Substitution of these values in the equations of condition gave the residuals shown in Table 21.

TABLE 21

Residuals of least-squares solution

| Date | | | res. in α | res. in δ |
|---|---|---|---|---|
| | | | ″ | ″ |
| 1946 | Aug. | 24 | +2.02 | + .73 |
| 1947 | Jan. | 17 | −1.27 | −2.21 |
| | Feb. | 21 | +2.33 | −1.66 |
| | Mar. | 30 | +1.66 | + .17 |
| | April | 20 | +1.08 | + .97 |
| | May | 28 | +1.83 | +1.40 |
| | June | 18 | + .58 | − .02 |
| | ,, | 28 | − .02 | − .18 |
| | July | 17 | −1.20 | − .01 |
| | Aug. | 12 | −1.78 | −1.79 |
| | Sept. | 2 | −1.39 | +2.17 |
| | Oct. | 6.1 | −1.99 | −1.42 |
| | Nov. | 5.2 | −2.32 | −2.93 |
| | Dec. | 9.0 | + .41 | −3.21 |
| 1948 | May | 9.4 | −4.06 | −3.59 |
| | July | 26.4 | +2.46 | −3.22 |
| | Sept. | 5.2 | − .49 | − .06 |
| | Oct. | 3.1 | −3.25 | +2.44 |

The mean error of a normal place of unit weight is $\pm 2''.63$. The corrections to the preliminary elements, with their mean errors are:

| | | |
|---|---|---|
| $d\Omega'$ | + °.00542 | ± °.00057 |
| $di'$ | + °.00481 | °.00043 |
| $d\omega'$ | + °.03862 | °.00109 |
| $de$ | + .000 7942 | .000 0207 |
| $dq$ | + .000 3579 | .000 0108 |
| $dT$ | + .06474 | .00198 |

and the final equatorial elements are:

| | | |
|---|---|---|
| $T$ | 1946 Oct. 26.76997 | |
| $\Omega'$ | 254°.61470 | 1950 |
| $i'$ | 47.26070 | 1950 |
| $\omega'$ | 293.19056 | 1950 |
| $e$ | 1.000 7942 | |
| $q$ | 1.136 2217 | |

with the osculation date 1946, Dec. 16.

The $P'$ and $Q'$ corresponding with these elements are:

| | | | |
|---|---|---|---|
| $P'_x$ | − .705 9479 | $Q'_x$ | + .013 8021 |
| $P'_y$ | − .214 1715 | $Q'_y$ | − .957 1634 |
| $P'_z$ | − .675 1059 | $Q'_z$ | + .289 2192 |

The original value of $1/a$ may be found in the note by H. A. PELS-KLUYVER (1960).

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# THE ORIGINAL VALUES OF 1/*a* FOR 17 COMETARY ORBITS

BY E. H. BILO AND I. VAN HOUTEN—GROENEVELD

The article describes the determination of the original values of the reciprocal semi-major axes for 17 comets for which sufficiently accurate orbits are known. The calculations were made with the method discussed in the first paper of this Bulletin. The final results are collected in Table 2.

The results of computations of original values of $1/a$ for comets in nearly parabolic orbits are presented. The selection of the comets was made according to the following conditions: The orbit should be nearly parabolic. The mean error in $1/a_{osc}$ should be smaller than ±0.000 100, or, if no mean error was given, the definitive orbit should have been determined by an arc of at least half a year. The theory has been described in the paper by BILO and VAN DE HULST (1960, which will be referred to here as Paper I). The fundamental equations used in the computations and a discussion of the errors are also given in that paper. The present article gives the results obtained and some remarks concerning the practical computational work. The computations were started by one of us (E.H.B.) in 1952 and continued by the second author. Eleven comets were investigated by the first author, six (1863 I, 1882 I, 1898 VIII, 1912 II, 1932 VI, and 1944 IV) by the second author, who also computed additional perturbations of Venus and Earth, and Venus, Earth, Mars for the comets 1917 III, 1919 V, 1930 IV and 1937 VI; she also computed the barycentre reduction in the manner discussed in Paper I for all 17 comets.

The method used was as follows. We limited ourselves to the computation of the reciprocal semi-major axis, $1/a$. We followed the comet as if it would move in an undisturbed parabolic orbit. Taking this parabolic orbit we determined, for each time interval, the perturbation $d(1/a)$ for each of the planets. The method requires only one numerical integration. Generally, the integration of the perturbations was extended over about 10 years, or till the heliocentric distance was about 20—26 A.U. Regardless of whether or not such perturbations had been used for the definitive orbit, we always computed the perturbations by Jupiter, Saturn, Uranus and Neptune for the extension into the past. If the perihelion distance $q$ was smaller than 1 A.U., the perturbations by the Earth and Venus were included. Moreover, the perturbations by Venus, Earth and Mars were computed for some other comets.

