THE ASTROPHYSICAL JOURNAL, 365:230–238, 1990 December 10


# INTERSTELLAR CIRCULAR POLARIZATION AND THE DIELECTRIC NATURE OF DUST GRAINS

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*Received 1987 December 29; accepted 1989 October 25*

## ABSTRACT

The relationship between interstellar linear and circular polarization is reexamined. Results of Mie theory calculations show that the observed coincidence of the crossover wavelength of circular polarization and the peak wavelength of linear polarization is independent of the optical constants assumed for the interstellar grains and is therefore not a critical constraint on the grain properties. This result agrees with the general argument derived in several studies from the Kramers-Kronig relations. It is shown that the apparent upper limit on the visual absorptivity of grains derived from earlier Mie theory calculations, which has been regarded as one of the principal constraints on grain properties, resulted from a violation of the Kramers-Kronig relations in the assumed optical constants of the particles.

The properties of dielectric core-mantle grains in the diffuse interstellar medium are discussed. The optical constants derived from available laboratory data indicate that the mantle material should be a dielectric with a low-energy gap ($\sim 1$ eV) and relatively high absorptivity at visible wavelength (the imaginary part of the index of refraction at least as high as 0.15 at 5500 Å). The model which incorporates the silicate core–organic refractory mantle grains, as well as smaller particles required to account for the ultraviolet extinction, is shown to satisfy all available constraints on the properties of grains in the visual.

*Subject headings:* interstellar: grains — polarization

## I. INTRODUCTION

Observations of interstellar circular polarization are regarded as one of the essential constraints on theoretical models of interstellar grains. Although the measurements are difficult, and the data are limited to only a handful of stars, all of them consistently show that the sign of circular polarization changes at a wavelength $\lambda_c$, which is always close to the peak wavelength of linear polarization, $\lambda_{\max}$. This effect was initially discovered in several lines of sight with $\lambda_{\max}$ close to the general average value of 5500 Å (Kemp and Wolstencroft 1972; Martin, Illing, and Angel 1972) and was later demonstrated for lines of sight in which $\lambda_{\max}$ varied from 4000 Å to 8000 Å (Martin and Angel 1977). Mie theory calculations (Martin 1972, 1974) apparently indicated that the observed condition $\lambda_c \approx \lambda_{\max}$ could be satisfied only if the particles responsible for the interstellar polarization were almost entirely transparent; i.e., the imaginary part of their index of refraction ($m = m' - im''$) was low at visible wavelengths. Martin's criterion, which effectively imposed an upper limit on the value of $m''$ in the visible, appeared to have immediate implications for theoretical models of grains. A complementary restriction on the properties of particles at visual wavelengths can be derived from the observations of the visual albedo, which yield an average value of $\sim 0.6$ (Savage and Mathis 1979), and thus imply that at least one of the populations contributing to visual extinction must be significantly absorptive. A combination of these two criteria seemed to offer a hope of eliminating at least some of the proposed models of grains. In this paper, we shall focus our attention on two grain models, which differ clearly in their approach to explaining polarization, and should therefore be sensitive to observational criteria related to it. In the graphite + silicate model (Mathis, Rumpl, and Nordsieck 1977 [MRN]; Mathis 1986), graphite cannot be aligned so that polarization is attributed entirely to silicate particles, whose absorptivity at visible wavelengths is very low. Approximately 70% of the visual extinction ($\lambda_V = 5500$ Å) comes from strongly absorptive graphite grains (Draine and Lee 1984); consequently, the model encounters no difficulty in matching the low value of the observed albedo. In the three-population model (Greenberg 1978; Hong and Greenberg 1980), large core-mantle grains, which are the only source of polarization, account for as much as 90% of visual extinction, with the remaining 10% due to absorption by particles giving rise to the 2200 Å hump. Therefore, in order to match the observed visual albedo of the entire population of interstellar grains, the value for core-mantle grains alone must not be higher than $\sim 0.7$. Since about 90% of the volume of core-mantle grains is contained in the mantle (Hong and Greenberg 1980; Chlewicki 1985), the optical properties of the entire particle are determined by the index of refraction of the mantle material. The validity of the three-population model therefore depends on whether the upper limit on the value of $m''$ for polarizing grains derived from the analysis of circular polarization is consistent with the required low albedo. Martin's original interpretation of his Mie theory calculations indicated that the highest allowed value of $m''$ in the visual was $\sim 0.1$, with a corresponding value of the albedo of polarizing grains of 0.7 (Martin 1974); i.e., within the range required by the three-population model. On the other hand, using Martin's calculations Mathis (1983) derived (for $m' = 1.5$) an upper limit of $m'' = 0.03$(!) and with this result ruled out the three-population model (Mathis

1986) because of the unacceptably high visual albedo (at least 0.9 with Mathis's upper limit on $m''$).

In the present paper, we resolve this controversy by demonstrating that the interpretation of Martin's Mie theory calculations was incorrect and that the relationship between the observed $\lambda_{max}$ and $\lambda_c$ in fact provides no fundamental restriction on the optical properties of the constituent materials of interstellar grains. An alternative interpretation which, as we shall demonstrate, cannot be reconciled with the "circular polarization criterion" constraining the imaginary part of the visual index of refraction, is based on the application of Kramer-Kronig relations. This approach to interstellar circular polarization was first suggested by Kemp (1972) and Kemp and Wolstencroft (1972, 1974) and subsequently given a very clear and elegant formulation by Shapiro (1975), who also showed that a strongly absorbing conductor (magnetite with room temperature optical constants) could reproduce a coincidence of $\lambda_c$ with $\lambda_{max}$. It is surprising that—with very few exceptions—this argument has received little attention in subsequent publications on interstellar polarization, one exception being Martin (1975*b*). Shapiro's result for magnetite was attributed to the particular wavelength dependence of the index of refraction of the material close to 5500 Å, with the implication that magnetite would not reproduce the observed correlation between $\lambda_c$ and $\lambda_{max}$ in those lines of sight in which $\lambda_{max}$ differs from the overall average (Mathis 1983; Aannestad and Greenberg 1983). In this paper, we shall reexamine the implications of the Kramers-Kronig argument and illustrate its universality with examples derived from Mie theory calculations. The same calculations will also serve to make apparent the origin of the misreading of the effect of grain absorption on the wavelength-dependence of circular polarization calculated by Martin (1972, 1974).

