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Coulomb-Regulated Conductance Oscillations in a Disordered Quantum Wire

A.A.M. Staring[1,*], *H. van Houten*[1], *C.W.J. Beenakker*[1], *and C.T. Foxon*[2]

[1]Philips Research Laboratories, 5600 JA Eindhoven, The Netherlands
[2]Philips Research Laboratories, Redhill, Surrey RH1 5HA, UK
*Also at: Eindhoven University of Technology, NL-5600 MB Eindhoven, The Netherlands

Abstract Disordered quantum wires have been defined by means of a split-gate lateral depletion technique in the two-dimensional electron gas in GaAs–AlGaAs heterostructures, the disorder being due to the incorporation of a layer of beryllium acceptors in the 2DEG. In contrast to the usual aperiodic conductance fluctuations due to quantum interference, *periodic* conductance oscillations are observed experimentally as a function of gate voltage (or density). No oscillations are seen in the magnetoconductance, although a strong magnetic field dramatically enhances the amplitude of the oscillations periodic in the gate voltage. The fundamentally different roles of gate voltage and magnetic field are elucidated by a theoretical study of a quantum dot separated by tunneling barriers from the leads. A formula for the periodicity of the conductance oscillations is derived which describes the regulation by the Coulomb interaction of resonant tunneling through zero-dimensional states, and which explains the suppression of the magnetoconductance oscillations observed experimentally.

1. Introduction

Aperiodic conductance fluctuations due to quantum interference are commonly observed in disordered conductors small compared to the phase coherence length [1]. One characteristic aspect of these universal conductance fluctuations is the fundamental similarity between a conductance trace as a function of gate voltage (or density), and that as a function of magnetic field. Both traces represent a "fingerprint" of the sample-specific impurity potential. The origin of the duality between density and magnetic field is that both variables affect the *phase* of the conduction electrons, which for a particular closed trajectory depends on the Fermi wavelength (determined by the density, and thus by the gate voltage) and on the enclosed flux. This density–magnetic-field duality is quite general. For example, it also applies to the conductance quantization of a quantum point contact [2] and to the quantum Hall effect [3]. The experimental and theoretical results presented in this paper pertain to a new transport regime where gate voltage and magnetic field play an entirely different role, due to the effects of the charging energy associated with the addition

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of a single electron to a conductance-limiting segment of a disordered quantum wire.

Experimentally, we investigate a phenomenon first observed by Scott-Thomas et al. [4] in ultra-narrow channels defined in the electron inversion layer in silicon. They reported remarkable conductance oscillations periodic in the gate voltage (or the electron gas density), in the absence of a magnetic field. It was concluded that the periodicity of the oscillations corresponded to the addition of a single electron to a conductance-limiting segment of the narrow channel, with a length determined by the distance between two strong scattering centers. The effect was tentatively attributed to the formation of a charge density wave. A similar effect was seen subsequently in narrow channels in inverted GaAs-AlGaAs heterostructures [5], and was given the same interpretation. As an alternative explanation, it was proposed by two of us [6] that the characteristic features of the experiment might be due to the Coulomb blockade of tunneling [7] — a single electron effect studied extensively in metals where quantum interference effects are negligible. More recently, Wingreen and Lee [8] studied the interplay of the Coulomb blockade and resonant tunneling by a self-consistent solution of the Schrödinger and Poisson equation in a narrow channel geometry.

In the present paper we explore the relative importance of single-electron charging effects and of resonant tunneling by focusing on the different roles of gate voltage and magnetic field. As a novel experimental system for these investigations we use a conventional GaAs–AlGaAs heterostructure in which a layer of compensating impurities is incorporated in the 2DEG during growth. Such impurities were chosen because they are likely to form strongly repulsive scattering centers, which might act as tunnel barriers. We note that a certain degree of compensation was also present in the inversion layers of Ref. [4] and in the channels defined by lateral p-n junctions of Ref. [5]. In our system a narrow channel is defined electrostatically in the two-dimensional electron gas by means of a split gate on top of the heterostructure.

