High resolution radio interferometric observations of the planetary nebulae NGC 40, NGC 6543, and NGC 6720

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High Resolution Radio Interferometric Observations of the Planetary Nebulae NGC 40, NGC 6543, and NGC 6720

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Summary. Radio brightness maps of the planetary nebulae NGC 40, NGC 6543, and NGC 6720 are presented. The resolution is $6'' \times 6''$ cosec δ and the wavelength 6 cm. Densities are derived from simple cylindrical models. Estimates of the total ionized masses are obtained. Electron temperatures are calculated for NGC 6543 and NGC 6720. The temperature obtained for NGC 6720 from the radio data is low compared to that derived from the forbidden line ratios.

Key words: planetary nebulae — continuum aperture synthesis maps — electron densities and temperatures — masses

Introduction

Recently there have been a number of radio interferometer observations of planetary nebulae [Elsmore (1968), Webster *et al.* (1970), Miley *et al.* (1970); Wynn-Williams (1970), Kaftan-Kassim (1973), Scott (1973), Terzian *et al.* (1974)]. Only a few of these observations, however, have had sufficient resolution to reveal much detail in the objects studied. In this paper we present the results of high resolution radio observations of the planetary nebulae NGC 40, NGC 6543, and NGC 6720 made with the Westerbork Aperture Synthesis Telescope (WSRT)[1]) at a wavelength of 6 cm. The larger angular sizes, high declinations and high brightnesses of these nebulae make them especially suitable for observation with the WSRT at this wavelength.

Observing Procedure

The WSRT consists of an east-west line of twelve 25 m paraboloids of which the first ten are fixed 144 m apart and the remaining two are movable along a 300 m track. The radio energy received in each dish is detected by dipoles crossed at right angles. The brightness and polarized brightness distributions are obtained by appropriate correlation of the output of each dipole of the fixed dishes with that of each dipole of the movables. For a twelve hour observation this configuration gives a total of 20 simultaneous interferometers with baselines from 72 m to 1440 m in 72 m steps. The resulting synthesized beam is elliptical with half-power dimensions of 6″ in right ascension and 6″ cosec δ in declination.

Errors in the measurements arise from system noise, calibration errors, the missing zero spacing, and from sidelobe effects. In the present observations the latter are negligible. The missing zero spacing produced a depression in the zero level which is constant over the map. The size of the depression depends on the intensity and distribution of the emission and is small for the three nebulae discussed here. In the case of the strongest source, NGC 6543, the estimated depression was 2.2 m.f.u. (1 m.f.u. $= 10^{-29}$ Watt m^{-2}H$_3{}^{-1}$) and a correction of this size has been applied to the corresponding map. The standard Westerbork calibration procedure was employed. The radio sources 3C48, 3C147 and 3C309.1 are used as calibration standards with assumed flux densities taken from Kellerman *et al.* (1969). The error is about 5%. For a twelve hour observation the rms noise at the field centre is about 1.3 m.f.u. Baars *et al.* (1973), and Baars and Hooghoudt (1974), give a complete description of the WSRT.

The Contour Maps

Both of the observed fields containing NGC 40 and NGC 6720 also contain one apparently unrelated point source at the positions and intensities given in Table 1. No conspicuous objects are visible at these positions on the Palomar Sky Survey. These sources introduce grating rings which pass close by the objects of interest.

