Fluctuation conductivity and Ginzburg-Landau parameters in high-temperature superconductors above Tc: effect of strong inelastic scattering

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Fluctuation conductivity and Ginzburg-Landau parameters in high-temperature superconductors above T_c: Effect of strong inelastic scattering

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(Received 22 September 1993)

The normal state of the high-T_c superconductors near optimal doping is characterized by the presence of strong inelastic scattering, leading to anomalous properties, most prominently a linear-in-temperature resistivity over a very large temperature range. We study the effect of this scattering on the correction to the conductivity due to thermal fluctuations of the order parameter and on the Ginzburg-Landau parameters above T_c. The fluctuation conductivity is affected (reduced) by the inelastic scattering, as compared to the case with a constant pairbreaking scattering rate (magnetic impurities). This leads to a substantial enhancement of an effect that was recently proposed by Ioffe *et al.* to account for the observed upturn of the *c*-axis resistivity above T_c.

I. INTRODUCTION

The anomalous normal-state properties of the high-temperature superconductor near optimal doping, like the linear-in-temperature resistivity over a large temperature range and the linear-in-frequency width of the quasiparticle peaks in photoemission experiments, indicate that the imaginary part of the self-energy behaves as

$$\mathrm{Im}\Sigma(\omega,T)=(\lambda\pi/2)\max(\omega,T)\ . \tag{1}$$

When starting from the marginal-Fermi-liquid ansatz for the polarizability, as proposed by Varma *et al.*,[1] one obtains $\lambda=g^2N(0)^2$, g being a coupling constant and $N(0)$ the density of states at the Fermi energy.

This behavior, if still valid at zero temperature and frequency, a regime which actually is hidden due to the occurrence of superconductivity in the cuprates, would imply a "just-breakdown" of the quasiparticle concept, hence the name "marginal Fermi liquid." If, on the other hand, a small energy scale exists in the system, for instance due to low-lying spin fluctuations, it might well be possible that the anomalous normal-state properties are consistent with a Fermi-liquid-like picture below this small energy scale and thus do not imply a breakdown of the quasiparticle concept in the cuprates.[2]

In either case, however, strong inelastic scattering dominates the physics of the normal state. It has been pointed out that this inelastic scattering in the normal state largely affects the superconducting state: First of all the transition temperature is lowered substantially as a consequence of pair breaking.[3-6] Secondly, the suppression of coherence peaks and a steep behavior of the gap, with an enhanced value of $2\Delta(0)/k_BT_c$, might arise as consequences of a strong temperature dependence of the inelastic scattering rate below T_c.[3-6,8]

The high transition temperature, the two-dimensionality of the high-T_c cuprate superconductors, and the short coherence length enhance the thermal fluctuations of the order parameter near T_c in comparison to classical superconductors. It is therefore of relevance to study the effect of the strong inelastic scattering above the transition temperature on the correction to physical quantities due to these thermal fluctuations.

In this paper we shall analyze the Azlamasov-Larkin fluctuation conductivity[9] in the presence of the inelastic scattering that leads to the self-energy (1). We find an appreciable change in the Ginzburg-Landau parameters and the current vertex; this leads to a suppression of the fluctuation conductivity as compared to the case of a constant [i.e., independent of frequency, unlike (1)] pair-breaking scattering rate (e.g., magnetic impurities), which increases with decreasing dimensionality and increasing scattering strength.

Recently, it was shown by Ioffe *et al.*[10] that in the *c* direction of strongly anisotropic superconductors, fluctuation corrections to the resistivity first lead to a resistivity enhancement, as a consequence of electron scattering against virtual Cooper pairs, before the *zero-dimensional* Azlamasov-Larkin fluctuation correction lowers the resistivity close to T_c. The magnetic impurity type of pair breaking, made temperature but not frequency dependent, was used in Ref. 10, and our findings thus modify the results of Ref. 10. We find that their effect is substantially enhanced and might explain the upturn of the *c*-axis resistivity in the cuprates just above T_c upon lowering the temperature.[10]

II. FLUCTUATION CONDUCTIVITY IN A MARGINAL FERMI LIQUID ABOVE T_c

In the presence of strong pair breaking, the leading contribution to the fluctuation conductivity above the superconducting transition is the Azlamasov-Larkin correction. The Maki-Thompson contribution, which describes the scattering of a particle-hole pair into another particle-hole pair by exchange of a virtual Cooper pair, is largely suppressed by the pair breaking. We shall therefore concentrate on the Azlamasov-Larkin correction for a system which displays marginal Fermi-liquid behavior above the superconducting transition.

