fast particles (ΣE_{fp} in Table 8) is much higher than the energy radiated in 900 years by a star of the same photographic absolute magnitude. An additional reason is that a considerable amount of the energy is still stored in the relativistic protons and electrons contained in the nebula.

Compared to a source for which $J(\nu)$ is independent of ν (such as we would have if the light were due to free-free transitions) the intensity at λ 4770 would be a factor $\left(\frac{4770}{4260}\right)^{1.15} = 1.14$ higher than that at λ 4260, corresponding with a colour excess of $+ 0^m.14$ in the E_1 scale. BAADE has estimated that the early-type stars in the vicinity of the Crab nebula would have a colour-excess $E_1 = + 0^m.15$ [1]. Such an extinction would make the observed value of $J(\nu)$ proportional to $\nu^{-2.23}$ instead of $\nu^{-1.15}$.

An exact determination of the spectral gradient would give extremely valuable information on the energy spectrum of the particles with highest energies. Our knowledge of the interstellar extinction is insufficient to permit such a determination from the data available at present.

It is of interest to estimate the far-ultraviolet radiation. With the energy distribution (29) the radiation beyond the Lyman limit is found to be 1.3×10^{37} erg/sec. With the steeper energy distribution indicated by cosmic rays ($\gamma_1 = -1.8$) it would be 0.3×10^{37} erg/sec. These values are smaller than for ordinary O stars, but of the same order as for a main-sequence Bo star. The radiation suffices for the excitation of the shell of filaments. If the mass of this shell is supposed

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[1] *Ap. J.* **96**, 188 (1942); *Mt Wilson Contr.* No. 665.

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to be one tenth of the solar mass, the overall density in this shell would be about 2 hydrogen atoms per cm^3. The distribution in the shell is evidently very inhomogeneous; but even if the density in the individual filaments were as high as 10 000 atoms per cm^3, the ultra-violet radiation from the amorphous mass would be enough to ionize these filaments.

If the central star is supposed to have a surface temperature like that of an O5 star the amount of radiation beyond the Lyman limit emitted by this star would be about one hundredth of that coming from the amorphous mass. It is therefore probable that it has no appreciable influence on the state of the shell of filaments.

Before concluding this section we wish to consider briefly *the magnetic field induced by the fast particles.* We suppose that the original field is coupled to the massive expanding shell of filaments (cf. section 9). The fast particles shot out from the central star will tend to neutralize this outer field.

Consider one relativistic electron or proton moving perpendicular to the local magnetic field, and describing a circle with radius R. We want to discuss the magnetic field produced by this electron. Inside the circle this is opposite to the original field, while outside it has the same direction as the original field. The total magnetic flux due to the circulating particle, integrated over the entire plane, is zero. It is easily seen that the resultant flux within a circle of radius ρ_n around the centre of the electron's orbit is

$$\frac{eR}{2\rho_n}, \text{ for } \rho_n \geqslant R. \tag{32}$$

Let the angle between the local magnetic force and the direction of the average field in the whole nebula be ε, and let, further, ρ_n be the radius of the part of the nebula in which the average magnetic field appears to be more or less parallel, then the projection of the average magnetic force induced by one electron on the direction of the average field of the nebula is obtained by dividing the above expression by the cross section of the nebula, and multiplying by $\cos\varepsilon$. If we have $n(E)$ fast particles with energy in a unit interval around E, and moving under angles θ with the local field, these will yield an average magnetic force in the direction of $\overline{H}$

$$H_{fp}(E) = \frac{\overline{eR\sin\theta}\,\overline{\cos\varepsilon}}{2\pi\rho_n{}^3}\, n(E) = \frac{\cos\varepsilon}{2\pi\rho_n{}^3 H} E n(E), \tag{33}$$

in which the expression (2) has been substituted for R. Integration over E gives for the total magnetic field due to the fast particles

$$H_{fp} = \frac{\Sigma E\,\overline{\cos\varepsilon}}{2\pi\rho_n{}^3 H} = \tfrac{2}{3} E_{fp} H^{-1}\,\overline{\cos\varepsilon}, \tag{34}$$

E_{fp} being the energy density of the fast particles.