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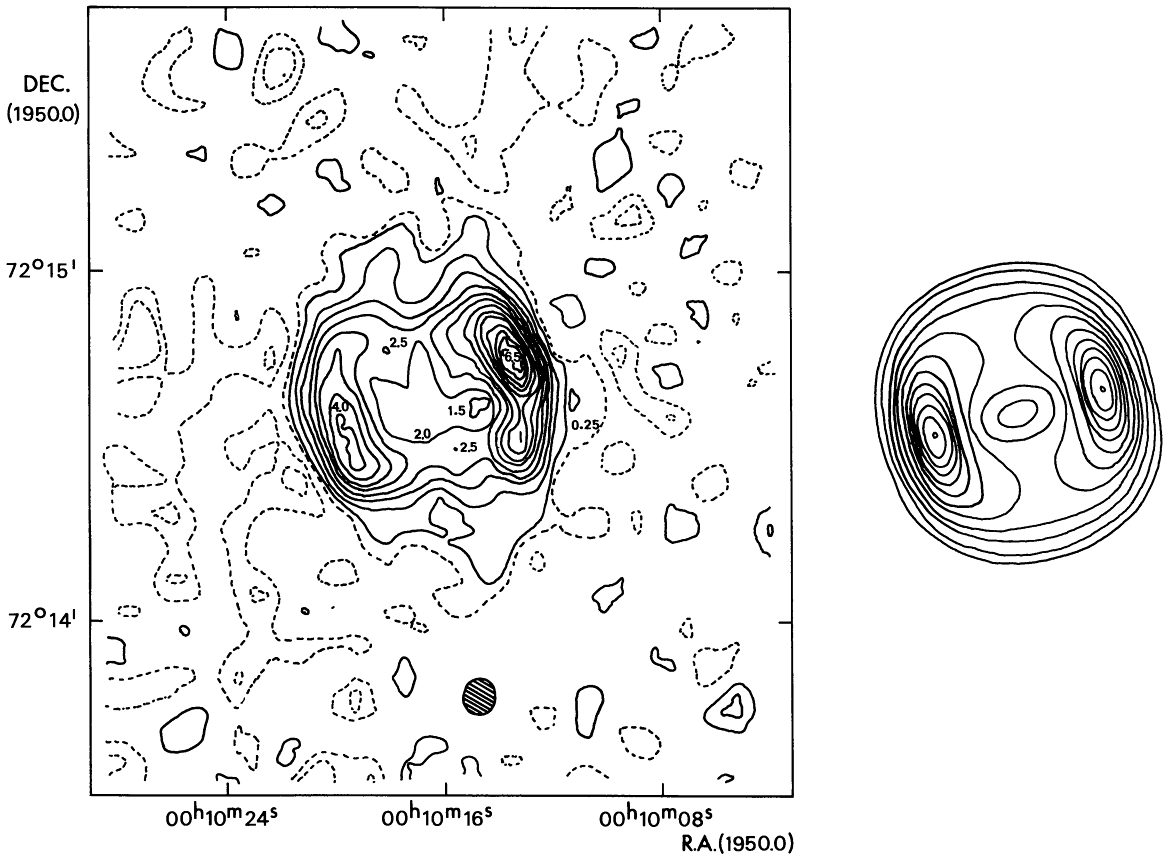


Fig. 1a and b. The synthesis map of NGC 40 at 4995 MHz is shown on the left (a). The intensity of the outer dashed contour is 1.25 m.f.u./beam and the first solid contour 2.5 m.f.u./beam. The contour interval is then 2.5 m.f.u./beam. The intensities marked are in units of 5 m.f.u./beam. The dotted contours are negative and the shaded ellipse represents the halfpower beam area. On the right (b) is the relative brightness distribution of the cylindrical model of NGC 40 convolved with the beam. The contour intervals are chosen to correspond to those in the observed map. See Table 4 for the model parameters

The effects of the sources have been subtracted from the maps discussed below.

The contour maps of NGC 40, NGC 6543, and NGC 6720 are shown in Figs. 1a, 2 and 3a. The observed brightness distribution of NGC 6543 differs greatly from those of NGC 40 and NGC 6720. The contours of the latter two nebulae bear a striking similarity to one another, both having two separate well-defined maxima, a central minimum, and approximately circular outer contours. These two maps exhibit a high degree of symmetry. In both, one peak is significantly more intense than the other. The similarity of the contours of NGC 40 and NGC 6720 is a strong indication that the emitting volumes in both nebulae have similar morphologies. The radio maps of these two nebulae are in good overall agreement with the Hα + [N II] isophotes of Aller (1956). The maxima in the radio maps are displaced inwards with respect to the optical peaks by about 4″ but shifts of this order are sometimes produced by the difference in resolution between the radio and optical observations. Within the limits set by the resolution, the radio and Hα + [N II] emissions appear to come from the same regions.