Theoretically, we extend previous work [9,10,11] by considering the combined effects of Coulomb interactions, gate voltage variations, and of a magnetic field on resonant tunneling through a quantum dot. This is relevant to our experiments (and to related experiments [4,5,12,13]) to the extent that one channel segment, delimited by two strong scattering centers, effectively limits the channel conductance. In addition, it is a model for experiments on the Aharonov-Bohm effect in individual quantum dots [14,15,16].

2. Experiments

The quasi one-dimensional electron gas used for the experiments described in this work is obtained by electrostatic confinement of the two-dimensional electron gas (2DEG) in a GaAs–AlGaAs heterostructure using a split-gate technique [17]. On top of the heterostructure, which is mesa-etched in the form of a Hall bar, a pattern of gold gates is defined using electron-beam lithography. The insets of Figs. 1 and 3 show a top view of the two geometries studied. At

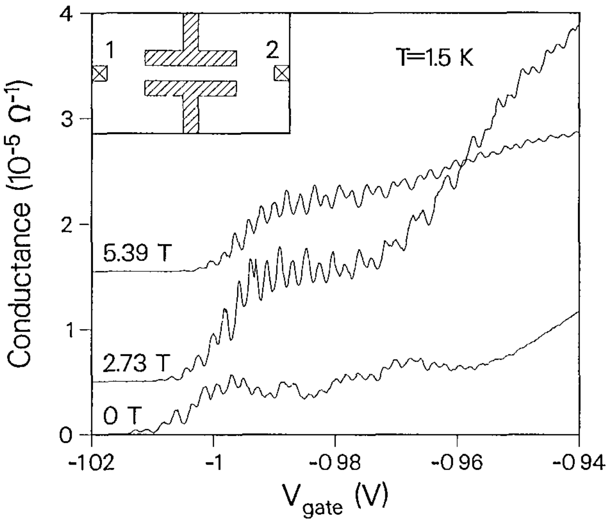

Figure 1: *Two-terminal conductance versus gate voltage at 1.5 K of a 3* μm *long split-gate quantum wire (inset, the shaded parts represent the gates while the contacts are labeled 1 and 2). The curves for different magnetic fields are offset vertically for clarity (zero conductance is reached at −1.02 V gate voltage).*

the depletion threshold of the 2DEG (-0.3 V), the quantum wires thus defined are nominally 0.5 μm wide, while their lengths vary from 1 μm to 16 μm; the side probes (if present) have a nominal width of 0.5 μm. Both the width and electron concentration of the wire decrease with gate voltage V_g. Pinch-off (as evidenced by the conductance) is typically reached at $V_g \approx -1$ V. One wire of 1 μm nominal width was also studied, having a pinch-off gate voltage on the order of −2 V. The results obtained with this wire were similar to those obtained with the 0.5 μm wires.

The heterostructure is of a conventional type and consists of the following layers, which are subsequently grown on top of a semi-insulating substrate by molecular beam epitaxy: A 1 μm thick GaAs buffer layer, a 20 nm undoped AlGaAs spacer layer, a 40 nm AlGaAs layer doped to 1.33×10^{18} cm^{-3} with Si, and an undoped 20 nm GaAs capping layer. The Al fraction in the AlGaAs layers is 33%. Disorder was introduced deliberately into the 2DEG by incorporating in the GaAs a planar doping layer of beryllium at 25 Å from the heterointerface, with a sheet concentration of 2×10^{10} cm^{-2}. The electron sheet concentration n_s of the wide 2DEG is 2.7×10^{11} cm^{-2}, with a mobility of about 8×10^4 cm^2/Vs (at 4.2 K). Contact to the 2DEG is made by alloyed AuGeNi ohmic contacts, located along the edges of the 1 mm × 0.3 mm Hall bar.