The relevant formulae are

$$M(v) = (t - T)\, q^{-3/2} \tag{1}$$

$$\mathbf{r} = q\mathbf{P}\left(1 - \tan^2\frac{v}{2}\right) + 2q\,\mathbf{Q}\tan\frac{v}{2}, \tag{2}$$

$$\mathbf{V} = \frac{d\mathbf{r}}{dt} = \frac{2k}{\sqrt{p}}\cos\frac{v}{2}\left(-\mathbf{P}\sin\frac{v}{2} + \mathbf{Q}\cos\frac{v}{2}\right), \tag{3}$$

where $p = 2q$,

$$-\frac{d(1/a)}{dt} = +2\mathbf{V}\mathbf{u} = \sqrt{\frac{2}{q}}\,k\left\{\mathbf{Q}(1+\cos v) - \mathbf{P}\sin v\right\}\sum_i m_i\left(\frac{\mathbf{r}_i - \mathbf{r}}{\rho_i^3} - \frac{\mathbf{r}_i}{r_i^3}\right). \tag{4}$$

The calculations were carried out in rectangular equatorial co-ordinates for the equinox 1950.0, except for 1892 II, which was computed in rectangular ecliptical co-ordinates, equinox 1900.0.

It is convenient to use $10^{-6}$ A.U./day$^2$ as unit for the acceleration by the perturbing force and $10^{-6}$ (A.U.)$^{-1}$ for the unit of $1/a$. For this reason we introduce

$$F_i = 10^6\, w\sqrt{\frac{2}{q}}\,k\,m_i$$

and

$$X_{K,i} = F_i\left(\frac{x_i - x}{\rho_i^3}\right) \qquad X_{P,i} = -F_i\frac{x_i}{r_i^3} \tag{5}$$

and similar expressions for $Y_{K,i}$, $Z_{K,i}$, $Y_{P,i}$ and $Z_{P,i}$, where $w$ is the integration interval. Further,

$$\begin{aligned} \xi &= (X_K + X_P)_{\text{Jup}} + (X_K + X_P)_{\text{Sat}} + \ldots \\ \eta &= (Y_K + Y_P)_{\text{Jup}} + (Y_K + Y_P)_{\text{Sat}} + \ldots \\ \zeta &= (Z_K + Z_P)_{\text{Jup}} + (Z_K + Z_P)_{\text{Sat}} + \ldots \end{aligned} \tag{6}$$

and, finally,

$$\begin{aligned} \alpha &= Q_x(1+\cos v) - P_x \sin v \\ \beta &= Q_y(1+\cos v) - P_y \sin v \\ \gamma &= Q_z(1+\cos v) - P_z \sin v. \end{aligned} \tag{7}$$

We then have at every step of time interval $w$ the perturbations of $1/a$:

$$-w\frac{d(1/a)}{dt} = \alpha\xi + \beta\eta + \gamma\zeta. \tag{8}$$

The quantities $F_i$ may be computed before starting the numerical integration. The values of $v$ and the rectangular components of **P** and **Q** follow from well-known formulae (STRACKE 1929). The components of $\mathbf{r}_i$ are taken from "Planetary Co-ordinates" for the years 1800—1960. The quantities $X_{P,i}$, $Y_{P,i}$, and $Z_{P,i}$ are

$$\frac{w}{10\,w_0^2\,k}\sqrt{\frac{2}{q}}$$

times the "solar attractions" given in "Planetary Co-ordinates", where $w_0$ is the time interval used in these tables.

The computation of $\frac{d(1/a)}{dt}$ should always start two or three steps beyond the date of osculation. The integration step $w$ used depends on the distance between the comet and the different disturbing planets and also on the velocities of the planets. The perturbations of the inner planets require an interval $w$ of 10 to 20 days; for Jupiter and Saturn we need $w = 40$ days up to $|t - T| = 2$ years; but around the osculation it is often better to use $w = 20$. For values of $|t - T|$ between 2 and 5 years we have used $w = 80^d$, and for still larger values of $|t - T|$, $w = 160^d$. For the perturbations of Uranus and Neptune we found it practical to use $w = 80^d$ for about 5 years and then $w = 160$ for $|t - T| > 5$ years; but here we computed only every second step and interpolated the intermediate ones. If the comet came close to one of these planets, the interpolation was not sufficiently accurate and the intermediate steps were computed exactly. The scheme of calculation may be illustrated by the following example of the computations of $\frac{d(1/a)}{dt}$ for comet 1930 IV for a step $w = 80^d$ centred on the epoch 1927 July 19.0. Index one corresponds to Jupiter, two to Saturn.

Scheme of computation of $\frac{d(1/a)}{dt}$ for comet 1930 IV

| | |
|---|---|
| $q$ | 2.078673 |
| $q^{-3/2}$ | 0.3336727 |
| $t-T$ | $-$1004.1212 |
| $M$ | $-$ 335.048 |
| $\tan\frac{v}{2}$ | $-$ 1.87570 |
| $1-\tan^2\frac{v}{2}$ | $-$ 2.51825 |
| $1+\tan^2\frac{v}{2}$ | + 4.51825 |
| $1+\cos v$ | + 0.44265 |
| $\sin v$ | $-$ 0.83028 |
| $r$ | 9.39197 |
| $r^2$ | 88.2091 |
| $x$ | + 3.24096 |
| $y$ | + 3.45101 |
| $z$ | $-$ 8.11146 |