The conclusions reached in this paper are independent of the specific characteristics of the grains, *so long as they provide the correct linear polarization.* However, numerical examples require specific assumptions about grain materials and have been calculated for a population of core-mantle particles included in the three-population model. The derivation of the optical constants for relevant materials is described in § II, with emphasis on general constraints on the electromagnetic response of an optical medium (Kramers-Kronig relations), which also provide the basis for the subsequent analysis of interstellar polarization. In § III, we summarize the formulae that relate the observed polarization to the properties of individual grains. The analysis of circular and linear polarization in the interstellar medium is presented in § IV. In the final section, we discuss the implications of this analysis for grain models.

## II. OPTICAL CONSTANTS OF ASTROPHYSICAL SOLIDS

The optical constants of most materials likely to be the constituents of interstellar grains cannot be derived directly from experimental measurements, and a "synthesis" of various laboratory results is usually required (Draine and Lee 1984). In this section, we derive the optical properties of materials which in the three-population model are assumed to form the cores and mantles of large dielectric grains. The mantle material will subsequently be used as a "generic" dielectric in the analysis of the influence of the optical properties of grains on the linear and circular polarization.

The cores of large grains in the three-population model are assumed to consist of silicates (Hong and Greenberg 1980). Since the mantle dominates the optical properties of the particles, any refractory dielectric could be assumed for the cores with no significant effect on Mie theory results. However, some form of amorphous silicate material is suggested by the observations of the 9.7 $\mu$m SiO feature. We have adopted the optical constants of silicates in the UV and in the visual from experimental measurements for olivine (Huffman and Stapp 1973), with a small fixed amount of absorption ($\epsilon'' = 0.12$) added in the visual to account for the amorphous structure of interstellar grains. In the near-infrared, we have used the data obtained for amorphous silicates by Day (1979). The resulting set of optical constants is very similar to the properties of "cosmic silicate" derived by Draine and Lee (1984).

The derivation of the optical constants for grain mantles is more complex, because there are as yet no direct laboratory measurements of the index of refraction similar to the data used for silicates. The mantles in the diffuse medium are assumed to consist of the refractory component of complex molecular mixtures initially accreted in molecular clouds and subsequently photoprocessed (Greenberg 1982*a*). The formation of such refractory residues in irradiated molecular ices has been shown experimentally; however, the doses of radiation applied in the laboratory have so far been limited to many orders of magnitude less than those received by grains in the diffuse medium. Materials studied in the laboratory are therefore representative of grain mantles which exist shortly after the particles emerge from molecular clouds (first-generation organic refractory), but their optical properties must differ from those of more highly photoprocessed mantle material in the diffuse medium. Existing laboratory measurements of refractory residues must consequently be taken only as general indications of the changes induced in organic refractory grain mantles by UV irradiation over times of $10^8$ yr.

We have assumed that the elemental composition of the mantles in the diffuse medium is likely to be close to that given by the ratios C:O:N:H = 2:1:1:2. The value of 1:1 adopted for the C:H ratio, which is particularly important in determining the optical properties of the mantle, is consistent with the number derived from the observed strength of the 3.4 $\mu$m feature (Greenberg 1982*a*), although this estimate depends on what fraction of carbon in the organic refractory is assumed to form aliphatic bonds. The high value assumed for the C:O ratio is based on the chemical analysis of laboratory samples of organic residues (Agarwal *et al.* 1985), which indicates an increase in the carbon-to-oxygen ratio from 1:4 in the initially accreted volatile material to more than 1:1 in first-generation organic refractory. The wavelength-dependence of electronic absorption is obviously determined by the specific molecular composition of the material; however, a survey of the spectra of complex organic molecules indicates that the strongest features are likely to occur in the far-UV; i.e., for photon energies greater than 7–8 eV (Calvert and Pitts 1966; Silverstein and Bassler 1967). The relative oscillator strength in the near-UV is determined by the fraction of carbon in $sp^2$ bonds, which will give rise to $\pi$-electron absorption below 6 eV. In the examples presented in this paper, we have assumed that the $\pi$-electron absorption in the mantles is relatively weak. The near-infrared properties (1 $\mu$m < $\lambda$ < 20 $\mu$m) have been derived from laboratory spectra of first-generation organic refractory (Schutte 1988); no attempt was made to derive the optical constants at longer wavelengths. We allowed the imaginary part of the dielectric constant ($\epsilon = \epsilon' - i\epsilon''$; $\epsilon = m^2$ so that $\epsilon' = m'^2 - m''^2$ and $\epsilon'' = 2m'm''$) at visible wavelengths to be varied in order to analyze the effects of various assumptions about the visual absorptivity of the material on the extinction and polarization. The calculations were simplified by assuming that $\epsilon''$ remained constant down to energies of $\sim 1$ eV; a more realistic model of a dielectric should include a decline of the value of $\epsilon''$ toward longer wavelengths.