We must consider only that region of the nebula in which the magnetic field shows pronounced parallelism. As such we have taken the inner sphere of 1′ radius.

In (32) it has been assumed that the particle considered moved in the centre of the nebula. For a particle at a distance ρ from the centre, only part of the circle with radius ρ_n around its orbit falls within the region of the nebula which we are considering. The induced field over that region may then become considerably higher. A rough estimate shows that for $\rho = 0.7\,\rho_n$ the expression (32) should be multiplied by a factor 2. Applying this factor to (34) and inserting the value of E_{fp} from Table 8, we obtain for $H = 10^{-3}$

$$H_{fp} = 0.10 \times 10^{-3}\,\overline{\cos\varepsilon}. \tag{35}$$

The average projection of the original field on the direction of the resultant field for the nebula is $10^{-3}\,\overline{\cos\varepsilon}$. We see therefore that about 10% of the „outside" field is compensated by the high-energy particles. This estimate indicates that we could not introduce many more particles without causing a drastic reduction of the original field and, consequently, a general escape of the electrons responsible for the optical radiation. The above result is in rough agreement with Table 8, where for $H = 10^{-3}$, the energy densities of the magnetic field and of the fast particles were found to be of the same order.

We realize that the above schematization may be inadequate. Among others we have neglected the effect of the particles moving in the surrounding parts of the nebula, and have taken no account of the fact that in the outermost regions the average field must have an opposite direction. Also we have neglected the pronounced local inhomogeneities in the magnetic field.

7. *Loss of energy by collision with particles and photons.*

For electrons with energies such as would be needed to obtain the emission at optical wavelengths the losses due to radiation greatly exceed those due to ionization or to collision with nucleons [1]). If high-energy nucleons are produced together with the electrons, these will not suffer any appreciable loss of energy, neither through ionization or encounters with free electrons nor through radiation, in a time of the order of the age of the Crab nebula. The principal loss of protons and other nucleons will be by escape from the field of the nebula.

The loss of energy of cosmic-ray particles through encounters with photons has been discussed by Feenberg and Primakoff [2]). At the very high particle

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[1]) Cf. Fermi, *Phys. Rev.* **75**, 1169 (1949), who states that for a medium of density 10^{-24} and with $H = 5 \times 10^{-6}$ the loss of energy is mainly due to ionization up to $E = 3 \times 10^8$ ev. In the Crab nebula, with its comparable mean density and higher magnetic field, the limit will be still lower.

[2]) 'Interaction of Cosmic-Ray Primaries with Sunlight and Starlight', *Phys. Rev.* **73**, 449 (1948).

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energies concerned, the particles loose energy by Compton scattering. The authors compute that a cylinder of 1 cm^2 cross section extending radially away from the sun, from the earth to infinity, contains 3×10^{20} photons, and that a cosmic-ray particle approaching the earth along such a cylinder will suffer 4×10^{-4} encounters with photons. The average loss of energy at a Compton collision is $4 \times 10^{-3} (1 - v^2/c^2)^{-1}$ for an electron, and $10^{-9} (1 - v^2/c^2)^{-1}$ for a proton. For the energies of the electrons responsible for the optical radiation of the Crab nebula this loss would be about 4×10^9 ev for a collision with a photon in the optical range. For a proton it would be negligible.

The Crab nebula emits 3.4×10^{21} erg/sec per c/s. The surface may be estimated at 2×10^{37} cm^2. If the density of the radiation in the outer part of the nebula is denoted by $u(\nu)$ we have

$$u(\nu) = \frac{3.4 \times 10^{21}}{2 \times 10^{37}\, c/2} = 1.1 \times 10^{-26} \text{ erg cm}^{-3} \text{ (c/s)}^{-1}.$$

This is for $\lambda = 4250$ A, or $\nu = 7.06 \times 10^{14}$. If $u(\nu)$ varies as $\nu^{-1.15}$, as indicated in (31), the number of photons per cm^3 with wavelengths between 1000 and 10000 A is found to be

$$\frac{1.1 \times 10^{-26}}{h} \int_{3 \times 10^{14}}^{3 \times 10^{15}} \left(\frac{\nu}{7.06 \times 10^{14}}\right)^{-1.15} \frac{d\nu}{\nu} = 3.6.$$