| | | | |
|---|---|---|---|
| $F_1$ | 1288.846 | $F_2$ | 385.671 |
| $x_1-x$ | + 1.6871 | $x_2-x$ | $-$ 7.2524 |
| $y_1-y$ | $-$ 3.9756 | $y_2-y$ | $-$ 11.9685 |
| $z_1-z$ | + 7.7659 | $z_2-z$ | + 4.7633 |
| $\rho_1^2$ | 78.961 | $\rho_2^2$ | 218.531 |
| $10^3\rho_1^{-3}$ | 1.4256 | $10^3\rho_2^{-3}$ | 0.3095 |
| $F_1\rho_1^{-3}$ | 1.8374 | $F_2\rho_2^{-3}$ | 0.1194 |
| $X_{K,\,\mathrm{Jup}}$ | + 3.10 | $X_{K,\,\mathrm{Sat}}$ | $-$ 0.87 |
| $Y_{K,\,\mathrm{Jup}}$ | $-$ 7.30 | $Y_{K,\,\mathrm{Sat}}$ | $-$ 1.43 |
| $Z_{K,\,\mathrm{Jup}}$ | +14.27 | $Z_{K,\,\mathrm{Sat}}$ | + 0.57 |
| $X_{P,\,\mathrm{Jup}}$ | $-$51.80 | $X_{P,\,\mathrm{Sat}}$ | + 1.55 |
| $Y_{P,\,\mathrm{Jup}}$ | + 5.51 | $Y_{P,\,\mathrm{Sat}}$ | + 3.29 |
| $Z_{P,\,\mathrm{Jup}}$ | + 3.63 | $Z_{P,\,\mathrm{Sat}}$ | + 1.29 |

| | |
|---|---|
| $\xi$ | $-$48.02 |
| $\eta$ | + 0.07 |
| $\zeta$ | +19.76 |
| $\alpha$ | $-$ 0.4630 |
| $\beta$ | + 0.0890 |
| $\gamma$ | + 0.8144 |
| $-w\frac{d(1/a)}{dt}$ | +38.33 |

Having computed $\frac{d(1/a)}{dt}$ for every step up to $|t-T|=10$ years, we performed the numerical integration of the perturbations of all disturbing planets for this time interval. When perturbations of Venus, Earth and Mars were computed, the numerical integration was extended for these planets to only $\frac{1}{2}$ to 1 year from the osculation date.

We next determine the term

$c(t)=\frac{1}{a'(t)}-\frac{1}{a(t)}$, the reduction to the barycentre.

Paper I contains an extensive discussion of the reduction to the barycentre.

We prefer to discuss the inner and the outer planets separately. We define:

$t_3$ = the date of osculation;

$t_2$ = the time to which the perturbations of the inner planets are computed, starting from $t_3$. In some cases $t_2=t_3$ (no perturbations computed for inner planets);

$t_1$ = the time to which the perturbations of the outer planets are computed.

If we did not calculate the perturbations of the inner planets, we determined $c_b(t_3)$ right at the start for the four inner planets with the formula

$$c_b(t)=\frac{2}{r}\left\{\frac{\mathbf{r}\,\mathbf{r}_b}{r^2}+\frac{2\,\mathbf{V}\,\mathbf{V}_b}{V^2}+\sum_{i=1}^{4}m_i\right\}, \tag{9}$$

where $\mathbf{r}$ and $\mathbf{V}$ are the parabolic position and velocity of the comet, and the barycentre is defined by

$$\mathbf{r}_b=\frac{1}{1+\sum_{i=1}^{4}m_i}\sum_{i=1}^{4}m_i\mathbf{r}_i$$

and a corresponding expression for $\mathbf{V}_b$.

For the barycentre co-ordinates of all comets later than 1924 we followed the rule as applied in "Planetary Co-ordinates 1960—1980", in which Mercury was omitted. For the earlier comets we also omitted the influence of Mars. The error arising in $\mathbf{r}_b$ caused by the omission of Mars is less than $5\times10^{-7}$. Whenever we calculated the perturbations of Venus, Earth, Mars, or Venus and Earth for a certain time interval, we computed $c_b(t_2)$ with formula (9) at the end $t_2$ of this interval for all four inner planets together.

The second group is formed by the outer planets. We computed for all 17 comets the perturbations by Jupiter and Saturn in one numerical integration and those by Uranus and Neptune in another integration, but always up to the same time $t_1$. For the co-ordinates and velocities of the centre of mass we used the values published in *Astronomical Papers Am. Eph.* **13**, part 4, by G. M. Clemence, which are defined by

$$\mathbf{r}_w=\frac{\sum_{i=5}^{9}m_i\mathbf{r}_i}{1+\sum_{i=1}^{9}m_i}.$$

The velocities are determined by numerical differentiation.