General physical relationships, which must be satisfied in any optical medium, can be used to derive a consistent set of optical constants from qualitative predictions about the characteristics of astrophysical solids, such as those outlined above for the organic refractory mantles. The most important of these general relations is contained in the Kramers-Kronig formulae, which for the dielectric constant take the form

$$\epsilon'(\omega) - 1 = \frac{2}{\pi} \int_0^\infty \frac{\omega' \epsilon''(\omega')}{\omega'^2 - \omega^2}\, d\omega' \tag{1}$$

$$\epsilon''(\omega) = \frac{2}{\pi} \int_0^\infty \frac{\omega[\epsilon'(\omega') - 1]}{\omega'^2 - \omega^2}\, d\omega' . \tag{2}$$

The Kramers-Kronig formulae applied to any frequency-dependent function which describes the response of a physical system to an oscillating signal ($\epsilon - 1 \propto \chi$, the dielectric susceptibility) are equivalent to the preservation of causality in the system's time-dependent response (Titchmarsh 1937). The discussion of interstellar circular polarization provides an example of the importance of this equivalence. Although the index of refraction does not represent directly the electromagnetic response of the medium ($m = \epsilon^{1/2}$), it satisfies the mathematical conditions necessary for the derivation of the Kramers-Kronig formulae, and equations (1) and (2) can be rewritten with $m$ substituted for $\epsilon$.

The Kramers-Kronig formulae lead to several further relations, some of which are of practical use in deriving the optical constants of materials. For $\omega = 0$, equation (1) yields

$$\epsilon'(0) - 1 = \frac{2}{\pi} \int_0^\infty \frac{\epsilon''(\omega')}{\omega'}\, d\omega' . \tag{3}$$

Since the static dielectric constant is often known for solids of astrophysical interest, equation (3) can serve as a useful constraint on the oscillator strength assumed for a specific material throughout the wavelength range of valence electron absorption.

At high frequencies, the valence electrons behave collectively as a free-electron plasma. The plasma frequency characterizing that behavior can be derived from an asymptotic form of the Kramers-Kronig relations. If $\epsilon''(\omega)$ converges sufficiently rapidly [$\epsilon''(\omega) \to 0$ faster than $(\omega^2 \ln \omega)^{-1}$ for $\omega \to \infty$], one obtains an important sum rule (Bassani and Altarelli 1983):

$$\frac{4\pi e^2}{m_e} n_e n_a = \frac{2}{\pi} \int_0^\infty \omega \epsilon''(\omega) d\omega , \tag{4}$$

where $n_e$ is the effective number of valence electrons per atom, $n_a$ is the volume density of atoms, $e$ and $m_e$ represent the electric charge and the mass of the electron. If the integration in equation (4) is carried out only up to some finite frequency, $\omega_0$, the sum rule yields the number of electrons that are optically active for $\omega \le \omega_0$. This makes it a powerful tool in assigning the absorptions measured in experiments to specific electronic transitions or—as in our study—in linking the optical constants of a material to its assumed chemical composition.

Equations (1)–(4) have been used in calculating the optical constants of the mantle material, one example of which will be shown in § IV. The contribution of electronic transitions in the UV to the polarizability of the material was calculated as a sum of 40 Lorentz oscillators. The imaginary part of the dielectric constant was derived from the Clausius-Mossotti equation. After setting the value of $\epsilon''$ in the visible, the real part of the dielectric constant was derived from equation (1). The total oscillator strength in all transitions (both given directly by the Lorentzians, and implicitly represented by the added absorption in the visible) was constrained by the sum rule in equation (4). Since the distribution of high-energy transitions is uncertain, the integration in equation (4) was carried out only up to 30 eV. In applying the sum rule, we have taken into account that in many materials, transitions above 30 eV may contribute as much as 20%–30% of the value of the integral at $\omega_0 = \infty$ so that for $\omega_0 = 30$ eV, the sum rule may be expected to recover approximately 75% of the total number of valence electrons. In using equation (4), we have assumed a specific gravity of 1.5 g cm$^{-3}$ for the mantle material, which corresponds to $n_a = 7.5 \times 10^{22}$ and the total number of valence electrons per cm$^3$ of $2.6 \times 10^{23}$. One of the most important implications of the Kramers-Kronig relations applied to the organic refractory material is the relatively high value of $m'$, which at visual wavelengths has a value of $\sim 1.7$ for moderately absorbing materials. Although this number is significantly higher than the values expected for saturated organic molecules, which are in the range 1.4–1.5 (*CRC Handbook of Chemistry and Physics* 1975), it is in good agreement with the index of refraction measured for amorphous CH with a similar number of valence electrons per unit volume (Fink *et al.* 1984).

### III. EXTINCTION AND POLARIZATION CROSS SECTIONS

We have derived the extinction and polarization cross sections from Mie theory solutions for infinite cylinders given by Lind and Greenberg (1966) for homogeneous particles and by Shah (1970) for the core-mantle case. These algorithms require significantly less computer time than Mie theory solutions for finite-size nonspherical grains and can therefore be used to study in detail the influence of the size distribution as well as the alignment mechanism on polarization. Comparison with results obtained for prolate spheroids shows that cross sections calculated for infinite cylinders provide a good approximation to the wavelength-dependence of extinction and polarization even for axial ratios as low as 2, although the polarization-to-extinction ratio can be overestimated (Rogers and Martin 1979).

We denote the maximum and minimum extinction cross section for linearly polarized light passing through a cloud of aligned particles by $Q_1$ and $Q_2$. These factors can be obtained by averaging the efficiencies derived directly from Mie theory (calculated as a function of the angle between the direction of propagation of the incident wave and the symmetry axis of the cylinder) over all orientations of the spinning particles. Geometrical projection factors for arbitrarily oriented particles have been derived by Greenberg (1968) and Hong and Greenberg (1980) and have been used in our calculations without substantial changes. The extinction and polarization for an initially unpolarized beam passing through a path length $L$ in the interstellar medium are given by

$$A_\lambda = 1.086 L n_d \sigma_d \frac{Q_1 + Q_2}{2} , \tag{5}$$

$$\Delta m_p = 1.086 L n_d \sigma_d (Q_1 - Q_2) , \tag{6}$$

where $n_d$ is the column density of the grains, $\sigma_d$ is the geometric cross section of the particles (by definition, $\sigma_d = 2al$ for a cylinder of radius $a$ and length $l$), $\Delta m_p$ is the difference in magnitudes between the values of extinction observed for the two orientations of the electric vector. Equation (6) is valid only for those lines of sight which have a constant orientation of the magnetic field and otherwise provides an upper limit on the observed polarization.

