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Stellar evolution with turbulent diffusion mixing

IV. Intermediate and low mass stars and $^{12}\text{C}/^{13}\text{C}$ ratio in giants of the first ascending branch

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Summary. We consider stellar evolution in low mass stars ($1\text{--}3 M_{\odot}$) near the main sequence with the hypothesis that mild turbulence is present within all the star. Turbulent transport of the elements is modeled by diffusion equations, where the diffusion coefficient is chosen to be $D = R_e^* \nu$, where ν is the kinematical viscosity and R_e^* is a Reynolds number. We consider the effects of the growth of the gradient of the mean molecular weight on turbulence. The main consequences of diffusion on stellar evolution are 1. an increase of the life time near the main sequence and 2. a change of the radial distributions of chemical species (^{12}C , ^{13}C , ^{14}N , ^{16}O). The inhibition of the turbulence, when the gradient of mean molecular weight reaches a certain critical value, allows the evolution towards the red giant branch. When stars evolve towards the giant branch, chemical species are dredged up to the surface. At this stage models with and without diffusion, predict substantially different surface abundances (in particular the $^{12}\text{C}/^{13}\text{C}$ and C/N ratios). Comparisons between models and the available data on giants during the first dredge-up show that abundance anomalies can be explained if turbulent mixing is present during the main sequence phase: a value $R_e^* \simeq 100$ satisfies the various observational constraints.

Key words: abundance – giants – stellar evolution – turbulence

I. Introduction

A number of arguments favour the suggestion that mild turbulence is present in the radiative zones of rotating stars. The effect of such turbulence is to modify the surface abundance of processed elements. Thus the measure of surface abundance can be used as a test of this physical process.

Young cluster stars exhibit a quasi exponential decrease for Li (Herbig and Wolff, 1966; Zappala, 1972; Duncan, 1981). In a cluster, Li decreases with the mass (for $M < 3 M_{\odot}$).

As the temperature at the bottom of the surface hydrogen convective zone increases when the mass decreases, Danziger (1969) has considered the possibility that the depletion time scale at the bottom of the convective zone could follow exactly the observed depletion time scale. However, this is in conflict with the models of the hydrogen convective zone since the lithium burning has a timescale varying like $T^{20.5}$ around $T = 2.4 \cdot 10^6$ K. The behaviour of the basis of the convective zone, which reaches

$T = 2.4 \cdot 10^6$ K for a certain value of the stellar mass, around $0.6 M_{\odot}$, explains the sudden disappearance of lithium (Reeves, 1974; Cayrel, 1983) but cannot explain the general behaviour as a function of mass.

The observations can be explained if Li is carried (with about the same characteristic time as the stellar life time) from the bottom of the convective zone towards the region of lithium destruction. For a diffusion process, the e -folding time scale is (h^2/D) , where h is the distance from the bottom of the convective zone to the lithium destruction level. The exact definition of this level is not very important due to the very fast increase of the nuclear destruction rate of lithium. The scale height of the $T^{20.5}$ law is of the order of $(1/8)$ of the pressure scale height H_p .

Observational evidences of diffusive transport of chemical density have been discussed by Schatzman (1977, 1981). He defined a coefficient of turbulent diffusion to describe the strength of the mixing already fairly well described by the theory of turbulent flow.

The same scaling of e -folding time explains the results concerning Beryllium. Beryllium is destroyed at a somewhat higher temperature ($3.5 \cdot 10^6$ K) existing in deeper layers. In the majority of the observed main sequence stars, the Be does not seem to have been destroyed after star formation (Boesgaard, 1976). This gives a constraint on the intensity of turbulent transport. Numerical simulations with Li, Be, B abundances give a determination of the diffusion coefficient (Vauclair et al., 1978) about 500 to $1000 \text{ cm}^2 \text{ s}^{-1}$ (i.e. 100 to 200 times the microscopic diffusion).

Li depletion has been observed in giants by Alschuler (1975) and has been discussed by Scalo and Miller (1980) who give a new scale of the mass of the giants probably more satisfactory than the one of Alschuler. Scalo and Miller (1980) conclude that Li must have been depleted in the main sequence and give an empirical time scale for its exponential depletion. Their time scale is in agreement with the theoretical interpretation given by Schatzman (1981) for lithium depletion in field stars as observed by Boesgaard (1977). It should be noticed that the time scale given by Scalo and Miller leads to a value of the turbulent diffusion coefficient which is in better agreement with the other determinations.

We conclude that Li abundance determination justifies the assumption of turbulent transport through the outer radiative stellar zone of stars.

The effect of such transport throughout the star has been studied for the Sun (Schatzman et al., 1981): the core is enriched with hydrogen and ^3He , and the resulting lowering of the central temperature solves the problem of the solar neutrino deficiency; the surface ^3He abundance grows with time and explains the

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observed surface abundance (Schatzman, 1983); according to the last result on cosmic abundance of ^2D (Laurent, 1983) $[\text{}^2\text{D}]/[\text{H}] \simeq 5 \cdot 10^{-6}$, ^2D burning cannot explain the surface abundance of ^3He .

Observations of the ($^{12}\text{C}/^{13}\text{C}$) abundance ratio (Lambert et al., 1980; Dearborn and Eggleton, 1976) show that many stars have a ratio well below the predicted range. As we are interested in the possible explanation of a ^{13}C excess by turbulent diffusion we shall not consider here post He flash stars, as the changes in their chemical composition can result from other effects. For the stars which we are considering here, we shall study the effect of a mild turbulence in the radiative zones during the H-core burning phase. As already shown by Genova and Schatzman (1979) turbulent diffusion can carry the ^{13}C formed near $M_r=0.4$ to the surface. Such a mechanism can lower the $^{12}\text{C}/^{13}\text{C}$ ratio to the observed values. However, the analytic models of Genova and Schatzman (1979) are not quite satisfactory. It is thus necessary to work out detail models, to study abundances and evolution any changes near the main sequence. The inner structure of diffusive stellar models differ from non-diffusive models in that, essentially, the core is enriched with hydrogen.

Let us describe the assumptions on which our ideas of the generation of turbulence are based. It is probable that turbulence is generated by the various instabilities associated with rotation (Tassoul, 1978). However, neither the level of turbulence which is reached nor its exact relation with the different instabilities is firmly established. Thus phenomenology remains for the time being a straightforward model. The results so far obtained should help establish a complete theory of the generation of turbulence in radiative stable zones.

II. Instabilities and turbulent mixing

The question of instabilities in rotating stars has been recently reconsidered by Knobloch and Spruit (1982, 1983), and Zahn (1983) has shown under which conditions a vertical turbulent mixing can take place. The main idea is that baroclinic instability can generate a 2- D turbulence, practically along the horizontal if the shear is sufficiently large as shown by Hopfinger et al. (1982). The cascade towards the small scale generates a 3- D turbulence, and its scale is determined by the condition that the inertial terms dominate over the Coriolis force,

$$(u/l) \gtrsim \Omega.$$

In order to obtain an order of magnitude of the level of turbulence which can be reached, Zahn considers the flow diagram: differential rotation \rightarrow 2- D turbulence \rightarrow 3- D turbulence. The kinetic energy of the differential rotation is produced by advection of the angular momentum, and is finally connected, in a stationary state, into turbulent energy. In this way, Zahn obtains a vertical turbulent diffusion coefficient

$$D = R_e^* \nu \quad (1)$$

$$R_e^* \simeq \frac{\kappa}{\nu} \frac{\Omega^2 r}{g} (\bar{V}_{\text{ad}} - \bar{V}_{\text{rad}})^{-1},$$

where κ is the thermal diffusivity and ν the microscopic viscosity. With this estimate Zahn recovers the order of magnitude of R_e^* , as found by Schatzman (1977, 1983) and Schatzman and Maeder (1981).

A vertical μ -gradient can prevent the 2- D turbulence from becoming three-dimensional. According to Zahn (1983), the criti-

cal condition on $\bar{V}\mu$ is given by the inequality:

$$\bar{V}\mu \gtrsim \frac{\kappa}{\nu} \left(\frac{H_p}{r} \right)^2 \left(\frac{\Omega^2 r}{g} \right) (\bar{V}_{\text{ad}} - \bar{V}_{\text{rad}} + \bar{V}\mu)^{-1} = (\bar{V}\mu)_{\text{crit}} \quad (2)$$

when this condition is fulfilled, we can think that the 3- D turbulence vanishes. The exact value depends on several factors which all have been taken of the order of 1. Therefore, we can say that we do not know the exact value of $(\bar{V}\mu)_{\text{crit}}$.