There are two ways to determine the reduction to the barycentre: (a) with the formulae (13), (14) and (15) of Paper I and (b) directly with formula (19) or (24) of Paper I. It is important to realize that in method (a) we need more decimal places for the position and velocity of the comet and of the centre of gravity. For the position of the comet we make use of Table 17 in Bauschinger-Stracke (1934), using the eccentricity on the date $t_1$. The velocity is found by well-known formulae (Bauschinger 1906, page 568). Formula (14) of Paper I thus becomes

$$\frac{1}{a'(t)}=\frac{2}{|\mathbf{r}-\mathbf{r}_w|}-\frac{|\mathbf{V}-\mathbf{V}_w|^2}{k^2(1+\sum_{i=5}^{9}m_i)}. \tag{10}$$

For method (b) we can work either with the nearly parabolic co-ordinates of the comet, as determined by (a) and then use the formula

$$c_w(t_1) = \frac{2\mathbf{rr}_w}{r^3} + \frac{1}{k^2}\left(2\mathbf{VV}_w + V^2 \sum_{i=5}^{9} m_i\right); \qquad (11)$$

or we can determine the parabolic co-ordinates and velocities of the comet and then apply formula (24) of Paper I:

$$c_w(t_1) = \frac{2}{r}\left(\frac{2\mathbf{rr}_w}{r^2} + \frac{2\mathbf{VV}_w}{V^2} + \sum_{i=5}^{9} m_i\right). \qquad (12)$$

The reduction to the barycentre never exceeds 1000 in the 7th decimal; it is therefore sufficient to calculate by method (b) with five significant figures throughout. Our final determination of the reciprocal value of the original semi-major axis thus becomes

$$\frac{1}{a(t_3)} + \int_{t_3}^{t_2} d(1/a)_{\text{(inner pl.)}} + c_b(t_2) + \int_{t_3}^{t_1} d(1/a)_{\text{(outer pl.)}} + c_w(t_1) = \frac{1}{a'}. \qquad (13)$$

In cases where no perturbations of the inner planets are computed, $t_3$ is identical with $t_2$.

The essential data for the 17 comets are given in Tables 1 and 2. Table 1 supplies fundamental values

TABLE 1

Data on definitive orbits

| comet | publication of definitive orbit[1]) | first and last observation | perturbations applied | date of osculation | $e_{osc}$ mean error | $q$ mean error |
|---|---|---|---|---|---|---|
| 1863 I Bruhns | A. v. FLOTOW 1902, *A.N.* **160**, 235. | 1862 Dec 1– 1863 Mar 12 | V E M J S | 1863 Feb 2.0 | 1.0000 470 ± 475 | 0.7947 631 ± 22 |
| 1882 I Wells | E. v. REBEUR-PASCHWITZ 1887, *A.N.* **117**, 285. | 1882 Mar 19– 1882 Aug 16 | small plan. pert. applied | [2]) | 0.9999 945 ± 12 | 0.0607 628 ± 33 |
| 1892 II Denning | L. STEINER 1898, *A.N.* **145**, 254. | 1892 Mar 19– 1893 Jan 12 | Me V E J S | 1892 May 5.0 | 1.0003 45 ± 64 | 1.9706 964 ± 359 |
| 1898 VIII Chase | M. WASNETZOFF 1914, *A.N.* **197**, 128 | 1898 Dec 2– 1899 May 9 | J S | 1898 Dec 10.0 | 0.9993 55 ± 1 86 | 2.2846 67 ± 1 18 |
| 1905 IV Kopff | G. PELS 1960, *B.A.N.* **15**, 129 (No. 499). | 1904 Jan 10– 1907 Jul 3 | J S | 1905 Sep 11.5 | 1.0014 806 ± 263 | 3.3398 867 ± 91 |
| 1910 III Metcalf | M. VIARO 1938, *Mem. di Mat. e di Sc. Fis. e Nat. della Soc. Ital. delle Sc.* Ser 3, **24**, 63. | 1910 Aug 10– 1911 Jun 23 | J | 1910 Sep 15.5 | 0.9998 121 ± 9 | 1.9480 09 ± 39 |
| 1912 II Gale | G. PEISINO and E. DE CARO 1931, *Publ. Triest* **2**, 128 (No. 6). | 1912 Sep 10– 1913 May 26 | V E J S | 1912 Sep 15.0 | 0.9995 145 — | 0.7161 144 — |
| 1914 III Neujmin | J. SVÄRDSON 1917, *Stockholm Iaktt.* **10**, No. 6, p. 16. | 1914 Jul 1– 1914 Dec 22 | J | 1914 Aug 30.5 | 1.0036 72 ± 2 96 | 3.7471 31 ± 2 43 |
| 1915 II Mellish | L. ROSENBAUM 1917, *Stockholm Iaktt.* **10**, No. 5, p. 18 | 1915 Feb 13– 1916 Jan 5 | E J | 1915 Feb 10.0 | 1.0002 35 ± 61 | 1.0053 38 ± 6 |
| 1917 III Wolf | J. C. DU PUI 1932, *Utrecht Dissertation* p. 58. | 1916 Apr 3– 1918 Jan 29 | Me V E M J S | 1917 Mar 6.0 | 0.9994 952 ± 104 | 1.6864 526 ± 22 |
| 1919 V Metcalf-Borrelly | A. PRZYBYLSKI 1939, *Acta Astr.* Ser. a **4**, 59. | 1919 Aug 24– 1920 Feb 3 | V E J S | 1919 Sep 18.5 | 1.0002 151 ± 629 | 1.1152 719 ± 207 |
| 1925 VII Shajn-Comas Solá | V. V. SAKK and D. K. KULIKOV 1951, *Bull. Inst. Theor. Astr.* **4**, 448. | 1925 Mar 22– 1927 Mar 4 | Me V E M J S | 1925 Mar 1.0 | 1.0019 407 ± 764 | 4.1806 902 ± 543 |
| 1927 IV Stearns | Z. SHEN and I. IMAI 1937, *V.J.S.* **72**, 364. | 1927 Mar 10– 1931 Feb | J S U | 1927 May 23.0 | 0.9980 650 — | 3.6837 30 — |
| 1930 IV Beyer | G. PELS 1947, *B.A.N.* **10**, 237 (No. 378). | 1929 Oct 9– 1931 Aug 13 | V E J S U N | 1930 Sep 24.0 | 1.0003 791 ± 264 | 2.0786 73 ± 7 |
| 1932 VI Geddes | G. VAN BIESBROECK 1937, *Publ. Yerkes* **8**, pt 3, p. 13. | 1931 Aug 14– 1934 Jul 19 | E M J S U N | 1932 Oct 23.0 | 1.0013 760 ± 22 | 2.3135 658 ± 70 |
| 1937 IV Whipple | G. CHIŞ 1950, *Publ. Cluj* No. 7, p. 4. | 1937 Feb 7– 1937 Oct 28 | Me V E M J S | 1937 Jun 19.0 | 1.0001 601 ± 345 | 1.7337 952 ± 109 |
| 1944 IV van Gent | A. PRZYBYLSKI 1956, *Acta Astr.* **6**, 125. | 1944 Jun 1– 1945 Aug 11 | V E M J S U N | 1944 Dec 16.0 | 1.0020 854 ± 354 | 2.2259 363 ± 194 |