During an estimated life of 180 years an electron traverses a distance of 1.7×10^{20} cm. A cylinder of this length and 1 cm^2 cross section would contain 6×10^{20} photons. According to the above estimate the electron would suffer an average of 8×10^{-4} Compton collisions during this time, so that no appreciable loss of energy through this process can occur. The same conclusion holds for the Compton scattering by lower-energy photons. In the radio domain $u(\nu)$ is about 10^3 times higher. With a range in ν corresponding to wavelengths from 1 mm to 10 metres, an electron would suffer about one photon collision in 30 years. However, the energy loss through Compton effect in one collision is entirely negligible for these soft photons. Accordingly we can safely disregard the effect of Compton scattering for the Crab nebula, even if we include the entire frequency region down to ν_0.

The same conclusion holds a fortiori for the protons and other nucleons in the nebula.

Using the data given by FEENBERG and PRIMAKOFF on the production of electron-positron pairs in proton-photon collisions, we see that for the energies with which we are concerned the number of such processes must be still very much smaller than that of Compton collisions.

8. *The source of the energy.*

We have found in section 6 that the total energy of the fast particles in the Crab nebula must be of the order of 10^{49} ergs, most of which is concentrated on particles with energies of the order of 10^{11} ev. This result cannot be lowered appreciably by assuming a different energy spectrum. We have already remarked that the total energy of these fast particles comprises a surprisingly large fraction (viz. about one ten-thousandth) of the total energy content that could have been contained in the supernova.

The energy produced in the form of fast particles is likewise large if we compare it with the more common forms of energy developed at the outburst of the supernova. The total amount of radiative energy emitted at such an outburst has been estimated to be of the order of 10^{48} ergs. The kinetic energy of the expanding shell of filaments would be 10^{47} ergs if it is assumed to have a mass equal to 1/100th of that of the sun.

It is therefore hardly probable that the high-energy particles could derive their energy from the kinetic energy contained in this shell. However, in view of the general difficulty of accounting for the high energies needed, we wish to consider this possibility somewhat further.

Both the character of the shell of filaments, and the local deviations found in the direction of the polarization, indicate that important irregularities exist. There are probably also considerable random motions superimposed upon the average expansional motion of the shell. As a high estimate we assume an average random velocity v of 500 km/sec, or half the velocity of expansion. We now consider the energy that the fast electrons or protons may gain by encounters with irregularities in the magnetic field, in the same way as suggested by FERMI for cosmic rays in the Galactic System [1]. Following FERMI we put the order of the average gain in energy at a collision with such an irregularity at

$$\delta E \approx (v/c)^2 E \approx 3 \times 10^{-6} E. \tag{36}$$

If this mechanism were to maintain the average energy of the fast electrons, it would have to provide an energy equal to the energy lost through radiation. With a half-life of 180 years (Table 8) this would mean that in 180 years δE must become equal to $E/2$. In this time the electron would therefore have to go through about 150 000 collisions with magnetic irregularities, i.e. about 1000 per year. In view of the scales of the irregularities actually found, which are of the order of $^1/_2{}'$, or $^1/_2$ light-year, it seems improbable that the electrons could regain by this process an appreciable fraction of the energy lost by radiation.

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[1] *Phys. Rev.* **75**, 1169 (1949).

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It seems, therefore, that the high energies of the electrons must either be remains of the explosion of the supernova, or that they must in some way be continually furnished by the remnant of the supernova, a possibility that was already suggested by the existence of the fast-moving light-ridges near the centre of the nebula [1]). In the former case the average magnetic field would have to be lower than the fields derived in section 6.

9. *The nature of the magnetic field.*

The observations described in section 2 show that in the brighter part of the nebula the resultant directions of polarization are fairly parallel. On the average, in the part within an average radius of $1'.2$, 13.8 % of the total light is polarized. This is for the whole area in which measures with the $^1/_2'$ diaphragm were made (cf. Figure 2). For the inner part of the nebula the average degree of polarization is about 20 %. In the outer region of the nebula, between roughly $1'.2$ and $2'.1$ from the centre, the resultant polarization is found to be negligible. The polarization in the inner regions will therefore have been reduced by the superimposed effect of the more outlying shells. If we try to estimate roughly the amount of this reduction we arrive at a true average polarization of 18 % for the part within $1'.2$ from the centre. The deviation from 100 % indicates a considerable variation in the direction of the magnetic field along the line of sight. This has to be rather larger than that between the integrated polarizations in the individual $^1/_2'$ regions, in which a part of the waviness of the magnetic field has been effaced by the integration over the line of sight.

