The constants of precession and of galactic rotation

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the weighted average was adopted as definitive corrections for these zones in both halves of the table.

If the material used is on the system of the *General Catalogue* the corrections are to be increased by the systematic differences FK3—GC. Applying the differences given by KOPFF [1]) we obtain for the corrections applicable to the GC system the quantities given in the last three columns of Table 2.

In cases where maximum systematic weight is wanted the data in Table 2 may be of some use. The corrections north of —20° are contradicted by studies on the motions of faint and distant stars, which agree better with the original GC. It should be remembered, however, that these results rest to some extent on assumptions concerning the apex.

TABLE 2.

Corrections (Δ_o) to the declination systems of the FK3 and the GC (upper part: northern catalogues; lower part: southern catalogues)

δ	*Dritter Fundamentalkatalog* (FK3)					*General Catalogue*		
	$\Delta_1\delta_{1900}$	$\Delta_1\mu_\delta$	$\Delta_0\delta_{1900}$	$\Delta_0\delta_{1950}$	$\Delta_0\mu_\delta$	$\Delta_0\delta_{1900}$	$\Delta_0\delta_{1950}$	$\Delta_0\mu_\delta$
° °	″	″/a	″	″	″/a	″	″	″/a
+ 80 to + 87·5	— ·01	+ ·0001	— ·01	— ·03	— ·0003	+ ·02	— ·01	— ·0008
+ 70 „ + 77·5	— 1	+ 17	— 1	+ 1	+ 5	+ 5	+ 14	+ 16
+ 60 „ + 67·5	— 2	+ 29	— 2	+ 2	+ 9	+ 5	+ 13	+ 18
+ 50 „ + 57·5	— 3	+ 43	— 3	+ 4	+ 16	+ 2	+ 19	+ 32
+ 40 „ + 47·5	— 5	+ 38	— 5	— 3	+ 4	— 2	+ 13	+ 30
+ 30 „ + 37·5	— 1	+ 62	— 1	+ 9	+ 20	+ 4	+ 25	+ 44
+ 20 „ + 27·5	— 1	+ 71	— 1	+ 10	+ 22	+ 3	+ 27	+ 49
+ 10 „ + 17·5	+ 1	+ 83	+ 2	+ 11	+ 19	+ 6	+ 31	+ 52
0 „ + 7·5	0	+ 82	— 1	+ 8	+ 19	+ 5	+ 25	+ 39
— 10 „ — 2·5	+ 2	+ 86	+ 1	+ 12	+ 22	+ 7	+ 23	+ 34
+ 10 to + 17·5	— ·16	— ·0079	+ ·02	+ ·11	+ ·0019	+ ·06	+ ·31	+ ·0052
0 „ + 7·5	— 23	— 78	— 1	+ 8	+ 19	+ 5	+ 25	+ 39
— 10 „ — 2·5	— 22	— 76	+ 1	+ 12	+ 22	+ 7	+ 23	+ 34
— 20 „ — 12·5	— 17	— 72	+ 1	+ 7	+ 12	+ 5	+ 14	+ 19
— 30 „ — 22·5	— 14	— 53	+ 1	+ 11	+ 20	+ 5	+ 29	+ 46
— 40 „ — 32·5	— 14	— 36	— 1	+ 12	+ 26	+ 7	+ 38	+ 63
— 50 „ — 42·5	— 8	— 42	+ 3	+ 6	+ 8	+ 7	+ 38	+ 68
— 60 „ — 52·5	— 6	— 39	+ 3	+ 3	0	+ 3	+ 38	+ 75
— 70 „ — 62·5	— 1	— 30	+ 4	+ 3	— 3	+ 1	+ 24	+ 45
— 80 „ — 72·5	+ 4	— 26	+ 8	+ 3	— 10	+ 1	+ 20	+ 38

The constants of precession and of galactic rotation, by *J. H. Oort*.

Summary.

The first two lines of Table 1 show the corrections needed to reduce precessional and rotational constants from the system of the GC to that of the FK3. The first line applies to stars distributed evenly over the whole sky, the second to stars situated on the galactic equator and evenly distributed over all longitudes. In the second part of the table a summary is given of different determinations of precession and rotation. The last line shows what are now estimated to be the best averages for the various constants, in so far as they can be determined from proper motions. Table 2 contains the corrections to be applied to the GC and FK3 for precession and motion of the equinox. The correction needed by NEWCOMB's constant of precession is + ″/a·0101 ± ·0010 (m.e.).