During stellar evolution, $\bar{V}\mu$ grows with time, and instabilities are inhibited when according to (2), $\bar{V}\mu$ exceeds a critical value $(\bar{V}\mu)_{\text{crit}}$. $(\bar{V}\mu)_{\text{crit}}$ depends on the velocity $v(z)$ and changes from one star to another. So we shall examine the consequences of inhibition of turbulence for different values of $(\bar{V}\mu)_{\text{crit}}$; various numerical values are tested below.

III. Numerical method

Our numerical simulations are performed with the program of Maeder [1976, 1981; opacities are obtained by spline interpolation from tables of Cox and Stewart (1970) using Henyey's code] in which nuclear changes between two time steps are replaced by a special treatment of the "diffusion-reaction" equations (Schatzman et al., 1981).

$$\rho \frac{\partial X}{\partial t} = \text{div}(\rho D \nabla X) + \rho f. \quad (3)$$

ρ is the density and f includes the nuclear reaction rates.

In the $1.5 M_{\odot}$ model we solve this equation with mass as a variable instead of the radius as with the other models. Thus the conditions of regularity at the center are satisfied. This method is similar to that of Cloutman and Eoll's one (1975).

We use detailed CNO chains (^{12}C , ^{13}C , ^{14}N , ^{16}O abundances). All β decay is assumed to take place instantaneously. ^{15}N and ^{17}O which have very short lifetimes near the center are omitted. The reaction rates are taken from Fowler et al. (1975) and weak screening is considered (Graboske et al., 1973). Initial compositions are $\text{H}=0.73$; $\text{He}=0.25$, $\text{Z}=0.02$, $^{12}\text{C}=0.0036$, $^{13}\text{C}=0.00004$, $^{14}\text{N}=0.0013$, $^{16}\text{O}=0.0108$.

IV. Stellar models with and without turbulent diffusion

On the main sequence, hydrogen in the core is processed into helium, through the $p-p$ and the CNO nuclear chains. Since the nuclear reaction rates are high in the core, the abundances of elements involved in the $p-p$ and CNO chains ($M < 2 M_{\odot}$) or CNO ($M > 2 M_{\odot}$) soon reach those of the stationary state. In the outer part of the star, initial abundances remain constant. Consequently in the core of $1 M_{\odot}$ $^{12}\text{C}/^{13}\text{C} \sim 3.5$ and $^{12}\text{C}/^{14}\text{N} \sim 1/100$. The surface abundances for the Sun are $^{12}\text{C}/^{13}\text{C}=89$ and $^{12}\text{C}/^{14}\text{N}=4.8$. Between these extreme values is a transition region where elements are partially processed. This transition region is defined by the destruction or formation time of the element considered, this time being equal to the present age of the star; therefore this region progressively moves towards the surface. ^{13}C is destroyed faster than its parent element ^{12}C , but the large abundances of ^{12}C in the transition region produce a sharp peak of the ^{13}C abundances.

Our numerical simulations without diffusion agree with previous ones (Iben, 1977; Dearborn et al., 1976). We shall just

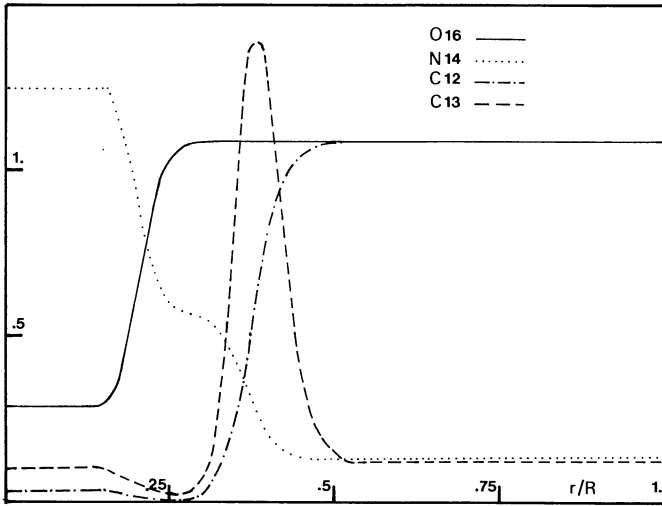


Fig. 1. ^{12}C , ^{13}C , ^{14}N , ^{16}O abundances distribution (in mass fraction) in a $3 M_{\odot}$ stellar model at the end of the life near the main sequence ($T = 281 \cdot 10^6$ yr) without diffusion ($R_e^* = 0$). The scale factor is 10^2 for ^{16}O and $3 \cdot 10^2$ for ^{14}N , $3 \cdot 10^2$ for ^{12}O and $3 \cdot 10^3$ for ^{13}C

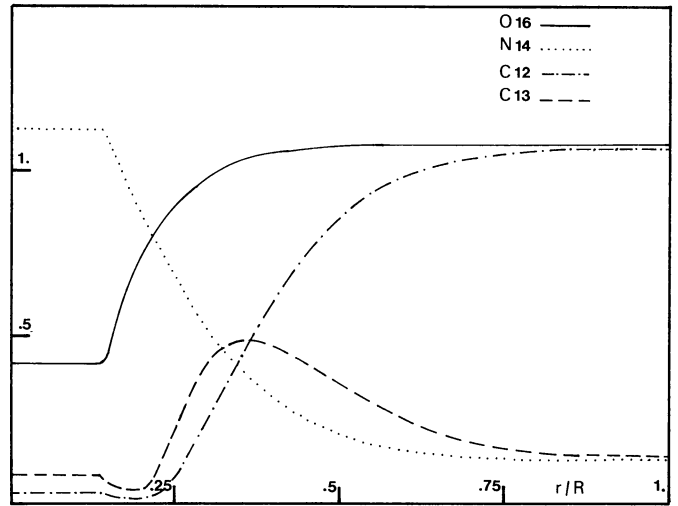


Fig. 2. As in Fig. 1 for a $3 M_{\odot}$ model with diffusion ($R_e^* = 100$) at $T = 323 \cdot 10^6$ yr

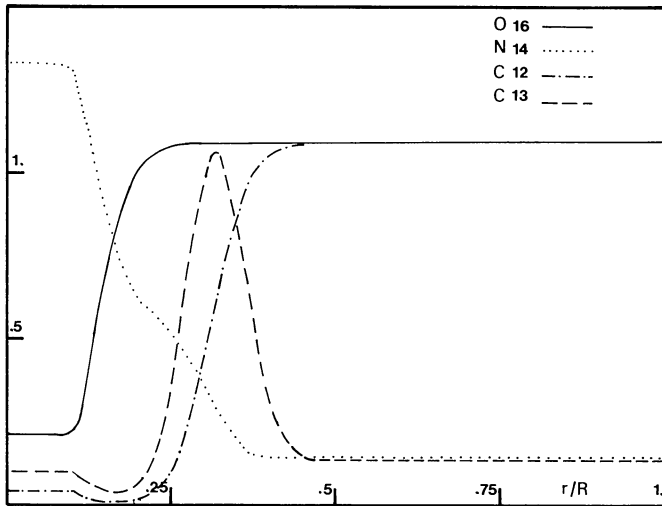


Fig. 3. As in Fig. 1 for $2 M_{\odot}$ model without a diffusion ($R_e^* = 0$) at $T = 0.89 \cdot 10^9$ yr

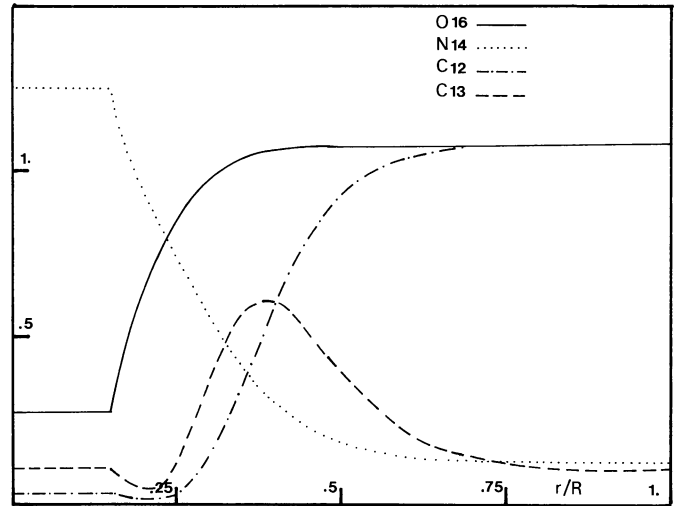


Fig. 4. As in Fig. 1 for a $2 M_{\odot}$ model with diffusion ($R_e^* = 100$) at $T = 1.13 \cdot 10^9$ yr