If the difference in optical depths for the two perpendicular orientations of the electric vector is small (this condition is satisfied for most of observed lines of sight), the percentage polarization, $p$, is given by

$$p = Ln_d \sigma_d \frac{Q_1 - Q_2}{2} . \tag{7}$$

The ratio of maximum percentage polarization to extinction, $p_{\max}/A_V$, which can be compared directly with observations, can then be calculated from

$$\frac{p_{\max}}{A_V} = \frac{1}{1.086} \frac{(Q_1 - Q_2)_{\max}}{(Q_1 + Q_2)_V} . \tag{8}$$

We note that the polarization on average has its maximum close to $\lambda_V$, which is about where the extinction exhibits a change in curvature (Greenberg 1978).

Circular polarization results from linear birefringence of the interstellar medium; i.e., the dependence of the phase lag sustained by the transmitted wave on the orientation of the electric vector. The phase lag of the transmitted wave is given by the imaginary part of the forward scattering amplitude (van de Hulst 1957) and can be formally included in the extinction cross section defined as a complex quantity through the optical theorem (van de Hulst 1957):

$$C_{\text{ext}} = \frac{4\pi}{k^2} S(0) , \tag{9}$$

where $S(0)$ is the complex forward scattering amplitude and $k = 2\pi/\lambda$. In this approach, Re $[Q_1 - Q_2]$ represents the linear dichroism of the interstellar medium, and Im $[Q_1 - Q_2]$ its linear birefringence.

The ratio of Stokes parameters, $V/I$, which is normally used as a measure of the observed circular polarization, is given by the expression (Martin 1972; Serkowski 1962)

$$\frac{V}{I} = G n_d^2 \sigma_d^2 \frac{\text{Re}\,[Q_1 - Q_2]}{2} \frac{\text{Im}\,[Q_1 - Q_2]}{2} , \tag{10}$$

where the factor $G$ accounts for the path length as well as geometric effects of the variation of the direction of alignment along the line of sight.

Equations (5)–(10) can easily be generalized for a size distribution of particles. The distribution adopted in our calculations is of the form

$$n(a) = n_d f_0 \exp\left[-5\left(\frac{a - a_c}{a_i}\right)^2\right] , \tag{11}$$

where $n_d$ is the total number density of grains, $a$ is the particle radius, and $n(a)da$ is the number of particles per unit volume with radii in the range $[a, a + da]$. The parameter $a_i$ describes the "cutoff" of the distribution, $a_c$ is the size of the core, which is assumed to be constant for a given distribution. The normalizing factor, $f_0$, and the average particle size, $\langle a \rangle$, are given by the formulae (Greenberg 1968)

$$f_0 = \frac{3 \times 5^{1/3}}{\Gamma(1/3) a_i} = \frac{1}{0.5222 a_i} , \tag{12}$$

$$\langle a \rangle = a_c + \frac{\Gamma(2/3)}{\Gamma(1/3) 5^{1/3}} a_i = a_c + 0.2956 a_i . \tag{13}$$

We have used the same distribution for homogeneous grains, with $a_c$ treated as an arbitrary parameter that could be adjusted to provide the best match to the observed wavelength dependence of extinction and linear polarization.

We have considered two cases of grain alignment: (1) perfect spinning (PSA), (2) partial Davis-Greenstein alignment (DG) due to paramagnetic relaxation with collisional spin-up. The degree of particle alignment due to paramagnetic relaxation is specified by the distribution of the angular momentum vectors of particles about the direction of the magnetic field, which is given by (Jones and Spitzer 1967):

$$f_\Omega(\beta) = \frac{1}{4\pi} \frac{\xi}{(\xi^2 \cos^2 \beta + \sin^2 \beta)^{3/2}} , \tag{14}$$

where $f_\Omega$ represents the relative number of particles per unit solid angle, $\beta$ is the angle between the angular momentum vector and the magnetic field. The degree of alignment is determined by the parameter $\xi$, which is defined by the expression

$$\xi^2 = \frac{1 + \delta(T_d/T_g)}{1 + \delta} , \tag{15}$$

where $T_d$ is the temperature of the grains, $T_g$ is the temperature of the gas, and $\delta$ is the ratio of the collisional damping time, $t_d$, to the magnetic relaxation time, $t_r$[1] (Jones and Spitzer 1967). This ratio is inversely proportional to the particle radius, $a$, and for a sphere the product $\delta_0^{(s)} = \delta \times a$ is given by

$$\begin{aligned} \delta_0^{(s)} &= \left(\frac{\pi}{2 m_H k_B}\right)^{1/2} \frac{\chi'' B^2}{\omega n_H} \\ &= 2.046 \times 10^{-2} \left(\frac{K}{2.5 \times 10^{-12}}\right) \left(\frac{B}{1\ \mu\text{G}}\right)^2 \left(\frac{1\ \text{cm}^{-3}}{n_H}\right) \\ &\quad \times \left(\frac{10\ \text{K}}{T_d}\right) \left(\frac{100\ \text{K}}{T_g}\right)^{1/2} , \end{aligned} \tag{16}$$

where $B$ is the magnetic field $n_H$ is the density of the gas, $m_H$ represents the mass of a hydrogen atom, and $k_B$ is the Boltzmann constant. Equation (16) implicitly assumes that the imaginary part of the magnetic susceptibility of grain material can be expressed as $\chi'' = K\omega/T_d$, where $K$ is a constant.

