



ELSEVIER

# Quantum fluctuations in $d^9$ model

L.F. Feiner <sup>a,\*</sup>, A.M. Oleś <sup>b</sup>, J. Zaanen <sup>c</sup>

<sup>a</sup> Philips Research Labs, Prof. Holstlaan 4, NL-5656 AA Eindhoven, The Netherlands

<sup>b</sup> Institute of Physics, Jagellonian University, Reymonta 4, PL-30059 Kraków, Poland

<sup>c</sup> Lorentz Institute for Theoretical Physics, Leiden University, P.O.B. 9506, NL-2300 RA Leiden, The Netherlands

## Abstract

We study the phase diagram and excitations of an effective spin–orbital model derived for  $d^9$  transition metal ions in the neighbourhood of an orbital degeneracy. RPA indicates a very strong renormalisation of the classical order parameters near the zero temperature multicritical point, suggesting the existence of a novel quantum liquid.

Transition metal oxides are perhaps the best known examples of charge transfer insulators [1]. As the electronic correlations are typically strong, the multiband models relevant for the doped systems may be replaced by effective models of the  $t$ – $J$  variety. Examples are the effective strong-coupling models for NiO [2],  $\text{CuO}_2$  planes in high- $T_c$  superconductors [3], and the  $t$ – $J$  model itself. In general the carrier propagation in such models may be more complex than in the usual  $t$ – $J$  model due to excitonic excitations [4].

This raises the question whether already in the *undoped* systems the (often neglected) orbital degrees of freedom, responsible for the excitons, could lead to a behaviour different from that of a Heisenberg anti-ferromagnet. The urgent questions are of course: (i) to what extent the properties of the antiferromagnetic (AF) phases are modified by the orbital degrees of freedom, and (ii) whether some qualitatively new physics could be realised in this enlarged space. We will address these issues by considering the spin–orbital model describing  $d^9$  transition metal ions like  $\text{Cu}^{2+}$ .

We start with the multiband model realised in  $\text{CuO}_2$  planes of high- $T_c$  superconductors [5]. In the limit where the Coulomb interaction  $U$  and charge-transfer energy are large compared to the  $d$ – $p$  hybridisation, one first integrates out the oxygen orbitals to derive subsequently the superexchange interactions by considering the virtual fluctuations to high-energy configurations,  $d_i^9 d_j^9 \rightleftharpoons d_i^8 d_j^{10}$ . The excited  $d_i^8$  configuration may be either one of the three singlets, or a triplet. The latter high-spin state has the lowest energy in this manifold, if the Hund's rule exchange interaction  $J_H > 0$  is present. Thus, a part of the

derived superexchange interactions is ferromagnetic (FM) and frustrates the usual AF interactions. As a result one finds a rather complex Hamiltonian [5,6], called in what follows the  $d^9$  model. The leading part is the superexchange interaction between two  $3d_{x^2-y^2} \sim x$  orbitals,  $J = 4t_{xx}^2/U$ . The remaining interactions,  $x$ – $z$  and  $z$ – $z$  ( $3d_{3z^2-r^2} \sim z$ ), are reduced by factors of  $\alpha_0^{-1}$  and  $\alpha_0^{-2}$ , respectively, with  $\alpha_0 \approx 3$ , due to the large differences between the involved hybridisation elements. In addition to the on-orbital spin operators, like  $S_{ixx} = \{d_{ix\uparrow}^\dagger d_{ix\downarrow}, d_{ix\downarrow}^\dagger d_{ix\uparrow}, (n_{ix\uparrow} - n_{ix\downarrow})/2\}$ , one finds as well the interorbital ones,  $S_{ixz}$ , with their transverse parts,  $S_{ixz}^+ = d_{ix\uparrow}^\dagger d_{iz\downarrow} + d_{iz\uparrow}^\dagger d_{ix\downarrow}$ , describing spin flips with orbital flips. The second part of the Hamiltonian is Ising-like and describes the interaction between the orbital degrees of freedom, with possible orbital (excitonic) excitations. Finally, the crystal-field term represents the difference between the hole energies in the  $d_{x^2-y^2}$  and  $d_{3z^2-r^2}$  orbitals,  $E_z = \epsilon_z - \epsilon_x$ . The characteristic feature is that, although the spins are still  $\text{SU}(2)$  invariant, the rotational invariance in the orbital sector is explicitly broken by the lattice.

First we solve the classical problem using mean-field theory (MFT). It is usually assumed that the spin and orbital degrees of freedom decouple,

$$c_{i\sigma}^\dagger = \cos \theta_i d_{ix,\sigma}^\dagger + \sin \theta_i d_{iz,\sigma}^\dagger \quad (1)$$

We considered uniform phases with two sublattices, i.e.  $\theta_i = \theta_A$  if  $i \in A$ , and  $\theta_i = \theta_B$  if  $i \in B$ . As expected, due to the exclusively AF character of the (super) exchange at  $J_H = 0$ , one finds pure AF phases if  $|E_z|$  is large, i.e.  $\theta_A = \theta_B$ , with either  $d_{x^2-y^2}$  (AFxx), or  $d_{3z^2-r^2}$  (AFzz) orbitals occupied for  $E_z > 0$  and  $E_z < 0$ , respectively. A trivial consequence of the explicit symmetry breaking in the orbital sector is that the transition moves from  $E_z = 0$  to  $E_z = -E_z^0 = -((\alpha_0^2 - 1)/\alpha_0^2)J$ . Note that the orbital mixing is totally suppressed in the AF phases ( $\theta_i = 0$  or

\* Corresponding author. Fax: +31-40-743365; email: feiner@prl.philips.nl.