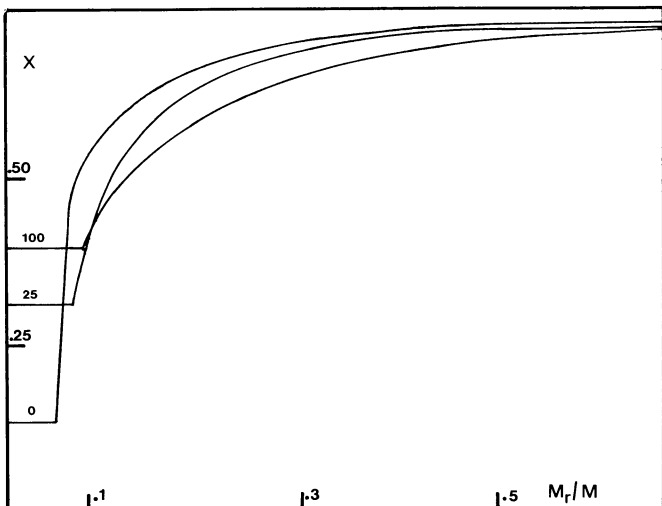


Fig. 5. Hydrogen abundance (in mass fraction) in $1.5 M_{\odot}$ models at time $T = 1.69 \cdot 10^9$ yr for different mixing strength. $R_e^* = 0, 25, 100$

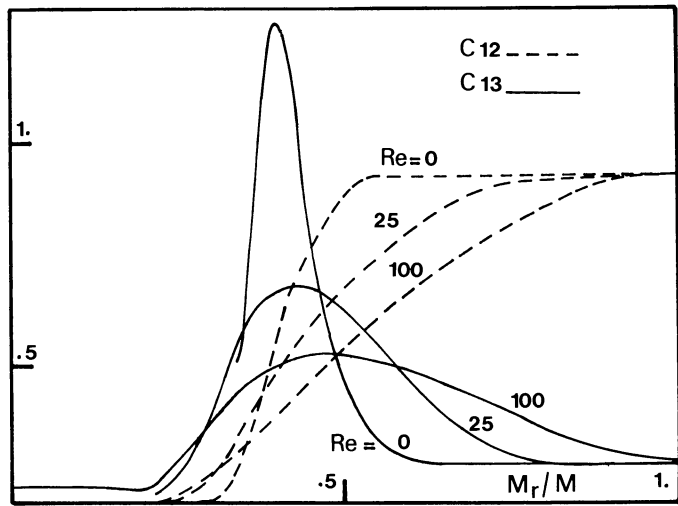


Fig. 6. ^{12}C and ^{13}C abundances (in mass fraction) in $1.5 M_{\odot}$ models at $T = 1.69 \cdot 10^9$ yr for different mixing strength $R_e^* = 0, 25, 100$

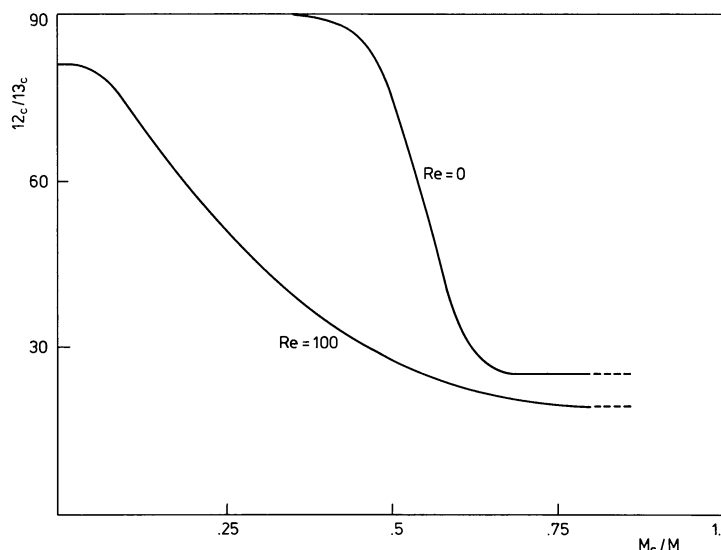


Fig. 7. $^{12}\text{C}/^{13}\text{C}$ abundance ratio (in mass fraction) predicted at the surface of a $1.5 M_{\odot}$ red giant on the first ascending branch, depending on the mass M_c of the convective zone

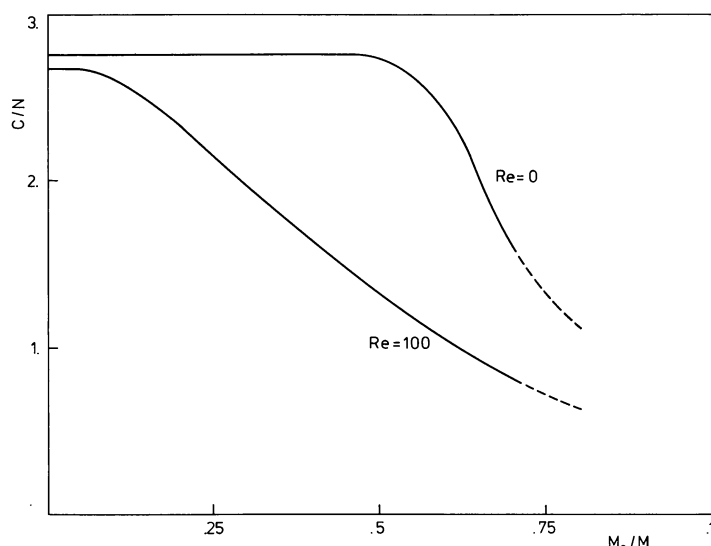


Fig. 8. As in Fig. 7 for the C/N abundance ratio (in mass fraction)

Table 1. Lifetime in years near the main sequence according to classical evolution ($R_e^*=0$) and to evolution with diffusion ($R_e^*=100$)

	$1.5 M_{\odot}$	$2 M_{\odot}$	$3 M_{\odot}$
$R_e^* = 0$	$1.9 \cdot 10^9$	$890 \cdot 10^6$	$280 \cdot 10^6$
$R_e^* = 100$	$2.9 \cdot 10^9$	$1100 \cdot 10^6$	$320 \cdot 10^6$

concentrate here on the differences which we observe in our simulations between the cases with and without diffusion.

Turbulence mixes chemical species over the star. ^{12}C (and ^{16}O if $M > 2 M_{\odot}$) are carried from the core towards the surface; ^{14}N from the surface to the core; ^{13}C is carried from its concentration peak towards, both, core and surface. Diffusion time is large

$\tau \sim R_0^2/D \sim 10^{10}$ yr for $1 M_{\odot}$ and $R_e^* = 100$. ^{12}C , ^{13}C are slowly transported inwards, but their fast destruction cannot modify the core abundances. In the envelope, the diffusion time is less than the nuclear reaction times and therefore the abundances are modified by the diffusion. Hydrogen is transported from the envelope to the core, increasing the amount of hydrogen fuel which can be burnt. This transport becomes important when the H-gradient is large on the core-boundary. With $M \lesssim 1.5 M_{\odot}$ the flow if not inhibited by the μ -gradient, could be sufficiently large to prevent H-exhaustion of the core and thus evolution towards the Red Giant Branch; in any cases larger masses (2 and $3 M_{\odot}$) would evolve (albeit slowly) towards the R.G.B.

In Figs. 1–4 the age corresponds to a quasi H-exhaustion of the core. ^{16}O abundances are modified only in the $2 M_{\odot}$ case. The destruction time of ^{16}O in the core is equal to the life time of the star and the ^{16}O core-depletion is slowed down by diffusion. The $3 M_{\odot}$ model has the shorter lifetime. Diffusion transport does not act long enough to strongly change the gradients of the abundance curves. From 1 to $3 M_{\odot}$ the main effects of diffusion are:

(i) Depletion of ^{12}C , and enrichment of ^{14}N , ^{13}C in the regions exterior to the H-burning shell.

(ii) The strong gradients of the $^{12}\text{C}/^{14}\text{N}$ (^{16}O for $2 M_{\odot}$) concentrations computed in the standard case without diffusion ($R_e^* = 0$) are softened in the case with diffusion ($R_e^* = 100$).