The Azlamasov-Larkin diagram (Fig. 1) represents a contribution to the current

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$$\mathbf{j}_{\rm AL}(i\Omega_l)=\frac{4e^2}{m^2}T\sum_{i\Omega_m}\int\frac{d\mathbf{Q}}{(2\pi)^D}\mathbf{V}(\mathbf{Q},i\Omega_m,i\Omega_l)^2\Gamma(\mathbf{Q},i\Omega_m)\Gamma(\mathbf{Q},i\Omega_m-i\Omega_l)\mathbf{A}(i\Omega_l)\ , \tag{2}$$

where $\mathbf{A}$ is the vector potential. The "vertex function" $\mathbf{V}$ is given by

$$\mathbf{V}(\mathbf{Q},i\Omega_m,i\Omega_l)=-2T\sum_{i\omega_n}\int\frac{d\mathbf{p}}{(2\pi)^D}2\mathbf{p}G(\mathbf{p},i\omega_n)G(\mathbf{p},i\Omega_l+i\omega_n)G(\mathbf{Q}-\mathbf{p},i\Omega_m-i\Omega_l-i\omega_n) \tag{3}$$

and $\Gamma(\mathbf{Q},i\Omega_m)$ is the pair propagator.

First, the pair propagator $\Gamma(\mathbf{Q},i\Omega_m)$ is calculated. It is given by the sum of the geometric series (Fig. 2), $\Gamma=-V/(1-X)$, where V is the usual BCS model interaction, which is constant and attractive up to an energy ω_0, and X is given by

$$X(\mathbf{Q},i\Omega_m)=VT\sum_{|\omega_n|<\omega_0}\int\frac{d\mathbf{k}}{(2\pi)^D}G(\mathbf{k}+\mathbf{Q},i\omega_n+i\Omega_m)G(-\mathbf{k},-i\omega_n)\ . \tag{4}$$

The temperature Green function in the normal state is

$$G(\mathbf{k},i\omega_n)=[i\omega_n-\epsilon(\mathbf{k})-\Sigma(i\omega_n)]^{-1}\ . \tag{5}$$

From analytic continuation of the self-energy $\Sigma(\omega)$, it follows that at the Matsubara frequencies

$$\Sigma(i\omega_n)=-i\lambda T\arctan\left[\frac{T}{\omega_n}\right]-\frac{1}{2}i\lambda\omega_n\ln\left[\frac{\omega_n^2+\omega_c^2}{\omega_n^2+T^2}\right]\ . \tag{6}$$

Here ω_c is an upper cutoff, estimated to be at least 0.5 eV.

We thus consider a system in which the *dominant* interaction leads to marginal-Fermi-liquid behavior. This does not rule out that a different, weaker interaction (V) causes superconductivity, provided retardation effects play a role such that the different interactions operate on different time scales. (A high transition temperature might in such a case be due to a large scale ω_0 or to the presence of a van Hove singularity in the density of states.[12])

The simple BCS model form that we assume for V makes an explicit calculation of the pair propagator $\Gamma(\mathbf{Q},i\Omega_m)$, in the limit of small $\mathbf{Q}$ and small Ω, possible. In the limiting case $\lambda=0$ one finds the well-known result

$$\Gamma^{-1}(\mathbf{Q},i\Omega_m)=-N(0)\left[\eta Q^2+\alpha\Omega_m+\frac{T-T_c^0}{T_c^0}\right]\ , \tag{7}$$

where T_c^0 is the usual BCS transition temperature, defined as the temperature where $\Gamma(0,0)$ diverges,

$$T_c^0=1.13\omega_0\exp\left[-\frac{1}{N(0)V}\right] \tag{8}$$

and where the coefficients η and α follow from a small $\mathbf{Q}$ and small Ω expansion of (4), giving

$$\eta=\frac{7\xi(3)v_F^2}{16D\pi^2(T_c^0)^2} \tag{9}$$

and $\alpha=\pi/8T_c^0$.