[1]) The page number refers to the page on which the elements are given.
[2]) No osculation date published; the perihelion passage 1882 Jun 11 was adopted as date of osculation.

of the definitive orbit. The column headings are: name of the comet, references to the publication of the definitive orbit, the dates of the first and the last observation used, perturbations applied in the definitive orbit, date of osculation, and the eccentricity $e$ and the perihelion distance $q$ with their mean errors.

The time is always expressed in Mean Time before 1925 Jan. 1 and in Universal Time afterwards.

Table 2 gives the results of our computations in the order of the terms of formula (13). The second column gives $1/a_{osc}$ with the mean error, which in practice depends only on the mean error in $e$. The third column shows the time $t_2$ up to which the perturbations of the inner planets were computed. If these perturbations were omitted, the date of osculation $t_3$ is given. The value of the "inner perturbation" is followed by capitals indicating those inner planets whose perturbations were applied. For the "outer perturbations", the perturbations of Jupiter, Saturn, Uranus and Neptune were computed throughout. The 5th column gives the time $t_1$, the end of the computation of the outer perturbations.

For all but two of the comets the value of the reciprocal semi-major axis changes in the direction of a more elliptical value. The exceptions are comet 1898 VIII, for which $1/a_{osc} = +0.000282$ and $1/a_{orig} = +0.000010$, and comet 1917 III, for which $1/a_{osc} = +0.000299$ and $1/a_{orig} = +0.000021$. Comet 1914 III remains hyperbolic, but the mean error of $1/a$ is larger than the value itself and it is also the largest of our series.

Table 3 shows a comparison between the $1/a'$ determined by other authors and our results. The column headings are: comet, publication, applied perturbations, time $t_1$ and the original reciprocal semi-major axis $1/a'$; the next columns show our value for $1/a'$ and the corresponding time $t_{1,\,B,\,vH}$. In some cases we redetermined the value of $1/a'$ for a

TABLE 2

Different terms for the determination of the original value of $1/a$ (in units of the 6th decimal)