(iii) The region of formation of ^{13}C is depleted in ^{12}C (when $R_e^* = 100$) and the maximum concentration of ^{13}C is lowered. Diffusion spreads the ^{13}C peak towards the surface.

Table 1 gives the lifetime near the main sequence until the central hydrogen is only a few percent of its initial abundance. Figure 5 gives the hydrogen distribution in a $1.5 M_{\odot}$ model at $T = 1.69 \cdot 10^9$ yr for $R_e^* = 0, 25, 100$.

In standard evolution models the time from the end of the main sequence to the giant branch is short compared to the main sequence lifetime. Thus it is justified to suppose that the distribution of the elements is not changed during this phase. When the star reaches the giant branch, a large outer convective zone is formed. This zone has its maximal extension before the top of the first ascending giant branch. The observations of Li depletion (Lambert et al., 1980) and of Be abundance (Boesgaard and Chesley, 1976) are explained by this outer convective zone (Iben, 1977). The ^{13}C is dredged up to the atmosphere of red subgiant stars by this convective zone. However standard models ($R_e^* = 0$) do not provide sufficiently low $^{12}\text{C}/^{13}\text{C}$ ratios to be in agreement with the observations.

Figure 6 gives the distribution of ^{12}C and ^{13}C in $1.5 M_{\odot}$ models with $R_e^* = 0, 25, 100$. Without diffusion ($R_e^* = 0$) the lower value of $^{12}\text{C}/^{13}\text{C}$ at the red giant phase is 25; for $R_e^* = 25$ (weak diffusion) this ratio is lowered only to 23 while for $R_e^* = 100$ the $^{12}\text{C}/^{13}\text{C}$ is lowered to 18. For the same models we give $^{12}\text{C}/^{13}\text{C}$ and C/N ratios at the surface, depending on the depth of the convective zone (Figs. 7 and 8).

V. Semi-stationary models with inhibition of the diffusion

We next study the effect of diffusion and its inhibition. A series of models, including only the main hydrogen reactions, were generated for $1.6 M_{\odot}$ and $1 M_{\odot}$. Only the element or abundances were allowed to vary with time [according to Eq. (3)]. The temperature and densities were initially fixed. The inhibition of the mixing was introduced at different steps in a sequence of models. Initial models are homogeneous $\text{H} = 0.74$, $\text{He} = 0.25$, $^{12}\text{C} = 0.0036$, $^{13}\text{C} = 0.00004$, $^{14}\text{N} = 0.0013$. We stop the computations when t is

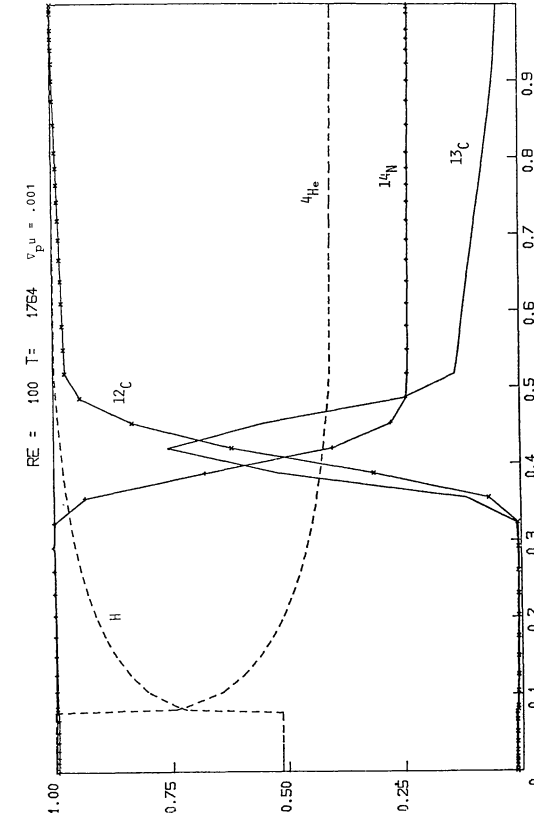
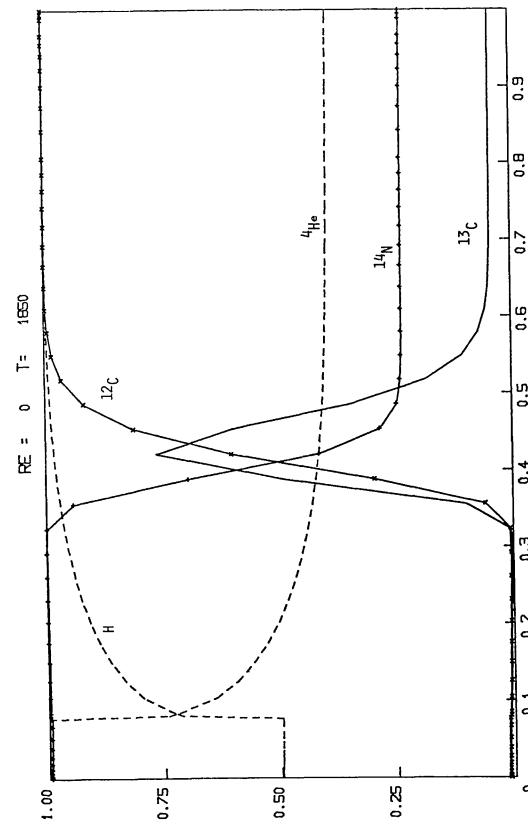
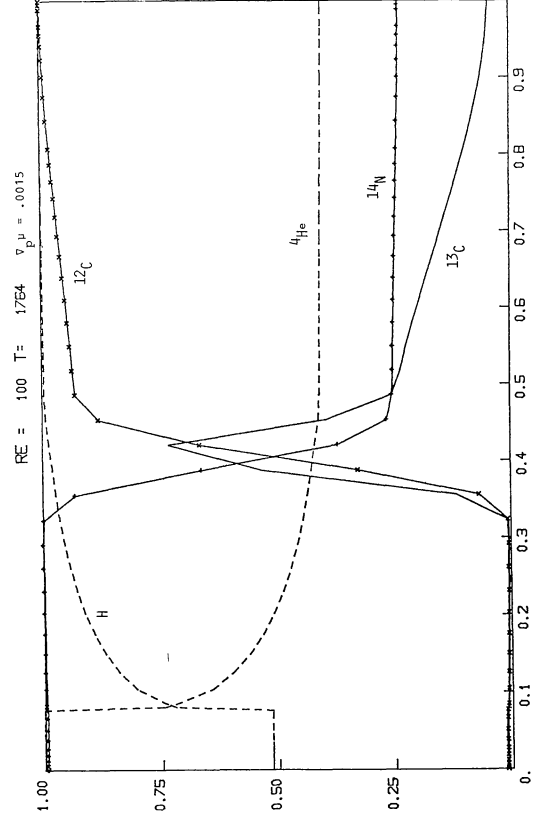
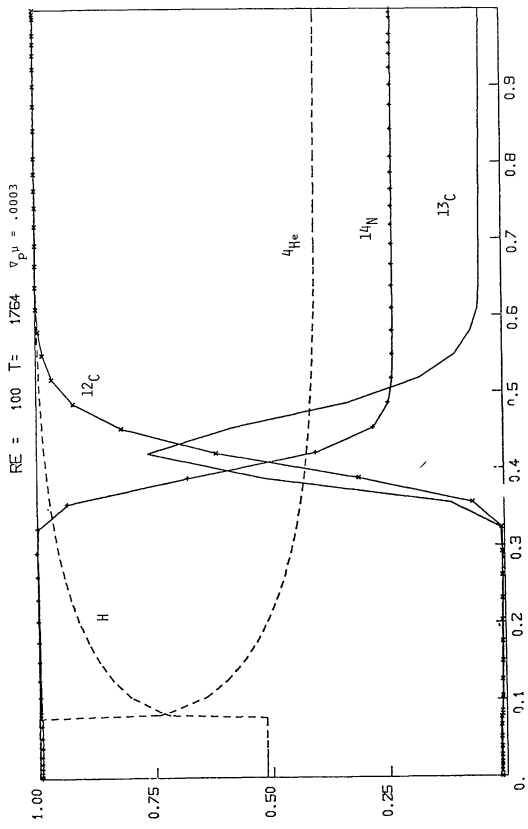