The measurements were performed with the samples in the mixing chamber of a dilution refrigerator at temperatures between 50 mK and 1.5 K. A conventional double ac lock-in technique, with an excitation voltage kept below kT/e in order to avoid electron heating, was used to determine the conductance of the quantum wires as a function of gate voltage and magnetic field. The field was oriented perpendicular to the 2DEG and had a maximum strength of 7.5 T. The gate voltage was swept at a rate of 10^{-4} V/s or less.

We now give an overview of the main results of our experiments, concentrating on the phenomenology, and defer a discussion of a mechanism which can account for these results to the next section. Fig. 1 shows the two-terminal conductance of a 6 μm long quantum wire at a temperature of 1.5 K for three different magnetic fields. Periodic oscillations as a function of the gate voltage can be seen in these traces. Calculations of Laux et al. [18] for a similar geometry indicate that the 1D electron density (per unit length) depends approximately linearly on the gate voltage. We thus conclude that the oscillations are periodic in the 1D electron density. The fact that it is still possible to observe the oscillations at the relatively high temperature of 1.5 K, in combination with their number (there are about 30 oscillations with a period of 2.2 mV resolved), will prove to be an important clue to their origin, as will be detailed in the next section. The *period* is insensitive to a magnetic field. Nevertheless, a magnetic field is seen to have a variety of effects. The amplitude of the oscillations in strong fields is enhanced above the zero-field case, as is the average conductance. The pinch-off gate voltage is shifted towards zero. The conductance peaks, in this particular sample, have a tendency to regroup in a doublet-like structure consisting of a stronger and a weaker peak.

On lowering the temperature to 50 mK the oscillations are better resolved, as is shown in Fig. 2. The insets show the Fourier transforms of the corresponding conductance traces, and clearly demonstrate that the dominant oscillation has a B–independent frequency of 450 V^{-1} (the trace at 7.47 T has a slightly increased frequency of 500 V^{-1}). Additionally, a second peak in the Fourier transform emerges at about half the dominant frequency as the field is increased. This second peak corresponds to the amplitude modulation of the peaks, which is most clearly seen in the trace at 5.62 T where high and low peaks alternate in a doublet-like structure.

Fig. 3 displays the dependence of the conductance oscillations on the magnetic field for the middle section of a device of the geometry shown in the inset. This particular sample does not exhibit periodic oscillations in the absence of a magnetic field, but only for $B \gtrsim 1$ T. Remarkably, very pronounced oscillations are seen at 5 T, in sharp contrast to the weak random conductance fluctuations in zero field. Between 2 T and 3 T short-period (0.5 mV) oscillations are observed in this sample, in addition to the slower dominant oscillations with a period of 2.2 mV which persist over the entire magnetic field range from 1 T up to 7.5 T. At high magnetic fields, traces of these short-period oscillations return.

The period of the oscillations does not correlate with the length of the quantum wire. We conclude this from measurements on a number of wires with lengths varying from 1 μm up to 16 μm. Sometimes the oscillations were not quite periodic, even in a magnetic field. An example of this behavior is

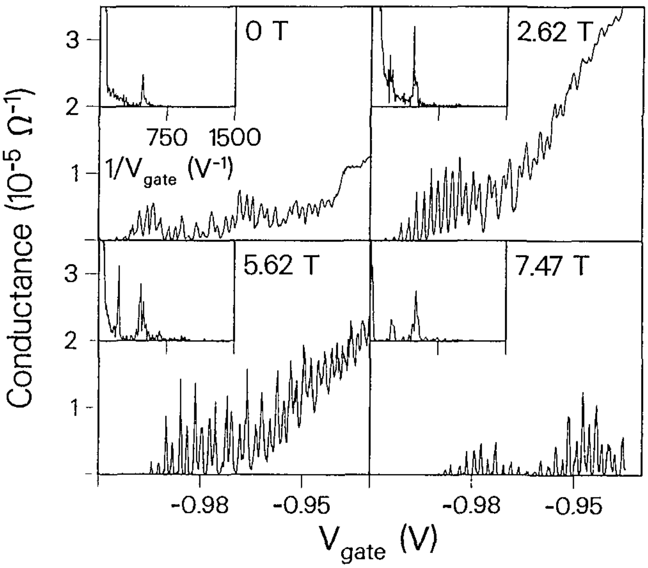

Figure 2: *Conductance versus gate voltage at 50 mK of the same device as in Fig. 1. Insets: Fourier transforms of the data, with the vertical axes of the 0 T and 7.47 T curves magnified by* 2.5×, *relative to the 2.62 T and 5.62 T traces.*