The central region of NGC 6543 is just resolved into two peaks (Fig. 2). The extension of the outer contour some 40″ towards the southwest is clear evidence that gas radiating weakly at 6 cm exists for a considerable distance from the bright central region. Millikan's

Table 1. Sources subtracted from the observed fields

Field	R.A.	Dec.	Flux (m.f.u.)
NGC 40	$00^h09^m32^s$	72°15′29″	62.0
NGC 6720	$18^h51^m36^s$	33°07′09″	16.7

Table 2. Total flux densities

Nebula	Galactic designation	Flux density (f.u.) (fringe amplitude on shortest baseline)	Flux density (f.u.) single dish
NGC 40	120 + 9° 1	0.457 ± .030	0.464 ± .041
NGC 6543	96 + 29° 1	0.873 ± .060	0.910 ± .096
NGC 6720	63 + 13° 1	0.343 ± .020	0.354 ± .054

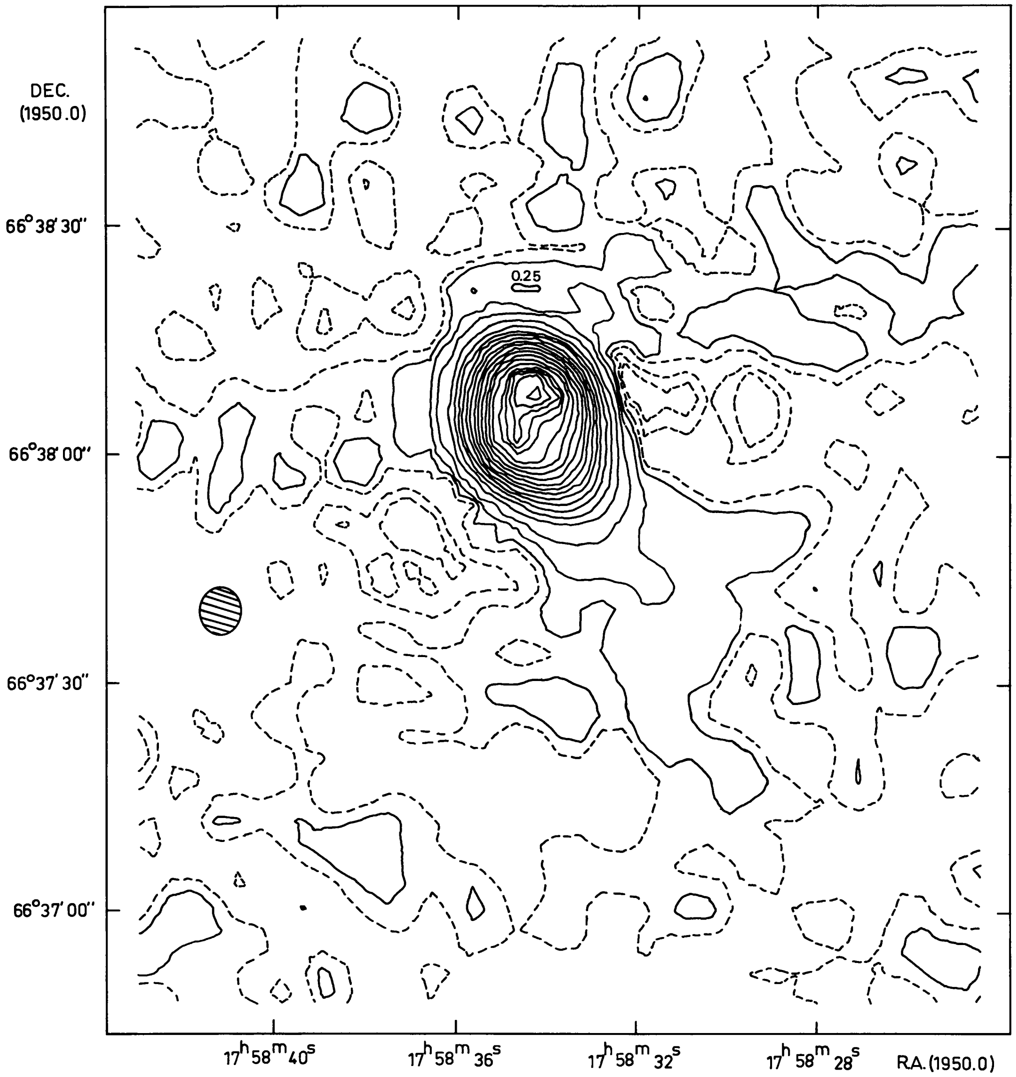


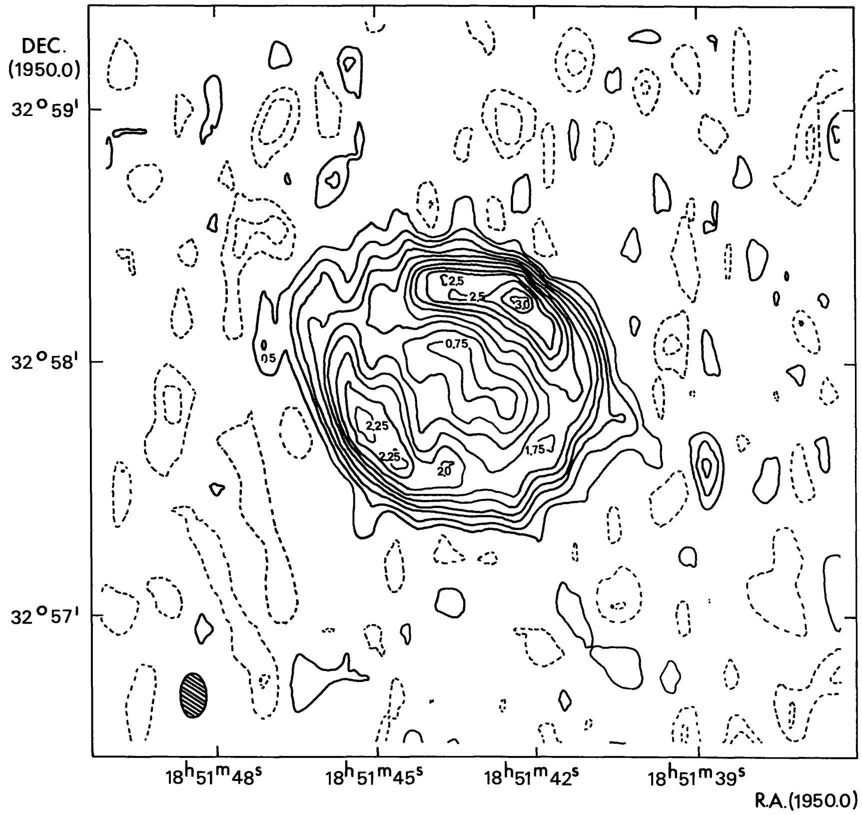
Fig. 2. The synthesis map of NGC 6543 at 4995 MHz. The outer contours are at 1.25, 2.5, 5 and 10 m.f.u./beam, then increasing in steps of 10 m.f.u./beam until the last three contours which have an interval of 5 m.f.u./beam. The central peak is 155 m.f.u./beam. The dotted contours are negative and the shaded ellipse represents the halfpower beam area

(1972) sensitive photograph of NGC 6543 shows the intricate structure within the 5.3 arc minute nebulosity, already noted by Czyzak *et al.* (1968), surrounding the bright optical nebula.

Because the shortest base line employed is 72 m, the visibility of any smooth structure greater in extent than about 1 arc minute will be greatly reduced. The fringe amplitudes for each nebula on the shortest baseline is given in Table 2. Also shown are the averages of the single dish measurements compiled by Higgs (1971). That the two sets of flux densities agree fairly well indicates that there is probably little contribution to the total flux density from smooth large scale structures. The averages given in column 4 have been corrected for the contributions of the point sources given in Table 1.