An impression of the scale of the waviness and the irregularities in the magnetic field may be obtained from the patches with practically 100 % polarization that were noted in section 2, as well as from the fact that considerable variations in amount and direction of polarization are observed from one field of $^1/_2'$ diameter to the next. This indicates that the diameters of the spaces where the field is practically homogeneous are not likely to be *much* smaller than $^1/_2'$ [2]).

As we have seen in section 5, the radiation from an electron is restricted to a small cone around the direction of its velocity. For $E_{\rm med} = 2 \times 10^{11}$ ev (corresponding to $H = 10^{-3}$) we find from (11) that the average angle between the emitted radiation and the velocity is 2.5×10^{-6} radians or $0''.5$. For a given angle θ between the velocity and H, the radiation will therefore be practically restricted to the mantle of a cone with semi-aperture θ around the lines of force. However, if θ is distributed more or less at random, as we have assumed in the preceding, we shall receive radiation from all parts of the nebula. The inclination of the magnetic field to the plane of the sky will then be of little influence, except if it is near 90°. From a given element of volume, where H makes an angle α with the line of sight, we observe only those electrons for which $\theta = \alpha$.

If the primary magnetic field has its seat in the shell of filaments it is improbable that large variations in this primary field could occur in a decade. The changes in the amorphous nebulosity discovered by LAMPLAND (cf. section 3) must accordingly be ascribed to slow variations in the eruptive activity of the central star rather than to changes in the primary magnetic field.

The origin of the magnetic field is still unknown. The total magnetic flux through that part of the nebula where the resultant polarizations are more or less parallel may be estimated to be of the order of 10^{33} gauss cm^2. As the magnetic flux through a star of the size of the sun is $1.5 \times 10^{22}\, H$, if H is the average magnetic field strength in this star, it is evident that for any conceivable value of H this remains far below the flux through the Crab nebula [3]). SHKLOVSKY and GINZBURG suggest that the magnetic field is of inter-

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[1]) Measures of such a moving ripple, which were recently made by Mr L. WOLTJER on 200-inch plates kindly loaned to us by BAADE, show convincingly that its radiation is polarized, thereby confirming that it is made up of relativistic particles. The mean resultant polarization is in the same general direction as the mean polarization in the central part of the nebula, but it appears to change in direction from one end of the ripple to the other. The wisp is elongated in the main direction of the magnetic field, but its apparent motion is approximately at right angles to it. More detailed measurements will be needed before a satisfactory interpretation of the moving ripples can be given.

[2]) Since the above was written, this has been fully confirmed by plates taken by BAADE with the 200-inch Hale telescope, and described by him in the concluding note of this *Bulletin*. Figure 15 (facing p. 289) gives a rough sketch of the magnetic lines of force in so far as these could be uniquely determined from BAADE's polarization plates. These plates indicate very directly that, in general, the field has a scale of almost the same order as the diameter of the nebula.

In the more central parts several loops of the field are superimposed. In the outer parts the lines of force may easily be followed. They run parallel to the structural features. Moreover, they show convincingly that the striking dark indentions in the continuous mass, the most pronounced of which is the large bay in its eastern edge, represent real structure of this mass, and are not due to absorption by superimposed dark clouds, as has sometimes been surmised.