| comet | $1/a_{osc}$ | $t_2$ | $\sum \frac{d(1/a)}{dt}$ inner planets | $c_b(t_2)$ | $t_1$ | $\sum \frac{d(1/a)}{dt}$ outer planets | | $c_w(t_1)$ | $1/a'$ |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | J + S | U + N | | |
| 1863 I | − 59 ± 60 | 1861 Jan 24.5 | − 2 VE | + 2 | 1852 Jul 10.5 | +507 | +3 | + 77 | + 528 |
| 1882 I | + 90 ± 20 | 1878 Dec 1.5 | +10 VE | + 1 | 1872 Jun 15.5 | +101 | +2 | − 61 | + 144 |
| 1892 II | −175 ± 32 | 1892 May 5.0 | — | − 2 | 1882 Oct 25.0 | +950 | +2 | + 78 | + 853 |
| 1898 VIII | +282 ± 81 | 1897 Aug 23.5 | + 7 VE | + 7 | 1886 Oct 20.5 | −170 | +1 | −119 | + 10 |
| 1905 IV | −443 ± 8 | 1905 Sep 11.5 | — | + 2 | 1896 Mar 21.5 | +283 | +1 | +203 | + 45 |
| 1910 III | + 96 ± 5 | 1910 Sep 15.5 | — | +10 | 1901 May 15.5 | +378 | +2 | − 12 | + 473 |
| 1912 II | +678 — | 1911 Aug 11.5 | +12 VE | + 2 | 1901 Sep 12.5 | +518 | −0 | +147 | +1356 |
| 1914 III | −980 ± 79 | 1914 Aug 30.5 | — | + 7 | 1906 May 29.5 | +910 | +0 | − 3 | − 66 |
| 1915 II | −234 ± 61 | 1915 Feb 10.0 | — | + 2 | 1908 Feb 28.5 | +305 | +1 | + 66 | + 140 |
| 1917 III | +299 ± 6 | 1916 Feb 26.5 | + 9 VE | − 1 | 1906 Dec 15.5 | −199 | −2 | − 86 | + 21 |
| 1919 V | −193 ± 56 | 1919 May 1.5 | − 3 VE | + 7 | 1910 May 8.5 | − 48 | −3 | +258 | + 18 |
| 1925 VII | −464 ± 18 | 1925 Mar 1.0 | — | + 2 | 1915 Oct 29.5 | +221 | +2 | +312 | + 73 |
| 1927 IV | +525 — | 1927 May 23.0 | — | + 4 | 1918 Feb 15.5 | −187 | +3 | +308 | + 653 |
| 1930 IV | −182 ± 13 | 1929 Mar 10.0 | − 5 VEM | + 2 | 1920 Jul 14.5 | +666 | +3 | + 41 | + 525 |
| 1932 VI | −595 ± 1 | 1932 Apr 3.0 | + 2 VEM | + 8 | 1920 Dec 21.5 | +331 | +1 | +325 | + 72 |
| 1937 IV | − 92 ± 20 | 1937 Jan 7.0 | +16 VEM | + 0 | 1928 Apr 24.0 | +189 | +3 | − 71 | + 45 |
| 1944 IV | −937 ± 16 | 1943 Nov 12.0 | − 5 VEM | + 9 | 1932 Nov 29.0 | +731 | +3 | +199 | + 0 |

TABLE 3

Comparison of results of $1/a'$ from different authors (in units of 6th decimal)

| comet | publication | perturbations applied | $t_1$ | $1/a'$ | $1/a'$ B, vH | $t_1$ B, vH |
|---|---|---|---|---|---|---|
| 1863 I | M. G. FAYET 1910, *Paris Ann. Mém.* **26**, A 89 | J | | +532 | +528 | 1852 Jul 10.5 |
| 1882 I | M. G. FAYET 1910, *Paris Ann. Mém.* **26**, A 89 | J | | + 89 | +144 | 1872 Jun 15.5 |
| 1892 II | M. G. FAYET 1910, *Paris Ann. Mém.* **26**, A 90 | J | | +823 | +853 | 1882 Oct 25.0 |
| 1898 VIII | M. G. FAYET 1910, *Paris Ann. Mém.* **26**, A 90 | J | | −115 | + 10 | 1886 Oct 20.5 |
| 1914 III | I. V. GALIBINA 1958, *Bull. Inst. Theor. Astr.* **6**, 657 | JSUN<br>JSUN | 1905 Sep 15.9<br>1907 Aug 1.5 | − 88<br>− 84 | − 66 | 1906 May 29.5 |
| 1919 V | A. PRZYBYLSKI 1957, *Acta Astr.* **7**, 251 | VEMJSUN<br>JS | 1899 Mar 6.5<br>1899 Mar 6.5 | + 16<br>+ 9 | + 18 | 1910 May 8.5 |
| 1925 VII | I. V. GALIBINA 1953, *Bull. Inst. Theor. Astr.* **5**, 418 | MeVEMJSU | 1916 Sep 13 | + 69 | + 73 | 1915 Oct 29.5 |
| | M. YA. SHMAKOVA 1953, *Bull. Inst. Theor. Astr.* **5**, 427 | MeVEMJSUN | 1919 Feb 11 | + 56 | + 71 | 1919 Feb 10.5 |
| | O. N. BARTENOVA 1955, *Bull. Inst. Theor. Astr.* **6**, 254 | JSUN | 1917 Jun 9 | + 68 | + 73 | 1917 Jun 9.5 |
| 1930 IV | M. A. DIRIKIS 1954, *Soviet Astr. Journal* **31**, 464 | E JS | 1921 Dec 16.5 | +515 | +527 | 1921 Dec 16.5 |
| | I. V. GALIBINA 1958, *Bull. Inst. Theor. Astr.* **6**, 663 | JSUN | 1920 Dec 24 | +540 | | |
| 1932 VI | G. VAN BIESBROECK 1937, *Publ. Yerkes* **8**, pt 3, p. 14 | JS | 1927 Dec 6 | + 44[1]) | + 68 | 1927 Dec 6 |

[1]) This value was published by SINDING (1948); VAN BIESBROECK did not give this original value.

time $t_1$ close to or identical with the time $t_1$ used by the other author.