It is shown that both the determination of the constant B of galactic rotation and the precessional constant are almost unaffected by the systematic errors of the proper motions, if, for each δ, the stars are evenly distributed in α. The mean error in B and Δn caused by the uncertainty in the system is then only of the order of ± ″/a·0003.

A solution of corrections to the precessional and rotational constants from the FK3 has been made by GLIESE [1]). This is however of low weight, as the bright stars of the FK3 with their large proper motions are not particularly suited for a determination of such tiny systematic motions. All other solutions of the last 15 years have been based on proper motions from the GC, or on motions referred to a system closely resembling that of the GC.

As was pointed out in the first article of this Bulletin, the FK3 system is likely to be the more

[1]) *Abh. Preuss. Ak.*, Math.-naturw. Kl. No. 18, 1939 and *A.N.* **269**, 160, 1939 (*Mitt. Coppernicus-Inst.* **5**, Nos. 5 and 4).

[1]) *A.N.* **270**, 127, 1940; *Mitt. Coppernicus-Inst.* **5**, No. 9.

trustworthy of the two for the proper motions in declination as well as for the errors $\Delta(\mu_\alpha)_\alpha$. It is therefore of some importance to inquire which changes would be brought about in the precessional corrections and the constants of galactic rotation if we change over from the system of the GC to that of the FK3 except for the errors $\Delta(\mu_\alpha)_\delta$ of the motions in right-ascension depending on δ [1]). In order to investigate this, systematic differences FK3—GC were formed as follows. The parts depending on δ were taken from Table I on p. 417 of this Bulletin; the terms depending on α were derived directly from KOPFF's comparison [2]), smoothed by averaging three consecutive hours of right-ascension. It can be shown that for systems resting on the same equinox (as GC and FK3) the errors $\Delta(\mu_\alpha)_\delta$, which are independent of α, have no influence on the determination of any of the quantities we are interested in, if the stars considered are evenly distributed in right-ascension. For the stars in general we can thus just as well change over entirely to the FK3 system; the solution in the first line of Table I refers to this total change. For objects strongly concentrated toward the galactic plane the condition of even distribution over α for each δ is not fulfilled, and in this case the solution, shown in the second line of the table, has been made from the *partial* differences FK3—GC mentioned, formed for various points on the galactic equator.

The independent quantities solved for were

$$x_1 = \Delta k + {\cdot}465\,B$$
$$x_2 = \Delta n + {\cdot}163\,B$$
$$x_3 = {\cdot}870\,B$$
$$x_4 = A$$

in which A, B are corrections to the constants of galactic rotation, Δk is the total average difference of proper motion in right-ascension between the two systems ($\Delta k = \Delta p_1 \cos\varepsilon - \Delta e - \Delta\lambda$; Δp_1 being the correction to the constant of lunisolar precession, ε the inclination of the ecliptic, Δe the correction required as a consequence of an error in NEWCOMB's motion of the equinox, $\Delta\lambda$ the correction to the constant of planetary precession), while $\Delta n = \Delta p_1 \sin\varepsilon$. If these corrections, as well as the motions, are expressed in seconds of arc per annum, the equations of condition are of the form

$$\mu_\alpha \cos\delta = x_1 \cos\delta + x_2 \sin\alpha \sin\delta + x_3 \cos\alpha \sin\delta + x_4 \{\cos 2(l-325) \cos b \cos\varphi + \tfrac{1}{2} \sin 2(l-325) \sin 2b \sin\varphi\}$$
$$\mu_\delta = x_2 \cos\alpha - x_3 \sin\alpha + x_4 \{\cos 2(l-325) \cos b \sin\varphi - \tfrac{1}{2} \sin 2(l-325) \sin 2b \cos\varphi\}$$

TABLE I.