Fig. 9a-h. ^{12}C , ^{13}C , ^{14}N abundances in $1.6 M_{\odot}$ stationary models with diffusion ($R_g^* = 100$) and inhibition. The conditions of inhibition of the mixing are defined by different critical values of V/μ

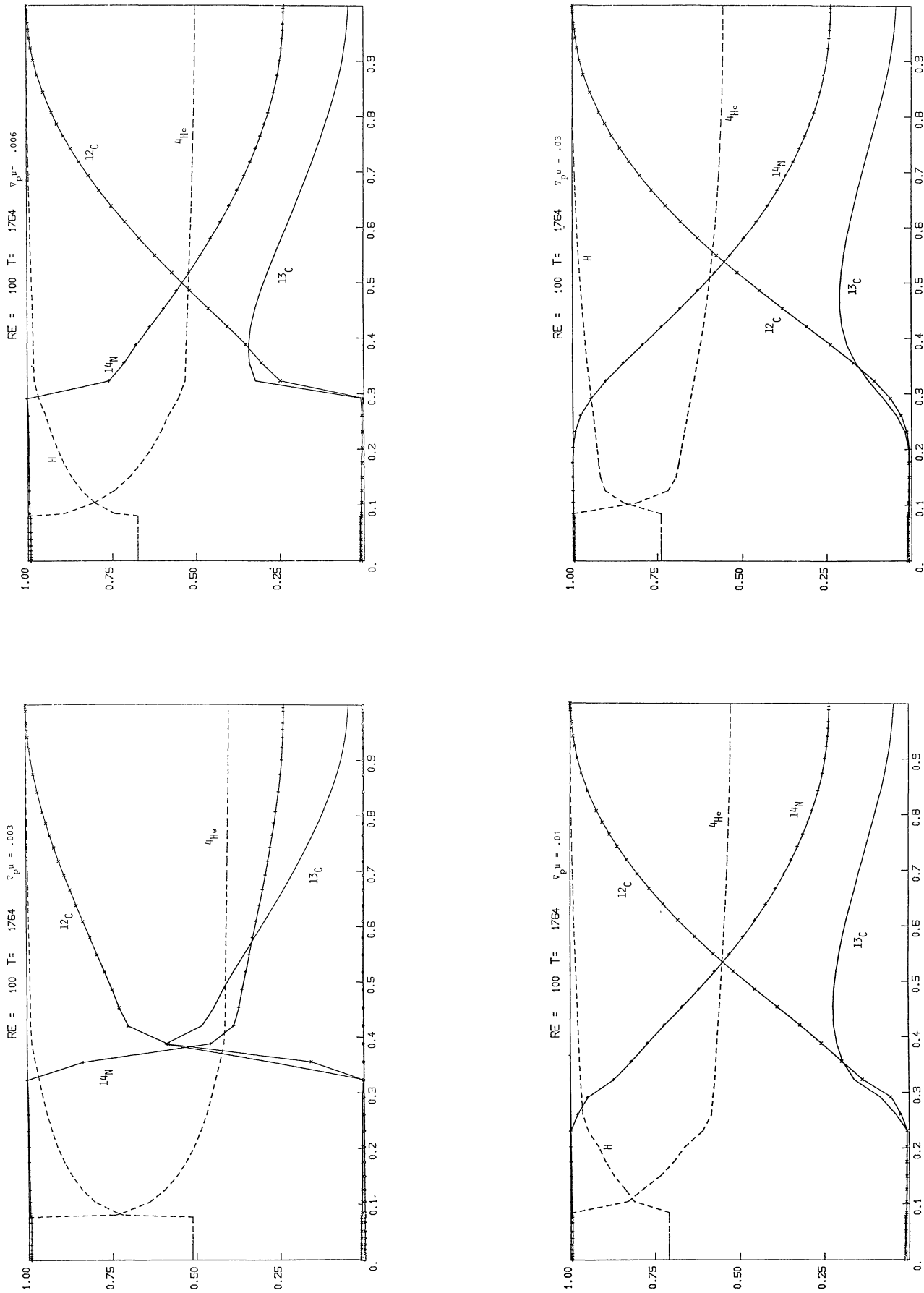


Fig. 9 (continued)

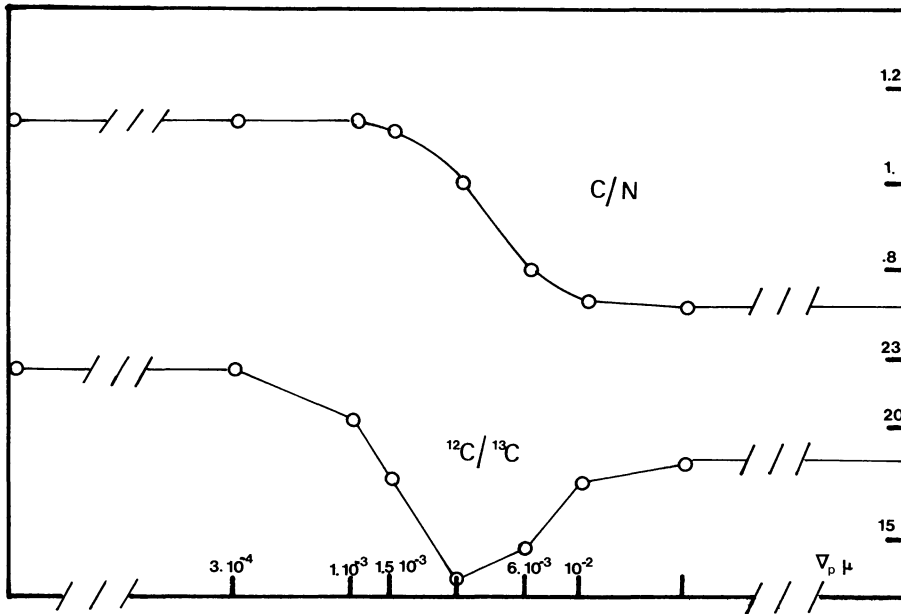


Fig. 10. $^{12}\text{C}/^{13}\text{C}$ and C/N ratios (in mass fraction) predicted at the top of the first AGB according to the $1.6M_{\odot}$ stationary models for the different conditions of inhibition of the diffusion

equal to the standard lifetime near the main sequence. However, due to the simplicity of the model, the H and He abundances are incorrectly described (H is depleted exponentially and not quasi linearly as in a realistic model). Thus the mean molecular weight μ and its variation $d\ln\mu/d\ln P$ computed there, are appreciably different from “true” values. The results of CNO abundances which are obtained are in reasonable agreement with those obtained by following nucleosynthesis in a completely time dependent model computed with a Henyey-code.

When $V\mu$ is greater than a critical value $(V\mu)_{\text{crit}}$, the turbulence is inhibited and diffusion is stopped. We consider the evolution of abundances for different values $(V\mu)_{\text{crit}}$. As H is depleted in the core, $V\mu$ grows. We stop the diffusion in the part of the star where the condition $(V\mu) > (V\mu)_{\text{crit}}$ is fulfilled and we keep the diffusion going on where $(V\mu) < (V\mu)_{\text{crit}}$. Consequently when the star is old enough a Stabilized Zone appears. In the Stabilized Zone (S.Z.) the diffusion can no longer smooth the H-gradient and $V\mu$ grows more quickly, $V\mu$ stays overcritical and the S.Z. remains stable. Near the S.Z., diffusion smoothes the H-gradient: if the local $V\tau$ is large enough (τ is the nuclear reaction time of H burning) $V\mu$ grows and the S.Z. extends. If $V\tau$ is too weak, the extension of the S.Z. stops.

When $(V\mu)_{\text{crit}}$ is small, the S.Z. appears very soon, when the H-gradient is still weak. Diffusion cannot prevent the nuclear reactions of increasing the $V\mu$. The size of the S.Z. grows and its final extension will be great. [If $(V\mu)_{\text{crit}} = 0$ the S.Z. extends all over the star; it is the case without diffusion – Fig. 9].

With $(V\mu)_{\text{crit}}$ large the S.Z. appears when the H-gradient are large near the S.Z. The diffusion smoothes the H-gradient, decreases $V\mu$ and prevents the extension of the S.Z. If $(V\mu)_{\text{crit}}$ is too large, no stabilized zone appears; it is the case with diffusion and no inhibition (Fig. 9).