Some remarks may be made concerning Table 3. The original values of $1/a$ given for comets 1863 I, 1882 I, 1892 II and 1898 VIII by FAYET were determined by an approximate method, and only the perturbations by Jupiter were taken into account. A good agreement is therefore not to be expected. Furthermore, the elements of comet 1863 I used by FAYET are different from ours. We used the hyperbolic elements No. I determined by FLOTOW (see Table 1), although because of the relatively large errors, FLOTOW himself considered that parabolic elements represented the observations sufficiently well. For comet 1882 I, no date of osculation was published; we adopted the time of perihelion passage as date of osculation. For comet 1898 VIII, FAYET used provisional parabolic elements. The difference between GALIBINA's and our results for comet 1914 III are larger than expected. It is possible that GALIBINA did not apply the barycentric correction for the inner planets. This correction would make the difference 3 units smaller. The agreement for comets 1919 V and 1925 VII is relatively good, except for the value published by SHMAKOVA. The comparisons for comets 1930 IV and 1932 VI are given in more detail in Tables 4 and 5, respectively.

We tried to reproduce the value of $1/a'$ found by DIRIKIS for comet 1930 IV as follows. The perturbations by the inner planets are taken only up to the time $t_2$ = 1929 Mar. 10.0, those by the outer planets up to the same date $t_1$ as DIRIKIS, 1921 Dec. 16.5. But we now use for the position of the centre of gravity only the Sun and those planets whose perturbations are applied:

$$\mathbf{r}_b' = \frac{m_E \mathbf{r}_E}{1 + m_E} \qquad (14)$$

and

$$\mathbf{r}_w' = \frac{1}{1 + m_J + m_S} (m_J \mathbf{r}_J + m_S \mathbf{r}_S) , \qquad (15)$$

while corresponding expressions are used for the velocity. Table 4 gives the results for comet 1930 IV for the time 1921 Dec. 16.5. The first line shows the values computed by our method, in which all planets were taken into account. In the second line only perturbations of Earth, Jupiter and Saturn were used; for the barycentre all planets were taken. The third line shows the result if the incorrect method, expressed by formulae (14) and (15), is applied. We may assume that DIRIKIS, who found virtually the same value, (+ 515, fourth line), used this method. The error made in this way is larger than that made by simply omitting the Uranus and Neptune perturbations, as is shown by the second line.

For comet 1932 VI, VAN BIESBROECK derived the

TABLE 4

Comparison of different methods; comet 1930 IV for 1921 Dec 16.5 (in units of 6th decimal)

| $1/a_{osc}$ | $c_b(t_3)$ | $\sum \frac{d(1/a)}{dt}$ inner planets | $c_b(t_2)$ | $\sum \frac{d(1/a)}{dt}$ outer planets | $c_w(t_1)$ | $1/a'$ |
|---|---|---|---|---|---|---|
| −182 | | −5 VEM | +2 MeVEM | +781 JSUN | −69 JSUNP | +527 |
| −182 | −1 MeVM | −3 E | +1 E | +779 JS | −69 JSUNP | +524 |
| −182 | | −3 E | +1 E | +779 JS | −78 JS | +516 |
| DIRIKIS | | E | | JS | | +515 |

original value of $1/a$ for 1927 Dec. 6.0, considering only the perturbations of Jupiter and Saturn. Following the plan used for Table 4, Table 5 gives in the first and second lines the data computed by our method, where $t_2 = 1932$ Apr. 3.0 for the perturbations by the inner planets. The third line gives the data as determined by means of formula (15). Here the term

$$\sum \frac{d(1/a)}{dt}_{JS} = +650 \times 10^{-6}$$ is derived from the data given by VAN BIESBROECK as difference $\frac{1}{a(t_1)} - \frac{1}{a(t_3)}$. The last line is the value published by SINDING (1948) and also by others. The conclusion drawn from this table is the same as that from Table 4.

Computations for comet 1932 VI have also served as a check that our method of determination of the perturbations in $1/a$ gives the same result as the exact method of Encke, used by VAN BIESBROECK (see also Paper I, section 6 D).

Table 6 gives for some comets the reduction to the barycentre for the inner planets at the time of osculation, the perturbations up to some other dates $t_2$ and, again for the four inner planets, the reduction to the barycentre at $t_2$. If the comet does not come close to one of the planets, no true perturbations occur and the sum $\sum \frac{d(1/a)}{dt} + c_b(t_2)$ is practically the same as $c_b(t_3)$. Comet 1912 II did come very close to the Earth (1912 Sep.) and Venus (1912 Oct.) and therefore it was important to determine the perturbations.

The conclusions which we may derive from Tables 4, 5 and 6 are the following (see also Paper I):

The reduction to the barycentre for the four inner planets is essential and may in general be made at the time of osculation. In cases where the comet passes one of these planets at a distance smaller than about 1 A.U., it is necessary to determine for this planet both the perturbations and the reduction term $c_b(t_2)$. For the remaining inner planets the reduction can be made at the date of osculation.