In the presence of scattering, e.g., as implied by (6) or in the presence of impurities, the form of (7) for Γ^{-1} is preserved, though the Ginzburg-Landau parameters α and η are renormalized.

It is instructive to first consider the interesting case of magnetic impurity scattering, with a constant scattering rate $1/\tau$ in the Green function G. The suppression of the transition temperature due to the pair breaking by the magnetic impurities is found to be[7]

$$\ln\left[\frac{T_c^0}{T_c^R}\right]=\psi\left[\frac{1}{2}+\frac{1}{4\pi T_c^R\tau}\right]-\psi\left[\frac{1}{2}\right]\ , \tag{10}$$

where $\psi(x)$ is the Digamma function. The renormalized Ginzburg-Landau coefficient η^R can be evaluated exactly to be[9]

$$\eta^R=-\frac{1}{D}v_F^2\left\{\tau^2\left[\psi\left[\frac{1}{2}+\frac{1}{4\pi T_c^R\tau}\right]-\psi\left[\frac{1}{2}\right]\right]-\frac{\tau}{4\pi T_c^R}\psi'\left[\frac{1}{2}\right]\right\}\ . \tag{11}$$

This expression is plotted as the dashed curve in Fig. 3.

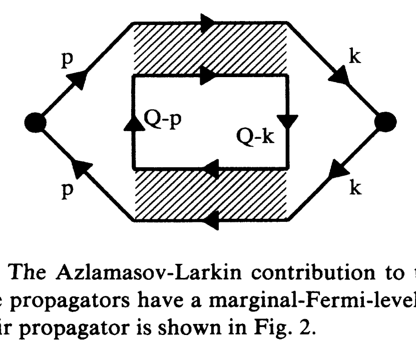

FIG. 1. The Azlamasov-Larkin contribution to the conductivity. The propagators have a marginal-Fermi-level self-energy and the pair propagator is shown in Fig. 2.

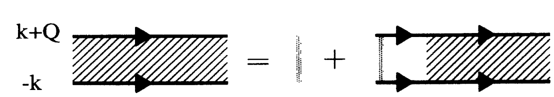

FIG. 2. The pair propagator $\Gamma(\mathbf{Q},i\Omega_m)$. The grey lines denote simple BCS interactions, the propagators have a marginal-Fermi-liquid self-energy.

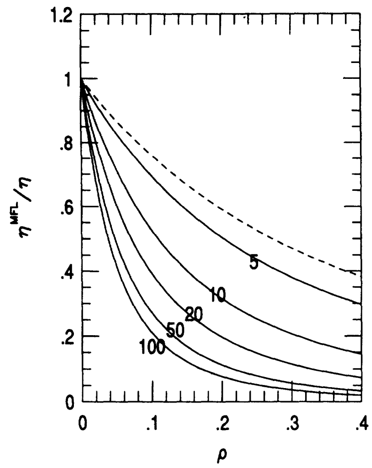

FIG. 3. The ratio η^R/η (dashed curve) and $\eta^{\rm MFL}/\eta$ (solid curves) for the case with a marginal-Fermi-liquid self-energy (solid curves) with ω_c/T_c as a parameter, both as a function of $\rho=1/(4\pi T\tau)$. For the marginal-Fermi-liquid case, with $1/2\tau=\mathrm{Im}\Sigma$ and $T>T_c>\omega=0, \rho=\lambda/4$.

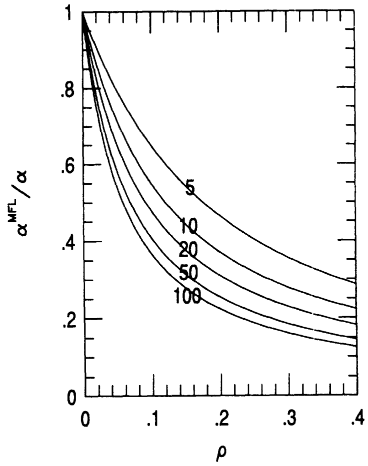

FIG. 4. Renormalization of the parameter α, normalized to the bare value $\pi/(8T_c)$, as a function of the scattering strength ρ.