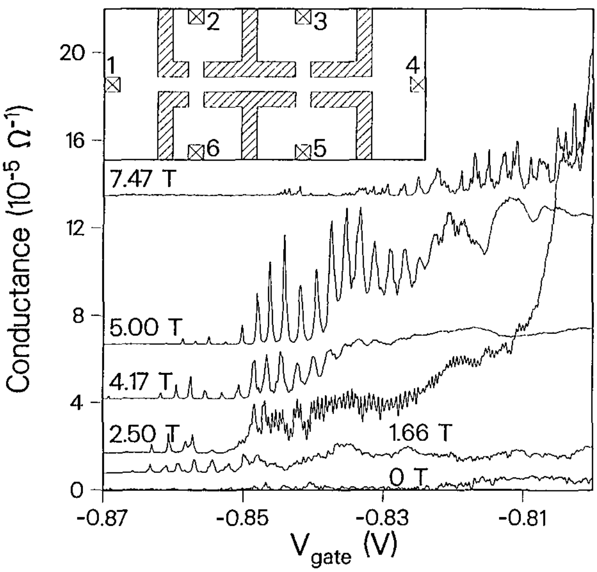

Figure 3: *Development of the conductance oscillations with magnetic field at 50 mK, for a device of the type shown in the inset. The current was passed through contacts 1 and 4, while the voltage was measured between contacts 2 and 3. The curves are offset vertically for clarity.*

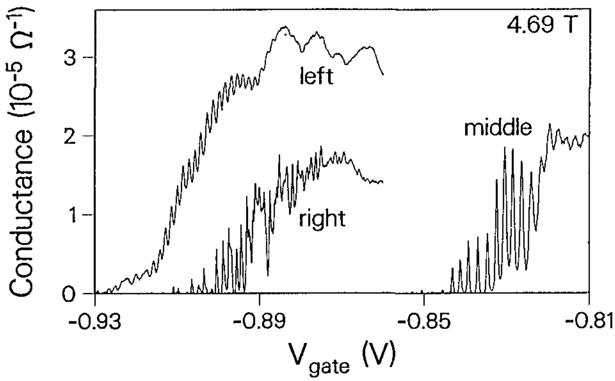

Figure 4: *Conductance at 4.69 T of the three sections of the device shown in Fig. 3 with lengths of 2* μm *(left), 6* μm *(middle) and 4* μm *(right). The current and voltage contacts used were, respectively, (1,2) and (1,6) (left), (2,3) and (6,5) (middle), and (3,4) and (5,4) (right).*

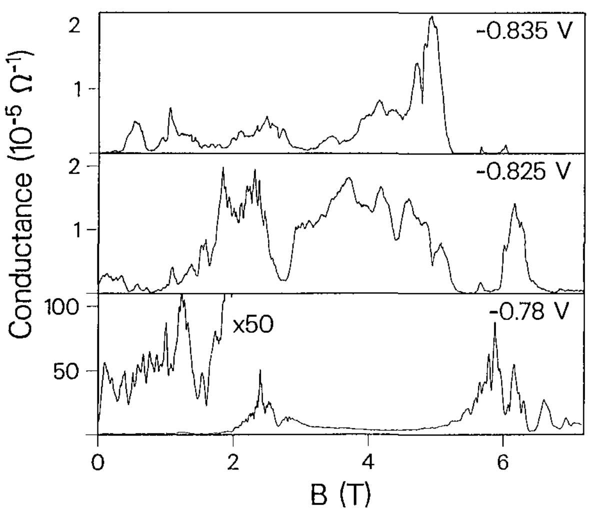

Figure 5: *Magnetoconductance of the device shown in Fig. 3, again using contacts 1 and 4 as current source and drain, and 2 and 3 as voltage probes.*

shown in Fig. 4 (right). It is also clear from this Figure that the middle section of this device determines the total two-terminal conductance (G_{14}).