We now consider the outer planets. Often the comet does not come close to Uranus and Neptune; for our 17 comets the absolute values of the perturbations by Uranus and Neptune were never larger than $3.3 \times 10^{-6}$, the mean being $1.9 \times 10^{-6}$. This means that the perturbations by Uranus and Neptune together are about 0.5 per cent of the perturbations by Jupiter and Saturn together.

The question may be asked whether there would be an advantage in calculating the barycentric correction due to Uranus and Neptune at the date of osculation as well. But this appears not to be feasible. The correction $c(t)$ in this case is very large, much larger than the sum of the computed perturbations, up to, say, 10 A.U., and the barycentric correction

TABLE 5

Comparison of different methods; comet 1932 VI for $t_1$ = 1927 Dec 6.0 (in units of 6th decimal)

| $1/a_{osc}$ | $\sum \frac{d(1/a)}{dt}$ inner planets | $c_b(t_2)$ | $\sum \frac{d(1/a)}{dt}$ outer planets | $c_w(t_1)$ | $1/a'$ |
|---|---|---|---|---|---|
| −595 | +2 VEM | +8 MeVEM | +648 JSUN | + 5 JSUNP | +68 |
| −595 | +2 VEM | +8 MeVEM | +650 JS | + 5 JSUNP | +70 |
| −595 | | | +650 JS | −12 JS | +43 |
| VAN BIESBROECK, SINDING | | | JS | | +44.1 |

TABLE 6

Perturbations of inner planets for different dates (in units of the 6th decimal)

| comet | $t_3$ | $c_b(t_3)$ | $t_2$ | $\sum \frac{d(1/a)}{dt}$ planets used | $c_b(t_2)$ | $\sum$ | minimum distance $\rho_V$ | $\rho_E$ | $\rho_M$ |
|---|---|---|---|---|---|---|---|---|---|
| 1912 II | 1912 Sep 15.0 | +21 | | | | +21 | 0.66 | 0.92 | — |
| | | | 1911 Dec 29.5 | +11 V E | + 3 | +14 | | | |
| | | | 1911 Aug 11.5 | +12 V E | + 2 | +14 | | | |
| 1917 III | 1917 Mar 6.0 | + 8 | | | | + 8 | 1.89 | 1.89 | — |
| | | | 1916 Jul 25.5 | + 1 V E | + 7 | + 8 | | | |
| | | | 1916 Feb 26.5 | + 9 V E | − 1 | + 8 | | | |
| 1919 V | 1919 Sep 18.5 | + 5 | | | | + 5 | 1.97 | 2.05 | — |
| | | | 1919 Aug 9.5 | − 7 V E | +11 | + 4 | | | |
| | | | 1919 Jun 10.5 | − 9 V E | +12 | + 3 | | | |
| | | | 1919 May 1.5 | − 3 V E | + 7 | + 4 | | | |
| 1930 IV | 1930 Sep 24.0 | − 2 | | | | − 2 | 1.60 | 1.43 | 3.06 |
| | | | 1929 Sep 6.0 | − 8 V E M | +5 | − 3 | | | |
| | | | 1929 Mar 10.0 | − 5 V E M | +2 | − 3 | | | |
| 1932 VI | 1932 Oct 23.0 | +10 | | | | +10 | 2.11 | 2.00 | 2.76 |
| | | | 1932 Jul 12.0 | +12 V E M | − 2 | +10 | | | |
| | | | 1932 Apr 3.0 | + 2 V E M | + 8 | +10 | | | |
| 1937 IV | 1937 Jun 19.0 | +16 | | | | +16 | 1.43 | 1.27 | 1.00 |
| | | | 1937 Feb 16.0 | + 8 V E M | + 8 | +16 | | | |
| | | | 1937 Jan 7.0 | +16 V E M | + 0 | +16 | | | |
| 1944 IV | 1944 Dec 16.0 | + 5 | | | | + 5 | 1.57 | 1.93 | 0.72 |
| | | | 1944 Mar 31.0 | + 2 V E M | + 2 | + 4 | | | |
| | | | 1943 Nov 12.0 | − 5 V E M | + 9 | + 4 | | | |

at that distance, together. For instance, in formula (34) of Paper I the coefficients computed for Uranus at a distance of 1 A.U. become 16800, 282 and 870, respectively. This can be explained by the fact that in the barycentric system the Sun also acts as a perturbing body, which may be seen from formulae (16) and (17) from Paper I. Consequently, considerable transfers of energy from the Sun to the comet take place near the perihelion. In this respect the question remains at what distance from the Sun it is permissible to make a reduction to the barycentre and neglect the barycentric perturbations which follow at larger distances. This problem has been thoroughly discussed in Paper I, section 6 C. To be on the safe side it is desirable that the comet be at least outside the orbit of the perturbing planet. For practical purposes this is not usually done for the planet Neptune; Table 2 in Paper I suggests that the errors made in this way are not large.

It should be kept in mind that the original value of $1/a$ is no more accurate than the $1/a$ of the osculating orbit on which it is based. Consequently, if the osculating orbit is not very accurate, the omission of Uranus and Neptune gives no additional loss of accuracy; the result will be good enough for statistical purposes.

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