For nonspherical grains, the angular velocity and angular momentum are not necessarily aligned. If the angular momentum is always perpendicular to the symmetry axis of the particle, as approximated by Greenberg (1968) and demonstrated by Purcell (1979) as due to internal energy dissipation, the ratio $\delta^{(c)} = t_d/t_r$ for cylinders can be derived from the relation

$$\delta_0^{(c)} = \delta_0^{(s)} \zeta_e , \tag{17}$$

$$\zeta_e = \frac{1.2e}{(1/4 + e^2/3)(1 + 2e)} , \tag{18}$$

[1] $t_d$ is the time required for a given component of the angular momentum to decrease $e$-fold as a result of thermal collisions with atoms; $t_r$ is a similarly defined measure of the rate at which paramagnetic relaxation dampens out the component of the angular momentum perpendicular to the magnetic field.

where $e$, the elongation of the cylinder ($e$ = length/diameter), is assumed in most of our calculations to be in the range 3–4. Equation (18) has been derived assuming that the ratio of the collisional damping time to the time $t_m$, in which the grain is struck by its own mass of gas, is 0.6 (Purcell and Spitzer 1971).

The value of $\delta_0^{(c)}$ required in our Mie theory calculations to match the maximum observed polarization-to-extinction ratio of $\sim$0.03 (Serkowski, Mathewson, and Ford 1975), is close to 0.2. We have assumed that this value can be achieved, recognizing that it requires a relaxation mechanism orders of magnitude more efficient than that given by normal paramagnetism.

## IV. INTERSTELLAR CIRCULAR POLARIZATION

Examples of the wavelength-dependence of linear and circular polarization derived from Mie theory cross sections for infinite cylinders have been collected in Figures 1 and 2. Fig. 1*b* shows the results for core-mantle grains with moderately absorbing mantles [the visual absorptivity given by $m''(V) \approx 0.15$, as shown in Fig. 1*a*]. The size distribution which best matches the observed wavelength-dependence of linear polarization for $(\lambda_{max})^{-1} \approx 1.75\ \mu m^{-1}$ is given by $a_c = 0.03\ \mu m$ and $a_i = 0.15\ \mu m$ and has an average particle radius of 0.075 $\mu m$. Figure 1*b* shows that the difference between results for partial and complete spinning alignment is rather small. The larger width of the peak in the DG curve reflects the dependence of the degree of alignment on particle size. Since the parameter $\delta$ in equation (15) is inversely proportional to the radius of the grain, smaller particles in the size distribution are more strongly aligned and their relative contribution to polarization, which peaks at $\lambda < \lambda_{max}$ is increased above the PSA case. The somewhat larger value of $\lambda_c$ in the PSA calculations is a simple consequence of the reduced linear polarization at short wavelengths (see discussion below).

Calculations for grains with similar absorptivity in the visible carried out by Martin (1972, 1975*a*) indicated that $m''(V) \approx 0.1$ was close to the upper limit allowed by the

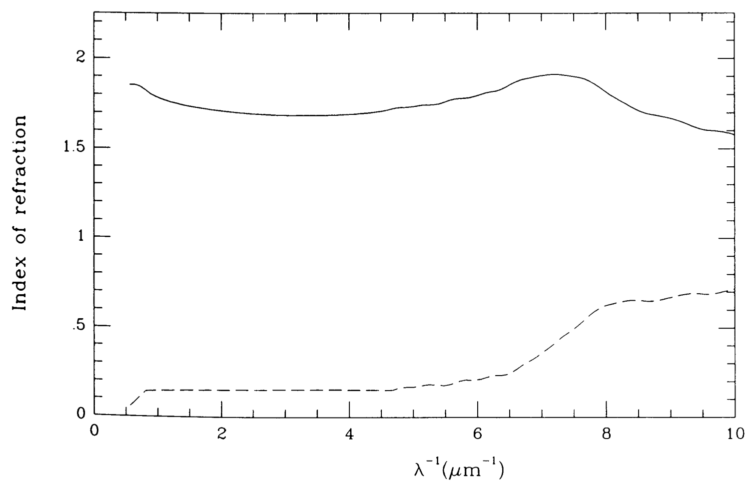


FIG. 1*a*

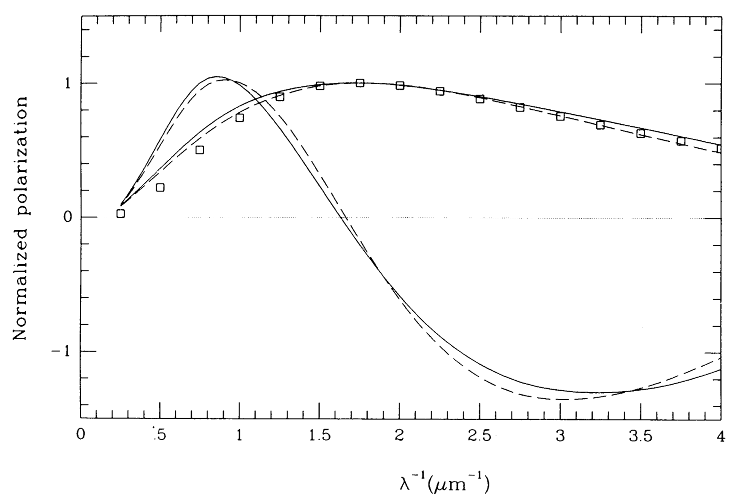


FIG. 1*b*

FIG. 1.—(*a*) The index of refraction of organic refractory with moderate visual absorptivity ($m''$ = 0.15). *Solid line*: real part, Re [$m$]; *dashed line*: imaginary part, Im [$m$]. The value of $m''$ = 0.15 in the visual is the lowest for which core-mantle grains match the observed albedo. (*b*) Linear and circular polarization derived from Mie theory calculations for core-mantle cylinders with optical constants of the mantle as in Fig. 1*a*. *Solid line*: Davis-Greenstein alignment; *dashed line*: perfect spinning alignment. The size distribution is given by $a_c = 0.04\ \mu m$, $a_i = 0.15\ \mu m$; the average size is 0.075 $\mu m$. The observed linear polarization (*squares*) is from Wilking, Lebofsky, and Rieke (1982) adjusted for $(\lambda_{max})^{-1} = 1.75\ \mu m^{-1}$.