Figure 10 gives $^{12}\text{C}/^{13}\text{C}$ and C/N versus $(V\mu)_{\text{crit}}$ just after the first dredge up. (In the figures $V\mu$ would be different if the models were completely time dependent; see above.) With $V\mu$ around $6 \cdot 10^{-3} - 10^{-2}$ the greatest ^{13}C storage takes place. In these models the edge of the S.Z. is on the outer wing of the ^{13}C concentration peak. There is still a strong maximum of ^{13}C , and consequently diffusion can carry a large amount of ^{13}C towards the surface.

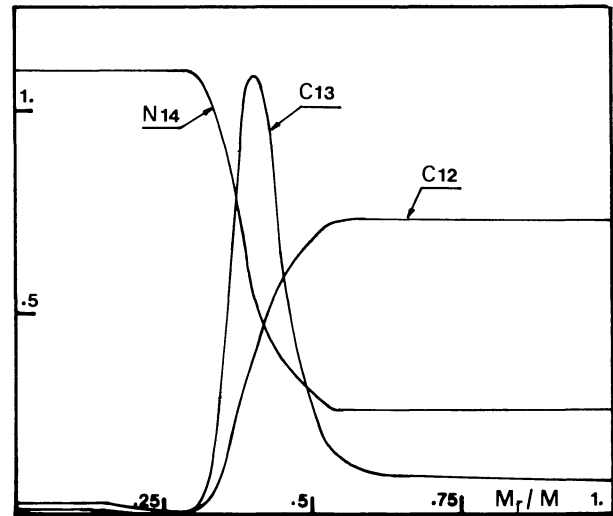


Fig. 11. ^{12}C , ^{13}C , ^{14}N abundance in a $1.5M_{\odot}$ model with $R_e^* = 0$ at $T = 1.9 \cdot 10^9$ yr. The scale factor is 200 for ^{14}N and ^{12}C and 2000 for ^{13}C

VI. A $1.5M_{\odot}$ model

We studied a $1.5M_{\odot}$ model with an Henyey-code and a late inhibition of the diffusion. We computed models with $R_e^* = 0$ and $R_e^* = 100$ (Figs. 11 and 12). Two models with inhibition ($R_e^* = 100$) were computed with the assumption that the S.Z. appears when the central H-abundance is, respectively, 0.50 and 0.25 [resp. $(V\mu)_{\text{crit}} = 0.37$ and 0.72]. In these models, when the S.Z. appears near the convective core, the central hydrogen depletion takes place faster. The later the diffusion is inhibited, the longer the lifetime of the star is (Fig. 13). The main characteristics of these models have been previously described. The S.Z. forms a barrier between the core and the envelope, and its mass represents only a few percent of the stellar mass. At the same age, for each of the three models with diffusion, ^{12}C , ^{13}C , and ^{14}N are the same. At the end of the main sequence lifetime, the abundance differences between the three models ($R_e^* = 100$) must be attributed to the age differences.

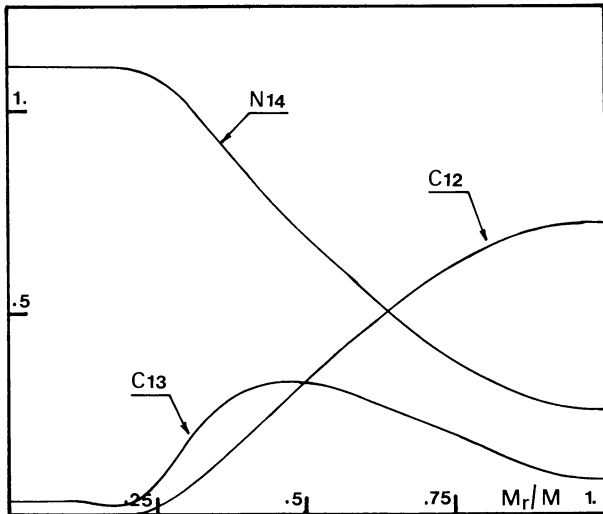


Fig. 12. As in Fig. 11 with $R_e^* = 100$ at $T = 2.09 \cdot 10^9$ yr

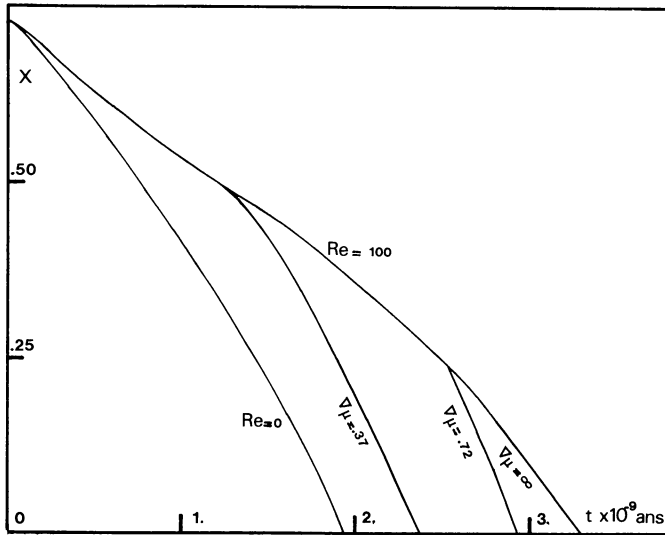


Fig. 13. Time-evolution of Central Hydrogen abundance in $1.5 M_{\odot}$ models. The " $R_e = 0$ " curve is standard evolution without diffusion. " $R_e = 100$ " curve gives evolution with diffusion; inhibition has been imposed into two models when $\bar{v}\mu$ is greater than 0.37 and 0.72

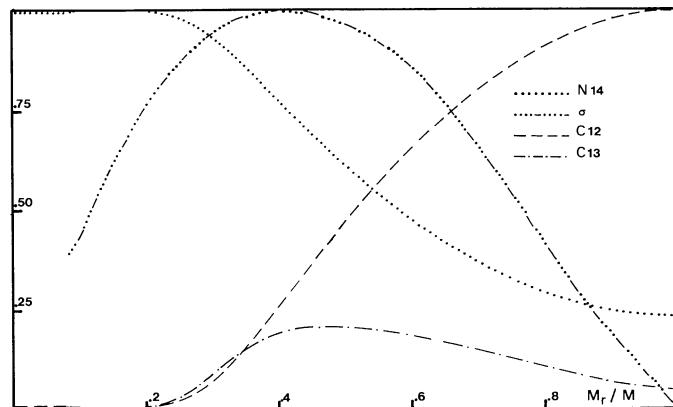


Fig. 14. Abundances in a $1.6 M_{\odot}$ model with $R_e = 100$ at $T = 1.8 \cdot 10^9$ yr. $\sigma = r^2 \rho^4 v$ is not constant over the star (see discussion)

The effects of the inhibition are:

- (i) A termination of the H-transport towards the core, allowing evolution towards the red giant branch.
- (ii) A termination of the ^{12}C , ^{14}N exchanges between the core and the envelope.
- (iii) A modification of the lifetime on the main sequence, depending upon the lateness of the epoch of the interruption of the diffusion. Note that the abundances of ^{12}C , ^{13}C , ^{14}N are similar to the case $R_e^* = 100$ without inhibition.
- (iv) No effect on the abundance ratios C/N and N/O for an early interruption. The abundance ratios are then similar to the case $R_e^* = 0$.
- (v) A larger storage of ^{13}C for an intermediate epoch of the interruption. The $^{12}\text{C}/^{13}\text{C}$ expected after the first dredge-up is 15 and not 20 as in the case where diffusion was not stopped.

VII. Discussion

We shall now discuss the few hypotheses which have been made in order to reconcile theoretical predictions of standard models and the observations of the low $^{12}\text{C}/^{13}\text{C}$ ratios in red giant stars.

A. The cosmological hypothesis assumes that the interstellar ^{13}C is more abundant than in the Sun at the time of young star formation. Observations (Wannier, 1980) do not justify this hypothesis. Actually Dearborn et al. (1976) show that to obtain a $^{12}\text{C}/^{13}\text{C}$ ratio as low as 15 or 20 it is necessary to have an interstellar $^{12}\text{C}/^{13}\text{C}$ ratio of 20–40.

B. Mass loss has been assumed to take place before the appearance of the outer convective zone. Dearborn et al. (1976) show that for 1 and $2 M_{\odot}$ stars $^{12}\text{C}/^{13}\text{C}$ ratios of 10 to 20 are achieved only with a 15% to 50% mass loss. This is excluded at such an evolutionary stage (Fusi-Peccini and Renzini, 1976).