The coefficient α is only slightly modified by the impurity scattering

$$\alpha^R=\frac{\pi}{8T_c^R}+\frac{\psi''(1/2)}{(4\pi T_c^R)^2\tau} \ . \tag{12}$$

In the limit of large τ the above expressions reduce to the unrenormalized coefficients.

In presence of the self-energy (1) the renormalized coefficients $\eta^{\rm MFL}$ and $\alpha^{\rm MFL}$ and the suppressed transition temperature cannot be calculated analytically due to the complicated summation over the Matsubara frequencies in (4). The solid curves in Fig. 3 show the result of a numerical evaluation of $\eta^{\rm MFL}$ as a function of the scattering strength λ, with ω_c/T_c as a parameter. It is seen that $\eta^{\rm MFL}$ is a decreasing function of λ, and for fixed λ the value of $\eta^{\rm MFL}$ decreases with increasing ω_c/T_c. It is clear that the marginal-Fermi-liquid self-energy leads to a reduced value of η, compared to the case of a constant scattering rate. Later we shall see what consequences this has for the Azlamasov-Larkin dc fluctuation correction.

The renormalization of α (Fig. 4) expresses how the Ginzburg-Landau relaxation time is renormalized by the inelastic scattering. Of course, the Ginzburg-Landau time is reduced as a result of the pair breaking by the inelastic scattering. This is of importance for dynamic responses above T_c, but, as we shall see, it also influences the dc limit. Likewise, the fact that $\eta^{\rm MFL}$ is reduced relative to η^R, implies a decrease in the correlation length above T_c. The suppression of the transition temperature was treated, in an approximate way, earlier by us.[5]

The second ingredient that is needed in the calculation of the fluctuation conductivity is the vertex function $\mathbf{V}(\mathbf{Q},i\Omega_m,i\Omega_l)$. We shall calculate only the dc limit of the conductivity, i.e., the case $\Omega_l=0$. $\mathbf{V}(\mathbf{Q},i\Omega_m)$ is evaluated in lowest nonvanishing order in frequency and momentum, since the pair propagator is strongly peaked at small values of its arguments and thus V contributes only significantly at small frequency and momentum. At zeroth order in Ω_m and first order in $\mathbf{Q}$ (the zeroth order in $\mathbf{Q}$ gives zero due to the vector nature of the current vertex) one finds that $\mathbf{V}=C\mathbf{Q}$, where

$$C=\frac{4T}{Dm}\sum_{i\omega_n}\int\frac{d\mathbf{p}}{(2\pi)^D}|\mathbf{p}|^2G^2(\mathbf{p},i\omega_n)G^2(-\mathbf{p},-i\omega_n) \ . \tag{13}$$

After some algebra it turns out that this expression is proportional to η,

$$C=8mN(0)\eta \ . \tag{14}$$

Using the results for Γ and $\mathbf{V}$ yields the Azlamasov-Larkin correction to the dc conductivity,[9]

$$\sigma_{\rm AL}=\frac{2^8e^2T_c}{D}\alpha\eta^2\int\frac{d\mathbf{Q}}{(2\pi)^D}\frac{Q^2}{[(T-T_c)/T_c+\eta Q^2]^3} \ , \tag{15}$$

where T_c, η, and α are the renormalized parameters. This expression is rewritten as

$$\sigma_{\rm AL}=B_D\alpha\eta^{1-D/2}\left[\frac{T_c}{T-T_c}\right]^{2-D/2} \tag{16}$$

and

$$B_D=\frac{2^8e^2k_BT_c}{\hbar D}\int_0^\infty dx\frac{x^{D+1}}{(x^2+1)^3} \ . \tag{17}$$

In the latter equation, we have reintroduced $\hbar$ and k_B explicitly. As (16) shows, the scattering rate dependence enters $\sigma_{\rm AL}$ through the coefficient $\alpha\eta^{1-D/2}$, and thus has a dimensionality dependent influence.