Estimation of the Nebular Temperatures

No significant polarization was detected in any of the nebulae. At the peak total intensities, the fractions of polarized radiation are less than 8%, 3% and 2% for NGC 40, NGC 6720 and NGC 6543 respectively. Clearly the limiting polarizations are higher at points on the contour maps where the total intensity is lower. Our evidence indicates, therefore, that at least the bulk of the radiation from the nebulae is due to thermal bremsstrahlung. The brightness temperatures at the peaks of NGC 40, NGC 6720 and NGC 6543 are 13 K, 10 K and 180 K, and so all the nebulae are optically thin at 6 cm. For an optically thin nebula, the radio emission depends strongly on the density but is practically independent of the temperature.



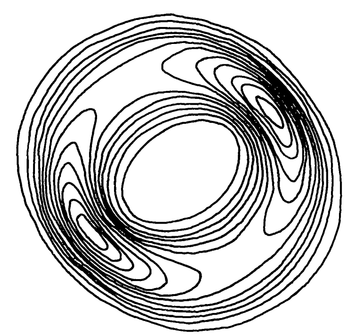


Fig. 3a and b. The synthesis map of NGC 6720 at 4995 MHz is shown on the left (a). The contour interval is 1.25 m.f.u./beam starting at 1.25 m.f.u./beam. The intensities marked are in units of 5 m.f.u./beam. The dotted contours are negative and the shaded ellipse represents the halfpower beam. On the right (b) is the relative brightness distribution of the cylindrical model of NGC 6720 convolved with the beam. The contour intervals are chosen to correspond to those in the observed map. See Table 4 for the model parameters

To estimate the electron temperature we use, therefore, a method that does not require a knowledge of the density. We assume that the temperature is uniform throughout the nebula. The method, described below, is a modified form of that used by Thompson (1967).