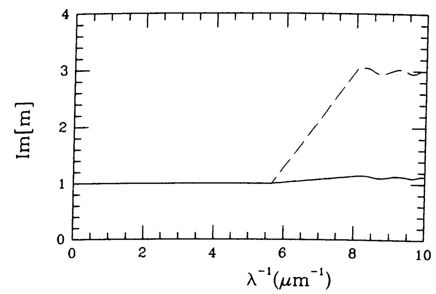


FIG. 2*a*

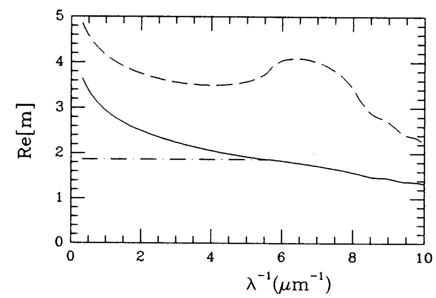


FIG. 2*b*

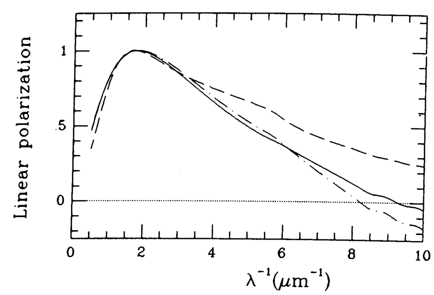


FIG. 2*c*

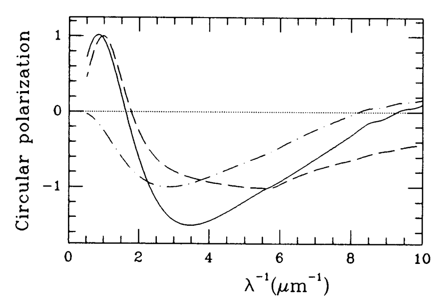


FIG. 2*d*

FIG. 2.—Kramers-Kronig interpretation of circular polarization. Different lines represent three sets of optical constants indicated in (*a*) and (*b*). The calculations of polarization are for homogeneous cylinders with the size distribution adjusted to match the observed average value of the peak wavelength of linear polarization.

observed coincidence of $\lambda_c$ and $\lambda_{max}$ and that already for this relatively low value of $m''$, $\lambda_c$ was beginning to shift away from $\lambda_{max}$ toward longer wavelengths. The results shown in Figure 1*b* clearly do not support this conclusion. The frequently adopted interpretation of Martin's calculations in terms of a "circular polarization criterion" becomes even more discrepant with our results as the value of $m''$ is increased further. We have carried out calculations for values of $m''(V)$ as high as 1.0, and the results consistently show that as long as the observed linear polarization curve is approximately matched, the correlation between $\lambda_c$ and $\lambda_{max}$ is preserved (see Fig. 2).

These numerical results illustrate the fact that the linear dichroism and birefringence of the interstellar medium are automatically connected by a fundamental physical relationship, which was first recognized by Kemp (1972) as being provided by the Kramers-Kronig relations. The argument presented in the fullest form by Shapiro (1975) and summarized here as a basis for interpreting our calculations of circular polarization is based on the fact the index of refraction, $\tilde{m}$, of a medium filled with scattering particles is related to the extinction cross section of individual grains (van de Hulst 1957) by

$$\tilde{m}' - 1 = \frac{1}{2k} N \operatorname{Im}\,[C_{ext}]\,, \tag{19}$$

$$\tilde{m}'' = \frac{1}{2k} N \operatorname{Re}\,[C_{ext}]\,, \tag{20}$$

where $N$ represents the number density of scatterers. The dichroism and birefringence of the interstellar medium can therefore be expressed in terms of the index of refraction, by introducing the differential index, $\Delta\tilde{m} = \tilde{m}_1 - \tilde{m}_2$, where the subscripts refer to the two perpendicular orientations of the electric vector defined in § III. The imaginary part of $\Delta\tilde{m}$ describes the linear dichroism of the interstellar medium, and the real part represents its linear birefringence. Equations (7) and (10) imply that $p \propto \operatorname{Im}\,[\Delta\tilde{m}]$ and $V/I \propto \operatorname{Re}\,[\Delta\tilde{m}] \times \operatorname{Im}\,[\Delta\tilde{m}]$. Since the index of refraction of any optical medium automatically satisfies the Kramers-Kronig relations, Equation (1) may equivalently be written also for $\Delta\tilde{m}$:

$$\Delta\tilde{m}'(\omega) = \frac{2}{\pi}\int_0^\infty \frac{\omega'\,\Delta\tilde{m}''(\omega')}{\omega'^2 - \omega^2}\,d\omega'\,. \tag{21}$$

The observed polarization suggests that the shape of the theoretical polarization curve given by $\lambda^{-1}\Delta\tilde{m}''$ corresponds to an isolated, rather broad "resonance" in a dispersive medium. Consequently, the real part of $\Delta\tilde{m}$ must exhibit a region of anomalous dispersion in which it decreases with increasing frequency. Characteristically then, at a frequency near the peak of the absorption, $\Delta\tilde{m}'$ crosses the "background" value it would have had with no local resonance. For the grain population adopted in our calculations, there are no further "resonances" in $\operatorname{Im}\,[\Delta\tilde{m}]$ in the UV, and the background value of $\operatorname{Re}\,[\Delta\tilde{m}]$ must therefore equal 0. The wavelength at which $\operatorname{Re}\,[\Delta\tilde{m}]$ changes sign is the crossover wavelength of circular polarization. The Kramers-Kronig relations therefore require that the crossover should occur at a wavelength close to $\lambda_{max}$—the observed nearly exact coincidence of the two wavelengths results from the particular shape of the interstellar linear polarization curve.