[3]) During a discussion in Guanajuato, CHANDRASEKHAR suggested the idea that the star which became the supernova was on the verge of stability on account of its internal magnetic energy being equal to its gravitational energy. This is a criterion for the stability of magnetized stars that was found by CHANDRASEKHAR and FERMI (cf. CHANDRASEKHAR, *M.N.R.A.S.* **113**, 667, 1953). In a private communication Dr CHANDRASEKHAR writes the following about this possibility: „If this be supposed, the magnetic energy of the original star should be $H^2/8\pi = C\, GM_\odot^2/R_\odot$, where C is a constant which may be taken to be 2. For a star of solar dimensions this gives $H^2/8\pi = 7.56 \times 10^{48}$ ergs. If this energy

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stellar origin, increased to a value of about 10^{-4} gauss through small-scale turbulence caused by the expanding shell. This mechanism does not seem very likely to us. The apparent parallelism and the large amount of polarization in the central part seem to preclude such a high degree of twisting and intertwining of the lines of force as would be needed to raise the interstellar field by the factor of at least hundred that is required to obtain the field strength of 10^{-3} found in section 6. The observation that there seems to be nearly complete polarization whenever a region localized in space is measured shows, moreover, that the field cannot in general be due to turbulence on a very small scale.

It seems probable that the field is connected with the expanding shell of filaments. If the supernova was rotating rapidly, the rotation of the shell might be of the order of 1 cm/sec. We have not attempted a calculation of the fields that might arise from possible separation of charge. The rough correspondence between the size of the amorphous mass and that of the system of filaments supports the inference that the field is connected with the filaments. For, apart from this possible connection through the magnetic field, there would be absolutely no relation between these two components of the Crab nebula. The velocities of the particles in the amorphous mass are at least a hundred times higher than the velocity of the expansion of the shell, so that they might otherwise well have spread over a very much larger volume.

A further indication of a connection between the system of filaments and the magnetic field might be seen in the fact that in the most northerly part of the nebula, where the filaments show a distinctly more twisted appearance than near the other borders, the polarizations are also abnormal in direction, as well as exceptionally weak.

Inspecting Figures 1 and 2 of the last article of this *Bulletin*, we may also note that the brightest parts of the continuous mass co-incide more or less with the regions where the brightest filaments are found. In the south-eastern corner, for instance, where the bright filaments extend no further than a third of the way from the central star to the edge of the filamentary system, the continuous light is also very much weaker than at the same distance from the centre in the north-west corner, where the bright filaments reach nearly to the outer edge of the nebula.

An observable relation may exist between the run of the emission filaments and the structure of the magnetic field that is partly depicted in Figure 15. A possibly significant phenomenon is, that in three of the dark „bays" found on the eastern and southern borders of the continuous mass, a strong emission filament runs centrally through the bays. These filaments are schematically indicated by full-drawn lines in Figure 15. The exact situation of the filaments with respect to the bright continuous rims around the bays may be seen from Figures 1 and 2 of BAADE's note on page 312.

The material available at the time of writing was insufficient to search successfully for a more general relation between the emission filaments and the magnetic field. The first thing one would wish to know, is how the principal emission filaments are located in space. Part of this space picture can already be inferred from a comparison of the spectra obtained by MAYALL in 1937 [1]) with enlargements of BAADE's plates showing the „shell" of filaments. But some more spectra, as well as transverse motions, will be required to delineate the whole structure.

We have made rough measurements of the transverse motion of a sharp filament belonging to the *continuous mass*. This filament, marked p in Figure 15, is so strong and well-defined that it can easily be recognized on the older plates. Its complete polarization shows that it is part of the continuous mass and must therefore correspond with a concentration of magnetic lines of force. The transverse motion was found to be practically the same as that of the ordinary, emission filaments. This would seem to confirm that the magnetic lines of force expand with a velocity roughly equal to that of the shell of emission filaments.

The present data do not enable us to say whether the field extends in appreciable strength beyond the shell of filaments, nor whether it would resemble a dipole field if considered on a sufficiently large scale. Beyond $\rho = 2'$ the optical surface brightness becomes so small that it may remain impossible to observe it.

10. *The emission at radio frequencies.*

As SHKLOVSKY has already remarked, the great strength of the emission at radio frequencies as compared to the optical radiation, no longer presents a problem if we accept the synchrotron mechanism for the radiation. In fact, we have used this in section 6 to arrive at an estimate of the energy spectrum of the relativistic electrons.

We have seen in section 2 that there is a large difference in scale between the distribution of the intensity at metre waves and that in the optical domain. Such a difference may easily be explained as a consequence

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is now „diluted" in a sphere of radius 1 parsec, then the resulting mean magnetic field will be given by

$$H' = \left[\frac{8\pi \times 7.56 \times 10^{48}}{(4/3)\pi \times (3 \times 10^{18})^3}\right]^{1/2} = 1.2 \times 10^{-3} \text{ gauss.}$$

This is very nearly the value for the field you find for the Crab Nebula. It is possible that this agreement is not an accident".