C. Endal and Sophia (1981) study the evolution of a rotating solar model, slowing the rotation with a wind from the surface, and including instabilities that carry angular momentum. Their transport coefficients are crude estimations. They conclude that they must have been underestimated near the surface. Their abundance curves ^{12}C , ^{13}C , ^{14}N at the present solar age are similar to ours with a coefficient $R_e^* = 25$.

D. Dearborn et al. (1976) discuss the great dependence of the strength of the meridional circulation with rotation. If a star rotates sufficiently swiftly to reduce $^{12}\text{C}/^{13}\text{C}$ to 22 (25 without rotation) a similar star rotating 2.25 faster would reduce this ratio to 3.5 and $^{12}\text{C}/^{14}\text{N}$ to 1/200. If the lowest observed values of the $^{12}\text{C}/^{13}\text{C}$ ratios, of the order 3.5, are due to the most efficient mixing in star rotations with the break-up velocity v_{crit} , then a star must rotate with an equatorial velocity larger than $v_{\text{crit}}/4$ in order to change its $^{12}\text{C}/^{13}\text{C}$ ratio at all. For a $1.5 M_{\odot}$ star, v_{crit} is around 600 km s^{-1} while on the MS $v \approx 30 \text{ km s}^{-1}$ for spectral type F5.

E. According to Dearborn and Eggleton (1977) meridional circulation is supposed to be turbulent and they solve the transport problem of chemical contaminants with a diffusion equation $\partial x / \partial t = \partial / \partial m (\sigma \partial x / \partial m)$, where σ is constant (zero in the core). However, if D is the diffusion coefficient, we have $\sigma \sim r^4 \rho^2 D$ (Fig. 14) and no model of turbulent diffusion can be in agreement with σ constant. Assuming a constant σ leads to an overestimate of the diffusion near the surface and an underestimate near $M_r = 0.4$. Such a poor modelisation may explain their difficulties in obtaining C/N and $^{12}\text{C}/^{13}\text{C}$ ratios in agreement with the observations.

F. Sweigart and Mengel (1979) assume internal rotation on the MS around $10^{-4} \text{ rad s}^{-1}$ and evolution towards the ascending

red giant branch with conservation of angular momentum. They show that near the top of the first ascending branch, when the convective zone is retracting, the rotational velocity near the H-burning shell induces a meridional circulation which transports the elements C and N, so that the observed anomalies in R.G. could be explained. Beyond the great rotational velocities which are necessary, a detailed computation would lead to the same difficulties as the models of Dearborn and Eggleton (1977). Further this explanation can work only with sufficiently evolved stars.

G. Scalo and Miller (1978) compute the fraction X of observed G, K giants (corrected for selection effects) having anomalously low $^{12}\text{C}/^{13}\text{C}$ ratios. They compute theoretically the ratio $f(M_L)$ of giants with mass greater than M_L . If any explanation of the low $^{12}\text{C}/^{13}\text{C}$ ratios requires a lower limit M_L to the stellar mass, then $f(M_L)$ and X must be of the same magnitude. Scalo and Miller find M_L less than or equal to $1.5 M_\odot$ and exclude the meridional circulation or a strong mass loss as an explanation of $^{12}\text{C}/^{13}\text{C}$ ratios. These mechanisms could be efficient only for greater masses.

H. Spallation at the surface, could produce an increase of the ^{13}C abundance, but would result also in a strong Li abundance. Lambert et al. (1980) observed Li in stars with known $^{12}\text{C}/^{13}\text{C}$ ratios. They reject the spallation mechanism as they observed a ^{13}C enrichment associated with a Li depletion.

VIII. Observations and our models

A. The general trend, discovered by Lambert and confirmed by a diagram by Vigroux et al. (1976) using data from various sources, is that the $^{12}\text{C}/^{13}\text{C}$ ratio decreases with increasing luminosity.

A simple explanation was that the convective zone becomes deeper during evolution on the first AGB, dredging up ^{13}C (Iben, 1977). However, the stars which are plotted on a diagram like that of Vigroux et al. are a mixture of stars at different evolutionary stages (Fig. 15). Standard evolutionary tracks suggest that those stars are low mass stars. The introduction of turbulent diffusion is not likely to change the mass estimate of these stars. In the same area of the HR diagram, we find subgiants, clump stars (post He-flash) and stars of the second AGB. Since the abundance changes which might take place during the further evolutionary stages (clump stars and second AGB) are beyond the scope of this paper we shall make a detailed comparison between the observations and our models only for the subgiants.

B. A diagram like the diagram by Lambert et al. (1970) in which the Li abundance is plotted as a function of the $^{12}\text{C}/^{13}\text{C}$ ratio gives a possibility of a first discrimination between different evolutionary stages (Fig. 16, Table 2). We take, for the lithium deficiency as a function of the stellar mass, the same empirical relation as Lambert did. We consider for two different values of the R_e^* number the abundance ratios $^{12}\text{C}/^{13}\text{C}$ which we have calculated. For $R_e^*=0$ our results agree with the models of Dearborn (1976). The value of the $^{12}\text{C}/^{13}\text{C}$ ratio corresponds to the case of a convective zone with a maximal depth; we then obtained the curves of Fig. 16. We note that our $R_e^*=0$ curve disagrees with the corresponding one by Lambert et al. (1980). The discrepancy seems to be due to the fact that they used the high abundance ratio $^{12}\text{C}/^{13}\text{C}$ derived of the interstellar medium; let us recall the remark of Dearborn et al. (1978) that "these quantities do not necessarily correspond to the surface abundances at any stages of evolution". Using the above sketched discussion we obtain Table 2 which gives the most probable

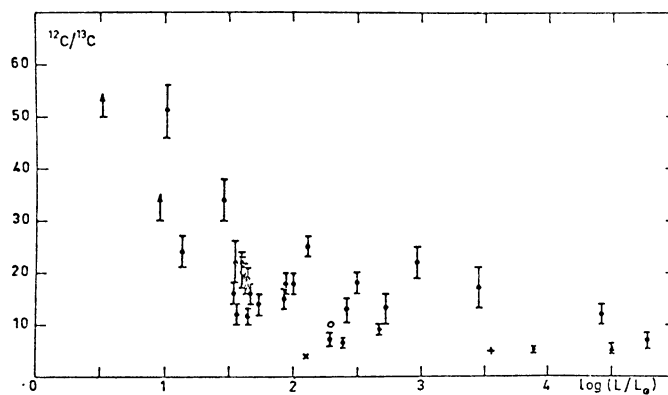


Fig. 15. $^{12}\text{C}/^{13}\text{C}$ ratios observed in giant stars against the stellar luminosity. From Vigroux et al. (1976)

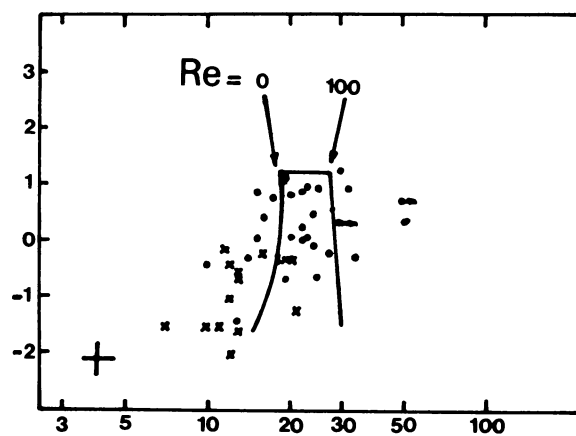


Fig. 16. The lithium abundance vs. the $^{12}\text{C}/^{13}\text{C}$ ratio for G and K giants (from Lambert et al., 1980). Upper limits to the Li abundance are given by crosses. Theoretical predictions for red giant models are given by the vertical curves for two hypothesis of mixing ($R_e^*=0$ and 100)

Table 2. Most probable subgiant stars among the sample of G-K giant observed by Lambert and Dominy (1980) and Lambert and Ries (1981). Data of $^{12}\text{C}/^{13}\text{C}$ and C/N ratio and (C/N) errors are from these references

Name	$^{12}\text{C}/^{13}\text{C}$	C/N
η Cep	> 30	—
δ Eri	> 50	5.13 (0.63)
ν^2 CMa	51	2.34 (0.67)
γ Cep	24	2.95 (0.33)
53 Eri	34	—
μ Aql	44	2.19 (0.31)
ψ UMa	32	1.18 (0.15)
η Psc	30	—
κ Gem	28	—
χ Vir	28	—