The Kramers-Kronig argument can be used to explain the origin of the incorrect interpretation of Martin's Mie theory results as a constraint on the optical properties of grain materials. In Figure 2 we have collected the results of several numerical experiments, which can serve as a basis for analyzing the connection between the index of refraction of grains and the linear and circular polarization. Figures 2*a* and 2*b* show several sets of optical constants, all of which have $m'' = 1$ in the visual. Case 1 is characterized by a rather low far-ultraviolet absorption, with values of $m''$ in the far-UV similar to those in the visual. Case 2 is for a much stronger UV absorption and consequently a larger $m'$ in the visual. For case 3, $m''$ is everywhere the same as for case 1, but $m'$ has been *kept constant* throughout the visual, *deliberately* violating the Kramers-Kronig relations. All the calculated linear polarization curves can, by appropriate size parameterization, be made to resemble the " normal " polarization curve (Fig. 2*c*). However, the circular polarization satisfies $\lambda_c = \lambda_{max}$ *only* for those particles whose optical constants satisfy the Kramers-Kronig relations (for case 2, the wavelength dependence of $m'$ in the near-UV broadens the linear polarization peak and consequently shifts the circular polarization crossover to somewhat higher frequencies). The circular polarization curve for case 3 is precisely the kind of curve obtained by Martin (1972) for particles with a high imaginary part of the index of refraction. The similarity of these results is significant—in Martin's calculations the real and imaginary part of the index of refraction were kept constant, violating the Kramers-Kronig relations exactly as in our case 3. An intrinsic feature of the Kramers-Kronig relations is that a constant value of $m''$ over a range as extensive as that in Martin's (1972) calculations, *demands* a *rise* in $m'$ toward the red, which is just what is needed to bring up the value of the circular polarization at long wavelengths. This rise (nonconstancy) of the value of $m'$ is a local effect; i.e., due to visible optical properties alone and cannot be modified by *any* addition of the far-UV absorption which, as is well known, adds a *constant* amount to the visual $m'$. For sufficiently small values of $m''$, the constancy of $m'$ is indeed a not unreasonable assumption (see Fig. 1*a*) but as seen in Fig. 2*b*, a large value of $m''$ in the visual leads to a rapidly rising $m'$ toward longer wavelengths. It is important to note that the use of constant indices of refraction in scattering calculations has a long tradition and generally leads to qualitatively correct results. The violation of the Kramers-Kronig relations resulting from such an assumption when $m'' \neq 0$ does not significantly affect quantities dependent only on the amplitude of the scattered wave, such as extinction and linear polarization (in Fig. 2*c* the linear polarization curves for case 1 and case 3—which violates the Kramers-Kronig relations—are nearly identical). However, the effect cannot be neglected for quantities derived from the phase of the scattered wave, including the linear birefringence of the interstellar medium, which gives rise to circular polarization (note the difference between cases 1 and 3 in Fig. 2*d*). Consequently, Mie theory calculations assuming constant indices of refraction, although acceptable in many of their predictions, should not be used in interpreting the phase-dependent effect of interstellar circular polarization.

In a discussion of circular polarization it is important to bear in mind the crucial but often confusing distinction between the index of refraction of grain constituents and that of the interstellar medium. As shown by equations (19) and (20), the index of refraction of the interstellar medium depends on the particle size as well as on the optical constants of their constituent materials; i.e., in some cases the optical properties of the particles are determined mainly by their size. One such phenomenon is illustrated in Figure 2*c*. For particle sizes required to match the visual peak in linear polarization, the saturated far-UV extinction inevitably approaches the same asymptotic value independent of the polarization of the incident wave. The linear polarization in the far-UV is consequently close to 0 for *any* set of optical constants, although the shape of the polarization peak is affected to some extent by the wavelength-dependence of the index of refraction of the grains. The Kramers-Kronig interpretation of circular polarization indicates only that the correlation between linear and circular polarization in the visual cannot be used to constrain the *index of refractions of grain materials.* However, it does not invalidate the constraints that can be placed on the *differential index of refraction of the interstellar medium* outside the visual range. The application of Kramers-Kronig relations to such an interpretation of interstellar polarization has been pointed out and discussed extensively by Martin (1975*b*). In Figure 3 we have illustrated the significance of constraints on the linear polarization in the far-UV derived from the condition $\lambda_c = \lambda_{max}$. The circular polarization curves shown in Figure 3 have been calculated using a technique introduced by Martin (1975*a*), in which the circular polarization is obtained directly from the assumed wavelength-dependence of linear polarization through Kramers-Kronig relations. The addition of a linear polarization feature in the far-UV with a magnitude similar to that of the visual peak results in a 10% shift in the value of $\lambda_c$, which would be barely detectable. However, because of the dominant effect of nearby features on the Kramers-Kronig integral (eq. [1]), $\lambda_c$ would shift further away from the Serkowski curve (Figs 2*c* and 2*d*). It appears therefore that the measurements of the visual circular polarization provide rather strict constraints on the linear polarization immediately outside the visible range but can only be used to rule out very strong features in the far-UV.

## V. IMPLICATIONS FOR GRAIN MODELS

Although the arguments we have discussed remove any previously derived restrictions on the visual absorptivity of polarizing grains, it still appears highly unlikely that interstellar polarization is due to conducting particles. The most convincing indication that the polarizing grains are dielectric comes from the IR polarimetry which shows substantial polarization in both the silicate 9.7 $\mu$m feature and 3.07 $\mu$m water ice band (Lonsdale *et al.* 1980). Another, albeit less direct, argument against conductors can be drawn from the general wavelength-dependence of polarization. Mie theory calculations, including those presented in § IV, show that the observed shape of the interstellar linear polarization curve can be matched relatively easily by a simple size distribution of dielectric grains, whose index of refraction is a smooth function of the wavelength and has no sharp features in the visual. For conducting materials, there is likely to be much more structure in the wavelength-dependence of the index of refraction, which would be reflected in the linear polarization curve. Apart from the free-electron term, which causes a rise toward near-infrared wavelength, sharp features at energies as low as 2–3 eV can be caused by interband transitions whose early onset is a typical feature of cosmically abundant metals (Nilsson 1974).