The relationship between the nebular temperature, T_e, and the brightness temperature, $T_b(\nu)$, at every point on the i^{th} contour may be written as

$$T_b(\nu) = T_e(1 - \exp[-\tau_i(\nu)]), \tag{1}$$

where ν is the frequency in MHz and $\tau_i(\nu)$ is the optical depth on the i^{th} contour. Since the brightness temperatures at the frequency of observation, ν_0, are known, substitution of an assumed value of T_e in Eq. (1) will yield an estimate of $\tau_i(\nu_0)$. Next from the $\tau_i(\nu_0)$, values of the optical depth at other frequencies are derived by means of the relation

$$\tau_i(\nu) = \tau_i(\nu_0)\left(\frac{\nu_0}{\nu}\right)^2 \frac{\ln(49.5\, T_e^{3/2}/\nu)}{\ln(49.5\, T_e^{3/2}/\nu_0)}. \tag{2}$$

The expected flux density from the whole nebula at the frequency ν and the assumed T_e is then given by

$$S(\nu, T_e) = \frac{2k\nu^2}{c^2} \sum_i T_b(\nu)\, \Delta\Omega_i \tag{3}$$

where $\Delta\Omega_i$ is the solid angle associated with the i^{th} contour and $T_b(\nu)$ is obtained from $\tau_i(\nu)$ by substitution in Eq. (1). Because the finite resolution of the telescope causes the observed contours to appear larger than they should, we calculate the $\Delta\Omega_i$ from the cylindrical models described in the next section before convolution of the brightness distribution with the antenna pattern. A similar procedure was employed by Scott (1973). By comparing the expected flux density with that measured in observations at other frequencies the appropriate value of T_e can be found. In practice low frequency measurements must be used since $S(\nu, T_e)$ is sensitive to T_e only at low enough frequency. For NGC 6720 we use the 195 MHz flux density of Terzian (1966) for comparison and for NGC 6543 the recent accurate 610 MHz determination using the WSRT (Willis *et al.*, in preparation). For NGC 40 the low frequency spectrum (Higgs, 1971) is not sufficiently well determined to be useful. The electron temperatures for the nebulae are given in Table 3.

The temperature calculated for NGC 6543 from the radio data is in good agreement with that of 8200 K obtained from the [O III] forbidden line ratios by Aller *et al.* (1968). Schmitter and Millis (1967) find a forbidden line temperature of 9900 K for NGC 6720.

Table 3. Temperatures, adopted distances, densities and masses

Nebula	T_e(K)	D(Kpc)	n_e(cm^{-3})	$M/M_\odot$	$M_c/M_\odot$[a])
NGC 40	—	1.78	1100	0.30	0.42
NGC 6543	7900 ± 300	1.11	3900	0.05	0.07
NGC 6720	3500 ± 2000	1.26	600	0.23	0.32

[a]) The masses corrected for helium content have been computed on the assumption that the helium is singly ionized, from the expression

$$M_c = M_H(1+4A_{He})(1+A_{He})^{-1},$$

where A_{He} is the abundance of helium relative to hydrogen by number and M_H is the uncorrected mass. Note that the factor $(1+A_{He})^{-1}$ arises because the total bremsstrahlung in the presence of helium is obtained by summing the contributions due to H^+ and He^+.

The corresponding temperature derived from the radio data is therefore much lower. Since the determination of the temperature for NGC 6720 depends on only one low frequency measurement which may contain substantial error, it is possible that the true temperature lies above the estimated maximum value of 5500 K. However the discrepancy is so large that it seems possible that the difference could be due to physical causes. Low temperatures have sometimes been derived from radio continuum data by other observers and a discussion of the problem can be found in Thompson (1967, 1968). It is suggested there that at least a part of the disparity can be attributed to temperature variations in the nebulae. In a well resolved nebula the observed radio flux density, and hence the estimated T_e, is proportional to $\int n_e^2 T^{-.35} e^{-\tau} dl$ where τ is the optical depth to the gas element situated l cm below the surface along the line of sight, $n_e^2 T^{-.35}$ is proportional to the generation rate of radio energy and T is the electron temperature at l. Because the integrand depends on $T^{-.35}$, T_e is weighted in favour of regions with lower temperature. The weighting will be enhanced because the cooler gas tends to be compressed by the hotter surrounding medium thereby increasing the local n_e. The procedure outlined above for determining T_e, therefore, will tend to yield the temperature of the cooler parts of the nebulae. Measurement of, for example, the [O III] line ratios on the other hand gives the temperature of the hotter regions.