[1]) *Publ. A.S.P.* **49**, 101 (1937).

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of the difference between the energy spectra in the two domains. We see from (26) that, if one of the terms in this equation preponderates, $J(\nu)$ is proportional to $H^{1-{}^1\!/\!_2\Gamma}$, if Γ is the exponent of the energy spectrum corresponding to this term. If two terms contribute in comparable amounts, we can easily calculate the corresponding exponent of H. We thus find that in the optical region $J(\nu)$ would be proportional to $H^{2.15}$ for values of H between $\frac{1}{2} \times 10^{-3}$ and 10^{-2}. In the domain of the metre waves $J(\nu)$ is similarly found to be proportional to $H^{0.62}$ near $H = 10^{-3}$. If we suppose that the falling off in light is due to a decrease in the average magnetic-field strength, coupled with a decrease of electron density proportional to H, the light-density u_l would be proportional to $H^{3.15}$, while the density of radio emission, u_r, would be proportional to $H^{1.62}$. Hence,

$$u_l = a\,u_r^{3.15/1.62} = a\,u_r^{1.94}, \qquad (37)$$

where a is a constant. With the alternative supposition that the electron density is constant over the whole nebula, we would get

$$u_l = a\,u_r^{2.15/0.62} = a\,u_r^{3.5}. \qquad (38)$$

A rough comparison of the estimated space distribution of light with that of the emission at 75 cm inferred from occultation observations, indicates that (37) may well represent the observations up to 1′.5 from the centre. At larger distances, however, u_l appears to fall off much more rapidly relative to u_r than would follow from this formula, or even from (38). If the radiation is indeed due to the synchrotron mechanism, this would mean that in the outer regions the density of electrons with „optical" energies has been reduced relative to that of the less energetic particles. A plausible explanation of such a decrease might be sought in the escape of the electrons of highest energies from the outermost parts of the nebula [1]).

As regards the radio spectrum we find from (26) and (29) the following dependence on frequency

ν	λ	$J(\nu)$
10^8	300 cm	$1.19 \times 10^{-27} k_1$
10^9	30 cm	1.88 ,,
10^{10}	3 cm	2.16 ,,

The emission varies little over this range, which is in agreement with available observations. The small increase between $\nu = 10^8$ and 10^9 shown by the above numbers can easily be made to disappear by a slight increase in the slope of the energy spectrum at low energies.

If the radio emission of the Crab nebula comes from fast electrons moving in a magnetic field, as we have supposed, the radiation at these frequencies should also show polarization. It has not yet been possible to establish the existence of such polarization. Measures have been made with the 21-cm equipment at Kootwijk by WESTERHOUT, and at metre waves at Jodrell Bank (private communication). Both gave negative results. The measures at 21 cm, which are described in the following paper, show that the polarization, if present, must be less than roughly 1% of the total radiation.

The results refer to the total radiation of the nebula, the beams being much larger than the dimensions of the nebula. It is doubtful whether under these circumstances an appreciable amount of polarization should be expected.

We have estimated above that the average degree of polarization of the light within a sphere of radius 1′.2 would be 18 %. For the radio-frequency radiation the polarization in the same sphere will be somewhat less, because of the greater relative intensity in the outer parts of the sphere, where the degree of polarization is less. A rough computation gives 15 % for the polarization of the radio emission. Outside this sphere the average polarization in the direction of the polarization shown by the inner parts is practically zero, as may be seen from the last line of Table 4. The polarization of the total radiation from the surface with 1′.2 radius will be much less than 15 %, because of the superposition of non-polarized radiation from outer shells. If we suppose that the space distribution of the radiation is similar to the distribution of surface intensity we find that the amount of superimposed radiation would be 1.8 times that in the sphere. As, according to the optical results, the radiation from regions more than 1′.2 from the centre has no polarization in the direction of that in the central sphere, the resultant polarization would be 5.5 %.