III. DISCUSSION

We have seen that the use of the Green functions with the marginal-Fermi-liquid self-energy in the calculation of the dc Azlamasov-Larkin fluctuation conductivity leads to a change of the parameters η and α, which appear in the final result (16) as the prefactor $\alpha\eta^{1-D/2}$. In Fig. 5 we have plotted the enhancement factor of the Azlamasov-Larkin contribution due to the marginal-Fermi-liquid effects in $D=3$, 2, 1, and 0 compared to the case of a constant magnetic impurity scattering rate as a function of $\rho=1(4\pi T_c\tau)$. For the case of the marginal-Fermi-liquid self-energy above T_c, $1/\tau=\lambda\pi T/2$, $\rho=\lambda/4$. It is seen that the frequency dependence of the pair-breaking scattering rate enhances the effect of the pair breaking and thus further reduces the fluctuation conductivity. From resistivity measurements it can be estimated that λ varies roughly between 0.25 and 1 in the different cuprates.

Despite the reduction of the fluctuation conductivity, fluctuation effects are observable up to high temperatures due to the small bare conductivity σ_0. The resistance ratio

$$\frac{R}{R_0}=\frac{1}{1+\sigma_{\mathrm{AL}}/\sigma_0} \tag{18}$$

deviates over a large temperature range from 1 when σ_0 is small.

Recently, Ioffe *et al.*[10] have shown that in strongly anisotropic materials fluctuation effects initially lead to an increase of the c-axis resistivity above T_c upon lowering the temperature, as a consequence of electron scattering against Cooper pairs, before it drops to zero. The decrease of the conductivity near T_c is first driven by *zero-dimensional* fluctuations and even closer to T_c by three-dimensional fluctuations. This observation might help to understand the observed upturn of the c-axis resistivity near T_c in the cuprate superconductors.[11] While it is difficult to extract parameters from the experiments accurately, the authors of Ref. 10 estimate this effect to be too small to fully account for the observed upturn. Moreover, the resistivity minimum appears too close to T_c (see the dashed curve in Fig. 6).

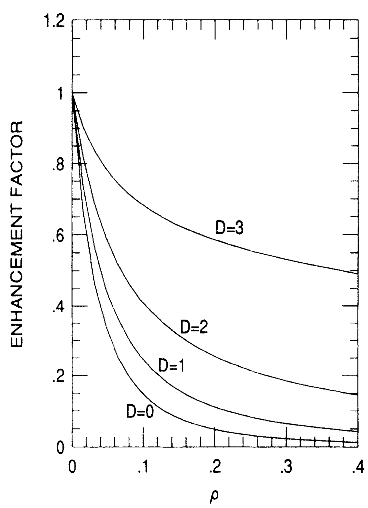

FIG. 5. The enhancement factor $(\alpha^{\mathrm{MFL}}/\alpha^{R})(\eta^{\mathrm{MFL}}/\eta^{R})^{1-D/2}$ of the Azlamasov-Larkin contribution due to the marginal-Fermi-liquid effects in $D=3$, 2, 1, and 0 compared to the case of a constant scattering rate as a function of $\rho=1/(4\pi T\tau)$.

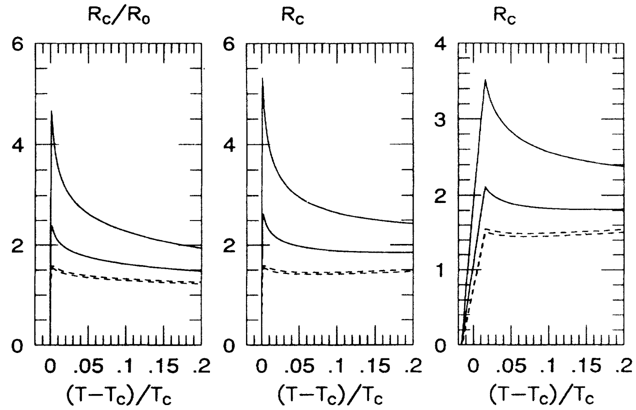

FIG. 6. The c-axis resistivity. The left figure shows the ratio R/R_0, where R_0 is the bare resistivity, the middle figure shows R with $R_0 \propto T$. The rightmost figure was obtained with a box distribution of T_c's with a $3K$ width. The dashed curves are obtained by using the expression derived by Ioffe *et al.*,[10] the solid curves are obtained taking the effects of the marginal-Fermi-liquid self-energy into account. The upper dashed curve corresponds to $\lambda=0.6$ and the lower dashed curve to $\lambda=0.3$. The same parameters are used for the solid curves.