An alternative mechanism has been offered by Salem and Seaton (1974). They point out that if the interferometer does not resolve all the significant detail inside the nebula an overestimate of the $\Delta\Omega_i$ will result. The above procedure will then underestimate the value of T_e. The low derived temperature for NGC 6720 could, therefore, indicate that important structural details are unresolved in these observations. In view of the small scale structure known to exist in some planetary nebulae this effect is probably by far the more important.

Models and Estimated Densities

It has already been pointed out that the maps of NGC 40 and NGC 6720 exhibit a high degree of symmetry. The ring of emission, most pronounced in NGC 6720, and the central minimum in their brightness distributions is typical of emission from a shell source. Moreover, the presence in each of these maps of two-resolved maxima coinciding with peaks in the optical isophotes shows that the radiating gas is not distributed with spherical symmetry. A simple morphology which accounts for the main characteristics of the contour maps of NGC 6720 and NGC 40 is that of a cylindrical shell uniformly filled with ionized gas and inclined to the line of sight. The cylindrical shell is regarded as an approximation to the true gas distribution in the nebula, and can, in fact, be used to approximate several different morphologies; for example, it approximates an ellipsoidal shell or a toroid. Apart from this, the main advantage of using a cylindrical shell is the ease with which computations can be performed on the model. The model can be made to fit the observed contours by adjusting the dimensions of the cylindrical shell, the angle of inclination of its axis and convolving the resulting brightness distribution with the antenna pattern. Similar models have been employed by Khromov and Kohoutek (1968) in a discussion of the optical isophotes of planetary nebulae. Scott (1973) interpreted the radio contours of NGC 7027 in terms of emission from such a shell. The map of NGC 6543 does not contain the well-separated maxima of the other two nebulae and is not characteristic of a clearly resolved shell source. Accordingly, we have used a uniformly filled cylinder to represent this nebula. Although a satisfactory approximation to the intermediate contours can be made, the inner and outer contours cannot be represented well in this case.

Figures 1b and 3b show the closest approximations to the contours of NGC 40 and NGC 6720. The correspondence between the observed and calculated central contours of NGC 6720 would be improved if some material were present inside the hollow cylindrical shell. The parameters defining the shells are given in Table 4. It can be seen that except for NGC 6543 the required shell thickness is small compared to the radius.

Table 4. Model parameters

Nebula	Outer diameter	Inner diameter	Length	Tiltangle (θ)	Shell thickness
NGC 40	39″	27″	30″	41°	6″
NGC 6543	16″	0″	24″	30°	—
NGC 6720	60″	46″	54″	20°	7″

If D kpc is the distance to each nebula then the *rms* density is given by

$$n_e = 1920\, F^{1/2}\, T_e^{.17}\, V^{-1/2}\, D^{-1/2}\,. \tag{4}$$

where V is the volume of the model in cubic arc seconds and F is the flux density in m.f.u. Errors in the four quantities determining n_e will produce smaller fractional errors in n_e. In particular errors in V due to an incorrect assumed morphology will not affect the value of n_e too greatly. The value of n_e is very insensitive to T_e. We have taken $T_e = 10^4$ K which is close to the values usually obtained from the forbidden line ratios, and we have used the distances of Cudworth (1973). The calculated densities are given in Table 3. These r.m.s. densities are low compared to those of, for example, Kaler (1970). Table 3 also contains the radiating masses obtained from the r.m.s. densities assuming the nebulae are pure hydrogen and also corrected to allow for a 15% helium content. Since the nebulae probably contain density fluctuations these are upper limits to the true ionized masses. Because the masses derived above are proportional to $D^{5/2}$, uncertainties in the distance will result in large errors in the mass. The masses found here are, however, in agreement with the average mass of $0.14\, M_\odot$ given by O'Dell (1962) and of $0.17\, M_\odot$ given by Seaton (1968) and Higgs (1973).

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