The only remaining restriction on the visual optical properties grain materials is provided by the observed albedo. Model calculations whose results have been collected in Figure 4 show that the $m''(V) = 0.15$ marks an approximate lower limit on the absorptivity of grain mantles allowed by the observations (the albedo at $\lambda_V$ is 0.7 for the core-mantle grains alone and 0.6 after averaging over all three populations). The albedo obtained for the three-population model using core-mantle

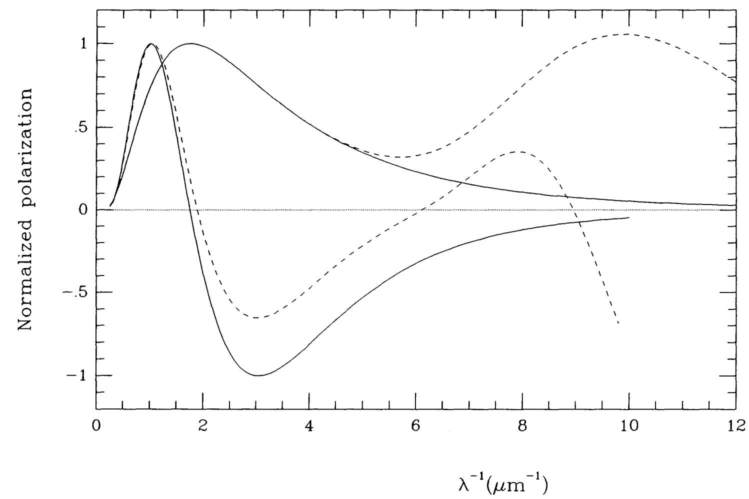


FIG. 3.—The effects of linear polarization in the far-ultraviolet on the observed circular polarization crossover. The solid line is the standard Serkowski curve, the dashed line includes an additional feature corresponding to the far-UV rise in extinction. The circular polarization for each case has been derived from the Kramers-Kronig relations.

spheres (in order to avoid the artificially high albedo produced by two-dimensional infinite particles; Greenberg, van de Bult, and Allamandola 1983) is not significantly different in the visual from the prediction of the MRN model which has also been included in Figure 4.

One of the consequences of adopting the optical constants of grain mantles described in § II is a reduction in the total of volume of core-mantle grains relative to what is required for mantles with a lower index of refraction. The difference is due to an increase in the volume efficiency of extinction, $\alpha$, which is defined as the ratio of the extinction cross section to the volume of the particle, $\alpha = C_{ext}/V$, and is essentially independent of particle shape (Greenberg and Hong 1974). The value of $\alpha$ at 5500 Å is $1.3 \times 10^5$ cm$^{-1}$ for an organic refractory mantle and only $6.8 \times 10^4$ cm$^{-1}$ for particles with an index of refraction of 1.45 which appears to be typical of a material consisting of saturated organic molecules. Since the specific gravity of the two types of materials is similar, and carbon makes up an almost identical fraction of the mass in each of them (the difference in the optical properties is due mainly to the change in the C:H ratio), the predicted depletion of carbon in core-mantle grains drops almost in inverse proportion to the value of $\alpha$. For $m' = 1.45$, the mantles must contain $\sim 45\%$ of cosmic carbon (using cosmic abundances tabulated by Greenberg 1978), whereas for the organic refractory the carbon depletion is only 26%. Since as much as 25% of carbon is required to match the observed strength of the 2200 Å hump—if it is due to the $\pi$-electron plasmon in graphite spheres—the total depletion of carbon required by the three-population model is reduced from an uncomfortably high value of $\sim 70\%$ to only $\sim 50\%$, which agrees much better with the low depletion of carbon suggested by the observations (see Whittet 1987 and references therein). It is perhaps worth noting that in spite of the low required depletion of carbon, all the polarization and most of the extinction are produced by the major carbon-bearing grains in our model.

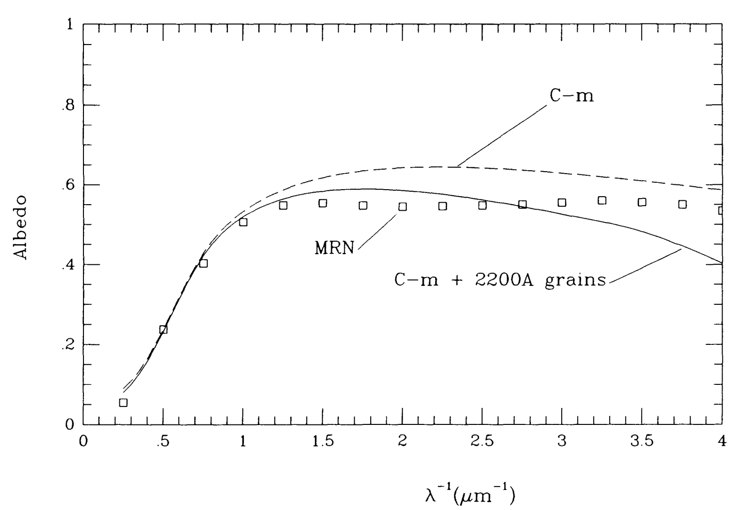


FIG. 4.—Visual albedo of grains for the three-population model (*solid line*) and for the MRN distribution (*squares*). Calculations have been carried out for mantles with $m''(V) = 0.15$. The size distribution is the same as in Fig. 1.

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