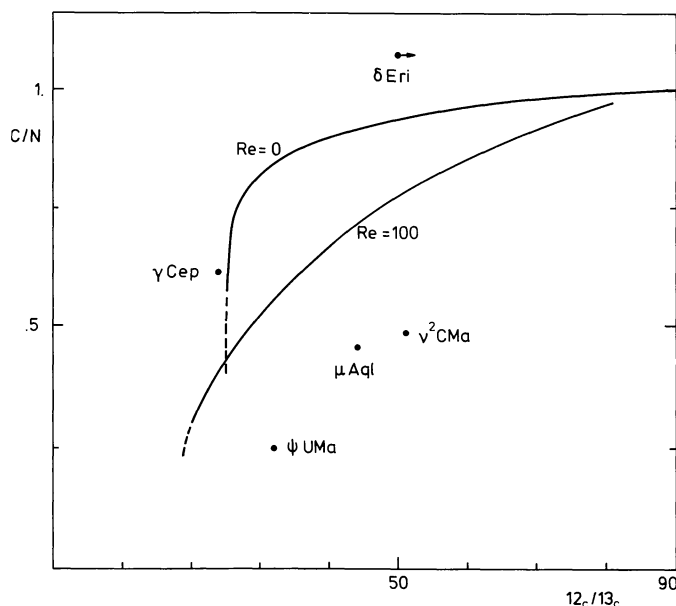


Fig. 17. Evolution tracks from the end of the main sequence to the top of the AGB for our $1.5 M_{\odot}$ models. The tracks give C/N normalized ratios vs. $^{12}\text{C}/^{13}\text{C}$ ratios that would be observed at the surface of $1.5 M_{\odot}$ stars for two hypothesis of mixing ($R_{\epsilon}^* = 0$ and 100). Stars plotted are subgiants observed by Lambert et al. (1981)

subgiant stars (stars plotted at the right of the curve $R_{\epsilon}^* = 0$) for which we carry the discussion of the abundances.

C. One way of deciding the evolutionary stage of a given star is to consider its luminosity. For stars in the range of masses we are interested in, standard models give for the giants (post He-flash) a luminosity [$\log(L/L_{\odot})$] greater than 1.6. Consequently the stars η Cep, δ Eri, v^2 CMa, γ Cep, 53 Eri, μ Aql whose luminosities are smaller are very likely subgiants. Still they have a large $^{12}\text{C}/^{13}\text{C}$ ratio. The convective zone of these stars has not yet reached its maximal extension and so the observed abundances are those of the external layers. They should have evolved according to standard evolution with the surface abundance only slightly changed.

The position of the stars in the HR diagram also gives an estimate of the masses (between 1 and $2 M_{\odot}$). The lower limits of the $^{12}\text{C}/^{13}\text{C}$ ratio for δ Eri and η Cep (respectively, 50 and 30) are just compatible with models with diffusion.

D. C/N ratios have been measured in a few subgiants whose $^{12}\text{C}/^{13}\text{C}$ are known. So we intend in this section to compare the theoretical curves giving (C/N) vs. the ($^{12}\text{C}/^{13}\text{C}$) ratio to the observed values for subgiants.

The stars observed by Lambert and Ries (1981) are plotted in Fig. 17 with the evolutionary tracks derived from our $1.5 M_{\odot}$ models with two values of R_{ϵ}^* (0 and 100). We have normalized the (C/N) ratio assuming a solar initial ratio [$(\text{C/N})_{\odot} = 4.8$].

Stars with a high $^{12}\text{C}/^{13}\text{C}$ ratio (subgiants) have clearly low C/N ratio if we exclude δ Eri. The position of the star γ Cep is in agreement with the standard evolutionary track " $R_{\epsilon}^* = 0$ ". Stars like μ Aql, v^2 CMa, and ψ UMa are certainly mixed. However, with the initial $(\text{C/N})_{\odot} = 4.8$ ratio strong turbulent diffusion, with $R_{\epsilon}^* \simeq 200$ or more is required. If, on the main sequence, μ Aql and v^2 CMa had $[\text{C/N}] = 2.6$ and ψ UMa had $[\text{C/N}] = 2.0$, these stars would be on the "standard" curve $R_{\epsilon}^* = 100$. However, such low ratios are excluded by Clegg's observations (1981). The high C/N

ratio of δ Eri close to the solar initial ratio shows that no dredge-up has yet taken place. We can conclude that its outer convective zone is thin, in agreement with the lower limit of the ratio $^{12}\text{C}/^{13}\text{C} > 50$ showing also that no dredge-up has taken place. Data of δ Eri are just compatible with both models $R_{\epsilon}^* = 0$ or 100.

Kjaergaard et al. (1982) also have measured C and N abundances in red giants. Some of their stars were observed by Lambert et al. (1981). The C/N ratios obtained by Kjaergaard et al. are systematically higher (see their Fig. 16). The only subgiant observed by both groups (ψ UMa) seems to be well below the curve $R_{\epsilon}^* = 100$. Kjaergaard et al. gave possible explanations for the disagreement (cf. their discussion of Fig. 16). Lambert's data are in better agreement with a model in which the stars have a $[(\text{C} + \text{N} + \text{O})/\text{Fe}]$ which is solar and in which $[\text{C}/\text{Fe}]$ and $[\text{N}/\text{Fe}]$ were solar on the main sequence as suggested by the observations of Clegg (1981). Before any final conclusion concerning the value that R_{ϵ}^* can reach for such subgiants stars, new measurement of the (C/N) ratio should be made.

We can examine the other subgiant stars of Table 2, which have large $^{12}\text{C}/^{13}\text{C}$ ratios but no C/N observations: η Psc (30), 53 Eri (34), κ Gem (28), χ Vir (28).

χ Vir and 53 Eri are just ascending the branch giant. Small errors in T_{eff} or in luminosity give large uncertainties in the stellar mass and the size of the convective zone. Thus, the $^{12}\text{C}/^{13}\text{C}$ ratio of these two stars can be explained either by models with diffusion and a thin convective zone or by models without diffusion and a deep convective zone.

η Psc and κ Gem with higher T_{eff} ($\simeq 5000$ K) do not have an extended outer convective zone. Their $^{12}\text{C}/^{13}\text{C}$ ratios can be explained as the result of a mixing on the main sequence and a moderately deep convective zone: we could not interpret the observations by standard models ($R_{\epsilon}^* = 0$).

IX. Conclusion

We have presented in this paper an attempt to solve the puzzle of surface abundances (Li and Be in main sequence stars and $^{12}\text{C}/^{13}\text{C}$, C/N ratios in giants) by a unique physical process, *turbulent diffusion*.

There is little doubt that the various instabilities which can take place in a rotating star can generate mild turbulence in the radiative stable regions. There is the further constraint that the main sequence stars should evolve towards the red giant-branch. This is fulfilled by assuming that the building up of the gradient of mean molecular weight is able to stabilize the radiative zones, the turbulent transport disappearing when $\nabla\mu > \nabla\mu_{\text{crit}}$. As we do not have any theoretical hydrodynamical model giving the value of $\nabla\mu_{\text{crit}}$, we have used $\nabla\mu_{\text{crit}}$ as a second parameter.

Be and Li abundances observed in 1 to 3 solar mass stars near the main sequence (Boesgaard, 1976) can be explained by a turbulent transport in the outer radiative regions (Vauclair et al., 1978). This turbulent transport can be described by a diffusion equation where the diffusion coefficient is $D = R_{\epsilon}^* \nu$ (ν viscosity). $R_{\epsilon}^* = 100$ gives the best agreement with observations.

The $^{12}\text{C}/^{13}\text{C}$ ratios observed in red giants are not correctly predicted by standard models. Turbulent diffusion throughout the star during the main sequence phase produces a ^{13}C enrichment in the envelope. At the red giant phase, ^{13}C is dredged up to the surface. However we must when we include the stabilizing effect of the mean molecular weight gradient ($\nabla\mu$) which stops the turbulent diffusion so the stars evolve towards the RGB. The details of the stabilization affect the final $^{12}\text{C}/^{13}\text{C}$ ratio.

The comparison of our models with red giants at the first dredge-up confirms the idea of turbulent transport during the life near the main sequence. In particular the observed abundances in ν^2 CMa, μ Aql, and ψ UMa, which are subgiant stars with known values of the abundance ratios $^{12}\text{C}/^{13}\text{C}$ and C/N, could be explained by models with diffusion.

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