Taking the marginal-Fermi-liquid self-energy into account, the effect by Ioffe *et al.*, is substantially enlarged via the changes in η and α, as is illustrated by the solid lines in Fig. 6. With a linear-in-temperature bare resistivity, the results of Ioffe *et al.*, hardly produce an upturn of the c-axis resistivity. Especially when a variation of T_c through the sample is taken into account, a small shoulder rather than a clear upturn is produced, as shown in Fig. 6(c). Well above T_c the resistivity is linear in temperature. The enhancement of the upturn in R_c/R_0 in Fig. 6(a) leads to a clear minimum in the c-axis resistivity R_c in Fig. 6(b). The high temperature where the minimum in R_c occurs (above $1.2T_c$) is in agreement with the experimental observations of Ref. 11.

In conclusion, we have discussed the effect of a linear-in-temperature and linear-in-frequency scattering rate on the Azlamasov-Larkin fluctuation conductivity. The Ginzburg-Landau parameters and the current vertices renormalize appreciably due to the inelastic scattering. We find that the fluctuation conductivity is reduced as compared to the case of magnetic impurity scattering. This causes an enhancement of the effect which was proposed by Ioffe *et al.*, to account for the upturn of the resistivity in the c direction near optimal doping.

ACKNOWLEDGMENTS

M.H. and F.V. are supported financially by the Dutch Foundation for Fundamental Research on Matter (FOM).

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[1]C. M. Varma, P. B. Littlewood, S. Schmitt-Rink, E. Abrahams, and A. E. Ruckenstein, Phys. Rev. Lett. **63**, 1996 (1989); **63**, 1996(E) (1989).

[2]K. Levin, Ju H. Kim, J. P. Lu, and Qimao Si, Physica C **175**, 449 (1991).

[3]P. A. Lee and N. Read, Phys. Rev. Lett. **58**, 2691 (1987).

[4]C. Bandte, P. Hertel, and J. Appel, Phys. Rev. B **45**, 8026 (1992); C. Bandte, *ibid.* **47**, 5473 (1993).

[5]M. L. Horbach, F. L. J. Vos, and W. van Saarloos, Phys. Rev. B **48**, 4061 (1993).

[6]P. B. Littlewood and C. M. Varma, Phys. Rev. B **46**, 405 (1992).

[7]A. A. Abrikosov and L. P. Gorkov, Zh. Eksp. Teor. Fiz. **39**, 1781 (1961) [Sov. Phys. JETP **12**, 1243 (1961)]; K. Maki, in *Superconductivity,* edited by R. D. Parks (Dekker, New York, 1969), p. 1037.

[8]L. Coffey, Phys. Rev. Lett. **64**, 1071 (1990).

[9]L. G. Azlamasov and A. I. Larkin, Phys. Lett. A **26**, 238 (1968).

[10]L. B. Ioffe, A. I. Larkin, A. A. Varlamov, and L. Yu, Phys. Rev. B **47**, 8936 (1993).

[11]S. J. Hagen, T. W. Jing, Z. Z. Wang, J. Horvath, and N. P. Ong, Phys. Rev. B **37**, 7928 (1988); T. Penney, S. von Molnár, D. Kaiser, F. Holtzberg, and A. W. Kleinsasser, *ibid.* **38**, 2918 (1988).

[12]J. Labbé and J. Bok, Europhys. Lett. **3**, 1225 (1987); D. M. Newns, C. C. Tsuei, P. C. Pattnaik, and C. L. Kane, Comments Condens. Matter Phys. **15**, 273 (1992); C. C. Tsuei, C. C. Chi, D. M. Newns, P. C. Pattnaik, and M. Däumling, Phys. Rev. Lett. **69**, 2134 (1992).