The polarization of the radio-frequency radiation of the whole nebula may then be calculated by adding the radiation from the region outside the circle with radius 1′.2. From BALDWIN's measures this is found to be 4.6 times the radiation coming from within this circle. As the radiation from these outer parts will not be polarized, the resultant polarization is found to be 1.0 %. It is doubtful whether the provisional measures of polarization made so far are sufficiently accurate to establish so small a degree of polarization. Possibly the actual average polarization will be still smaller, for instance if the outer parts should have a resultant polarization perpendicular to that of the inner parts.

In order to establish whether or not the radiation of a radio source is due to the synchrotron mechanism, it would evidently be of eminent importance to con-

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[1]) We refrain from giving numerical data, because a much better discussion of this problem will become possible when the reductions of extensive optical measures made by one of us at the Observatoire de Haute Provence have been completed, and when these results have been compared with SEEGER's measures at 75 cm of the November 30 occultation of the nebula.

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firm the presence of polarization in the radio-frequency radiation. A larger percentage of polarization can only be expected when measuring with a very small beamwidth. For instance for a beamwidth of 3′.5 the estimated polarization would be about 3 %.

UNSÖLD and SEEGER have drawn our attention to the possibility that the polarization of the emitted radiation is wiped out by Faraday effects in the interstellar medium and in the filamentary shell of the nebula. If polarized light passes through a medium with electron density $N(r)$ in a field of H gauss, H making an angle ϑ with the line of sight, the electric vector is turned over an angle

$$\Omega = \frac{7490\pi}{\nu^2}\int H\cos\vartheta\, N(r)\, dr \text{ radians.} \tag{39}$$

r is the distance from the observer; the integral has to be extended from the observer to the object. Assuming that the shell of emission filaments in the Crab nebula has a mass of one tenth of that of the sun, concentrated in a shell of 1/3 pc thickness, and taking $H \cos \vartheta = 10^{-3}$, we obtain 14 radians for the average Faraday effect in the shell at a wavelength of 21 cm. Because the field is likely to vary greatly over the shell, the initial polarization may be largely obliterated. If we assume that the average interstellar density is one atom per cm³, that 10% of this is ionized, and that $H \cos \vartheta$ averages 3×10^{-6} gauss, the passage through the interstellar medium gives a rotation Ω of 7 radians. This is of the same order as the value derived for the shell, but as it is likely to vary only little over the surface of the nebula, it may not greatly decrease the amount of polarization. The effects of shell and interstellar medium will probably be negligible for wavelengths smaller than 3 cm.

11. *The synchrotron mechanism as a general source of radio-frequency emission.*

It is tempting to suppose that the radiation of other radio sources is likewise due to the synchrotron mechanism. Two conditions must then be fulfilled. There must be a source of high-energy electrons, and there must be a magnetic field. The latter may either be connected with an expanding gas shell as in the Crab nebula, or it may be interstellar, of the same type as the interstellar fields found in the Galactic System. This latter case is likely to apply to extra-galactic sources. In the galactic sources the high-energy particles probably originate in a special type of stars. In the radio sources identified with colliding galaxies they may have derived their energies from acceleration in electric fields set up by the relative motion of large-scale magnetic fields associated with each of the two colliding systems, as has been suggested by J. HEIDMANN¹).

In all other known radio sources (except the ordinary extra-galactic nebulae) the ratio of continuous light to radio emission is much smaller than in the Crab nebula. This might well be due to a weaker magnetic field in other radio sources, in which case the relative intensity of the light-emission can be very much smaller, as we have seen for the outer parts of the Crab nebula. In extra-galactic sources, such as Cygnus A, where we are presumably dealing with ordinary interstellar fields, these are likely to be much weaker than the 10^{-3} gauss found in the Crab nebula.

For the same reason considerable differences in dimension and structure may be expected between the radio sources and their optical counterparts, differences such as were found in Cygnus A, for example, and probably also in Virgo A.

Whether or not observable polarization effects occur, will depend on the existence of an average resultant magnetic field of the entire source. Only when this resultant field is not too small compared to the fields occurring in more localized regions of the source, can we expect to find polarization in the integrated radiation.