	Δk	m.e.	Δn	m.e.	A	m.e.	B	m.e.
	"/a	"/a	"/a	"/a	"/a	"/a	"/a	"/a
FK3—GC (general)	− ·0010		·0000		+ ·0008		+ ·0012	
FK3—GC (gal. circle)	− 19		− 19		+ 9		+ 16	
galactic stars	− ·0013	± ·0006	+ ·0050	± ·0008	+ ·0022	± ·0008	− ·0039	± ·0008
General Catalogue	− 24	± 5	+ 37	± 5	+ 26	± 7	− 25	± 5
faint stars	− 3	± 6	+ 47	± 6	+ 30	± 9	− 31	± 8
mean on GC system	− 13	± 4	+ 44	± 4	+ 26	± 7	− 30	± 5
mean on FK3 system	− 26		+ 40		+ 34		− 17	

The "galactic stars" solution refers to various types of stars showing strong galactic concentration. It represents the unweighted average of results found by OORT [3]), PLASKETT and PEARCE [4]), R. E. WILSON and H. RAYMOND [5]) and ALI [6]). The values derived for the different types of galactic objects were fairly accordant. In the first two investigations the precessional corrections rest exclusively on proper motions in galactic latitude. The mean errors have been computed from the data given by the various authors. As part of the stars are common to all four investigations the weight of the average for Δk was assumed to be twice that of one of the investigations. In the case of Δn, A and B, where the attainable accuracy is limited by the large systematic errors of the type $\Delta(\mu_\alpha)_\delta$ and $\Delta(\mu_\delta)_\delta$, the mean error of a single determination was also assumed for the average of the four. The "*General Catalogue*" solution is from the cited article by WILSON and RAYMOND [5]). The mean errors correspond to the probable errors given by the authors; for A, where the authors did not give a probable error, it is a rough estimate. The "faint stars" are the 18000 stars of average photovisual magnitude around 11·5, for which VAN DE KAMP

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[1]) In a more restricted sense this problem has also been treated by Frl. NOWACKI in *A.N.* **245**, 63, 1932. She has given an application to the proper motions of the Lund AG zone.
[2]) *A.N.* **269**, 160, 1939; *Mitt. Coppernicus-Inst.* **5**, No. 4 (Tab. 7).
[3]) *B.A.N.* **4**, 79, 1927 (Tables 5 and 6, Groups I, II, III).
[4]) *Victoria Publ.* **5**, 297–299, 1936.
[5]) *A.J.* **47**, 49, 1938. For the "galactic stars" solution that given for "all B stars" in Table III was used. The results in the line "*General Catalogue*" refer to stars of all types together; they are the values that were finally adopted by WILSON and RAYMOND; the authors have excluded large proper motions. The small errors due to an incompleteness of WILSON and RAYMOND's equations (c.f. EMMA T. R. WILLIAMS, *A.J.* **48**, 84, 1939) have been neglected.
[6]) *M.N.* **101**, 324, 1941.

and VYSSOTSKY have derived proper motions on the GC system [1]). The 341 regions used are fairly regularly distributed over the sky north of — 25° declination. The values given are simple averages of VAN DE KAMP and VYSSOTSKY's determination, a later revision by OORT [2]), and, for Δk and Δn, additional results derived by VAN DE KAMP and VYSSOTSKY from the proper motions in right-ascension of the comparison stars used in the Allegheny and Yale parallax programmes.

The agreement between the three sets of values in the table, referring to groups of stars radically different in space distribution, indicates that the "standard of rest" as defined by the fixed stars leaves little to be desired.

In forming the averages in the next to last line equal weights were given to the three sets of values, except for Δn and B, where the "galactic stars" solution was given half weight (see below). In estimating the final mean errors I have tried to take account of the effects of systematic errors [3]).

The results have been reduced to the FK3 system by applying the corrections in the first line to the results for the "*General Catalogue*" and the "faint stars" (though the reductions do not strictly apply to the latter, as these are confined to declinations north of — 25°); to the "galactic stars" the corrections of the second line of the table have been applied, reducing these values to the FK3 system except for the errors $\Delta(\mu_\alpha)_\delta$. The average of the corrected values is given in the last line of the table.

The resulting final value for A, + ″/a·0034, or + ·016 km/sec.ps ± ·003 (m.e.), agrees well with the average of the values derived from radial velocities, for which we may adopt + ·018 km/sec.ps. The final value for B is — ″/a·0017 or — ·008 km/sec.ps.