Whereas the conductance as a function of gate voltage at a fixed magnetic field shows periodic oscillations, no such behavior is observed in the opposite case where the magnetic field is varied and the gate voltage is fixed. As is shown in Fig. 5 the magnetoconductance shows essentially random fluctuations, in contrast to the periodic oscillations seen in Figs. 1–4. Note the extreme sensitivity of these magnetoconductance fluctuations to a small shift in the gate voltage.

3. Theory and Discussion

A theory able to account quantitatively for all of the experimental observations is likely to require a full treatment of the electron–electron interactions. The charge density wave phenomenon [4,5,12] may play a role in such a theory, which however does not yet exist. Our present goal, in the spirit of Ref. [6], is to investigate to what extent the remarkable periodicity of the oscillations as a function of gate voltage, and the absence of regular oscillations in the magnetoconductance, may be explained in terms of single-electron tunneling.

Since quantum effects are known to be important in semiconductor nanostructures [19], it is natural to first consider whether resonant tunneling through zero-dimensional states in a "quantum dot", defined by a conductance-limiting segment of the channel (see Fig. 6), might by itself be able to account for the gate-voltage periodic oscillations. Field et al. [12] argued against such a mechanism, because of the absence of the expected spin-splitting of the peaks in a strong magnetic field, and also because the peaks would most likely not be periodic in V_g. We arrive at the same conclusion, and put forward an additional compelling argument. At a temperature as high as 1.5 K we still find clear oscillations (see Fig. 1), although some thermal smearing is evident in the data (compare with Fig. 2). The width of the thermal smearing function at this temperature is $4kT \approx 0.5$ meV, so that the energy level separation in the case of resonant tunneling would have to be somewhat larger, say around 2 meV. Since each conductance peak would correspond to the depopulation of a single discrete level, the Fermi energy $E_F = 10$ meV at channel definition would then imply a maximum number of about 5 peaks in the full gate-voltage range from definition to complete pinch-off. Clearly, a much larger number of peaks is observed in our experiments, thereby demonstrating that resonant tunneling can not by itself account for the conductance oscillations.

We now discuss, following Ref. [20], how the charging energy associated with the transfer of single electrons modifies the mechanism of sequential res-

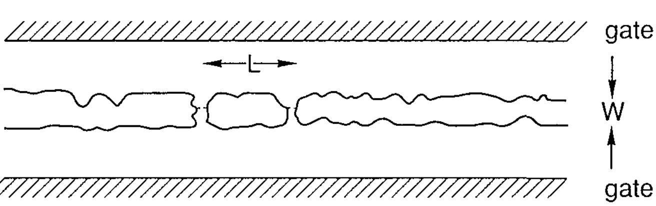

Figure 6: *Schematic diagram of a quantum conductance-limiting segment of quantum wire, of width W and length L, separated from the remainder of the wire by tunneling barriers (dotted lines). The short segment can be regarded as a quantum dot, with discrete energy levels.*

onant tunneling through zero-dimensional states. As shown schematically in Fig. 6, we model the conductance limiting segment by a "quantum dot", separated by tunneling barriers from the leads. The single-electron levels in this dot are denoted by E_p ($p = 1, 2, \ldots$), measured relative to the local conduction band bottom. These levels, which can each contain only one electron of given spin, depend on V_g and B, but are assumed to be independent of the number of electrons N in the dot [21]. The ground state energy of the dot contains a contribution from the occupied single-electron levels, and from the electrostatic energy $\int_0^{-Ne} \phi(Q)dQ$. Here $\phi = Q/C + \phi_{ext}$ is the potential difference between the dot and the leads due to a charge Q on the dot and due to an external potential ϕ_{ext} from the gate electrode and from the ionized donors in the heterostructure. The capacitance C of the dot to the leads is in our geometry dominated by the dot–gate capacitance. The ground state energy becomes:

$$U(N) = \sum_{p=1}^{N} E_p + \frac{(Ne)^2}{2C} - Ne\phi_{ext}. \tag{1}$$

Tunneling through the dot requires the transfer of a single electron with Fermi energy E_F from one of the leads into the dot. In the absence of electron-electron interactions, the resulting change in energy of the dot is simply the energy of the lowest unoccupied energy level, E_{N+1}. On resonance $E_{N+1} = E_F$, and tunneling can proceed without increasing the ground state energy of the system (leads plus dot). This picture changes, however, because of the effects of the charging energy. The condition for resonant tunneling now becomes [20]

$$U(N+1) - U(N) = E_F, \tag{2}$$

which is the general condition for equality of the electro-chemical potential $\Delta U/\Delta N$ in dot and leads. Combining Eqs. (1) and (2), we find (replacing N by $N-1$)

$$E_N^* \equiv E_N + \frac{e^2}{C}(N - \frac{1}{2}) = E_F + e\phi_{ext}. \tag{3}$$

The left hand side of Eq. (3) defines a renormalized energy level E_N^*. The renormalized level spacing relevant for transport $\Delta E^* = \Delta E + e^2/C$ is enhanced above the bare level spacing by the charging energy e^2/C. A comparison between the bare energy levels and the renormalized energy levels is shown in Fig. 7, from which it is clear that the latter are much more regularly spaced than the former.

Experimentally, the conductance peaks are spaced by $\delta V_g \approx 2$ mV. This is interpreted as the gate voltage change needed to induce a charge of one electron in the dot. The dot–gate capacitance is thus $e/\delta V_g \approx 10^{-16}$ F, which we assume to be approximately the same as the dot–lead capacitance C. Consequently, the renormalized level spacing $\Delta E^* \gtrsim e^2/C \approx 2$ meV, an energy which is consistent with the temperature dependence of the conductance, discussed in the previous section. The length L of the quantum dot may be estimated from the gate-voltage range between channel definition and pinch-

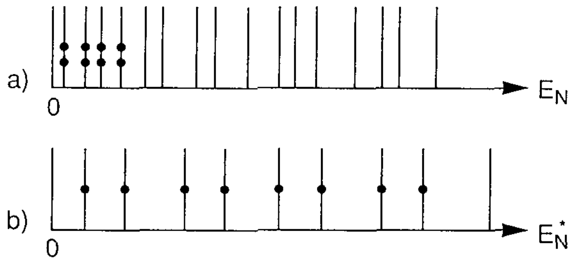

Figure 7: *Diagram of the bare energy levels (a) and the renormalized energy levels (b) for the case $e^2/C \approx 2\langle\Delta E\rangle$. The renormalized level spacing is much more regular than the bare level spacing. Note that the spin degeneracy of the bare levels is lifted by the charging energy.*

off: $\Delta V_g = en_s W_0 L/C \approx 1$ V, where W_0 and n_s are the width and electron concentration in the channel at definition. From the above estimate for C we find $L \approx 500$ nm. The width of the dot is estimated to be about $W \approx 40$ nm in the gate voltage range of interest. The bare level spacing for a dot of this area is $\Delta E \approx (mLW/\pi\hbar^2)^{-1}$, with $m = 0.065\, m_e$. Consequently, $\Delta E \approx 0.2$ meV, a full order of magnitude smaller than the elementary charging energy e^2/C, and two orders smaller than E_F [22]. This difference between the bare and renormalized level spacing explains how a large number of peaks in a trace of conductance as a function of gate voltage can be reconciled with the weak temperature dependence noted in the previous section. In addition, it accounts for the regularity of the conductance oscillations: since $e^2/C \gg \Delta E$, the renormalized level spacing ΔE^* is constant. Gate-voltage periodic peaks result from Eq. (3), provided that the 1D electron density varies linearly with V_g. The absence of peak splitting in a strong magnetic field is explained similarly: $\Delta E_{spin} = g\mu_B B \ll e^2/C$, so that the spin degeneracy is removed at $B = 0$ by the charging energy, see Fig. 7.