In most of the radio sources identified with optical objects the light is mostly due either to stars, or to line emission, so that there may be little chance of finding polarization in the optical radiation. The most prominent exception may be the straight jet near the centre of NGC 4486 (Virgo A), the light of which is largely continuous. This jet may well be a powerful source of relativistic electrons, which emit radio radiation in magnetic fields associated with the entire stellar system.

We have made an attempt to investigate whether the light from this wisp is polarized, by photographing it through a birefringent prism, which gave two images polarized in perpendicular directions. Two good plates were obtained, on the first of which the directions of polarization are parallel and perpendicular to the jet, while on the second they make angles of 45° with it. On the first plate there is an indication of several percent polarization with the electrical vector parallel to the jet, but this will have to be confirmed by further exposures before it can be accepted as real²).

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¹) Private communication.

²) In March 1956 BAADE discovered with the 200-inch telescope strong polarization in each of the three bright condensations which are visible in the picture of the jet published in *Ap. J.* **119**, 215, Fig. 8 (BAADE and MINKOWSKI). The direction of the polarization in one of the knots differs greatly from that in the others. This probably explains why our plates, on which the condensations are not separately visible, failed to show any striking effect. A detailed report will be published by BAADE in the May number of the *Astrophysical Journal.* We are greatly indebted to him for his permission to refer to his discovery prior to publication.

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12. *Production of cosmic rays.*

If the synchrotron theory of the radiation of the Crab nebula is correct, and if this nebula may be considered as representative of supernovae in general, the supernovae, as has been pointed out by SHKLOVSKY [1]), will furnish at least an important fraction of the cosmic rays observed in the Galactic System. This may be confirmed by the following admittedly tentative computation.

The energy density of cosmic rays has been estimated to be of the order of 10^{-12} erg/cm^3. If we put the volume of the principal part of the Galactic System at 3×10^{11} pc^3, or roughly 10^{67} cm^3, the total energy of the cosmic rays in this System would be 10^{55} ergs. According to UNSÖLD [2]) the life of a cosmic-ray particle in the Galactic System may be between 10^6 and 10^7 years. With the higher of these estimates an energy of about 10^{54} ergs in the form of cosmic rays should be produced in a million years.

The supernova remnant in the Crab nebula, according to Table 8, has produced between 10^{48} and 10^{49} ergs in the 900 years of its existence. It is likely to continue with this production during a longer interval, but for a minimum estimate we shall assume that this is all it will produce. All of the fast nucleons in the Crab nebula will ultimately escape into the Galactic System. Assuming that there is one supernova in the entire Galactic System each century, and that each produces 10^{49} ergs, we then arrive at a production of 10^{53} ergs of cosmic-ray energy per million years. Considering the great uncertainties in the estimates, the accordance with the value of 10^{54} ergs cited above is remarkably close.

If this theory is correct, the Crab nebula might well be the nearest direct source of cosmic rays. Clearly, it would be very important if a direct proof could be obtained that the Crab nebula is a source of cosmic rays. However, only particles of the most extreme energies might be expected to get through without too much deflection. If the average interstellar field between the Crab nebula and us is 10^{-6} gauss, the energies must exceed 10^{19} ev in order not to be deflected by more than a tenth of a radian. The chances of finding a direct correlation seem therefore rather slight.

If intergalactic magnetic fields are extremely small it would be conceivable that some of the strongest extra-galactic radio sources might be found to be direct sources of very energetic cosmic rays.

The above article was written in August 1955. The new information provided by BAADE's observations with the 200-inch telescope in September made it desirable to add a few passages and notes in the proof. We have likewise inserted subsequent references to the results of a recent inspection of old plates, and to radio-frequency observations of the two occultations of the nebula which took place in November.

We are greatly indebted to Professor DANJON and Dr LALLEMAND for the photomultiplier used in this investigation. Professor VAN DE HULST has kindly read the manuscript. We wish to thank him for a number of constructive suggestions.

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[1]) *Doklady Akad. Nauk USSR* **91**, 475 (1953); 'Les Processus Nucléaires dans les Astres' (5e Colloque International d'Astrophysique, Liège), 515 (1953). Cf. also GINZBURG, *Doklady Akad. Nauk USSR* **92**, No. 6 (1953).

[2]) 'Physik der Sternatmosphären', 2te Aufl., p. 779.

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