The result for Δn corresponds to a correction of + ″/a·0040/sin ε = + ″/a·0101 ± ·0010 (m.e.) to NEWCOMB's constant of precession. From Δk = — ″/a·0026 we then derive — Δe — $\Delta\lambda$ = — ″/a·0119 ± ·0010 (m.e.). These values are derived from the FK3 system, but those from the GC system are very nearly identical. The discordance between the result for — Δe — $\Delta\lambda$ and the directly observed, respectively computed, values for Δe and $\Delta\lambda$ has previously been commented upon [1]). Adopting the average of the two recent studies of the equinox by MORGAN and KAHRSTEDT, giving Δe = — ″/a·0007 with an estimated mean error of ± ·0021, and DE SITTER and BROUWER's [2]) value for $\Delta\lambda$, viz. + ″/a·0002 ± ·0019 (m.e.), we would get — Δe — $\Delta\lambda$ = + ″/a·0005 ± ·0028 (m.e.). Acceptance of these latter values would, however, imply either that all the stars in Table 1 have systematic motions equivalent to a rotation of ″/a·0121 approximately around the earth's axis, or else that both fundamental catalogues have systematic errors amounting to + ″/a·0051 cos α in μ_δ and to + ″/a·0051 sin α sin δ in μ_α cos δ. Both alternatives are utterly improbable. The rotational motion would be much too large to be taken into serious consideration, while, moreover, it would have to be largely perpendicular to the galaxy. I have investigated the other alternative with some care, and find that this is equally improbable. For this purpose I have roughly determined the coefficient of the term proportional with cos α in μ_δ from the tables of systematic corrections $\Delta\delta_\alpha$ given in the GC. As in the first article of this Bulletin, the investigation was confined to the catalogues used for the construction of the declination system of the FK3; the catalogues Heidelberg, Kasan and Nikolajew 1926, which do not occur in the GC tables, were omitted; weights were again taken from Table 21 of the introduction to the GC except that those higher than 5 were decreased to 5. It appeared that the terms proportional with cos α are always very small; the largest coefficient is ″·14, while the average for a catalogue of weight 5 is ± ″·035. From these residuals I have computed that the true mean error of Δn, as far as this is due to systematic errors of the catalogues, is only ± ″/a·00036. It should be noted that the determination of the constant B rests mainly on a sin α term in μ_δ, and is thus entirely equivalent to that of Δn; it will be as little affected by systematic error. It is due to this circumstance, which has perhaps not been always clearly realized, that useful knowledge of B could be obtained at all, notwithstanding the great systematic errors of the proper motion system on the whole. As the final values of Δn and B rest not only on μ_δ but to some extent also on μ_α, their final errors will be

1) *Publ. McCormick Obs.* **7**, 1937; *A.J.* **45**, 171, 1936.

2) *B.A.N.* **8**, 149, 1937.

3) It is to be noted that, for stars evenly distributed over the sky, the accuracy of x_1 cannot be influenced by systematic errors, for the only type of error which enters into x_1 is the motion of the equinox, and this is just what x_1 intends to determine. The accuracy of x_1 and to a large extent also that of Δk, is thus wholly determined by the systematic and peculiar motions of the stars and by the *accidental* errors of the motions. It will be shown below that the influence of systematic errors is likewise negligible in the case of Δn and B, as long as stars are considered which are evenly distributed.

For stars confined to the galactic belt the circumstances are slightly less favourable for Δk, because this will now be affected to some extent by systematic errors of the type $\Delta(\mu_\alpha)_\delta$. For Δn and B the circumstances are *much* less favorable in this case. They are mainly determined from terms in μ_δ which vary with α; but, if we are confined to the galactic circle, variation of α necessarily entails variation in δ, so that these terms will become affected by the large systematic errors depending upon δ. As regards the constant A, this is the least well-determined of the four constants in Table 1; in the case of the galactic stars it is influenced by both $\Delta(\mu_\alpha)_\delta$ and $\Delta(\mu_\delta)_\delta$, while in the other case it is still influenced by $\Delta(\mu_\delta)_\delta$.

1) See, for instance, *B.A.N.* **8**, 155, 1937, and *A.J.* **47**, 57, 1938.

2) *B.A.N.* **8**, 230, 1938.

TABLE 2.