One would expect to observe Aharonov-Bohm magnetoconductance oscillations for a singly-connected quantum dot in a strong magnetic field. The reason is that such a dot is effectively doubly connected if the magnetic length l_m is much smaller than the dot radius R, due to the presence of circulating edge states. The Aharonov-Bohm (AB) effect in such a dot may be interpreted as resonant tunneling through zero-dimensional states [14,23]. In the absence of Coulomb interaction, the period ΔB of the AB oscillations for a hard-wall dot of area LW is $\Delta B = h/eLW$ (it may be larger for a soft-wall confining potential [14]). Such oscillations have indeed been observed in large quantum dots [14,15,16], but in our experiment, at high magnetic fields, no clear oscillations with the estimated $\Delta B \approx 0.2$ T are found. While random quantum-interference effects in the remainder of the wire and the effect of the magnetic field on the tunneling rates may be of importance, we here want to discuss the role of the electrostatic charging energy, which is dominant in small quantum dots. As pointed out in Ref. [20], each AB oscillation corresponds to an increase of the number of electrons in the dot by one. One can show from

Eq. (3) that the period of the magnetoconductance oscillations is enhanced due to charging effects, according to [20]

$$\Delta B^* = \Delta B \left(1 + \frac{e^2}{C\Delta E}\right), \tag{4}$$

where ΔE represents the energy level spacing of the circulating edge states. Sivan and Imry [23] estimate $\Delta E \approx \hbar\omega_c l_m/2R$ for a hard-wall dot. Under the conditions of our experiment, taking $2R = \sqrt{LW}$ and $B = 3$ T, we estimate $\Delta E \approx 0.5$ meV, so that $\Delta B^* \approx 5\Delta B \approx 1$ T. This will be further enhanced by the softness of the confining potential. The rapid AB oscillations in the magnetoresistance are therefore suppressed, notwithstanding the fact that oscillations can still be observed easily in a conductance trace as a function of gate voltage. The insensitivity of the period of the latter oscillations to a strong magnetic field is explained by the fact that the renormalized level spacing $\Delta E^* \approx e^2/C$ is approximately B-independent.

4. Conclusions

One major conclusion of our study is that Coulomb effects *regulate* resonant tunneling through a single conductance-limiting segment in a disordered quantum wire. The occurrence of periodic conductance oscillations as a function of gate voltage is thus explained. In particular, it is clarified how a large number of oscillations can be reconciled with a weak temperature dependence. The absence of regular magnetoconductance oscillations is interpreted as a signature of a more general phenomenon: the violation of the duality between density and magnetic field due to Coulomb interaction. It remains to clarify the rich variety of effects of the magnetic field on the amplitude of the oscillations, which the present study has revealed, as well as the curious doublet structure induced in one of the samples by a magnetic field. We surmise that these may be related to the influence of the magnetic field on the tunneling rates through the barriers forming the conductance-limiting segment. Also, it is necessary to consider the role of spin in this context in more detail.

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References

[1] B.L. Al'tshuler, Pis'ma Zh. Eksp. Teor. Fiz. **41**, 530 (1985) [JETP Lett. **41**, 648 (1985)]; P.A. Lee and A.D. Stone, Phys. Rev. Lett. **55**, 1622 (1985).

[2] B.J. van Wees, H. van Houten, C.W.J. Beenakker, J.G. Williamson, L.P. Kouwenhoven, D. van der Marel, and C.T. Foxon, Phys. Rev. Lett. **60**, 848 (1988); B.J. van Wees, L.P. Kouwenhoven, H. van Houten, C.W.J. Beenakker, J.E. Mooij, C.T. Foxon, and J.J. Harris, PRB38, 3625 (1988); D.A. Wharam, T.J. Thornton, R. Newbury, M. Pepper, H. Ahmed, J.E.F. Frost, D.G. Hasko, D.C. Peacock, D.A. Ritchie, and G.A.C. Jones, J. Phys. C **21**, L209 (1988).