Corrections to be applied to the GC and FK3 for precession and motion of the equinox

	General Catalogue	*Dritter Fundamentalkatalog*
Δp_1	+ ″/a·0111 ± ″/a·0010 (m.e.)	+ ″/a·0101 ± ″/a·0010 (m.e.)
$-\Delta e - \Delta\lambda$	− ″/a·0114 ± ″/a·0010 ,,	− ″/a·0119 ± ″/a·0010 ,,
corr. to μ_α	+ s/a·000 09 − s/a·000 29 $\sin\alpha \operatorname{tg}\delta$	+ s/a·000 17 − s/a·000 27 $\sin\alpha \operatorname{tg}\delta$
corr. to μ_δ	− ″/a·0044 $\cos\alpha$	− ″/a·0040 $\cos\alpha$

still smaller [1]). This makes it extremely unlikely that the coefficients ″/a·0044 and ″/a·0040 found in the fundamental catalogues could be due to systematic errors. The fault, then, must be with the direct determinations of Δe or $\Delta\lambda$ [2]).

K. PILOWSKI has expressed serious doubt about the correctness of such a conclusion [3]). He argued that the necessity of introducing an empirical correction to the proper motions in right-ascension proves that it is impossible in practice to define an inertial system by means of the stars, as there might as well be systematic errors or motions equivalent to a rotation perpendicular to the equator, which could not be unravelled from the rotational motion of the stellar system. Referring to the above discussion and to previous articles we may now remark against this that, on the one hand, there is a sound a priori reason for expecting an especially large error common to all proper motions in right-ascension, while, on the other hand, the probability of other systematic errors giving rise to spurious rotations (either in the plane of the equator or in any other plane) has been shown to be extremely small.

If thus we adopt the point of view that the direct determination of $-\Delta e - \Delta\lambda$ is probably affected by considerable systematic error, the above determinations of Δk and Δn lead to the corrections shown in Table 2 above. As the mean epoch of the FK3 is around 1900, it follows that the FK3 positions for 1945, and thus also those given in the recent Almanacs, require corrections of $+$ s·008 − s·012 $\sin\alpha \operatorname{tg}\delta$ in α, and − ″·18 $\cos\alpha$ in δ.

In practice it will sometimes be desired to apply to proper motions not only these precessional corrections, but also to eliminate galactic rotation. New tables for effecting both corrections to the GC simultaneously have been given by SMART [1]); the constants used do not differ much from those derived in the present article.

It is a pleasure to thank Mr PELS for his co-operation. Most of the calculations on which the results of this Bulletin rest were made by him.

[1]) WILSON and RAYMOND have shown that the value of Δn derived from μ_α alone is closely comparable to that derived from μ_δ, which is an additional proof of its reality.

[2]) The possibility of serious errors in these quantities has been commented upon in previous articles (c.f. first footnote, p. 426, second column).

[3]) Thesis, *A.N.* **245**, 121, 1932; *Mitt. Astr. Rechen-Inst.* **3**, No. 3.

[1]) *M.N.* **101**, 40, 1941.

Note about galactic precession, by *A. J. J. van Woerkom.*

In this note I have calculated the precession of the invariable plane of the planetary system, and also of the planes of Neptune and Pluto.

For this I take a co-ordinate system with the origin in the centre of gravity of the planetary system, and of which the directions of the axes are invariable. The z axis is in the direction of the galactic pole and the x axis in the direction of the rotation of the galactic system. It is supposed that the orbits of the planets are all in the same plane, the ecliptic. The following symbols will be used:

m_i, a_i, n_i, c_i and l_i are, respectively, the mass, semi-major axis, angular motion, angular momentum and mean longitude of the $i$eth planet; C, K_0, $\frac{\partial K}{\partial \varpi}$, $\frac{\partial K}{\partial z}$, γ, Π, E and $\frac{2\pi}{3} - \alpha$ are, respectively, the angular momentum of the whole system, the attractive force of the galactic system in the origin, the gradient of the attractive force in the direction opposite to the direction of the centre of the galactic system, the gradient in the direction of the galactic pole, the inclination of the ecliptic on the galactic plane, the galactic longitude of the ascending node of the ecliptic on the galactic plane (the x axis being taken as zero direction), the angle between this ascending node and the point Aries, and the galactic longitude of the direction of the centre of the galactic system.

I have used the following formula:

$$\frac{d}{dt}\,(\text{angular momentum}) = \text{momentum of force},$$

which is valid with regard to the centre of gravity. The components of the angular momentum in the directions of the three axes are

$$c_x = c \sin\Pi \sin\gamma$$
$$c_y = -c \cos\Pi \sin\gamma$$
$$c_z = c \cos\gamma$$