[3] K. von Klitzing, G. Dorda, and M. Pepper, Phys. Rev. Lett. **45**, 494 (1980).

[4] J.H.F. Scott-Thomas, S.B. Field, M.A. Kastner, H.I. Smith, and D.A. Antoniadis, Phys. Rev. Lett. **62**, 583 (1989).

[5] U. Meirav, M.A. Kastner, M. Heiblum, and S.J. Wind, Phys. Rev. B **40**, 5871 (1989).

[6] H. van Houten and C.W.J. Beenakker, Phys. Rev. Lett. **63**, 1893 (1989).

[7] K.K. Likharev, IBM J. Res. Dev. **32**, 144 (1988), and references therein.

[8] N.S. Wingreen and P.A. Lee, presented at the *NATO Adv. Study Inst. on Quantum Coherence in Mesoscopic systems* (Les Arcs, 1990).

[9] L.I. Glazman and R.I. Shekhter, J. Phys. Condens. Matter **1**, 5811 (1989).

[10] D.V. Averin and A.N. Korotkov, Zh. Eksp. Teor. Fiz. [Sov. Phys. JETP] (to be published); A.N. Korotkov, D.V. Averin, and K.K. Likharev, in *Proc. 19th Int. Conf. on Low Temperature Physics* (Physica B, to be published).

[11] M. Amman, K. Mullen, and E. Ben-Jacob, J. Appl. Phys. **65**, 339 (1989).

[12] S.B. Field, M.A. Kastner, U. Meirav, J.H.F. Scott-Thomas, D.A. Antoniadis, H.I. Smith, and S.J. Wind, preprint.

[13] U. Meirav, M.A. Kastner, and S.J. Wind, preprint.

[14] B.J. van Wees, L.P. Kouwenhoven, C.J.P.M. Harmans, J.G. Williamson, C.E. Timmering, M.E.I. Broekaart, C.T. Foxon, and J.J. Harris, Phys. Rev. Lett. **62**, 2523 (1989).

[15] R.J. Brown, C.G. Smith, M. Pepper, M.J. Kelly, R. Newbury, H. Ahmed, D.G. Hasko, J.E.F. Frost, D.C. Peacock, D.A. Ritchie, and G.A.C. Jones, J. Phys. Condens. Matter **1**, 6291 (1989).

[16] D.A. Wharam, M. Pepper, R. Newbury, H. Ahmed, D.G. Hasko, D.C. Peacock, J.E.F. Frost, D.A. Ritchie, and G.A.C. Jones, J. Phys. Condens. Matter **1**, 3369 (1989).

[17] T.J. Thornton, M. Pepper, H. Ahmed, D. Andrews, and G.J. Davies, Phys. Rev. Lett. **56**, 1198 (1986); H.Z. Zheng, H.P. Wei, D.C. Tsui, and G. Weimann, Phys. Rev. B **34**, 5635 (1986).

[18] S.E. Laux, D.J. Frank , and F. Stern, Surf. Sci. **196**, 101 (1988).

[19] C.W.J. Beenakker and H. van Houten, *Quantum Transport in Semiconductor Nanostructures*, in *Solid State Physics*, H. Ehrenreich and D. Turnbull, eds., (Academic Press, New York), to be published.

[20] C.W.J. Beenakker, H. van Houten, and A.A.M. Staring, submitted to Phys. Rev. Lett.

[21] A. Kumar, S.E. Laux, and F. Stern, preprint.

[22] For an elongated dot it is more appropriate to assume that only one transverse mode is present, in which case $\Delta E = (h/4L)(E_{\mathrm{F}}/2\,m)^{\frac{1}{2}}$, assuming hard-wall boundary conditions. This leads to the same estimate for ΔE, however.

[23] U. Sivan and Y. Imry, Phys. Rev. Lett. **61**, 1001 (1988).